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Symmetry-breaking effects on radiative decays of mesons

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Symmetry-breaking effects on $VP\gamma$ decays are discussed assuming nonexoticity of intermediate states and V-P symmetry. The SU(3)-breaking predictions are in good agreement with all experimental values except for $\Gamma(K^{*0}\to K^0\gamma)$. The analysis demands an accurate measurement of $\Gamma(K^{*0}\to K^0\gamma)$. We also consider isospin breaking which can make $\Gamma(K^{*0}\to K^0\gamma)$ compatible with other vector-meson decay rates. Two-photon decay widths of pseudoscalar mesons are also calculated.

I. INTRODUCTION

Over the last few years, various attempts[1-8] have been made to understand radiative decays of mesons. It is now clear that the experimental data cannot be explained within the limit of exact internal symmetry with the conventional form of the electromagnetic (EM) current. Relaxing the nonet symmetry[2] and/or the introduction of an independent singlet piece[3] in the EM current has not proven to be helpful. The vector-meson dominance (VMD) mechanism also seems to be unable to alter the $(\rho\to\pi\gamma)/(\omega\to\pi\gamma)$ ratio.[8] The data seem to demand the introduction of symmetry breaking. However, the general Muraskin-Glashow[9] symmetry-breaking formalism introduces too many independent parameters to lead to useful information, and additional assumptions are required to reduce the number of parameters. Boson symmetry, i.e, symmetry between the vector meson and photon (which makes symmetry breaking compatible with the VMD scheme) does not solve the problem as the $\rho\to\pi\gamma$ and $\omega\to\pi\gamma$ amplitudes remain related through[5]

$$\langle\pi\gamma\,|\,\rho\rangle = 1/\sqrt{3}\,(\langle\pi\gamma\,|\,\omega\rangle\sin\theta + \langle\pi\gamma\,|\,\phi\rangle\cos\theta)\,, \qquad (1.1)$$

where θ is the ω-ϕ mixing angle. Also the symmetry-breaking scheme with nonet symmetry on the EM Hamiltonian, i.e.,

$$\langle P(9)\,||\,H_{\rm EM}(8)\,||\,V(9)\rangle = \langle P(9)\,||\,H_{\rm EM}(1)\,||\,V(9)\rangle \quad (1.2)$$

does not lead to a good fit with experiment.[6]

In this paper, we consider symmetry-breaking effects in a semidynamical approach. The symmetry breaking is assumed to arise through the scattering process $S+V\to P+\gamma$, where S is the symmetry-breaking spurion and the symmetry-breaking contributions are expressed in terms of reduced matrix elements corresponding to each intermediate state in s, t, and u channels. Constraints on the reduced amplitude are obtained by assuming (i) that the nonexotic intermediate states contribute dominantly and (ii) that the symmetry-breaking Hamiltonian preserves the symmetry between vector meson and pseudoscalar meson. Leaving aside $K^{*+}\to K^+\gamma$ then SU(3)-symmetry-breaking effects are expressed in terms of only two parameters. We observe that all the predicted $VP\gamma$ decay rates including the $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$ ratio are in good agreement with experiment, except the $K^{*0}\to K^0\gamma$ decay width which is predicted to be larger than the experimental value. Using a symmetry-broken $VP\gamma$ vertex, we calculate the two-photon decay widths of pseudoscalar mesons. The recently measured $\rho\to\pi\gamma$ decay width raises the $\pi^0\to\gamma\gamma$ decay width reasonably close to the experimental value.

Assuming that the measurement of $\Gamma(K^{*0}\to K^0\gamma)$ is correct, one may conclude that SU(3) breaking alone is unable to explain the radiative decays. It has been pointed out[7] that one can explain $K^{*0}\to K^0\gamma$ and $\rho\to\pi\gamma$ decay rates by including isospin breaking. With isospin breaking a satisfactory $\Gamma(K^{*0}\to K^0\gamma)$ can be obtained without disturbing other vector-meson decay widths, but this lowers the η'-meson decay ratio $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$ to an unacceptable value of 4 and gives very large $P\to\gamma\gamma$ decay widths. Using $\Gamma(\pi^0\to\gamma\gamma)$ to fix the isospin breaking, a satisfactory agreement between the calculated and the experimental values of $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$, $\Gamma(\eta\to\gamma\gamma)$, and $\Gamma(\eta'\to\gamma\gamma)$ is obtained.

We also extend our formalism to SU(4) symmetry to include charm particles. Okubo-Zweig-Iizuka-rule–violating ψ and η_c decays remain forbidden as a result of ideal mixing. In the presence of SU(4) breaking one may obtain a low $\psi\to\eta_c\gamma$ decay rate. We are unable to predict charm-particle decay rates, as at least one input is required to fix the relative strength of SU(4)-breaking interaction.

In Sec. II, preliminaries of the method are described. In Sec. III, we discuss the SU(3)-breaking effects on $VP\gamma$ decays. Isospin breaking is included in Sec. IV. In Sec. V, the formalism is extended to SU(4). Summary and conclusions are given in the last section.

