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Gauge model with light W and Z bosons

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We show how the standard electroweak gauge model can be naturally generalized to SU(2) $\times$ U(1) $\times$ SU(2)$'$, which has twice the number of gauge bosons of the standard model. All known fermions transform with respect to SU(2) $\times$ U(1); spontaneous symmetry breaking is achieved with a scalar quartet which links SU(2) and SU(2)$'$ and the usual scalar doublet. The interaction Hamiltonian at low energy reproduces the low-energy phenomenology of the standard model. The high-energy predictions of the model are explored. One W and one Z are less massive than the corresponding bosons of the standard model. These light bosons are relatively narrow, with total widths typically 50 MeV. Cross sections for light Z and W production in e^+e^- collisions and hadron collisions are given. A Z mass as low as 32 GeV is compatible with existing measurements; the most stringent present limit is set by the e^+e^- annihilation cross section.

I. INTRODUCTION

The standard SU(2)$\times$U(1) electroweak model[1] has successfully accounted for all low-energy weak-interaction measurements.[2] However, until the W and Z gauge bosons and the Higgs bosons are discovered, there is theoretical and experimental interest in alternative gauge models which are consistent with the low-energy phenomena. A class of models based on the gauge group SU(2) $\times$U(1)$\times G$, where G is an arbitrary group, can be constructed in which the structure of the interaction Hamiltonian at low energies is the same as the standard model in all tested aspects.[3-6] In such models there exist gauge bosons with masses below those of the W and Z of the standard model. If these low-mass gauge bosons exist, they could be observed at the higher energies that will soon be reached in e^+e^- and $\bar{p}p$ colliders. Even if light W or Z bosons are not found, the alternative models provide a basis of comparison by which the success of the standard model can be judged at energies below the predicted masses of its gauge bosons. Alternatives to the standard model discussed by other authors have modified only the neutral-boson sector.[3,4] Our purpose is to discuss a model[5] which has a light W boson as well as a light Z boson.

We shall present a systematic study of the electroweak gauge group SU(2)$\times$U(1)$\times$SU(2)$'$ with couplings g_0, $\frac{1}{2}g_1$, g_2, respectively. All known quarks and leptons are assumed to transform as in the standard model with respect to the SU(2) $\times$U(1) subgroup only. The additional SU(2)$'$ may couple to heavy fermions which play no role at presently available energies. To generate masses, we use a scalar doublet $\Phi=(\phi^+,\phi^0)$ in the representation $(\frac{1}{2}, 1, 0)$ and a scalar quartet $\eta=(\chi^+,\chi^0,\eta^0,\eta^-)$ in the representation $(\frac{1}{2},0,\frac{1}{2})$. Spontaneous symmetry breakdown occurs with nonvanishing vacuum expectation values $\langle\phi^0\rangle$ and $\langle\eta^0\rangle=\langle\chi^0\rangle$. The latter equality is guaranteed by the imposition of the discrete symmetries $\eta\to i\eta$, and $\eta\to\tau_2\eta^*\tau_2$ in the potential. This pattern of symmetry breaking reproduces the standard-model result for the ratio of neutral-current to charged-current coupling strengths at low energy. The details of the Higgs mechanism are given in Sec. II.

The physical gauge fields are related to the mass eigenstates in Sec. III. The five basic parameters of the model g_0, g_1, g_2, $\langle\phi^0\rangle$, and $\langle\eta^0\rangle$ are expressed in terms of G_F, e, the weak angle of the standard model, and the masses of the two neutral gauge bosons.

In Sec. IV the phenomenology of the model is presented and compared with corresponding results of the standard model. The total widths and branching fractions of the gauge bosons are evaluated. The weak contribution to the muon anomalous magnetic moment of the muon is found to be small and does not require any significant restriction on the gauge boson masses. Predictions of cross sections and asymmetry in e^+e^- collisions are made. From the PETRA data we infer a lower limit of 32 GeV on the light-Z-boson mass. For the Drell-Yan process in hadron collisions, we give the transverse-momentum distributions and dilepton mass spectrum due to light W and Z production.

II. THE HIGGS-BOSON STRUCTURE

The covariant derivative for the SU(2)×U(1)×SU(2)′ gauge group is given by

$$D_\nu = \partial_\nu - ig_0\vec{T}\cdot\vec{W}_\nu - ig_1\frac{Y}{2}B_\nu - ig_2\vec{T}'\cdot\vec{W}'_\nu , \tag{1}$$

where $\vec{W}'_\nu$ are the new gauge bosons of SU(2)′ which are not coupled to the known fermions. The electric charge operator is $Q = T_3 + Y/2 + T'_3$. The gauge-symmetry breaking is accomplished by the usual Higgs-boson doublet ($T=\frac{1}{2}$, $Y=1$, $T'=0$)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2a}$$

and a Higgs-boson quartet ($T=\frac{1}{2}$, $Y=0$, $T'=\frac{1}{2}$)

$$\eta = \begin{array}{cc} \begin{array}{cc} -\frac{1}{2} & +\frac{1}{2} \end{array} & \\ \begin{pmatrix} \eta^0 & \chi^+ \\ \eta^- & \chi^0 \end{pmatrix} & \begin{array}{c} +\frac{1}{2} \\ -\frac{1}{2} \end{array} \end{array} , \tag{2b}$$

where the rows correspond to different values of T_3, and the columns to different values of T'_3. We note that

$$\vec{T}\eta = \frac{\vec{\tau}}{2}\eta ,$$
$$\vec{T}'\eta = -\eta\frac{\vec{\tau}}{2} . \tag{3}$$

Under SU(2) rotations, η transforms into $U\eta(U')^\dagger$. The charge-conjugate state $\tilde{\eta}$ which transforms in the same way as η is

$$\tilde{\eta} = \tau_2\eta^*\tau_2 = \begin{pmatrix} \chi^0 & -\eta^- \\ -\chi^+ & \eta^0 \end{pmatrix} . \tag{4}$$

We require that the Higgs Lagrangian be invariant under two discrete symmetry transformations

$$\eta \to i\eta ,$$
$$\eta \to \tilde{\eta} . \tag{5}$$

The most general Lagrangian invariant under these discrete symmetries is

$$\mathcal{L} = (D_\nu\Phi)^\dagger(D_\nu\Phi) + \mathrm{Tr}(D_\nu\eta)^\dagger(D_\nu\eta) - V(\eta,\Phi) , \tag{6}$$

where the Higgs potential is

$$\begin{aligned} V = {} & \mu^2\Phi^\dagger\Phi + \lambda|\Phi^\dagger\Phi|^2 + m^2\mathrm{Tr}\,\eta^\dagger\eta \\ & + h_0(\mathrm{Tr}\,\eta^\dagger\eta)^2 - h_1|\mathrm{Tr}\,\eta^\dagger\tilde{\eta}|^2 \\ & - h_2\mathrm{Re}(\mathrm{Tr}\,\eta^\dagger\tilde{\eta})^2 + f(\Phi^\dagger\eta\eta^\dagger\Phi + \Phi^\dagger\tilde{\eta}\tilde{\eta}^\dagger\Phi) . \end{aligned} \tag{7}$$

Here Tr denotes the trace. All other quadratic combinations can be reduced to the above forms. Expanding this expression for the potential in terms of the component fields, we obtain

$$\begin{aligned} V = {} & \mu^2[|\phi^0|^2 + |\phi^+|^2] + \lambda[|\phi^0|^2 + |\phi^+|^2]^2 + m^2[|\eta^0|^2 + |\chi^0|^2 + |\eta^-|^2 + |\chi^+|^2] \\ & + h_0[|\eta^0|^2 + |\chi^0|^2 + |\eta^-|^2 + |\chi^+|^2]^2 - 4h_1|\eta^0\chi^0 - \eta^-\chi^+|^2 - 4h_2\mathrm{Re}(\eta^0\chi^0 - \eta^-\chi^+)^2 \\ & + f[|\eta^0|^2 + |\chi^0|^2 + |\eta^-|^2 + |\chi^+|^2][|\phi^0|^2 + |\phi^+|^2] . \end{aligned} \tag{8}$$

