Reexamination of radiative decays of vector mesons

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New data have been used to reexamine radiative decays of mesons from the viewpoint of SU(3) symmetry. Most former difficulties have been resolved, and further tests must await improvement of measurements of $\phi\to\eta\gamma$ and $K^{*0}\to K^0\gamma$.

In spite of many theoretical attempts,[1-5] progress in the understanding of radiative decay rates of ordinary mesons has been meager; this has been primarily because of the previously measured small value of $\rho\to\pi\gamma$.[6] In contrast to the success in interpreting $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$,[7] the vector-dominance model (VDM), SU(3) symmetry, and the simple quark model (QM) could not account for the old ratio of $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$. Comparison of values for $\Gamma(\rho\to\pi\gamma)$, $\Gamma(\omega\to\pi\gamma)$, and $\Gamma(\phi\to\pi\gamma)$ indicated a breaking of SU(3) that could not be explained simply by varying somewhat the ω-ϕ mixing angle.[5] In addition, data for $\Gamma(K^{*0}\to K^0\gamma)$ (Ref. 8) and $\Gamma(\phi\to\eta\gamma)$ (Refs. 9 and 10) were not consistent with $\Gamma(\omega\to\pi\gamma)$ in the framework of the simplest theoretical ideas.

The recent remeasurement of $\Gamma(\rho\to\pi\gamma)$,[11] yielding almost twice the previous value, has greatly reduced the difficulties in the overall picture of the radiative transitions of ordinary mesons. Also, the first measurement of $\Gamma(K^{*-}\to K^-\gamma)$ (Ref. 12) and of $\Gamma(\eta'\to\gamma\gamma)$ (Ref. 13) [the latter is important because it can be used to deduce the absolute values of $\Gamma(\eta'\to\rho\gamma)$ and $\Gamma(\eta'\to\omega\gamma)$] provide new additional information that can be used for confronting various models. Because many previous attempts to understand radiative-decay systematics were encumbered with the small value of $\Gamma(\rho\to\pi\gamma)$, we felt that it would be useful to reexamine the theoretical situation in light of the availability of the new data.

Couplings for radiative transitions of vector and pseudoscalar mesons can be written as follows[5,14,15]:

$$
\begin{aligned}
g(\rho\to\pi\gamma) &= \frac{a}{2}\,,\\
g(\omega\to\pi\gamma) &= \frac{\sqrt{3}}{2}(a\sin\theta_V + d\cos\theta_V)\,,\\
g(\phi\to\pi\gamma) &= \frac{\sqrt{3}}{2}(a\cos\theta_V - d\sin\theta_V)\,,\\
g(\rho\to\eta\gamma) &= \frac{\sqrt{3}}{2}(a\cos\theta_P - e\sin\theta_P)\,,\\
g(\omega\to\eta\gamma) &= \tfrac{1}{2}(-a\sin\theta_V\cos\theta_P + d\cos\theta_V\cos\theta_P - e\sin\theta_V\sin\theta_P)\,,\\
g(\phi\to\eta\gamma) &= \tfrac{1}{2}(-a\cos\theta_V\cos\theta_P - d\sin\theta_V\cos\theta_P - e\cos\theta_V\sin\theta_P)\,,\\
g(\eta'\to\rho\gamma) &= \frac{\sqrt{3}}{2}(a\sin\theta_P + e\cos\theta_P)\,,\\
g(\eta'\to\omega\gamma) &= \tfrac{1}{2}(-a\sin\theta_V\sin\theta_P + d\cos\theta_V\sin\theta_P + e\sin\theta_V\cos\theta_P)\,,\\
g(\phi\to\eta'\gamma) &= \tfrac{1}{2}(-a\cos\theta_V\sin\theta_P - d\sin\theta_V\sin\theta_P + e\cos\theta_V\cos\theta_P)\,,\\
g(K^{*0}\to K^0\gamma) &= -a\,,\\
g(K^{*\pm}\to K^\pm\gamma) &= \frac{a}{2}\,,
\end{aligned}
\tag{1}
$$