II. PRELIMINARIES

Firstly, we discuss radiative decays in the uncharmed sector and so work in an SU(3)-symmetry framework. The SU(3)-symmetric EM Hamiltonian is taken to transform like

$$2/3T^1_1-1/3T^2_2-1/3T^3_3 \tag{2.1}$$

components of octet representation. SU(3)-symmetric contributions are obtained from the contraction

$$A[(P^m_a V^b_m+P^b_m V^m_a)-\tfrac{2}{3}\,\delta^b_a(P^m_n V^n_m)]T^a_b\,, \tag{2.2}$$

where T^a_b is the EM Hamiltonian and P^a_b and V^a_b are tensors representing nonet representation of pseudoscalar and vector mesons, respectively. In matrix form P^a_b and V^a_b are given as

$$P^a_b=\begin{bmatrix} \frac{1}{\sqrt{2}}\,(\pi^0+\eta\cos\theta_P+\eta'\sin\theta_P) & \pi^- & K^- \\ \pi^+ & \frac{1}{\sqrt{2}}(-\pi^0+\eta\cos\theta_P+\eta'\sin\theta_P) & \bar{K}^0 \\ K^+ & K^0 & -\eta\sin\theta_P+\eta'\cos\theta_P \end{bmatrix}, \tag{2.3}$$

$$V^a_b=\begin{bmatrix} \frac{1}{\sqrt{2}}\,(\rho^0+\omega\cos\theta_V+\phi\sin\theta_V) & \rho^- & K^{*-} \\ \rho^+ & \frac{1}{\sqrt{2}}\,(-\rho^0+\omega\cos\theta_V+\phi\sin\theta_V) & \bar{K}^{*0} \\ K^{*+} & K^{*0} & -\omega\sin\theta_V+\phi\cos\theta_V \end{bmatrix}, \tag{2.4}$$

where

$$\begin{aligned} &\theta_P\simeq 45^\circ \text{ (quadratic mass formula) (Ref. 10)}\,,\\ &\theta_V=0 \text{ (ideal mixing)}\\ &\quad\simeq 5^\circ \text{ (quadratic mass formula)}\,. \end{aligned} \tag{2.5}$$

In general, symmetry breaking to $VP\gamma$ decays can be obtained by introducing matrix elements $\langle P|T(H_{\rm EM},H')|V\rangle$, where H' is the symmetry-breaking Hamiltonian[9] at the EM vertex. Various possible contractions for these matrix elements are

$$\begin{aligned} H_{\rm SB}=\;&b_1(P^n_b V^m_n T^a_m H^b_a)+b_2(P^a_n V^n_m T^m_b H^b_a)+b_3(P^a_b V^n_m T^m_n H^b_a)\\ &+b_4(P^n_m V^a_b T^m_n H^b_a)+b_5(P^n_b V^a_m T^m_n H^b_a)+b_6(P^a_m V^n_b T^m_n H^b_a)+b_7(P^m_n V^n_b T^a_m H^b_a)+b_8(P^n_m V^a_n T^m_b H^b_a)+b_9(P^n_m V^m_n T^a_b H^b_a)\,, \end{aligned} \tag{2.6}$$

while there is one relation among these nine parameters. C invariances of the EM interaction reduces these to five through the following:

$$\begin{aligned} &b_1=b_2\,,\\ &b_5=b_6\,,\\ &b_7=b_8\,. \end{aligned} \tag{2.7}$$

We obtain further constraints on these parameters by considering the scattering process

$$S+V\to P+\gamma\,, \tag{2.8}$$

where S is the symmetry-breaking spurion. The transition amplitude for the process is expressed in terms of reduced matrix elements corresponding to each intermediate state $|m\rangle$ in s, t, and u channels. Different reduced matrix elements are defined as

$$\begin{aligned} &A^s_m=\langle P||\gamma||m\rangle\langle m||S||V\rangle\\ &\text{for } s \text{ channel } (S+V\to m\to P+\gamma)\,,\\ &A^t_m=\langle\gamma||S||m\rangle\langle m||P||V\rangle\\ &\text{for } t \text{ channel } (V+P\to m\to\gamma+S)\,,\\ &A^u_m=\langle P||S||m\rangle\langle m||\gamma||V\rangle\\ &\text{for } u \text{ channel } (V+\gamma\to m\to P+S)\,, \end{aligned} \tag{2.9}$$

where the superscripts and subscripts denote the channel and intermediate states, respectively. The possible intermediate states $|m\rangle$ in these channels belong to 1, 8, 10, 10*, and 27 representations. The correspondence between the various reduced matrix elements A's and the parameter b's in Eq. (2.6) for each channel is given in the Appendix. In order to obtain constraints on the reduced matrix elements we assume the following.

(1) Nonexotic intermediate states $(\bar{q}q)$ contribute dominantly to the symmetry-breaking interaction, i.e.,

$$A^{s}_{10,10^*,27} = A^{t}_{10,10^*,27} = A^{u}_{10,10^*,27} = 0 . \qquad (2.10)$$

(2) The symmetry-breaking Hamiltonian preserves the symmetry between pseudoscalar and vector meson[11] [see Eq. (2.2)], i.e.,

$$\langle P|\gamma|V\rangle_{SB} = \langle V|\gamma|P\rangle_{SB} . \qquad (2.11)$$

The effective Hamiltonian for symmetry-breaking interaction is then reduced to

for the t channel:

$$a_1(P^n_b V^m_n T^a_m H^b_a + P^a_n V^n_m T^m_b H^b_a + P^m_n V^n_b T^a_m H^b_a + P^n_m V^a_n T^m_b H^b_a) + a_2(P^n_m V^m_n T^a_b H^b_a) ; \qquad (2.12)$$

for the s and u channels:

$$a_3(P^n_b V^a_m T^m_n H^b_a + P^a_m V^n_b T^m_n H^b_a) . \qquad (2.13)$$

Notice that an effective independent singlet piece is generated in the EM Hamiltonian through the second term in (2.12).