With μ and m imaginary, the Higgs phenomenon occurs with vacuum expectation values

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} , \quad \langle\eta\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u & 0 \\ 0 & u' \end{pmatrix} . \tag{9}$$

For vacuum stability, we must impose the conditions

$$h_0 \geq 2(h_1 + h_2) ,$$
$$h_0 \geq 0 . \tag{10}$$

Necessary conditions for a minimum of the potential with nonvanishing u, u', and v are

$$\begin{aligned} & u' = u , \\ & -|\mu^2| + 2\lambda v^2 + f u^2 = 0 , \\ & -|m^2| + 2h_0u^2 - 2(h_1+h_2)u^2 + f v^2 = 0 . \end{aligned} \tag{11}$$

The equality of u' and u is forced by the h_1 and h_2 terms in V. Redefining the scalar fields as $\phi \to \phi + \langle\phi\rangle$ and $\eta \to \eta + \langle\eta\rangle$, we obtain the Higgs-boson mass-squared matrix from the quadratic terms of the potential.

In the charged boson sector

$$V = 2u^2(h_1+h_2)|\eta^{-*} + \chi^+|^2 . \tag{12}$$

Thus the combination $\eta^{-*} + \chi^+$ acquires mass. The $\eta^{-*} - \chi^+$ and ϕ^+ states can be gauged away to become the longitudinal components of the charged vector gauge bosons.

The quadratic terms in the potential involving $\mathrm{Im}\,\eta^0$ and $\mathrm{Im}\,\chi^0$ become

$$V = 4u^2h_2(\mathrm{Im}\,\eta^0 + \mathrm{Im}\,\chi^0)^2 . \tag{13}$$

In this case $(\mathrm{Im}\,\eta^0 - \mathrm{Im}\,\chi^0)$ and $\mathrm{Im}\,\phi^0$ are absorbed into longitudinal components of the two neutral gauge bosons.

The mass-squared matrix of the remaining Higgs-boson states $\mathrm{Re}\,\phi^0$, $\mathrm{Re}\,\eta^0$, $\mathrm{Re}\,\chi^0$ is

$$\mathcal{V}=\begin{bmatrix} 4v^2\lambda & \sqrt{2}\,vuf & \sqrt{2}\,vuf \\ \sqrt{2}\,vuf & 2u^2h_0 & 2u^2(h_0-2h_1-2h_2) \\ \sqrt{2}\,vuf & 2u^2(h_0-2h_1-2h_2) & 2u^2h_0 \end{bmatrix}. \tag{14}$$

To ensure that all observable Higgs bosons have positive mass-squared, we require $h_1+h_2>0$, $h_2>0$, and $f>0$ in addition to Eq. (10).

III. MASS EIGENSTATES OF THE GAUGE FIELDS

The mass term for the gauge bosons in the Lagrangian of Eq. (6) is

$$\mathcal{L}=\langle\Phi\rangle^\dagger\left[g_0\frac{\vec{\tau}}{2}\cdot\vec{W}_\nu^\dagger+\frac{g_1}{2}B_\nu^\dagger\right]\left[g_0\frac{\vec{\tau}}{2}\cdot\vec{W}_\nu+\frac{g_1}{2}B_\nu\right]\langle\Phi\rangle$$
$$+\mathrm{Tr}\left[\langle\eta\rangle^\dagger g_0\frac{\vec{\tau}}{2}\cdot\vec{W}_\nu^\dagger-g_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\nu'^\dagger\langle\eta\rangle^\dagger\right]\left[g_0\frac{\vec{\tau}}{2}\cdot\vec{W}_\nu\langle\eta\rangle-g_2\langle\eta\rangle\frac{\vec{\tau}}{2}\cdot\vec{W}_\nu'\right]. \tag{15}$$

In the neutral sector the mass-squared matrix in the $W^{(3)}$, $W'^{(3)}$, and B basis is

$$\mathfrak{M}_Z^2=\frac{1}{2}\begin{bmatrix} g_0^2(v^2+u^2) & -g_0g_2u^2 & -g_0g_1v^2 \\ -g_0g_2u^2 & g_2^2u^2 & 0 \\ -g_0g_1v^2 & 0 & g_1^2v^2 \end{bmatrix}. \tag{16}$$

The eigenvalues λ_i of $\mathfrak{M}_Z^2$ are 0, $M_{Z_1}^2$, and $M_{Z_2}^2$, where

$$M_{Z_i}^4-\tfrac{1}{2}[(g_0^2+g_1^2)v^2+(g_0^2+g_2^2)u^2]M_{Z_i}^2+\tfrac{1}{4}u^2v^2g_0^2g_1^2g_2^2\left[\frac{1}{g_0^2}+\frac{1}{g_1^2}+\frac{1}{g_2^2}\right]=0\,, \tag{17}$$

for $i=1,2$. From the cofactors of the first row of the matrix $(\mathfrak{M}_Z^2-\lambda)$, we obtain the form of the matrix $\mathfrak{R}$ for which $\mathfrak{R}\mathfrak{M}_Z^2\mathfrak{R}^\dagger=\lambda$:

$$\mathfrak{R}=\begin{bmatrix} \dfrac{n_0}{g_0} & \dfrac{n_0}{g_2} & \dfrac{n_0}{g_1} \\ \dfrac{n_1}{g_0}\left(1-\dfrac{g_1^2v^2}{2M_{Z_1}^2}\right) & -\dfrac{n_1}{2M_{Z_1}^2}g_2u^2\left(\dfrac{2M_{Z_1}^2-g_1^2v^2}{2M_{Z_1}^2-g_2^2u^2}\right) & -\dfrac{n_1g_1v^2}{2M_{Z_1}^2} \\ \dfrac{n_2}{g_0}\left(1-\dfrac{g_1^2v^2}{2M_{Z_2}^2}\right) & -\dfrac{n_2}{2M_{Z_2}^2}g_2u^2\left(\dfrac{2M_{Z_2}^2-g_1^2v^2}{2M_{Z_2}^2-g_2^2u^2}\right) & -\dfrac{n_2g_1v^2}{2M_{Z_2}^2} \end{bmatrix}. \tag{18}$$

Here n_0, n_1, and n_2 are normalizations to be chosen such that $\mathfrak{R}\mathfrak{R}^\dagger=\mathfrak{R}^\dagger\mathfrak{R}=1$:

$$n_0^2=\left[\frac{1}{g_1^2}+\frac{1}{g_0^2}+\frac{1}{g_2^2}\right]^{-1},$$
$$\frac{n_1^2}{M_{Z_1}^2}+\frac{n_2^2}{M_{Z_2}^2}=\frac{2}{v^2}, \tag{19}$$
$$\frac{n_1^2}{M_{Z_1}^4}+\frac{n_2^2}{M_{Z_2}^4}=\frac{4}{v^4}\left[\frac{g_0^2+g_2^2}{g_0^2g_2^2+g_1^2(g_0^2+g_2^2)}\right].$$

The primordial fields are related to the mass eigenstates by

$$\begin{bmatrix} W^{(3)} \\ W'^{(3)} \\ B \end{bmatrix}=\mathfrak{R}^\dagger\begin{bmatrix} A \\ Z_1 \\ Z_2 \end{bmatrix}, \tag{20}$$

where A is the photon field. In terms of these eigenstates, the covariant derivative of Eq. (1) for the neutral sector is

$$D_\nu^0=\partial_\nu-in_0QA_\nu-i\sum_{i=1,2}n_i\left[T^{(3)}-Q\left(\frac{g_1^2v^2}{2M_{Z_i}^2}\right)+T'^{(3)}\left(\frac{g_1^2v^2-g_2^2u^2}{2M_{Z_i}^2-g_2^2u^2}\right)\right]Z_{i\nu}. \tag{21}$$