where a is the coupling constant for the vector octet (V_8)–pseudoscalar octet (P_8)–photon (γ) vertex, d is the coupling constant for the vector singlet (V_1)–P_8–γ vertex, and e describes the V_8–pseudoscalar singlet (P_1)–γ coupling. θ_V and θ_P are the octet-singlet mixing angles, with ideal mixing and the quadratic mass formula taken for vector and pseudoscalar mesons, respectively. [Using instead the SU(3) relation for two-photon decays of pseudoscalars, $g_{\eta'\gamma\gamma}\sin\theta_P + g_{\eta\gamma\gamma}\cos\theta_P = g_{\pi\gamma\gamma}/\sqrt{3}$, and the values $\Gamma(\pi^0\to 2\gamma) = 7.86\pm 0.54$ eV,[16] $\Gamma(\eta\to\gamma\gamma) = 323\pm 46.4$ eV,[16] and $\Gamma(\eta'\to\gamma\gamma) = 5.66\pm 1.45$ keV,[13] yields $\theta_P = -8.0°\pm 2.5°$, which is consistent with out choice for θ_P of $-11°$.] Radiative decay widths can be expressed in terms of the above couplings as[2,3,15,17]

$$
\begin{aligned}
\Gamma(V\to P\gamma) &= \frac{|k|^3}{12\pi}|g(V\to P\gamma)|^2\,,\\
\Gamma(P\to V\gamma) &= \frac{|k|^3}{4\pi}|g(P\to V\gamma)|^2\,,
\end{aligned}
\tag{2}
$$

where k is the momentum of the photon in the rest frame of the decay.

Previous phenomenological studies have not been consistent in choosing the most reliable data available. We, consequently, list all our sources and indicate specifically which data are not utilized and offer reasons for our procedure:

(1) $\Gamma(\rho\to\pi\gamma)$. We use the latest and most reliable measurement of 67 ± 7 keV.[11]

(2) $\Gamma(\omega\to\pi\gamma)$. The Particle Data Group[16] lists the branching ratio $B(\omega\to\pi\gamma)=8.8\pm0.5\%$. This is obtained from an overall fit to several decay modes, and does not necessarily yield the best value for $\Gamma(\omega\to\pi\gamma)$. For the ratio $\Gamma(\omega\to\pi\gamma)/\Gamma(\omega\to\pi^+\pi^-\pi^0)$, we have used a value of $8.72\pm0.98\%$. This has been obtained from an average of the three experiments[18] which directly measure this ratio. (The Particle Data Group value is $9.8\pm0.5\%$.) Although $\Gamma(\omega\to\pi\gamma)$ is correlated with the other decay modes in the fit, our choice will not seriously affect the Particle Data Group value of $B(\omega\to\pi^+\pi^-\pi^0)$. We have therefore used $B(\omega\to\pi^+\pi^-\pi^0)=89.9\pm0.56\%$ as quoted by the Particle Data Group, which yields $\Gamma(\omega\to\pi\gamma)=789\pm92$ keV. We wish to note that including the $\rho\to\pi^0\gamma$ contribution in the extraction of $\omega\to\pi\gamma$ may reduce $\Gamma(\omega\to\pi\gamma)$ by about 10%. (See Benaksas *et al.*, Ref. 18.)

(3) $\Gamma(\phi\to\pi\gamma)$. We use the average of the only two experiments available.[19]

(4) $\Gamma(\rho\to\eta\gamma)$. The only absolute measurement of $\Gamma(\rho\to\eta\gamma)$ is given in Ref. 10. However, the reported value has to be corrected for the $\rho\to\pi\gamma$ subtraction that was used in the normalization procedure in that experiment. Because of a two-solution ambiguity in the data, we have not used the values of $\Gamma(\rho\to\eta\gamma)$ as inputs in our fits.

(5) $\Gamma(\omega\to\eta\gamma)$. We use 8.3 ± 35.2 keV, based on the measured ratio $\Gamma(\omega\to\eta\gamma)/\Gamma(\omega\to\pi\gamma)=(1.05\pm4.46)\%$.[20] We do not use the result of Andrews *et al.* (Ref. 10) because of the ambiguity mentioned under item (4).

(6) $\Gamma(\phi\to\eta\gamma)$. We use the average of all measurements,[9,10] again correcting Andrews *et al.* for the $\rho\to\pi\gamma$ ratio rate as in items (4) and (5) to obtain 67.7 ± 9.5 keV.

(7) $\Gamma(\eta'\to\rho\gamma)$. By averaging the two measurements of the total width,[13] we estimate as $\Gamma(\eta'\to\text{all})=289.7\pm73.4$ keV. In addition, using the average of $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\pi^+\pi^-\gamma)=1.082\pm0.077$ (Ref. 21) and the Particle Data value for $B(\eta'\to\pi^+\pi^-\gamma)$ (Ref. 16) yields $\Gamma(\eta'\to\rho\gamma)=93.1\pm25.1$ keV, where the large error comes mainly from the uncertainty in the total width.