Obtaining symmetry-broken decay amplitude $A_{VP\gamma}$ in this manner, the radiative decay widths are calculated from

$$\Gamma(V \to P\gamma) = \frac{1}{96\pi}\left(\frac{m_V^2 - m_P^2}{m_V}\right)^3 |A_{VP\gamma}|^2 , \qquad (2.14)$$

$$\Gamma(P \to V\gamma) = \frac{1}{32\pi}\left(\frac{m_P^2 - m_V^2}{m_P}\right)^2 |A_{PV\gamma}|^2 . \qquad (2.15)$$

Using these radiative decay amplitudes, one may also calculate the symmetry-breaking effects on two-photon decays of pseudoscalar mesons since the two processes can be related through the VMD mechanism,[8] i.e.,

$$P \to \gamma + \gamma \equiv P \to V + \gamma \;, \quad V \to \gamma$$

or

$$A_{P\gamma\gamma} = \frac{e}{g_\rho}\left(A_{\rho^0 P\gamma} + \tfrac{1}{3}A_{\omega P\gamma} - \sqrt{2}/3 A_{\phi P\gamma}\right) , \qquad (2.16)$$

where $g_\rho^2/4\pi = 2.93$ is the $\rho\pi\pi$ coupling constant. The $P \to \gamma\gamma$ decay widths are then calculated from

$$\Gamma(P \to \gamma\gamma) = \frac{1}{64\pi} m_P^3 |A_{P\gamma\gamma}|^2 . \qquad (2.17)$$

III. SU(3)-SYMMETRY-BREAKING EFFECTS

SU(3)-symmetric contributions to $VP\gamma$ decay amplitudes, obtained from (2.2), are given in the second column of Table I. SU(3)-breaking contributions can be obtained from (2.12) and (2.13) by choosing the symmetry-breaking Hamiltonian H^b_a to be the H^3_3 component of the octet. Such contributions to various decays arising in the s, t, and u channels are given in the third and fourth columns of Table I. Aside from $K^{*+} \to K^+\gamma$, all the

TABLE I. Symmetric and symmetry-breaking contributions to $VP\gamma$ decays.

	Symmetric Hamiltonian	SU(3) breaking: t channel	SU(3) breaking: s and u channels	SU(2) breaking: t channel	SU(2) breaking: s and u channels
$\rho^- \to \pi^-\gamma$	$A/3$	$-a_2/3$	0	$(4a_1'+2a_2')/3$	$-a_3'/3$
$\rho^0 \to \pi^0\gamma$	$A/3$	$-a_2/3$	0	$(4a_1'+2a_2')/3$	$2a_3'/3$
$\rho \to \eta\gamma$	$A/\sqrt{2}$	0	0	$4a_1'/3\sqrt{2}$	$2a_3'/3\sqrt{2}$
$\omega \to \pi\gamma$	A	0	0	$4a_1'/3$	$2a_3'/3$
$\omega \to \eta\gamma$	$A/3\sqrt{2}$	$-a_2/3\sqrt{2}$	0	$(4a_1'+2a_2')/3\sqrt{2}$	$2a_3'/3\sqrt{2}$
$\phi \to \pi\gamma$	0	0	0	0	0
$\phi \to \eta\gamma$	$2A/3\sqrt{2}$	$(4a_1+a_2)/3\sqrt{2}$	$2a_3/3\sqrt{2}$	$-2a_2'/3\sqrt{2}$	0
$\phi \to \eta'\gamma$	$-2A/3\sqrt{2}$	$(-4a_1-a_2)/3\sqrt{2}$	$-2a_3/3\sqrt{2}$	$2a_2'/3\sqrt{2}$	0
$K^{*0} \to K^0\gamma$	$-2A/3$	$-(2a_1+a_2)/3$	$-a_3/3$	$2a_2'/3$	0
$K^{*+} \to K^+\gamma$	$A/3$	$-(2a_1+a_2)/3$	$2a_3/3$	$(4a_1'+2a_2')/3$	$-a_3'/3$
$\eta' \to \rho\gamma$	$A/\sqrt{2}$	0	0	$4a_1'/3\sqrt{2}$	$2a_3'/3\sqrt{2}$
$\eta' \to \omega\gamma$	$A/3\sqrt{2}$	$-a_2/3\sqrt{2}$	0	$(4a_1'+2a_2')/3\sqrt{2}$	$2a_3'/3\sqrt{2}$

TABLE II. Meson radiative decay widths in keV (ideal ω-ϕ mixing and physical η-η' mixing). Values in parentheses are input.

Decay	SU(3) breaking		SU(3) + SU(2) breaking		Experiment
$\rho^-\pi^-\gamma$	(35)	(63)	63 [a]	63 [a]	35 ± 10 (Ref. 12) 63 ± 8 (Ref. 13)
$\rho^0\pi^0\gamma$	35	63	380	71	
$\rho\eta\gamma$	54	54	54	54	50 ± 13 (Ref. 14) 76 ± 15
$\omega\pi\gamma$	(870)	(870)	(870)	(870)	870 ± 60 (Ref. 17)
$\omega\eta\gamma$	2.7	4.8	(29)	5.4	$3.0^{+2.5}_{-1.8}$ (Ref. 14) 29 ± 7
$\phi\pi\gamma$ [b]	0	0	0	0	5.7 ± 2.1 (Ref. 17)
$\phi\eta\gamma$	(65)	(65)	(65)	(65)	65 ± 15 (Refs. 14, 17)
$\phi\eta'\gamma$	0.3	0.3	0.3	0.3	
$K^{*0}K^0\gamma$	167	148	62	142	75 ± 35 (Ref. 17)
$K^{*+}K^+\gamma$	40 [a]	40 [a]	40 [a]	40 [a]	40 ± 15 (Ref. 13)
$\eta'\rho\gamma$	115	115	115	115	93.1 ± 25.1 (Ref. 4)
$\eta'\omega\gamma$	4.0	7.4	44	8.3	8.4 ± 2.7 (Ref. 4)
$\eta'\rho\gamma/\eta'\omega\gamma$	28	15.6	3.7	13.8	14.0 ± 3.4 (Ref. 14)
$\pi^0\gamma\gamma$ (eV)	5.4	6.8	18.6	(7.8)	7.8 ± 0.9 (Ref. 12)
$\eta\gamma\gamma$	0.46	0.49	0.65	0.49	0.323 ± 0.046 (Ref. 17)
$\eta'\gamma\gamma$	5.2	5.4	6.6	5.4	5.4 ± 2.1 (Ref. 15)