Hence n_0 is to be identified with the electric charge e. In the fermion sector the SU(2)′ group is inactive and we obtain the interaction Hamiltonian

$$\mathcal{H}=ej_\nu^{\mathrm{em}}A_\nu+\sum_{i=1,2}n_i\left(j_\nu^{(3)}-\frac{g_1^2v^2}{2M_{Z_i}^2}j_\nu^{\mathrm{em}}\right)Z_{i\nu}, \tag{22}$$

where

$$j_\nu^{\rm em}=\bar\psi\gamma_\nu Q\psi\,,$$
$$\vec{\rm j}_\nu=\bar\psi\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)\vec{\rm T}\psi\,, \tag{23}$$

for a fermion doublet field ψ. The effective weak neutral-current interaction at low energy is then

$$\mathcal{H}_{\rm eff}^{\rm NC}=\frac{1}{v^2}\left\{\left[j_\nu^{(3)}-\left(1-\frac{e^2}{g_1^2}\right)j_\nu^{\rm em}\right]^2+C[j_\nu^{\rm em}]^2\right\}, \tag{24}$$

where

$$C\equiv\frac{g_1^4v^6}{8}\sum_{i=1,2}\frac{n_i^2}{M_{Z_i}^6}-\left(1-\frac{e^2}{g_1^2}\right)^2. \tag{25}$$

To ensure that the first term of Eq. (24) reproduces the effective Hamiltonian of the standard model

$$\mathcal{H}_{\rm eff}^{\rm NC}=\frac{4G_F}{\sqrt2}(j_\nu^{(3)}-\sin^2\theta_W j_\nu^{\rm em})^2\,, \tag{26}$$

we require that

$$\frac{1}{4v^2}=\frac{G_F}{\sqrt2}\,,$$
$$g_1=\frac{e}{\cos\theta_W}\,. \tag{27}$$

In terms of the gauge-boson masses of the standard model,

$$M_W=\frac{e}{G_F^{1/2}2^{5/4}\sin\theta_W}\,,$$
$$M_Z=\frac{M_W}{\cos\theta_W}\,, \tag{28}$$

we have

$$\tfrac12 g_1^2v^2=M_Z^2\sin^2\theta_W\,. \tag{29}$$

Using the above relations, the right-hand side of Eqs. (19) can be replaced by observable quantities:

$$n_0=e\,,$$
$$\frac{n_1^2}{M_{Z_1}^2}+\frac{n_2^2}{M_{Z_2}^2}=\frac{8G_F}{\sqrt2}\,, \tag{30}$$
$$\frac{n_1^2}{M_{Z_1}^4}+\frac{n_2^2}{M_{Z_2}^4}=\frac{8G_F}{\sqrt2\,M_Z^2}\,.$$

The values of n_1 and n_2 which enter in the Hamiltonian of Eq. (22) can thereby be expressed in terms of the Z_1, Z_2 masses:

$$n_1=\left(\frac{8G_F}{\sqrt2}\right)^{1/2}\frac{M_{Z_1}^2}{M_Z}\left(\frac{M_{Z_2}^2-M_Z^2}{M_{Z_2}^2-M_{Z_1}^2}\right)^{1/2},$$
$$n_2=\left(\frac{8G_F}{\sqrt2}\right)^{1/2}\frac{M_{Z_2}^2}{M_Z}\left(\frac{M_Z^2-M_{Z_1}^2}{M_{Z_2}^2-M_{Z_1}^2}\right)^{1/2}. \tag{31}$$

Using Eqs. (31), the expression for C in Eq. (25) becomes

$$C=\sin^4\theta_W\frac{(M_{Z_2}^2-M_Z^2)(M_Z^2-M_{Z_1}^2)}{M_{Z_1}^2M_{Z_2}^2}\,. \tag{32}$$

The C term in the neutral-current Hamiltonian of Eq. (24) is absent in the standard model. If we label Z_1 and Z_2 such that $M_{Z_1}<M_{Z_2}$, the reality of n_1 and n_2 requires

$$M_{Z_1}<M_Z<M_{Z_2}\,. \tag{33}$$

In the charged sector the mass-squared matrix in the W, W' basis is

$$\mathfrak{M}_W^2=\frac12\begin{pmatrix} g_0^2(v^2+u^2) & -g_0g_2u^2\\ -g_0g_2u^2 & g_2^2u^2\end{pmatrix}. \tag{34}$$

The elements of $\mathfrak{M}_W^2$ are the same as those of $\mathfrak{M}_Z^2$ in the $W^{(3)}$, $W'^{(3)}$ submatrix. By dropping terms involving g_1 and n_0 in Eqs. (18) and (19), we immediately find the diagonalization matrix for $\mathfrak{M}_W^2$ is

$$\mathcal{R}=\begin{pmatrix} \dfrac{N_1}{g_0} & -N_1\left(\dfrac{g_2u^2}{2M_{W_1}^2-g_2^2u^2}\right)\\[2ex] \dfrac{N_2}{g_0} & -N_2\left(\dfrac{g_2u^2}{2M_{W_2}^2-g_2^2u^2}\right)\end{pmatrix}, \tag{35}$$

where

$$\frac{N_1^2}{M_{W_1}^2}+\frac{N_2^2}{M_{W_2}^2}=\frac{2}{v^2}=\frac{8G_F}{\sqrt2}\,,$$
$$\frac{N_1^2}{M_{W_1}^4}+\frac{N_2^4}{M_{W_2}^4}=\frac{4}{v^4}\left(\frac{1}{g_0^2}+\frac{1}{g_2^2}\right)=\frac{8G_F}{\sqrt2\,M_W^2}\,. \tag{36}$$

In terms of W_1 and W_2 masses the normalization constants are

$$N_1=\left(\frac{8G_F}{\sqrt2}\right)^{1/2}\frac{M_{W_1}^2}{M_W}\left(\frac{M_{W_2}^2-M_W^2}{M_{W_2}^2-M_{W_1}^2}\right)^{1/2},$$
$$N_2=\left(\frac{8G_F}{\sqrt2}\right)^{1/2}\frac{M_{W_2}^2}{M_W}\left(\frac{M_W^2-M_{W_1}^2}{M_{W_2}^2-M_{W_1}^2}\right)^{1/2}. \tag{37}$$

Labeling W_1 and W_2 such that $M_{W_1}<M_{W_2}$, the reality of N_1 and N_2 requires

$$M_{W_1}<M_W<M_{W_2}\,. \tag{38}$$

The charged-current Hamiltonian in the fermion sector is

$$\mathcal{H}=\frac{1}{\sqrt2}(N_1W_{1\nu}^{+}+N_2W_{2\nu}^{+})j_\nu^{(+)}+\text{H.c.}\,, \tag{39}$$

where $j_\nu^{(+)} = j_\nu^{(1)} + i j_\nu^{(2)}$ and $\vec{J}_\nu$ are given by Eq. (23).

The effective weak coupling strengths at low energy are $\frac{1}{2}(N_1^2/M_{W_1}^2 + N_2^2/M_{W_2}^2)$ for the charged current and $(n_1^2/M_{Z_1}^2 + n_2^2/M_{Z_2}^2)$ for the neutral current. By Eqs. (30) and (36) these strengths are in the same ratio as the standard model. This is a consequence of the equality $u = u'$ in the Higgs symmetry-breaking mechanism.