(8) $\Gamma(\eta'\to\omega\gamma)$. From the measurement of $\Gamma(\eta'\to\omega\gamma)/\Gamma(\eta'\to\eta\pi^+\pi^-)=(6.8\pm1.3)\%$,[7] we determine $\Gamma(\eta'\to\omega\gamma)=8.4\pm2.7$ keV.

(9) $\Gamma(K^{*0}\to K^0\gamma)$. We use the only value available at this time, namely 75 ± 35 keV.[8]

(10) $\Gamma(K^{*-}\to K^-\gamma)$. We use 40 ± 15 keV from Ref. 12.

Gaussian errors have been assumed for all averaging and consequently the errors may be somewhat optimistic.

The values of $\Gamma(\rho\to\pi\gamma)$, $\Gamma(\omega\to\pi\gamma)$, and $\Gamma(\eta'\to\rho\gamma)$ are relatively well determined and have, therefore, been used as input in the fit. The coupling constant a is determined by $\Gamma(\rho\to\pi\gamma)$ as well as

TABLE I. Comparison of radiative widths with predictions from SU(3) symmetry.

Decay process	Γ_{exp} (keV)	Γ_{calc} (keV) Three-parameter fit[a]	Two-parameter fit[b]
$\rho\to\pi\gamma$	67 ± 7[c]	67 ± 7	73.8 ± 5.5
$\omega\to\pi\gamma$	789 ± 92[c]	789 ± 120	684 ± 51
$\phi\to\pi\gamma$	6.5 ± 1.9[d]	11.1 ± 14.5	6.5 ± 2.0
$\rho\to\eta\gamma$	$\left\{\begin{matrix}52.5\pm13.7^{e}\\79.8\pm15.9^{f}\end{matrix}\right\}$	45.6 ± 21.7	47.8 ± 3.5
$\omega\to\eta\gamma$	$\left\{\begin{matrix}8.3\pm35.2\\3.2^{+2.6}_{-1.9}{}^{e}\\30.5\pm7.4^{f}\end{matrix}\right\}$	9.0 ± 2.5	4.4 ± 0.4
$\phi\to\eta\gamma$	67.7 ± 9.5	137 ± 18	135 ± 10
$\eta'\to\rho\gamma$	93.1 ± 25.1[c]	93.1 ± 24.1	79.9 ± 5.9
$\eta'\to\omega\gamma$	8.4 ± 2.7	8.4 ± 2.4	10.3 ± 0.8
$\phi\to\eta'\gamma$		0.7 ± 0.1	0.6 ± 0.04
$K^{*0}\to K^0\gamma$	75 ± 35	147 ± 16	162 ± 12
$K^{*-}\to K^-\gamma$	40 ± 15	37.5 ± 4.0	41.3 ± 3.1

[a] For the three-parameter fit we have used ideal mixing ($\theta_V=35.3°$) and the quadratic mass formula ($\theta_P=-11°$) for vector and pseudoscalar mesons, respectively. We have not imposed nonet symmetry. The input data are labeled with a superscript c.

[b] For the two-parameter fit we imposed nonet symmetry and kept θ_V as a free parameter. The input data are labeled with superscripts c and d.

[c] Input data.

[d] Additional input data for two-parameter fit.

[e] This corresponds to the first solution of Ref. 10.

[f] This corresponds to the second solution of Ref. 10.

by $\Gamma(\omega\to\pi\gamma)$ and $\Gamma(\eta'\to\rho\gamma)$. The couplings d and e can be extracted from $\Gamma(\omega\to\pi\gamma)$ and $\Gamma(\eta'\to\rho\gamma)$, respectively. Using the three input widths we obtain the couplings $a=0.437\pm0.023$, $d=0.734\pm0.063$, and $e=0.688\pm0.078$, all in units of GeV^{-1}. The predicted widths for the other decay modes a are shown in the third column of Table I; quoted errors are calculated using the uncertainties in the fitted couplings. Figure 1 shows a comparison between the predicted decay couplings and those deduced from the experimental results [using Eq. (2) and the sign of g that is consistent with SU(3) symmetry].