[a] These can be fitted independently of others.
[b] Nonzero $\Gamma(\phi\to\pi\gamma)$ can be obtained with nonideal ω-ϕ mixing.

decay amplitudes involve three unknown parameters, which we fix by using $\rho\pi\gamma$, $\omega\pi\gamma$, and $\phi\eta\gamma$ decay widths as inputs. The calculated decay widths are displayed in the second and third columns of Table II. To discuss the predictions more explicitly, we look into the decay amplitudes. We obtain the following simple sum rules in the case of ideal ω-ϕ mixing:

$$\langle\pi\gamma|\phi\rangle = 0\,, \tag{3.1}$$

$$\langle\rho\gamma|\eta'\rangle = \langle\pi\gamma|\omega\rangle/\sqrt{2} = \langle\eta\gamma|\rho\rangle$$

$$9.97\pm0.29 \quad 9.59\pm1.25 \text{ (constructive-interference solution)}$$

$$11.82\pm1.16 \text{ (destructive-interference solution)}\,, \tag{3.2}$$

$$\langle\omega\gamma|\eta'\rangle = \langle\pi^0\gamma|\rho^0\rangle/\sqrt{2} = \langle\pi^-\gamma|\rho^-\rangle/\sqrt{2} = \langle\eta\gamma|\omega\rangle$$

$$2.06\pm0.34^{*} \quad 2.18\pm0.80 \text{ (constructive-interference solution)}\,,$$

$$2.76\pm0.17^{**} \quad 6.78\pm0.81 \text{ (destructive-interference solution)}\,, \tag{3.3}$$

$$-\langle\eta'\gamma|\phi\rangle = \langle\eta\gamma|\phi\rangle$$

$$4.14\pm0.48\,, \tag{3.4}$$

$$\langle K^0\gamma|K^{*0}\rangle = \tfrac{1}{2}(\langle\pi^-\gamma|\rho^-\rangle - \langle\pi\gamma|\omega\rangle - \sqrt{2}\langle\eta\gamma|\phi\rangle)$$

$$-8.51\pm0.83^{*}$$

$$-5.68\pm1.25 \quad -8.02\pm0.66^{**}\,. \tag{3.5}$$

Relations (3.2) and (3.3) give the η' decay ratio as

$$\frac{\langle\rho\gamma|\eta'\rangle}{\langle\omega\gamma|\eta'\rangle} = \frac{\langle\pi\gamma|\omega\rangle}{\langle\pi\gamma|\rho\rangle}$$

$$3.43\pm0.42 \quad 4.85\pm0.96^{*}$$

$$3.61\pm0.38^{**}\,. \tag{3.6}$$

Where single- and double-asterisk values correspond to 35 ± 10 keV (Ref. 12) and 63 ± 8 keV (Ref. 13) values of $\rho \to \pi\gamma$ decay width, respectively, and all the decay amplitudes are in the units of $(96\pi \times 10^{-11})^{1/2}$ MeV^{-1}. The $\phi \to \pi\gamma$ decay rate vanishes even in the presence of symmetry breaking and a nonvanishing value can be obtained by varying the ω-ϕ mixing slightly from its ideal value. Since this does not produce a significant change in other numbers, we keep the ideal ω-ϕ mixing in discussing other decays. Notice that relation (1.1) is not valid in the present analysis and the $\rho \to \pi\gamma$ decay width can be fixed independently of that of $\omega \to \pi\gamma$. In fact, in relation (3.3) one finds that the constructive-interference solution for $\omega \to \eta\gamma$ is in good agreement with low $\Gamma(\rho \to \pi\gamma) = 35 \pm 10$ keV,[12] though $\rho \to \eta\gamma$ satisfies relation (3.2) for both solutions.[14] Through relation (3.2) the $\Gamma(\omega \to \pi\gamma)$ decay rate predicts $\Gamma(\eta' \to \rho\gamma) = 115 \pm 8$ keV which agrees with 93.1 ± 25.1 keV obtained[4] from known branching fractions $B(\eta' \to \rho\gamma)$, $B(\eta' \to \pi^+\pi^-\gamma)$ and total decay width $\Gamma\eta' = 280 \pm 100$ keV observed in a recent $\pi - p \to n+$ missing mass experiment.[15] But the low $\rho \to \pi\gamma$ decay width remains in conflict with the $\eta' \to \rho\gamma/\eta' \to \omega\gamma$ ratio and predicts a large value for $\Gamma(K^{*0} \to K^0\gamma)$.