In terms of the eigenvalues M_{W_1}, M_{W_2}, the trace and determinant of $\mathfrak{M}_W^2$ give

$$
\begin{aligned}
&M_{W_1}^2 + M_{W_2}^2 = \tfrac{1}{2} g_0^2 (v^2 + u^2) + \tfrac{1}{2} g_2^2 u^2 , \\
&M_{W_1}^2 M_{W_2}^2 = \tfrac{1}{4} g_0^2 g_2^2 v^2 u^2 .
\end{aligned}
\tag{40}
$$

The corresponding results in the Z_1, Z_2 sector are, from Eq. (17),

$$
\begin{aligned}
&M_{Z_1}^2 + M_{Z_2}^2 = \tfrac{1}{2} g_0^2 (v^2 + u^2) + \tfrac{1}{2} g_2^2 u^2 + \tfrac{1}{2} g_1^2 v^2 , \\
&M_{Z_1}^2 M_{Z_2}^2 = \tfrac{1}{4} g_0^2 g_2^2 v^2 u^2 \frac{g_1^2}{e^2} .
\end{aligned}
\tag{41}
$$

From Eqs. (40), (41), and (27) we derive the mass relations

$$
\begin{aligned}
&M_{W_1} M_{W_2} = M_{Z_1} M_{Z_2} \cos\theta_W , \\
&M_{W_1}^2 + M_{W_2}^2 + M_W^2 \tan^2\theta_W = M_{Z_1}^2 + M_{Z_2}^2 .
\end{aligned}
\tag{42}
$$

From Eq. (42) and the inequalities in Eqs. (33) and (38), it can be shown that the masses satisfy constraints

$$
\begin{aligned}
&M_{Z_1} \cos\theta_W < M_{W_1} < M_{Z_1} , \\
&M_{Z_2} \cos\theta_W < M_{W_2} < M_{Z_2} , \\
&M_{W_1} < M_{Z_1} < M_{W_2} < M_{Z_2} .
\end{aligned}
\tag{43}
$$

The other parameters of the model are related to the masses as follows:

$$
\begin{aligned}
&g_0^2 = \frac{e^2}{\sin^2\theta_W} \left[1 + \frac{(M_{Z_2}^2 - M_Z^2)(M_Z^2 - M_{Z_1}^2)}{M_Z^4 \cos^2\theta_W} \right] , \\
&g_2^2 = \frac{e^2}{\sin^2\theta_W} \left[1 + \frac{M_Z^4 \cos^2\theta_W}{(M_{Z_2}^2 - M_Z^2)(M_Z^2 - M_{Z_1}^2)} \right] , \\
&u^2 = \frac{e^2}{\sin^2\theta_W} \frac{2 M_{Z_1}^2 M_{Z_2}^2}{g_0^2 g_2^2 M_Z^2} .
\end{aligned}
\tag{44}
$$

The covariant derivative of Eq. (21) for the neutral sector becomes

$$
D_\nu^0 = \partial_\nu - ieQA_\nu - i \sum_{i=1,2} n_i \left[T^{(3)} - Q \left(\frac{M_Z^2 \sin^2\theta_W}{M_{Z_i}^2} \right) + T'^{(3)} \left(\frac{M_{Z_{3-i}}^2 - M_Z^2 \sin^2\theta_W}{M_{Z_{3-i}}^2 - M_Z^2} \right) \cdot \frac{M_Z^2}{M_{Z_i}^2} \right] Z_{i\nu} .
\tag{45}
$$

The ZWW vertex is

$$
T[Z_{i\mu} \to W_{j\nu}(p) W_{k\lambda}^\dagger(q)] = -i f_{ijk} [g_{\nu\lambda}(q-p)_\mu + g_{\mu\nu}(2p+q)_\lambda - g_{\lambda\mu}(2q+p)_\nu] ,
\tag{46}
$$

where

$$
f_{ijk} = \frac{e \cot\theta_W (-1)^{i+1} M_{Z_{3-i}}^2}{[(M_{Z_{3-i}}^2 - M_{Z_i}^2)(M_{Z_{3-i}}^2 - M_Z^2)]^{1/2}} \left(\delta_{jk} - \frac{\sqrt{2} N_j N_k}{8 G_F M_{Z_{3-i}}^2 \cos^2\theta_W} \right) ,
\tag{47}
$$

with $i, j, k = 1$ or 2. The AWW vertex is obtained from Eq. (46) with $f_{ijk} = e$ and $j = k$.

IV. PHENOMENOLOGY OF LOW-MASS GAUGE BOSONS

In this section we consider the experimental implications of Z_1 and W_1 gauge bosons which have masses below those of the standard model. As the two free parameters of the model, we use the masses M_{Z_1} and M_{Z_2}. The masses M_{W_1} and M_{W_2} are then determined through Eq. (42). Figure 1 shows a mapping from Z masses to W masses. Since $\cos\theta_W \simeq 0.88$ (from $\sin^2\theta_W = 0.23$), the mass of W_1 is close to that of Z_1 and the mass of W_2 is similar to that of Z_2.

The couplings to fermions can be expressed in terms of the gauge boson masses using Eqs. (22), (23), (29), (31), (37), (39), and (45). For the doublets (ν, e) and (u, d), the charged-current Hamiltonian is

$$
\mathcal{H}_{CC} = \frac{1}{2\sqrt{2}} [\bar{\nu}\gamma_\mu(1-\gamma_5)e + \bar{u}\gamma_\mu(1-\gamma_5)dc] \sum_{i=1,2} N_i W_{i\mu}^\dagger ,
\tag{48}
$$

where the N_1, N_2 are given in Eqs. (37). The neutral-current Hamiltonian is

$$
\mathcal{H}_{NC} = \sum_{i=1,2} (g_V^i \bar{\psi}\gamma_\mu\psi + g_A^i \bar{\psi}\gamma_\mu\gamma_5\psi) n_i Z_{i\mu} ,
\tag{49}
$$

where n_1, n_2 are given in Eq. (31) and

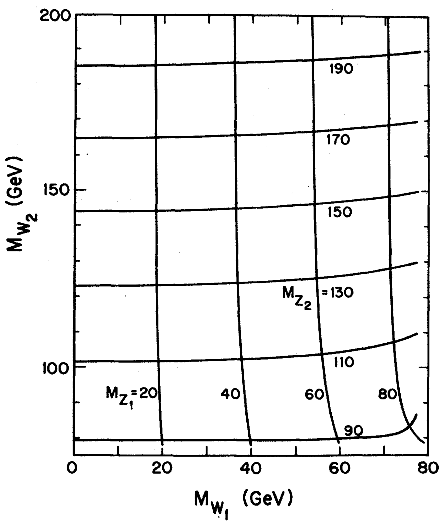

FIG. 1. Mapping from M_{Z_1}, M_{Z_2} to M_{W_1}, M_{W_2} determined by Eq. (42).

$$
\begin{aligned}
&g_V^i(\nu)=\tfrac{1}{4}, \quad g_A^i(\nu)=-\tfrac{1}{4},\\
&g_V^i(e)=-\tfrac{1}{4}+\frac{\sin^2\theta_W M_Z^2}{M_{Z_i}^2}, \quad g_A^i(e)=\tfrac{1}{4},\\
&g_V^i(u)=\tfrac{1}{4}-\frac{2\sin^2\theta_W M_Z^2}{3M_{Z_i}^2}, \quad g_A^i(u)=-\tfrac{1}{4},\\
&g_V^i(d)=-\tfrac{1}{4}+\frac{\sin^2\theta_W M_Z^2}{3M_{Z_i}^2}, \quad g_A^i(d)=\tfrac{1}{4}.
\end{aligned}
\tag{50}
$$

In Eq. (48), d_C denotes the Cabibbo-rotated combination $d_C=d\cos\theta_C+s\sin\theta_C$. The couplings of other quark or lepton doublets are identical to those in Eqs. (48)–(50).

