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Dip and kink structures in hadron-nucleus and hadron-hadron diffraction dissociation

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Using the idea in the geometrical model that $\Omega e^{-\Omega}$ is approximately the "source" distribution for the outgoing particle in diffraction dissociation, we calculate differential cross sections for such dissociation in πA and pp collisions and show that there are dips which agree generally in positions with the observed kink structures in several such experiments. The computation of the dip position contains no adjustable parameters, justifying the picture that the dip and kink structures are geometrical in origin.

INTRODUCTION

It is well known that in high-energy elastic pp and np scatterings[1] there is a dip at $|t| \sim 1.2$ to 1.4 $(\mathrm{GeV}/c)^2$. The existence of such a dip was predicted in a geometrical model.[2,3] For diffraction dissociation[4] processes such as

$$\pi^- A \to (\pi\pi\pi)A, \quad A = \mathrm{Cu\ or\ Pb} \tag{1}$$

and

$$\begin{aligned} &pp \to p(n\pi^+)\,, \\ &pp \to (p\pi^+\pi^-)(p\pi^+\pi^-)\,, \\ &np \to (p\pi^-)p\,, \\ &pn \to (p\pi^+\pi^-)(p\pi^-)\,, \\ &nn \to (p\pi^-)(p\pi^-)\,, \end{aligned} \tag{2}$$

recent experiments[5,6] indicate that there are also dip or shoulder structures. It is natural to ask whether the geometrical model also gives an explanation of these dip structures.

We shall show in this paper that the answer to this question is in the affirmative. The basic conceptual modification necessary to change from a description of elastic scattering to diffraction dissociation is to change the "source" of the outgoing wave from

$$1 - e^{-\Omega} \tag{3}$$

for the elastic case to

$$(\mathrm{constant})\,\Omega e^{-\Omega} \tag{4}$$

for the diffraction dissociation case.

ARGUMENT FOR $\Omega e^{-\Omega}$

In (3) and (4) $e^{-\Omega(b)}$ describes the absorption factor of an incoming hadron at an impact parameter b. Expression (3) is the basic concept in the geometrical model.[2] That (4) is a natural zeroth approximation for diffraction scattering is obvious since at an impact parameter b, the dissociation process may take place at any point P (Fig. 1) during the traversal. The probability of the process occurring is approximately proportional in the first instance to the thickness of the material traversed, $\Omega(b)$. (We note here that the diffraction dissociation cross section is very small compared with the elastic cross section.) There is absorption of the incoming amplitude before P and of the outgoing amplitude after P. Assuming equal mean free path for incoming amplitude and outgoing amplitude we obtain (4). This approximation was first discussed in Ref. 7. It was given the name "coherent droplet model" and was applied to the reaction $\pi^- p \to \pi^0 n$. See also Ref. 8.

Of course the beam in its traversal clearly switches coherently back and forth between many possible states.[9] Equation (4) is thus only a simple zeroth approximation.

Using (4) we have

$$\left(\frac{d\sigma}{dt}\right)_{AB\to AB^*, A^*B^*} = (\mathrm{constant})\,\left|(\Omega e^{-\Omega})_{2D\,FT}\right|^2, \tag{5}$$

where $(\,)_{2D\,FT}$ means two-dimensional Fourier transform from the two-dimensional variable $\vec{b}$ to the two-dimensional momentum transfer $\vec{k}$:

$$(X)_{2D\,FT} = (2\pi)^{-1}\int d^2b\, X(b)\, e^{-i\vec{k}\cdot\vec{b}}\,,$$
$$-t = k^2\,.$$

We also have[2]

$$\left(\frac{d\sigma}{dt}\right)_{el} = \pi\left|(1 - e^{-\Omega})_{2D\,FT}\right|^2 \tag{6}$$

and

$$\sigma_{total} = 2\int d^2b\,(1 - e^{-\Omega}). \tag{7}$$

π-NUCLEUS COLLISION

We shall calculate the t distribution for diffraction dissociation (1) through Eq. (5). It is also