Recently, it has been suggested by Kamal and Kane[16] that in the analysis of the Primakoff-effect experiments for measuring radiative decay widths, it is important to include the A_2-exchange amplitude for high-Z nuclei. With the inclusion of A_2-exchange effects the experimental value for $\rho \to \pi\gamma$ can be expected to be as high as 66 keV in the experiment of Gobbi *et al.*[12] A more recent experiment[13] has yielded a rate $\Gamma(\rho \to \pi\gamma) = 63 \pm 8$ keV. With this value for $\rho \to \pi\gamma$, the η' decay-width ratio is predicted to be 15, which agrees well with the experimental value 14.1 ± 3.4.[17] Within experimental errors the $\omega\eta\gamma$ decay remains compatible with the larger $\Gamma(\rho \to \pi\gamma)$ value. The $\Gamma(K^{*0} \to K^0\gamma)$ is somewhat lowered, although it still remains higher than the experimental value. The $\Gamma(K^{*+} \to K^+\gamma)$ can be fixed to be 40 ± 15 keV[13] by an independent choice of a_3, i.e., s- and u-channel contributions.[18] Predictions with $\Gamma(\rho \to \pi\gamma) = 63$ keV as input are given in the third column of Table II. Values of the parameters A, a_1, a_2, a_3 are found to be

$$A = 14.09, \quad a_1 = -2.27, \quad a_2 = 2.40, \quad a_3 = -1.97 . \tag{3.7}$$

Thus, symmetry-breaking contributions are about 15%. The a_1 term occurring in the t channel (2.12) corresponds to symmetry breaking due to quark mass, i.e., by taking the

$$\tfrac{2}{3}T_1^1 - \tfrac{1}{3}T_2^2 - \frac{m_u}{3m_s}T_3^3$$

form of EM Hamiltonian for $M1$ transitions. In fact, a_1 is related to the quark-mass ratio m_u/m_s through

$$\frac{m_u}{m_s} = 1 + \frac{2a_1}{A} , \tag{3.8}$$

predicting m_u/m_s to be 0.68 in agreement with other estimates.[19,20] The a_2 term corresponds, effectively, to an independent singlet piece.[3,4] As the other terms are of the same order as a_2, it is clear that the addition of a singlet piece alone in EM current cannot explain the data.

Using the vector-meson dominance[8] hypothesis, the radiative $VP\gamma$ decays can be related to other mesonic processes such as $V \to VP$, $V \to 3P$, and $P \to \gamma\gamma$. Fixing $g_\rho^2/4\pi$ from the $\rho^0 \to e^+e^-$ decay rate, O'Donnell has obtained[8] $\Gamma(\rho \to \pi\gamma) = 65$ keV and $\Gamma(K^{*+} \to K^+\gamma) = 36$ keV, in nice agreement with recent measurements.[13] But this choice implies $\Gamma(\omega \to \pi\gamma) = 723$ keV, lower than the world average value 870 keV.[17] This simple VMD scheme also gives large $\Gamma(\phi \to \eta\gamma) = 117$ keV and $\Gamma(K^{*0} \to K^0\gamma) = 144$ keV. Although a recent analysis by Ohshima claims $\Gamma(\omega \to \pi\gamma)$ to be 789 ± 92 keV, consistent with O'Donnell's prediction,[8] $\Gamma(\phi \to \eta\gamma)$ and $\Gamma(K^{*0} \to K^0\gamma)$ remain much larger in both schemes. Etim and Greco[18] have shown that a satisfactory description for most of the radiative meson decays, especially the low $K^{*0} \to K^0\gamma$ decay width, can be obtained with vector-meson-dominance and quark current-algebra constraints. However, they fail to explain the low $\rho \to \pi\gamma$ decay width. Even the recently measured value 63 ± 8 keV[13] for $\Gamma(\rho \to \pi\gamma)$ is far below the VMD prediction value of 95 keV.[8] Also, $K^{*+} \to K^+\gamma$ is predicted to be small in comparison to its experimental value 40 ± 15 keV.[13] Therefore, symmetry breaking may still be required by the data, as considered here. We also calculate symmetry-breaking effects on $\Gamma(P \to \gamma\gamma)$ decay widths, by relating these decays to $VP\gamma$ decays through the VMD mechanism (2.16). The calculated decay widths are given in Table II. Notice that $\Gamma(\rho \to \pi\gamma) = 63 \pm 8$ keV raises $\Gamma(\pi^0 \to \gamma\gamma)$ reasonably close to experiment. The $\eta' \to \gamma\gamma$ decay width is in good agreement with recent measurements of the η' decay width.[15]

Thus we find that all of the rates except for $K^{*0} \to K^0\gamma$ agree well with experiment in the presence of symmetry-breaking effects considered in the present analysis. The predicted value for $K^{*0} \to K^0\gamma$ decay differs by as much as 40% from the maximum possible value. However, the measurement of $\Gamma(K^{*0} \to K^0\gamma)$ has also been criticized by Kamal and Kane[16] and the true value could well be much higher. So either the measured rate is wrong or else one has to look for a theoretical explanation for the narrow width of $K^{*0} \to K^0\gamma$.

For this reason, we include isospin breaking in the next section.

IV. INCLUSION OF ISOSPIN BREAKING

It has already been pointed out that a good agreement for $K^{*0} \to K^0\gamma$ and $\rho \to \pi\gamma$ decay rates can be obtained by taking account of isospin and SU(3) breakings.[7] In other sectors also, such isospin-breaking effects have been looked for.[19,21] Theoretical estimates for d/u quark mass ratio ranges from 1.5 to[19,21] infinity. In some experiments, for instance $f \to K\bar{K}$ decays, large isospin violations have been observed.[22] In our formalism isospin-breaking contributions can be obtained by choosing the symmetry-breaking Hamiltonian to be the H_1^1 component of the octet. In the fifth and sixth columns of Table I we give isospin-breaking contributions. Primes over the a's indicate that reduced matrix elements are taken to be different from those for SU(3) breaking. Calculated decay widths in the presence of SU(2) as well as SU(3) breaking are compared in the fourth and fifth columns of Table II. One immediate consequence of including isospin breaking is that the $\rho^0 \to \pi^0\gamma$ rate is no longer equal to $\rho^- \to \pi^-\gamma$ (notice that this equality is broken only in s and u channels). Relations (3.1), (3.2), and (3.4) remain valid, others are modified to