In the limit $M_{Z_i}\to M_Z$ (and hence $M_{W_i}\to M_W$) for $i=1$ or 2, the other gauge bosons decouple from fermions and the standard model Hamiltonian is recovered. For this reason, it will be very difficult to exclude this model by experiments other than direct weak-boson mass determinations.

A. Decay widths and branching fractions

The partial widths for fermion-antifermion decays are

$$
\begin{aligned}
\Gamma(W_i\to f_1\bar f_2)&=\frac{cN_i^2}{24\pi M_{W_i}}\lambda^{1/2}(M_{W_i}^2,m_1^2,m_2^2)\\
&\quad\times\left[1-\frac{1}{2}\frac{(m_1^2+m_2^2)}{M_{W_i}^2}-\frac{1}{2}\frac{(m_1^2-m_2^2)^2}{M_{W_i}^4}\right],\\
\Gamma(Z_i\to f\bar f)&=\frac{cn_i^2}{12\pi M_{Z_i}}\lambda^{1/2}(M_{Z_i}^2,m^2,m^2)\\
&\quad\times\left[(g_V^i)^2\left[1+\frac{2m^2}{M_{Z_i}^2}\right]+(g_A^i)^2\left[1-\frac{4m^2}{M_{Z_i}^2}\right]\right],
\end{aligned}
\tag{51}
$$

where $c=1$ for leptons, $c=3$ for quarks, and $\lambda(x,y,z)=x^2+y^2+z^2-2xy-2xz-2yz$. In making estimates of the widths, we assume six flavors of leptons and quarks and use the mass assignments $m_\tau=1.79$, $m_{\nu_\tau}=0$, $m_u=m_d=0.3$, $m_s=0.5$, $m_c=1.5$, $m_b=4.7$, and $m_t=20$ or 30 GeV. The results are shown in Fig. 2. As M_{W_1} approaches M_W and M_{Z_1}, the W_2 and Z_2 bosons decouple from fermions. Thus the widths in Fig. 2 for $M_{W_1}=77.7$ GeV and $M_{Z_1}=88.6$ GeV correspond to the standard-model values. For W_1, Z_1 masses around 40 GeV, the widths become quite narrow, of order 50 MeV. This is about a factor of 50 narrower than the $\Gamma_W=2.4$ GeV, $\Gamma_Z=2.4$ GeV widths expected in the standard model. The branching fractions for $W\to\mu\nu$, $W\to$ hadrons, $Z\to\mu\bar\mu$, and $Z\to$ hadrons are

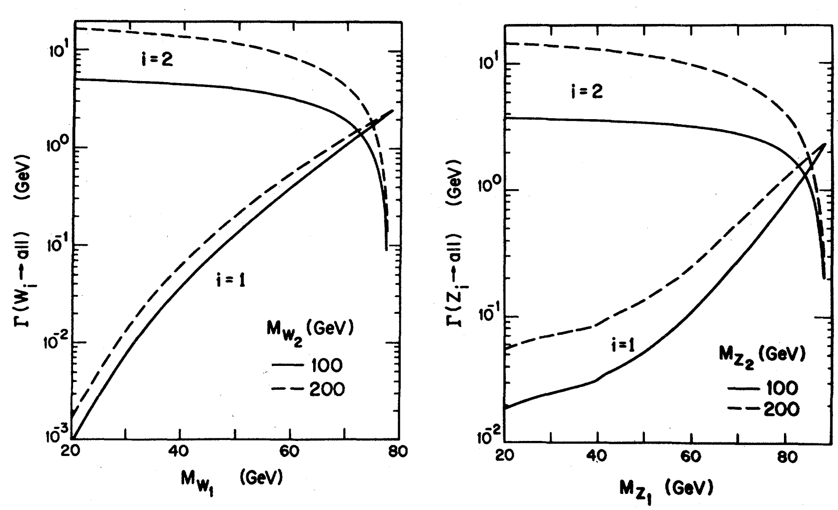

FIG. 2. Total widths of W_1, W_2 and Z_1, Z_2 gauge bosons for $m_t=20$ GeV.

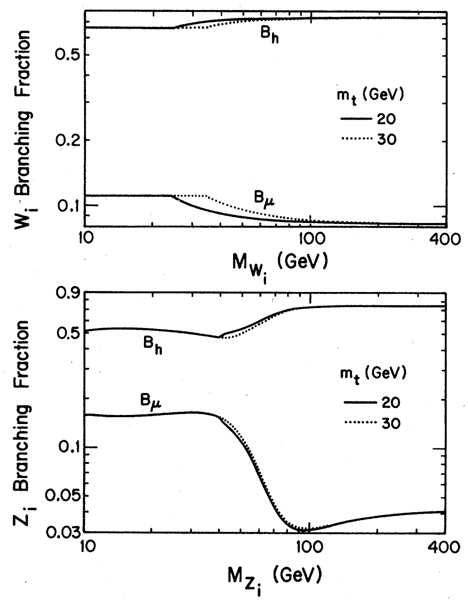

FIG. 3. Branching fractions for $W \to \mu\nu$, $W \to$ hadrons and $Z \to \mu\bar{\mu}$, $Z \to$ hadrons. Solid curves represent $m_t = 20$ GeV, dotted curves $m_t = 30$ GeV.

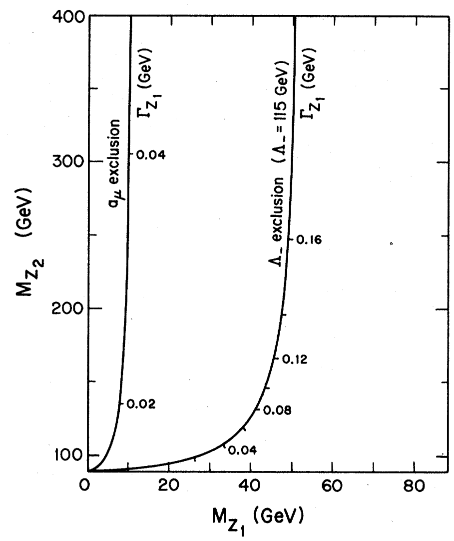

FIG. 4. Excluded regions (to the left of curves) for Z_1 and Z_2 masses from (i) the muon anomalous magnetic moment a_μ, allowing for one standard deviation from the mean experimental value, and (ii) the QED cutoff parameter Λ_- of $e^+e^- \to \mu^+\mu^-$ measurements.

shown in Fig. 3. These branching fractions and the total widths are insensitive to the value of m_t.

B. Muon anomalous magnetic moment

The weak contributions to $a_\mu = \frac{1}{2}(g_\mu - 2)$ from triangle diagrams are[7]

$$a_\mu = \frac{m_\mu^2}{12\pi^2}\left\{\frac{5}{8}\sum_{i=1,2}\frac{N_i^2}{M_{W_i}^2} + \sum_{i=1,2}\frac{n_i^2}{M_{Z_i}^2}\left[(g_V^i)^2 - 5(g_A^i)^2\right]\right\}. \tag{52}$$

The $W_{1,2}$ contribution is the same as in the standard model. The C term of Eq. (32) increases the contribution of the neutral sector. The result is

$$a_\mu = a_\mu^{\text{SM}} + \frac{\sqrt{2}G_F m_\mu^2 C}{3\pi^2}, \tag{53}$$

where the standard-model contribution is $a_\mu^{\text{SM}} = 1.9 \times 10^{-9}$. The discrepancy between the experimental value for a_μ and the theoretical electromagnetic contributions through eighth order is[8]

$$a_\mu(\text{exp}) - a_\mu(\text{em}) = (4 \pm 22) \times 10^{-9}.$$

This can be used to set a lower limit on M_{Z_1} for given M_{Z_2}. Allowing for one standard deviation in the discrepancy, the resulting bound on M_{Z_1} is not very restrictive (see Fig. 4).