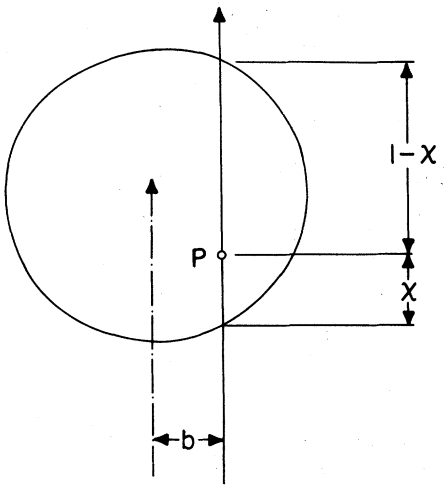

FIG. 1. Schematic drawing for a diffraction dissociation process. The dissociation may take place at any point P along the path.

necessary to estimate the cross section for

$$\pi^- A \to (\pi\pi\pi)A^* . \tag{1'}$$

We shall write the observed $d\sigma/dt$ as a sum due to these two processes

$$(d\sigma/dt) = (d\sigma/dt)_{(1)} + (d\sigma/dt)_{(1')} . \tag{8}$$

The second term can be estimated by comparison with

$$\pi^- A \to \pi^- A^* , \tag{9}$$

which is usually called quasielastic, and has been evaluated in Ref. 10, which reported on a measurement of the combined result of

$$\pi^- A \to \pi^- A \quad \text{and} \quad \pi^- A \to \pi^- A^* .$$

We assume that

$$\frac{(d\sigma/dt)_{(1')}}{(d\sigma/dt)_{(9)}} = \frac{(d\sigma/dt)(\pi p \to (\pi\pi\pi)p)}{(d\sigma/dt)(\pi p \to \pi p)}\bigg|_{t=0} . \tag{10}$$

We take[11]

$$d\sigma/dt(\pi p \to (\pi\pi\pi)p)\big|_{t=0} = 30\ \mu\text{b}\,(\text{GeV}/c)^{-2} \tag{11}$$

and

$$d\sigma/dt(\pi p \to \pi p)\big|_{t=0} = 29\ \text{mb}\,(\text{GeV}/c)^{-2} . \tag{12}$$

Then the differential cross sections for process (1′) can be calculated. The results are shown in Figs. 2(a) and 3(a).

To compute the amplitude for coherent production process (1), the nucleus is regarded as a sphere of radius of $1.12A^{1/3}$ fm. To find $\Omega(b)$ we need to calculate the damping of a coherent pion beam through nuclear matter per fermi of path. Now for π-nucleon collision, the blackness $\Omega_{\pi N}(b)$ was evaluated in Ref. 12. From that calculation we find

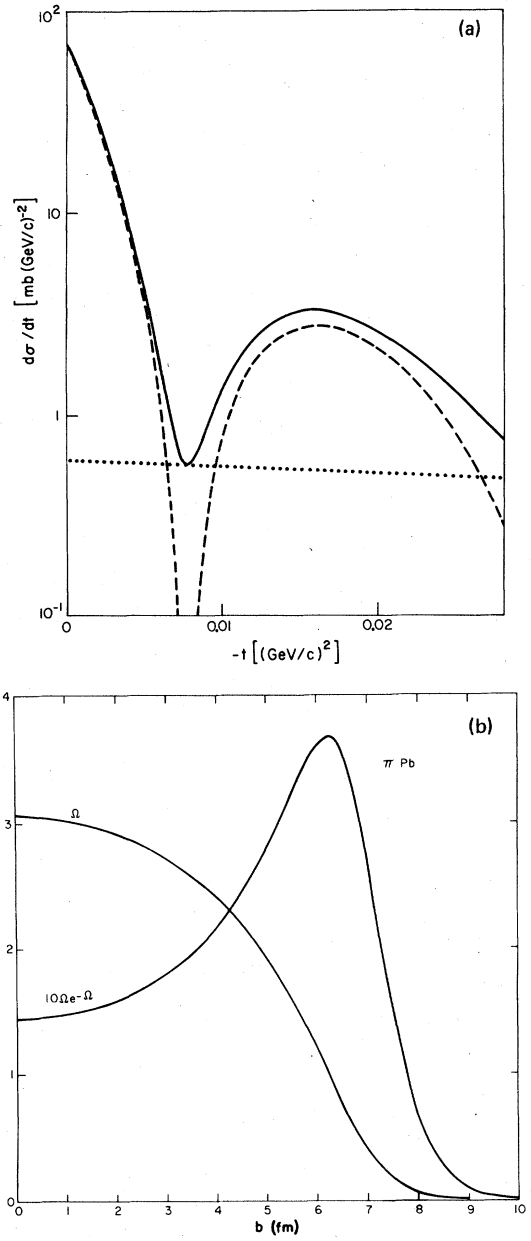

FIG. 2. (a) The calculated differential cross sections for the diffraction dissociation process $\pi\text{Pb} \to \pi^*\text{Pb}$ (dashed curve), and for the diffractive excitation process $\pi\text{Pb} \to \pi^*\text{Pb}^*$ (dotted curve). The solid line represents the sum of the two curves. (b) A plot of Ω and $\Omega e^{-\Omega}$ for πPb collision at 200 GeV/c.