$$\langle\omega\gamma\,|\,\eta'\rangle = \langle\pi^0\gamma\,|\,\rho^0\rangle/\sqrt{2} = \langle\eta\gamma\,|\,\omega\rangle$$

$$6.78 \pm 0.81 \text{ (destructive-interference solution)}$$

$$2.18 \pm 0.80 \text{ (constructive-interference solution)}, \tag{4.1}$$

$$\langle K^0\gamma\,|\,K^{*0}\rangle = \tfrac{1}{2}\left[\sqrt{2}(\langle\eta\gamma\,|\,\omega\rangle - \langle\eta\gamma\,|\,\phi\rangle) - \langle\pi\gamma\,|\,\omega\rangle\right]$$

$$-5.12 \pm 0.64 \text{ (destructive-interference solution)}$$

$$-5.68 \pm 1.25 \quad -8.4 \pm 1.10 \text{ (constructive-interference solution)}. \tag{4.2}$$

Because of relation (3.2), $\rho\eta\gamma$ remains compatible with $\omega\pi\gamma$ for both solutions and now $\rho^- \to \pi^-\gamma$ no longer discriminates between the two solutions for $\omega \to \eta\gamma$. The destructive-interference solution[14] for $\omega \to \eta\gamma$ predicts a narrow $K^{*0} \to K^0\gamma$ decay width. But this choice for the $\omega \to \eta\gamma$ decay rate lowers the $\eta' \to \rho\gamma/\eta' \to \omega\gamma$ decay-width ratio substantially to 3.7. From Table I it is easy to obtain the following relation [in the presence of SU(2) as well as SU(3) breaking]:

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\eta' \to \omega\gamma)} = 1.4\,\frac{\Gamma(\rho \to \eta\gamma)}{\Gamma(\omega \to \eta\gamma)} = 1.4\left(\frac{76 \pm 15}{29 \pm 7}\right), \tag{4.3}$$

which can give the η' decay-rate ratio as high as 5.8.

Evidently, there appear to be seven parameters, but, leaving aside $\rho^- \to \pi^-\gamma$ and $K^{*+} \to K^+\gamma$, SU(3)- and SU(2)-breaking contributions to $VP\gamma$ decays can be expressed in terms of only two parameters. Using $\omega \to \pi\gamma$, $\omega \to \eta\gamma$, and $\phi \to \eta\gamma$ as inputs, other decay widths are calculated as shown in the fourth column of Table II. Values of the effective parameters are

$$A + \tfrac{2}{3}(2a_1' + a_3') = 14.09\,,$$
$$2a_1 + a_3 = 2.02\,, \tag{4.4}$$
$$(2a_1' + a_3') - \tfrac{3}{4}(a_2 - 2a_2') = 11.00\,.$$

Now $\Gamma(\rho^- \to \pi^-\gamma)$ and $\Gamma(K^{*+} \to K^+\gamma)$ can be fixed independently to their desired values, leading to

$$a_1 = 0.57, \quad a_3 = 0.60, \quad a_3' = 5.68\,,$$
$$A + \tfrac{4}{3}a_1' = 10.29, \quad a_1' - \tfrac{3}{8}(a_2 - 2a_2') = 2.66\,. \tag{4.5}$$

If SU(3) breaking is suppressed,[7] these decay amplitudes becomes equal, i.e,

$$\langle K^+\gamma\,|\,K^{*+}\rangle = \langle\pi^-\gamma\,|\,\rho^-\rangle$$
$$4.10 \pm 0.77 \quad 3.90 \pm 0.24\,, \tag{4.6}$$

a well satisfied relation. Relation (4.2) breaks up into

$$\langle K^0\gamma\,|\,K^{*0}\rangle = -\sqrt{2}\,\langle\eta\gamma\,|\,\phi\rangle$$
$$-5.68 \pm 1.25 \quad -5.85 \pm 0.61\,, \tag{4.7}$$

$$\langle\pi\gamma\,|\,\omega\rangle = \sqrt{2}(\langle\gamma\eta\,|\,\omega\rangle + \langle\eta\gamma\,|\,\phi\rangle)\,,$$
$$14.09 \pm 0.41 \quad 15.44 \pm 0.90 \text{ (destructive-interference solution)}. \tag{4.8}$$

Thus large isospin violations, which tend to choose destructive-interference solutions for $\omega \to \eta\gamma$, can give K^* decay rates in good agreement with experiment. But this choice leads to unacceptable values for the η' decay ratio and predicts very large values for the $\rho^0 \to \pi^0\gamma$ decay width. Although there is no experimental measurement available for $\rho^0 \to \pi^0\gamma$, its larger value is seriously unfavored by two-photon decay widths of pseudoscalar mesons (fourth column of Table II). Fixing isospin-breaking strength by using $\Gamma(\pi^0\to\gamma\gamma)$ as input, we obtain $\Gamma(\omega\to\eta\gamma)=5.45$ keV, clearly favoring the constructive-interference solution. With this the agreement with other $V\to P\gamma$, including η' decays, is restored (fourth column of Table II). Various parameters are

$$A+\tfrac{4}{3}a_1'=13.91,\quad a_1=-2.14,\quad a_3=-1.83,\quad a_3'=0.26,\quad a_1'-\tfrac{3}{8}(a_2-2a_2')=-0.73\,. \tag{4.9}$$

The a_1 and a_1' are related to the quark-mass ratio as

$$\frac{m_u}{m_d}=1-\frac{4a_1'}{A}\,,\quad \frac{m_u}{m_s}=1+\frac{2a_1}{A}=0.67\,. \tag{4.10}$$

Although $\Gamma(K^{*0}\to K^0\gamma)$ is found to be large again, isospin breaking tends to lower it. We also obtain

$$\Gamma(\rho^0\to\pi^0\gamma)=1.12\Gamma(\rho^-\to\pi^-\gamma)\,. \tag{4.11}$$

The same value has also been predicted by Isgur *et al.*[23] based on isospin violation due to segregation into $d\bar{d}$ and $u\bar{u}$ mesons.