C. Production by e^+e^- colliding beams

In the approximation of zero lepton masses, the Mandelstam variables for the $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to e^+e^-$ reactions are $s = 4E^2$, $t = -s\sin^2(\theta_{\text{c.m.}}/2)$, and $u = -s\cos^2(\theta_{\text{c.m.}}/2)$, where E is the c.m. energy and $\theta_{\text{c.m.}}$ is the c.m. scattering angle of the outgoing μ^- or e^- relative to the incident e^- The unpolarized differential cross sections can be expressed in terms of the amplitudes

$$\begin{aligned} G_\pm(r) &= \frac{e^2}{r} + \sum_{i=1,2}\frac{n_i^2\left[(g_V^i)^2 \pm (g_A^i)^2\right]}{r - M_{Z_i}^2 + iM_{Z_i}\Gamma_{Z_i}}, \\ G_0(r) &= \sum_{i=1,2}\frac{n_i^2 2(g_V^i)(g_A^i)}{r - M_{Z_i}^2 + iM_{Z_i}\Gamma_{Z_i}}, \end{aligned} \tag{54}$$

where the argument r is either s or t and the g^i are given in Eq. (50). The cross sections have the forms

$$\begin{aligned} \frac{d\sigma}{dt}(e^+e^- \to \mu^+\mu^-) &= \frac{1}{8\pi s^2}\left[|G_-(s)|^2 t^2 + \left(|G_+(s)|^2 + |G_0(s)|^2\right)u^2\right], \\ \frac{d\sigma}{dt}(e^+e^- \to e^+e^-) &= \frac{1}{8\pi s^2}\left[|G_-(s)|^2 t^2 + |G_-(t)|^2 s^2 + \left(|G_+(t) + G_+(s)|^2 + |G_0(t) + G_0(s)|^2\right)u^2\right]. \end{aligned} \tag{55}$$

Figure 5 illustrates the ratio of the integrated $e^+e^- \to \mu^+\mu^-$ cross section to the pure QED value as a function of $\sqrt{s}$, for the case $M_{Z_1} = 45$ GeV, $M_{Z_2} = 100$ GeV. The standard model results are shown for comparison.

The integrated forward-backward asymmetry in the $e^+e^- \to \mu^+\mu^-$ reaction is given by

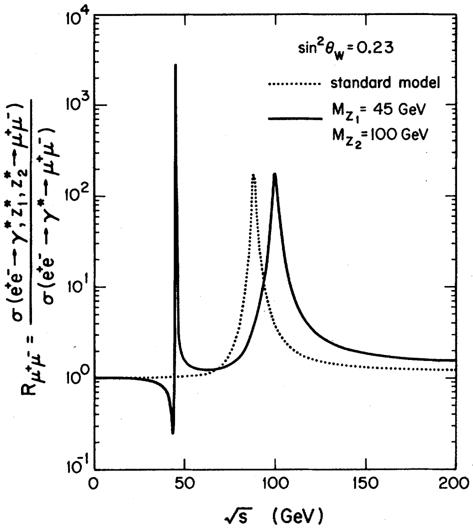

FIG. 5. Ratio of total $e^+e^- \to \mu^+\mu^-$ cross section to the pure QED value vs $\sqrt{s}$, for the case $M_{Z_1} = 45$ GeV, $M_{Z_2} = 100$ GeV.

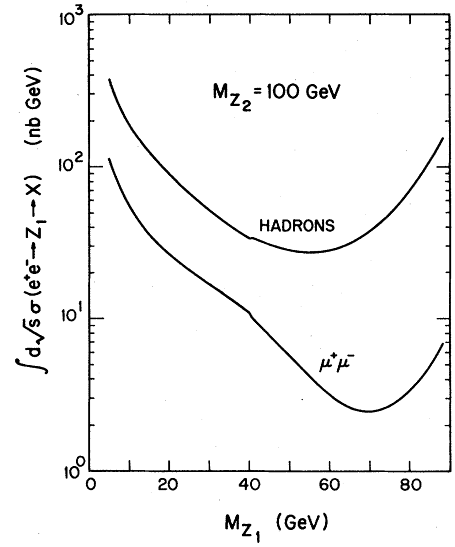

FIG. 7. Integrated Z_1 resonance contributions $\int d\sqrt{s}\,\sigma(e^+e^- \to Z_1 \to X)$ vs M_{Z_1}, for $X = \mu^+\mu^-$ and $X =$ hadrons.

$$A = \frac{1}{\sigma}\int_0^{\pi/2} [d\sigma(\theta_{\text{c.m.}}) - d\sigma(-\theta_{\text{c.m.}})]$$

$$= \frac{3}{4}\,\frac{-|G_-(s)|^2 + |G_+(s)|^2 + |G_0(s)|^2}{|G_-(s)|^2 + |G_+(s)|^2 + |G_0(s)|_2}. \quad (56)$$

Figure 6 shows the predicted asymmetry as a function of $\sqrt{s}$ for the case $M_{Z_1} = 45$ GeV, $M_{Z_2} = 100$ GeV. The SPEAR measurement[9] at 6.8 GeV places no constraint on M_{Z_1}.

The integrated resonance contributions above background are

$$\int d\sqrt{s}\,\sigma(e^+e^- \to Z \to \mu^+\mu^-) = 6\pi^2 B_\mu{}^2 \Gamma_Z / M_Z{}^2,$$
$$\int d\sqrt{s}\,\sigma(e^+e^- \to Z \to \text{hadrons}) = 6\pi^2 B_\mu B_h \Gamma_Z / M_Z{}^2, \quad (57)$$

where B_μ is the $Z \to \mu^+\mu^-$ branching fraction and B_h is the $Z \to$ hadrons branching fraction. Figure 7 shows the behavior of these integrated quantities for the Z_1 boson. The magnitudes are strongly dependent on the Z_1 mass. When M_{Z_2} approaches M_Z, the present model approaches the standard model, with the light Z_1 almost decoupled from all interactions. To avoid this limit, we require $M_{Z_2} > 90$ GeV. Figure 8 shows the cross section for $e^+e^- \to Z_1, Z_1\gamma$ vs $\sqrt{s}$ for $M_{Z_1} = 40$ GeV and M_{Z_2}

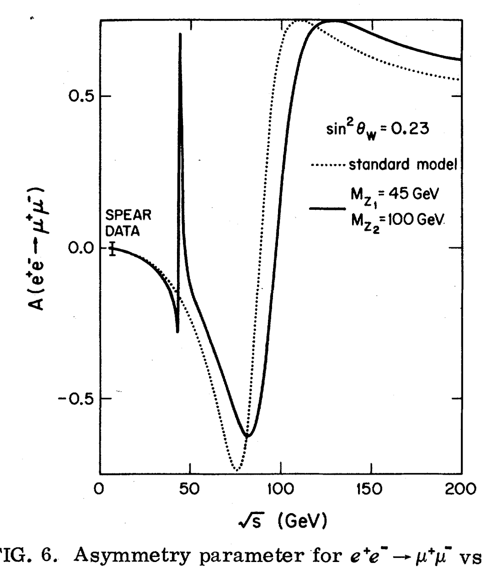

FIG. 6. Asymmetry parameter for $e^+e^- \to \mu^+\mu^-$ vs $\sqrt{s}$, for the case $M_{Z_1} = 45$ GeV, $M_{Z_2} = 100$ GeV.