$$\int \Omega_{\pi N}(b)\, d^2b = 14.34\ \text{mb}. \tag{13}$$

In nuclear matter the nuclear density ρ is known. The blackness per fermi of traversal in nuclear matter is thus

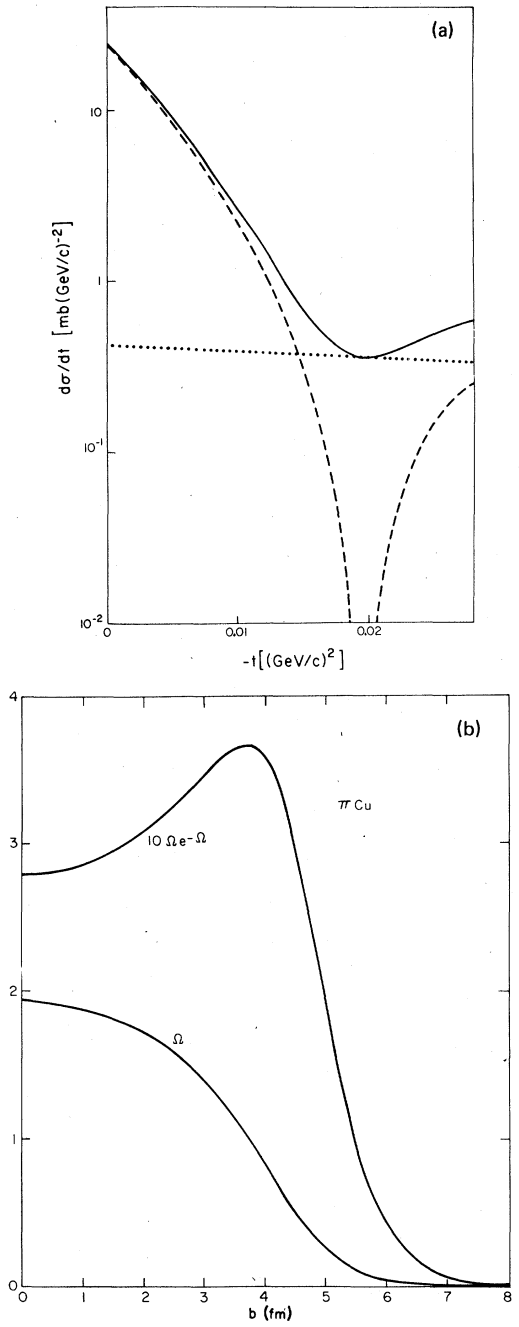

FIG. 3. (a) The calculated differential cross sections for the diffraction dissociation process $\pi\mathrm{Cu}\to\pi^*\mathrm{Cu}$ (dashed curve), and for the diffractive excitation process $\pi\mathrm{Cu}\to\pi^*\mathrm{Cu}^*$ (dotted curve). The solid line represents the sum of the two curves. (b) A plot of Ω and the $\Omega e^{-\Omega}$ for πCu collision at 200 GeV/c.

(14.3 mb) $\rho = 0.244$ fm^{-1} .

In other words we have, for homogenized nuclear matter, the following concept:

In passing through nuclear matter over a length of L fermis, the amplitude of an incoming pion wave is coherently damped by a factor of

$$\exp(-0.244L)\ . \tag{14}$$

The mean absorption length for the intensity of the coherent pion beam is thus

$$1/2(0.244) = 2.05 \text{ fm}. \tag{15}$$

The blackness distribution $\Omega(b)$ can be easily computed from (14) using a nuclear density of the Woods-Saxon shape

$$\rho(r) = \rho_0 \Big/ \left[1 + \exp\left(\frac{r - r_0}{a}\right)\right]$$

with the radius $r_0 = 1.12A^{1/3}$ fm, and the surface-thickness parameter $a = 0.5$ fm. We exhibit Ω and $\Omega e^{-\Omega}$ as functions of b in Figs. 2(b) and 3(b) for πPb and πCu collisions. The differential cross sections for coherent diffraction dissociation process can thus be completely determined from (5) except for the normalization constant which we shall estimate in the following way.