V. CHARM SECTOR

The formalism can be extended to SU(4) symmetry to include charm particles by choosing the electromagnetic Hamiltonian to transform as a

$$\tfrac{2}{3}T_1^1-\tfrac{1}{3}T_2^2-\tfrac{1}{3}T_3^3+\tfrac{2}{3}T_4^4 \tag{5.1}$$

component of $15\oplus 1$. Vector and pseudoscalar mesons now form the 16-plet of SU(4). In this scheme the form for the symmetry-breaking Hamiltonian remains the same as (2.13) and (2.14) for t and s-u channels, respectively. SU(4)-breaking contributions can be obtained by assigning H_b^a to be the T_4^4 component of 15. The uncharmed sector remains unaffected. The Okubo-Zweig-Iizuka-rule-violating[24] decays such as $\psi\to\pi(\eta,\eta')+\gamma$ and $\eta_c\to\rho(\omega,\phi)+\gamma$ are forbidden as a result of ideal mixing. Here also small observed values for ψ decays[25] can be obtained by varying ω,ϕ,ψ mixing slightly from its ideal value. In Table III we give the contribution to charm-particle decays arising from isospin-, SU(3)-, and SU(4)-breaking interaction. There appears to be three parameters for SU(4) breaking. However, one may start by assuming the general symmetry-breaking Hamiltonian to be a $T_1^1+xT_3^3+yT_4^4$ component of 15, thereby expressing the relative strength of SU(4) through the single parameter y. Since at present no experimental data is available for charm-particle radiative decays, we are unable to predict the decay rates.

VI. SUMMARY AND CONCLUSION

In this paper, we have studied symmetry-breaking effects on $VP\gamma$ decays in a semidynamical scheme. We consider the symmetry-breaking contribution to arise from the scattering process $S+V\to P+\gamma$ in s, t, and u channels and assume that the transition is dominated by nonexotic intermediate states and that the V and P exchange symmetry present in the symmetric case is respected by symmetry-breaking interaction. Such SU(3)-symmetry-breaking effects are able to explain all the $VP\gamma$ decay rates, including the η' decays, except that a large value for $\Gamma(K^{*0}\to K^0\gamma)$ is predicted. In view of the recent criticism[16] of the data analysis in the Primakoff-effect experiments, the measurement of $\Gamma(K^{*0}\to K^0\gamma)$ is suspect and a new experiment is very desirable. Another possibility to explain the rather narrow $\Gamma(K^{*0}\to K^0\gamma)$ is to include the isospin breaking.[7] The consequences of isospin breaking have also been studied in other areas,[19,21] leading to various estimates for m_d/m_u ratio. We observe that isospin breaking can account for $K^{*0}\to K^0\gamma$ and $K^{*+}\to K^+\gamma$ decays, but lowers the ratio $\eta'\to\rho\gamma\,|\,\eta'\to\omega\gamma$ substantially and gives larger values for $P\to\gamma\gamma$ decay widths. Determining isospin breaking by using $\pi^0\to\gamma\gamma$ decay width, the agreement with experiment is restored. $\Gamma(K^{*0}\to K^*\gamma)$ is also lowered to 142 keV which differs by as much as 30% from the maximum possible experimental value. Looking at the 20% uncertainty in other numbers, this disagreement may not be that serious. Another effect of isospin breaking is the nonequality[23] of charged and neutral modes of $\rho\to\pi\gamma$; we

TABLE III. Charm-particle decay amplitudes. The $\psi\to\pi|\eta|\eta'+\gamma$ and $\eta_c\to\rho|\omega|+\gamma$ decay amplitudes vanish.

	Symmetry	SU(4) breaking t channel	SU(4) breaking s-u channel	SU(3) breaking t channel	SU(3) breaking s-u channel	SU(2) breaking t channel	SU(2) breaking s-u channel
$D^{*0}D^0\gamma$	$4A/3$	$(4a_1''+2a_2'')/3$	$2a_3''/3$	$-a_2/3$	0	$(4a_1'+2a_2')/3$	$2a_3'/3$
$D^{*+}D^+\gamma$	$A/3$	$(4a_1''+2a_2'')/3$	$-a_3''/3$	$-a_2/3$	0	$2a_2'/3$	0
$F^{*+}F^+\gamma$	$A/3$	$(4a_1''+2a_2'')/3$	$-a_3''/3$	$(-2a_1-a_2)/3$	$2a_3/3$	$2a_2'/3$	0
$\psi\eta_c\gamma$	$4A/3$	$(8a_1''+2a_2'')/3$	$4a_3''/3$	$-a_2/3$	0	$2a_2'/3$	0

predict $\Gamma(\rho^0\to\pi^0\gamma)=1.12\Gamma(\rho^-\to\pi^-\gamma)$.