To incorporate the observed rate for $\phi\to\pi\gamma$ in the framework of the nonet ansatz,[22] we have to relax the restriction of ideal mixing for vector mesons, otherwise this decay would be forbidden. Thus, assuming nonet symmetry, we obtain the following relationship among the couplings: $\sqrt{2}\ a=d=e$.[1,4] Adding $\Gamma(\phi\to\pi\gamma)$ to the three previous input widths yields the following fitted parameters: $\sqrt{2}\ a=d=e=0.648\pm0.024\ \mathrm{GeV}^{-1}$ and $\theta_V=38.9\pm0.6°$. This value of θ_V lies about halfway between the θ_V angles obtained using a linear or quadratic mass formula. From this two-parameter fit we obtain the predicted widths given in column 4 of Table I.

From the results shown in Fig. 1 we see that the three-parameter fit provides predictions in excellent agreement with the values of $\Gamma(\phi\to\pi\gamma)$, $\Gamma(\eta'\to\omega\gamma)$, and $\Gamma(K^{*-}\to K^-\gamma)$. Also, the fit favors solution (a) for $\Gamma(\rho\to\eta\gamma)$ and $\Gamma(\omega\to\eta\gamma)$ of Ref. 10. In addition, the results of the fit for the ratios of $g(\omega\to\pi\gamma)/g(\rho\to\pi\gamma)$, $g(\rho\to\eta\gamma)/g(\omega\to\eta\gamma)$, and $g(\eta'\to\rho\gamma)/g(\eta'\to\omega\gamma)$, which are, respectively, 3.33 ± 0.36, 2.33 ± 0.64, and 3.17 ± 0.61, are consistent with the VDM prediction of (3). Only two widths appear to be different from the predicted values of our fit: $\Gamma(K^{*0}\to K^0\gamma)$ and $\Gamma(\phi\to\eta\gamma)$. The K^{*0} measurement has a very large relative error and its value is about two standard deviations away from the expected width. In fact, this measurement may be suspect because of the somewhat model-dependent assumptions used in extracting $\Gamma(K^{*0}\to K^0\gamma)$.[23] As for the $\phi\to\eta\gamma$ discrepancy, we point out that the four measurements are not very consistent ranging from 57.8 ± 12.7 keV (Ref. 10) to 301.5 ± 79.5 keV of Basile *et al.*[9] Nevertheless, the newer experiments yield a value of $\Gamma(\phi\to\eta\gamma)$ that is about one half of that we predict. If this result holds up in the future, it will indicate the presence of substantial SU(3) violation in ϕ decays. (The two-parameter fit, using the nonet ansatz, yields results that are in agreement with the other fit.)

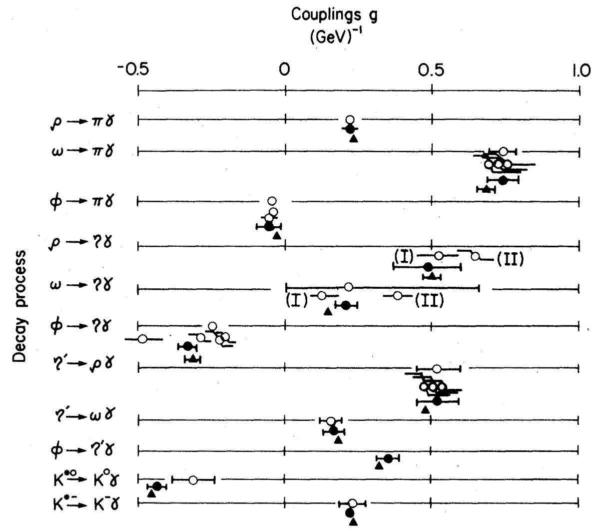

FIG. 1. Comparison between the predicted decay couplings and those deduced from experimental data. The symbols and their explanations follow. ⊢o⊣ on the same line as the decay process is the coupling deduced from Γ_{exp} of Table I. —o— below the line for the reaction show the coupling deduced from the individual data points used to get Γ_{exp}. [Points (I) and (II) correspond to the two solutions of Ref. 10.] ⊢•⊣ and ⊢▲⊣ represent the predicted decay couplings (deduced from Table I) for the three- and the two-parameter fits, respectively.

In conclusion, the new measurement of $\Gamma(\rho\to\pi\gamma)$ has removed a serious difficulty in the SU(3) approach for understanding radiative transitions. On the whole, SU(3) invariance, in spite of the approximate nature of the symmetry, seems to be valid for radiative decays of ordinary mesons. Additional experimental data for $\phi\to\eta\gamma$ and for $K^{*0}\to K^0\gamma$ decays would be valuable to check out to what extent SU(3) is violated.

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