For coherent diffraction dissociation of pions on proton and nuclei, the differential cross sections can be written, respectively, as

$$(d\sigma/dt)_{\pi p\to\pi^* p} = \pi K_{\pi p}\left|(\Omega_{\pi p}\, e^{-\Omega_{\pi p}})_{2D\,FT}\right|^2$$

and (16)

$$(d\sigma/dt)_{\pi A\to\pi^* A} = \pi K_{\pi A}\left|(\Omega_{\pi A}\, e^{-\Omega_{\pi A}})_{2D\,FT}\right|^2 .$$

It is reasonable to assume that the constants $K_{\pi A}$ are the same for all nuclei, and in particular $K_{\pi A} = K_{\pi p}$. With the knowledge of the measured value of $d\sigma/dt$ in (11) the constant $K_{\pi p}$ may be readily computed. We have thus

$$K_{\pi A} = K_{\pi p} = 1.37\times 10^{-3} = 0.533\times 10^{-3} \text{ mb (GeV}/c)^2 .$$

TABLE I. Sensitivity of parameters α and γ to dip position t_0 in πCu diffraction dissociation.

α	t_0 [(GeV/c)^2]
0.50	0.012
1.00	0.020
2.00	0.034
γ	t_0 [(GeV/c)^2]
0.25	0.025
0.50	0.023
0.75	0.021
1.00	0.020
1.25	0.018
1.50	0.017
2.00	0.016
3.00	0.014

The calculated differential cross sections for (1) and (1′) are exhibited in Figs. 2(a) and 3(a). In the same figures the resultant curves for (8) are also presented.

The data in Ref. 5 for πPb and πCu scattering are unnormalized. They show kink structures at $|t| \sim 0.009$ and 0.019 $(\mathrm{GeV}/c)^2$, respectively. These are approximately at the same position as the dips of Figs. 2(a) and 3(a). The general shape of the data points of Ref. 5 cannot be said to agree with the calculated curves, but that is not surprising since (4) is only a zeroth approximation.

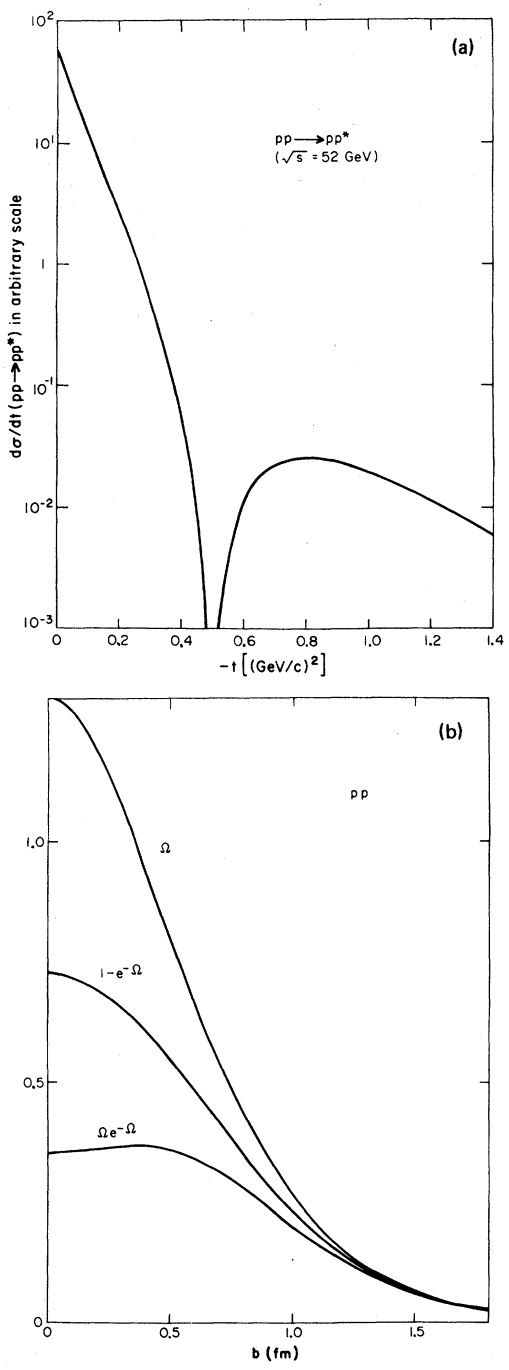

FIG. 4. (a) The calculated differential cross section for pp diffraction dissociation processes. (b) The plot of Ω, $\Omega e^{-\Omega}$, and $1-e^{-\Omega}$ as functions of b for the pp system at $\sigma_T = 42$ mb.