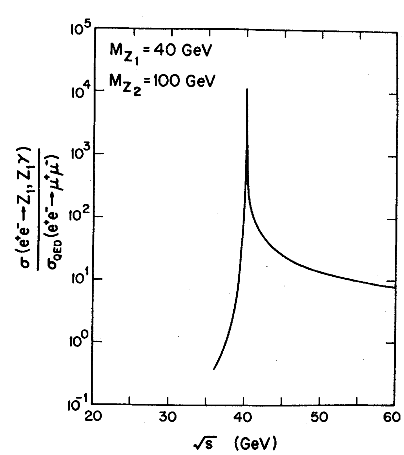

FIG. 8. Ratio of total $e^+e^- \to Z_1, Z_1\gamma$ cross section to the pure QED cross section for the process $e^+e^- \to \mu^+\mu^-$ vs $\sqrt{s}$, with $M_{Z_1} = 40$ GeV, $M_{Z_2} = 100$ GeV.

$=100$ GeV. The expression for the $e^+e^- \to Z_1\gamma$ cross section is

$$\sigma(e^+e^- \to Z_1\gamma) = \frac{\alpha n_1[(g_A{}^1)^2 + (g_V{}^1)^2]}{(s - M_{Z_1}{}^2)} \times \left[\left(1 + \frac{M_{Z_1}{}^4}{s^2}\right)\ln\frac{s}{m_e{}^2} - \left(1 - \frac{M_{Z_1}{}^2}{s}\right)^2\right]k, \tag{58}$$

where k is the soft-photon correction factor[10]

$$k = \left(1 - \frac{M_{Z_1}{}^2}{s}\right)^\delta \left[1 + \tfrac{13}{12}\delta + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{17}{18}\right)\right],$$

with

$$\delta = \frac{4\alpha}{\pi}\left(\ln\frac{M_{Z_1}}{m_e} - \frac{1}{2}\right).$$

The signal is large and would have been detected at PETRA if $M_{Z_1} < 32$ GeV.

Limits on deviations from QED of the $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to e^+e^-$ cross sections can be used to place a lower bound on M_{Z_1} for a given M_{Z_2}. The bounds on the deviations have been parametrized as

$$\left(1 - \frac{s}{\Lambda_-{}^2 - s}\right)^2 \leq \frac{\sigma}{\sigma_{\rm QED}} \leq \left(1 + \frac{s}{\Lambda_+{}^2 - s}\right)^2. \tag{59}$$

For small $s/\Lambda_\pm{}^2$, this becomes

$$-\frac{2s}{\Lambda_-{}^2} \leq \frac{\sigma - \sigma_{\rm QED}}{\sigma_{\rm QED}} \leq \frac{2s}{\Lambda_+{}^2}. \tag{60}$$

The integrated $e^+e^- \to \mu^+\mu^-$ cross section

$$\sigma = \frac{s}{24\pi}[|G_-(s)|^2 + |G_+(s)|^2 + |G_0(s)|^2] \tag{61}$$

for $s \ll M_{Z_1}{}^2$ becomes

$$\sigma \simeq \frac{4\pi\alpha^2}{3s}\left(1 - \frac{3G_F s}{\sqrt{2}\pi\alpha}[C + (\tfrac{1}{4} - \sin^2\theta_W)^2]\right). \tag{62}$$

Assuming that σ saturates the experimental lower bound in Eq. (60), the constraint on C is

$$C \leq \frac{\pi\alpha}{\sqrt{2}G_F\Lambda_-{}^2} - (\tfrac{1}{4} - \sin^2\theta_W)^2. \tag{63}$$

Using Eq. (32), the restriction on Z_1, Z_2 masses for $\Lambda_- = 115$ GeV (Ref. 11) are shown in Fig. 4.

At energies below the Z_1 mass, the QED Bhabha scattering differential cross section is modified by weak effects. Figure 9 shows typical modifications to the angular distribution for the case $M_{Z_1} = 35$ GeV with $M_{Z_2} = 150$ and 500 GeV.

Representative predictions for gauge-boson pair production by e^+e^- colliding beams are shown in Figs. 10 and 11. If M_{Z_2} is above $2M_{W_1}$, a large enhancement of the $W_1^+W_1^-$ cross section occurs as a consequence of the triple gauge-boson coupling Eq. (47). The Z_1Z_1 pair-production cross section proceeds solely through electron exchange. The cross-section formulas can be found in Ref. 12.

D. Drell-Yan production

In hadron-hadron collisions, quark-antiquark annihilation leads to the production of Z and W gauge bosons. The Z bosons can be observed

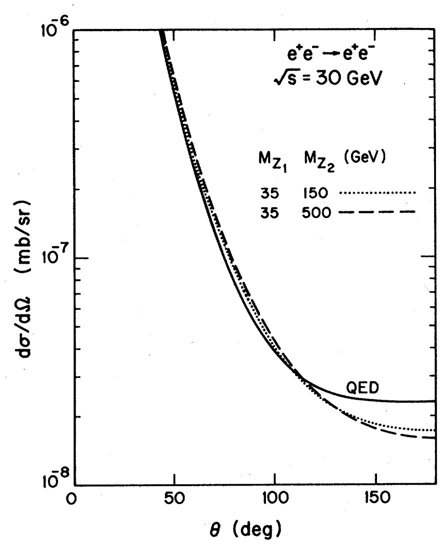

FIG. 9. Differential cross section for $e^+e^- \to e^+e^-$ vs scattering angle at $\sqrt{s} = 30$ GeV, for $M_{Z_1} = 35$ GeV with $M_{Z_2} = 150$ and 500 GeV.

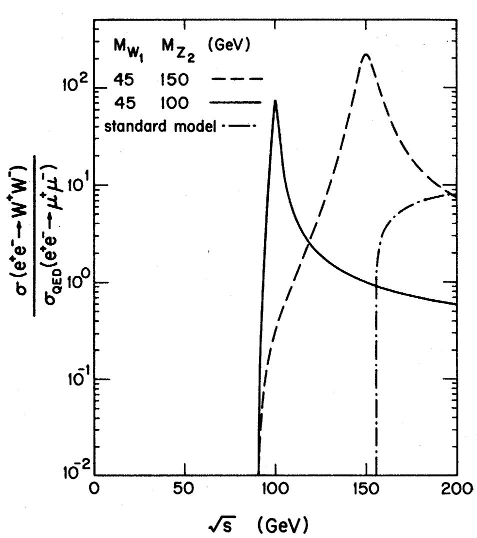

FIG. 10. Ratio of $e^+e^- \to W_1^+W_1^-$ cross section to the pure QED $e^+e^- \to \mu^+\mu^-$ cross section vs $\sqrt{s}$, for the case $M_{W_1} = 45$ GeV with $M_{Z_2} = 100$ and 150 GeV.

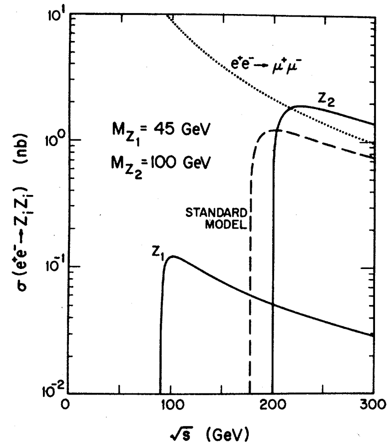

FIG. 11. Total $e^+e^- \to Z_iZ_i$ cross section vs $\sqrt{s}$ with $i=1,2$ for $M_{Z_1}=45$ GeV, $M_{Z_2}=100$ GeV. The dashed line represents Z-pair production in the standard model. The dotted line represents the pure QED value for the process $e^+e^- \to \mu^+\mu$.