Finally, we conclude that the measurement of $\rho^0\to\pi^0\gamma$ and $K^{*0}\to K^0\gamma$ decay rates and absolute values for η' decays are highly desirable for the proper understanding of radiative decays. Clearing the interference-phase problem between $\rho\to\eta\gamma$ and $\omega\to\eta\gamma$ will throw more light on the picture.

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APPENDIX

Various reduced matrix element A's are related to b's [parameter in (2.6)] as follows:

(i) For the s channel:

$$
\begin{aligned}
b_1&=(-A^s_{10}+A^s_{10^*}+A^s_{27}),\\
b_2&=(A^s_{10}-A^s_{10^*}+A^s_{27}),\\
b_3&=(-A^s_{10}-A^s_{10^*}+A^s_{27}),\\
b_4&=(A^s_1-\tfrac{4}{3}A^s_{8_{11}}+\tfrac{1}{10}A^s_{27}),\\
b_5&=(A^s_{8_{11}}+A^s_{8_{12}}+A^s_{8_{21}}+A^s_{8_{22}}+\tfrac{1}{3}A^s_{10}+\tfrac{1}{3}A^s_{10^*}-\tfrac{1}{5}A^s_{27}),\\
b_6&=(A^s_{8_{11}}-A^s_{8_{12}}-A^s_{8_{21}}+A^s_{8_{22}}+\tfrac{1}{3}A^s_{10}-\tfrac{1}{3}A^s_{10^*}-\tfrac{1}{5}A^s_{27}),\\
b_7&=(A^s_{8_{11}}-A^s_{8_{12}}+A^s_{8_{21}}-A^s_{8_{22}}-\tfrac{1}{3}A^s_{10}-\tfrac{1}{3}A^s_{10^*}-\tfrac{1}{5}A^s_{27}),\\
b_8&=(A^s_{8_{11}}+A^s_{8_{12}}-A^s_{8_{21}}-A^s_{8_{22}}-\tfrac{1}{3}A^s_{10}-\tfrac{1}{3}A^s_{10^*}-\tfrac{1}{5}A^s_{27}),\\
b_9&=(A^s_{10}+A^s_{10^*}+A^s_{27}).
\end{aligned}
\tag{A1}
$$

(ii) For the t channel:

$$
\begin{aligned}
b_1&=(A^t_{8_{11}}+A^t_{8_{12}}+A^t_{8_{21}}+A^t_{8_{22}}+\tfrac{1}{3}A^t_{10}+\tfrac{1}{3}A^t_{10^*}-\tfrac{1}{5}A^t_{27}),\\
b_2&=(A^t_{8_{11}}-A^t_{8_{12}}-A^t_{8_{21}}+A^t_{8_{22}}+\tfrac{1}{3}A^t_{10}+\tfrac{1}{3}A^t_{10^*}-\tfrac{1}{5}A^t_{27}),\\
b_3&=(-A^t_{10}-A^t_{10^*}+A^t_{27}),\\
b_4&=(A^t_{10}+A^t_{10^*}+A^t_{27}),\\
b_5&=(-A^t_{10}+A^t_{10^*}+A^t_{27}),\\
b_6&=(A^t_{10}-A^t_{10^*}+A^t_{27}),\\
b_7&=(A^t_{8_{11}}+A^t_{8_{12}}-A^t_{8_{21}}-A^t_{8_{22}}-\tfrac{1}{3}A^t_{10}-\tfrac{1}{3}A^t_{10^*}-\tfrac{1}{5}A^t_{27}),\\
b_8&=(A^t_{8_{11}}-A^t_{8_{12}}+A^t_{8_{21}}-A^t_{8_{22}}-\tfrac{1}{3}A^t_{10}-\tfrac{1}{3}A^t_{10^*}-\tfrac{1}{5}A^t_{27}),\\
b_9&=(A^t_1-\tfrac{4}{3}A^t_{8_{11}}+\tfrac{1}{10}A^t_{27}).
\end{aligned}
\tag{A2}
$$

(iii) For the u channel:

$$
\begin{aligned}
b_1&=(A^u_{8_{11}}+A^u_{8_{12}}-A^u_{8_{21}}-A^u_{8_{22}}-\tfrac{1}{3}A^u_{10}-\tfrac{1}{3}A^u_{10^*}-\tfrac{1}{5}A^u_{27}),\\
b_2&=(A^u_{8_{11}}-A^u_{8_{12}}+A^u_{8_{21}}-A^u_{8_{22}}-\tfrac{1}{3}A^u_{10}-\tfrac{1}{3}A^u_{10^*}-\tfrac{1}{5}A^u_{27}),\\
b_3&=(A^u_1-\tfrac{4}{3}A^u_{8_{11}}+\tfrac{1}{10}A^u_{27}),\\
b_4&=(-A^u_{10}-A^u_{10^*}+A^u_{27}),\\
b_5&=(A^u_{8_{11}}-A^u_{8_{12}}-A^u_{8_{21}}+A^u_{8_{22}}+\tfrac{1}{3}A^u_{10}+\tfrac{1}{3}A^u_{10^*}-\tfrac{1}{5}A^u_{27}),\\
b_6&=(A^u_{8_{11}}+A^u_{8_{12}}+A^u_{8_{21}}+A^u_{8_{22}}+\tfrac{1}{3}A^u_{10}+\tfrac{1}{3}A^u_{10^*}-\tfrac{1}{5}A^u_{27}),\\
b_7&=(A^u_{10}-A^u_{10^*}+A^u_{27}),\\
b_8&=(-A^u_{10}+A^u_{10^*}+A^u_{27}),\\
b_9&=(A^u_{10}+A^u_{10^*}+A^u_{27}).
\end{aligned}
\tag{A3}
$$

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