To test sensitivity with respect to the assumption of $\Omega e^{-\Omega}$ as the source, we repeated the calculation for $\pi\mathrm{Cu} \to (\pi\pi\pi)$ Cu, using, instead of $\Omega e^{-\Omega}$, first the expression $\Omega^{\alpha} e^{-\Omega}$. The dip position changes with respect to α as shown in Table I. Next we took $\alpha = 1$, but assumed that the absorption mean free path of $(\pi\pi\pi)$ in nuclear matter is $1/\gamma$ times that of π. Take $\Omega(b)$ to be the blackness at b for the pion. The absorption factor for the incoming pion amplitude for the process illustrated in Fig. 1 is $e^{-\Omega(b)x}$ while that for the outgoing $(\pi\pi\pi)$ is $e^{-\gamma\Omega(b)(1-x)}$. The total absorption factor is thus $e^{-x\Omega-\gamma(1-x)\Omega}$. Therefore $\Omega e^{-\Omega}$ should be replaced by

$$\Omega \int_0^1 e^{-x\Omega-\gamma(1-x)\Omega}\, dx = (e^{-\gamma\Omega} - e^{-\Omega})/(1-\gamma)\ . \qquad (17)$$

The dip-position dependence on the parameter γ is shown in Table I.

pp COLLISION

For pp collision, we take the Ω discussed in Ref. 2 (which was denoted there by $\tilde{\Delta}$), using the

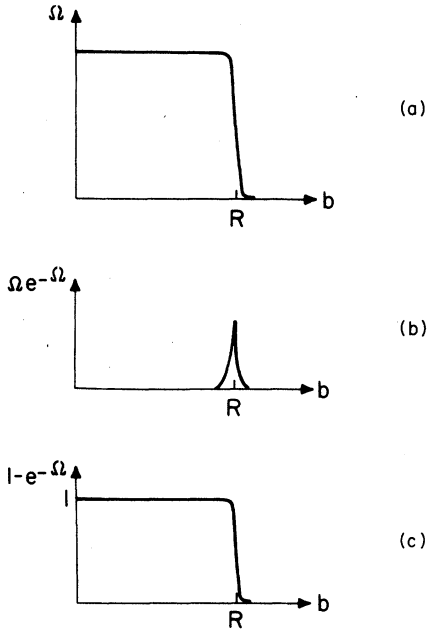

FIG. 5. Schematic drawing of (a) Ω, (b) $\Omega e^{-\Omega}$, and (c) $1-e^{-\Omega}$ as functions of the impact parameter b for an opaque disc.

TABLE II. Dip or kink positions [in $(\mathrm{GeV}/c)^2$] for elastic and diffraction dissociation processes in pp, πCu, and πPb collisions.

	Elastic scattering		Diffraction dissociation	
	Theory	Experiment (Ref. 14)	Theory	Experiment
pp	1.30	~1.36	0.50	0.4–0.5 (Ref. 15)
πCu	0.0283	0.0297 ± 0.0007	0.020	~0.019
πPb	0.0128	0.0135 ± 0.0008	0.0086	~0.009
	0.0419	0.0402 ± 0.0015		

dipole form factor $G_M/\mu = (1+|t|/0.71)^{-2}$ and $\sigma_T = 42.1$ mb. The computation of $(d\sigma/dt)_{el}$ through (6) yields an elastic $d\sigma/dt$ which is in very good agreement with experiments[13] for $-t$ lower than ~$2(\mathrm{GeV}/c)^2$. In particular, it successfully predicts the position of the first dip in pp elastic scattering. [Beyond $-t = 2(\mathrm{GeV}/c)^2$ the computation does not agree with CERN ISR experiments. That is not strange, since the approach to a (slowly varying) limit at such high $-t$ values is slow.[3]]

We apply this $\Omega(b)$ to (5) and evaluate $(d\sigma/dt)_{AB\to AB^*,A^*B^*}$. The result is exhibited in Fig. 4(a). We also plot Ω, $\Omega e^{-\Omega}$, and $1-e^{-\Omega}$ as functions of the impact parameter b in Fig. 4(b).