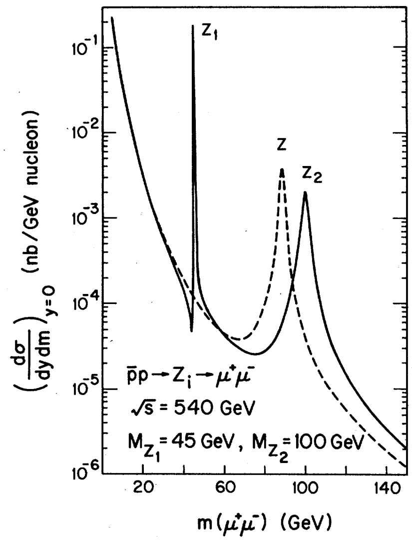

FIG. 12. Prediction for the cross section $d\sigma/dy\,dm$ at $y=0$ vs m of the Drell-Yan muon pair production process for $\bar{p}p$ colliding beams at $\sqrt{s}=540$ GeV; the case $M_{Z_1}=45$ GeV, $M_{Z_2}=100$ GeV is illustrated.

through muon pair production. The subprocess cross section for annihilation of quarks of flavor q_k with charge e_k in units of e is

$$\sigma(q_k q_{\bar{k}} \to \mu\bar{\mu}) = \frac{m^2}{24\pi}(|H_-|^2 + |H_+|^2 + |H_0|^2)\,, \qquad (64a)$$

$$\frac{d\sigma}{dt}(q_k q_{\bar{k}} \to \mu\bar{\mu}) = \frac{1}{8\pi m^4}[|H_-|^2 t^2 + (|H_+|^2 + |H_0|^2)u^2]\,, \qquad (64b)$$

where m is the muon-pair mass, and

$$H_\pm = \frac{e_k e^2}{m^2} + \sum_{i=1,2} \frac{n_i^2[g_V^i(\mu)g_V^i(q_k) \pm g_A^i(\mu)g_A^i(q_k)]}{m^2 - M_{Z_i}^2 + iM_{Z_i}\Gamma_{Z_i}}\,, \qquad (65)$$

$$H_0 = \sum_{i=1,2} \frac{n_i^2[g_V^i(\mu)g_A^i(q_k) + g_V^i(q_k)g_A^i(\mu)]}{m^2 - M_{Z_i}^2 + iM_{Z_i}\Gamma_{Z_i}}\,.$$

In calculating the inclusive production cross section $AB \to \mu^+\mu^- X$, $\sigma(q_k\bar{q}_k \to \mu\bar{\mu})$ must be folded with the momentum distributions of the quark k in the initial hadrons

$$\frac{d\sigma}{dy\,dm} = \frac{2x_+x_-}{3m}\sum_k f_k^A(x_+,m^2) f_{\bar{k}}^B(x_-,m^2)\sigma(q_k q_{\bar{k}} \to \mu^+\mu^-)\,. \qquad (66)$$

Here $f_k^A(x,m^2)$ is the fractional momentum distribution of quark k in particle A, $f_{\bar{k}}$ is defined similarly; the summation is over all quark and antiquark flavors. Also, y is the rapidity of the muon pair, $x_\pm = (m/\sqrt{s})\exp(\pm y)$, and $s=(p_A+p_B)^2$ is the c.m. energy squared. For an arbitrary nucleus $A=(N,Z)$, the up-quark distribution functions are $f_u^A = Zu + Nd$, $f_{\bar{u}}^A = Z\bar{u} + N\bar{d}$; for incident protons $f_u^p = u$, $f_{\bar{u}}^p = \bar{u}$, and for incident antiprotons $f_{\bar{u}}^{\bar{p}} = u$, $f_u^{\bar{p}} = \bar{u}$. We use the quantum-chromodynamics parametrization of Owens and Reya[13] for the parton distributions. Figure 12 shows the dimuon mass distribution at $y=0$ in $\bar{p}p \to \mu^-\mu^+X$ at $\sqrt{s}=540$ GeV, for the case $M_{Z_1}=45$ GeV, $M_{Z_2}=100$ GeV. Experimental resolution would broaden the sharp Z_1 resonance spike. Measurements[14] of the process $pA \to \mu^-\mu^+X$ at $\sqrt{s}=27.4$ GeV rule out[15] a Z_1 in the mass range $M_{Z_1} < 15$ GeV, since no hundredfold enhancements above background were observed.

For W production via the Drell-Yan process the distribution of interest[16] is the transverse momentum of a single lepton ($p_\perp$). The rate for the relevant subprocess $Q_k q_{\bar{k}} \to W^+ \to \mu^+\nu$ in terms of the invariant mass m of the $\mu^+\nu$ pair is

$$\frac{d\sigma}{dt}(Q_k q_{\bar{k}} \to \mu^+\nu) = \frac{1}{64\pi m^2}\left|\sum_i \frac{N_i^2 U_{Qq}}{m^2 - M_{W_i}^2 + iM_{W_i}\Gamma_i}\right|^2 \times [\epsilon_k t^2 + (1-\epsilon_k)u^2]\,, \qquad (67)$$

where $\epsilon_k = 1$ for quark k and $\epsilon_k = 0$ for antiquark k. The elements of the quark mixing matrix are $U_{ud} = \cos\theta_C$ and $U_{us} = \sin\theta_C$, where θ_C is the Cabibbo angle. For $\mu^-\nu$ production $\epsilon_k \to 1-\epsilon_k$ in Eq. (67). The inclusive cross section $AB \to \mu^+X$ is[17]

$$E\frac{d^3\sigma}{dp^3} = \frac{d\sigma}{p\,dp\,d\Omega} = \frac{1}{3\pi}\int dy' \sum_k x_k x_{\bar{k}} f^A(x_k) f^B(x_{\bar{k}}) \times \frac{d\sigma}{dt}(\text{subprocess})\,, \qquad (68)$$

where

$$x_k = \frac{p_\perp}{\sqrt{s}}(e^{y} + e^{y'}),$$
$$x_{\bar{k}} = \frac{p_\perp}{\sqrt{s}}(e^{-y} + e^{-y'}),$$
$$m^2 = x_k x_{\bar{k}} s, \qquad (69)$$
$$t = -p_\perp \sqrt{s}\, x_{\bar{k}} e^{y'},$$
$$u = -p_\perp \sqrt{s}\, x_k e^{-y'}.$$

The quantity y is the rapidity of the produced lepton. The integration limits for y' are $\pm\ln(\sqrt{s}/p_\perp - e^{\pm y})$. In Fig. 13, the cross section $d\sigma/dp_\perp d\Omega$ is plotted at $y=0$ ($\theta_{c.m.}=90°$, $p=p_\perp$) for $\bar{p}p$ collisions at $\sqrt{s}=540$ GeV. The $p_\perp$ spectrum of μ^+ from virtual γ and $Z_{1,2}$ are shown along with the spectrum from $W^+_{1,2}$. In this illustration the masses are $M_{W_1}=43$ GeV, $M_{Z_1}=45$ GeV, $M_{W_2}=91$ GeV, and $M_{Z_2}=100$ GeV. Notice that the single-lepton signal from W_1 (or Z_1) production is significantly above the γ^* background, even if M_{W_1} is as small as 30 GeV.

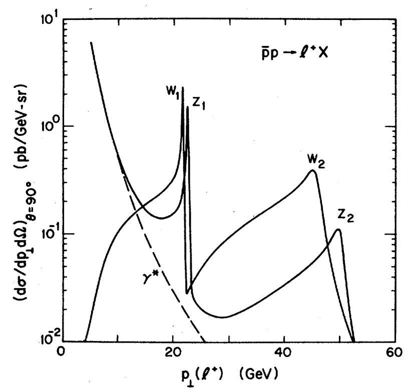

FIG. 13. Predicted transverse-momentum spectrum of the lepton (l^+) resulting from $W^+_{1,2}$ and $Z_{1,2}$ production $\bar{p}p$ collisions at c.m. energy $\sqrt{s}=540$ GeV. The case $M_{W_1}=43$ GeV, $M_{Z_1}=45$ GeV, $M_{W_2}=91$ GeV, and $M_{Z_2}=100$ GeV is illustrated.

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