There is a dip in Fig. 4(a) at $-t = 0.5$ $(\mathrm{GeV}/c)^2$. We believe this is the qualitative reason, without adjustable parameter, of the kinks in experiments (2). It is hard to make quantitative fits without additional adjustable parameters which would have to be introduced if we want to take into consideration spin effects, effects of the differences of mean free paths of p and p^* in hadronic matter, effects of switches back and forth between many coherent states during traversal, effects due to nonvanishing real parts of the scattering amplitudes, etc.

QUALITATIVE ARGUMENTS

It is qualitatively easy to understand the existence of the dip of Fig. 4(a). If the blackness is very large in the center and drops off suddenly at the edge, as illustrated in Fig. 5(a), then $\Omega e^{-\Omega}$ and $1-e^{-\Omega}$ assume the shapes shown in Figs. 5(b) and 5(c). Thus (5) and (6) lead to

$$(d\sigma/dt)_{AB\to AB^*,A^*B^*} \propto |J_0(\sqrt{-t}\,R)|^2 ,$$

$$(d\sigma/dt)_{\text{elastic}} \propto |J_1(\sqrt{-t}\,R)/\sqrt{-t}\,|^2 .$$

In other words both would exhibit many dips, the first dips occurring at t values bearing the following ratio:

$$\frac{(-t \text{ at first dip})_{\text{elastic}}}{(-t \text{ at first dip})_{AB\to AB^*,A^*B^*}} = \left(\frac{3.8}{2.4}\right)^2 = 2.5 . \tag{18}$$

On the other hand, if Ω is small we can expand the right-hand sides of (5) and (6) in powers of $(\Omega)_{2\mathrm{DFT}}$

$$(d\sigma/dt)_{AB\to AB^*,A^*B^*} = (\text{const})\,|(\Omega)_{2\mathrm{DFT}} - (\Omega)_{2\mathrm{DFT}} \otimes (\Omega)_{2\mathrm{DFT}} + \cdots|^2 , \tag{19}$$

$$(d\sigma/dt)_{\text{elastic}} = \pi\,|(\Omega)_{2\mathrm{DFT}} - \tfrac{1}{2}(\Omega)_{2\mathrm{DFT}} \otimes (\Omega)_{2\mathrm{DFT}} + \cdots|^2 . \tag{20}$$

In this case the dip occurs approximately at where

$$(\Omega)_{2\mathrm{DFT}} \simeq (\Omega)_{2\mathrm{DFT}} \otimes (\Omega)_{2\mathrm{DFT}} \quad \text{for } AB\to AB^* \text{ or } AB\to A^*B^* , \tag{21}$$

$$2(\Omega)_{2\mathrm{DFT}} \simeq (\Omega)_{2\mathrm{DFT}} \otimes (\Omega)_{2\mathrm{DFT}} \quad \text{for elastic collision.} \tag{22}$$

Since $(\Omega)_{2\mathrm{DFT}}$ and $(\Omega)_{2\mathrm{DFT}} \otimes (\Omega)_{2\mathrm{DFT}}$ are both roughly Gaussian for small $|t|$, with the former more peaked and narrower in width (in $-t$ variable), the dip in diffractive dissociation [from (21)] occurs again at a smaller $-t$ value than the dip in elastic scattering [from (22)]. Thus the existence of a dip in diffraction dissociation at a smaller $|t|$ than the dip position for elastic scattering is quite natural in the geometrical picture.

CONCLUSION

We list in Table II the dip positions for both elastic scattering and diffraction dissociation experiments in several reactions. Also exhibited are dip positions computed from (6) and (5). The computation of dip positions contains *no adjustable parameters*, since Ω is determined from electron scattering experiments together with πp and pp total cross section. The agreement between the computation and experiment is generally very good, *justifying the geometrical origin of the dips and kinks in the experimental data.* As remarked in an earlier section, the quantitative fit between the computed curves and data is not impressive. An admixture of some imaginary parts to the blackness $\Omega(b)$ serves to greatly improve the fit. Further computations along these lines are under way.

ACKNOWLEDGMENTS

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