PHYSICAL REVIEW D VOLUME 22, NUMBER 2 15 JULY 1980

Static meson potentials

M. Bolsterli
T Division, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545
(Received 5 September 1979)

The potentials between static sources of meson fields are computed for various cases of interest. For isoscalar mesons, the interaction via a scalar-meson field is the well-known Yukawa potential; the interaction via an isoscalar vector-meson field is shown to have the expected Yukawa form when an appropriate Hamiltonian is used. For isovector mesons, which interact through non-Abelian current operators, numerical computations of the potential are presented for vector-meson field interactions. In contrast to the Abelian cases, the non-Abelian interactions are not the same as the one-meson-exchange potentials.

I. SUMMARY AND RESULTS

Recently, renewed effort has been devoted to developing meson field theories of nuclear systems. One branch of this effort deals with covariant quantum field theories,[1] where the main tool must be perturbation theory. The other branch treats noncovariant Hamiltonians[2]; here variational methods can be applied.

The meson theoretic description of infinite nuclear matter is relatively simple. However, nuclear matter provides too few data to determine the parameters of a meson theory. The two-nucleon system gives a wealth of experimental data, but its description in terms of a meson theory has proved to be difficult; none of the meson theories yet gives a satisfactory picture of the deuteron.

As a first step toward describing two nucleons interacting through virtual meson exchange, the related problem of static sources interacting through virtual meson exchange is considered in this paper. For the case of an isoscalar scalar meson field, the interaction of static point sources is the Yukawa interaction $-\gamma e^{-mR}/R$, with R the source separation, m the meson mass, and $\gamma=g^2/4\pi$ the coupling. The derivation of this result is straightforward and well known; it follows directly from calculating the ground-state energy of the Hamiltonian H_{SS},

$$\begin{aligned} H_{SS} &= \int \omega(k) a^\dagger(k) a(k) dk \\ &\quad - \int [u(k)a^\dagger(k) + u^*(k)a(k)]dk \,, \\ \omega(k) &= (k^2+m^2)^{1/2} \,, \\ u(k) &= \frac{g\tilde{\rho}_S(k)}{[16\pi^3\omega(k)]^{1/2}} \,, \\ \tilde{\rho}_S(k) &= \int e^{-ik\cdot r}\rho_S(r)dr \,. \end{aligned} \tag{1.1}$$

Here $a(k)$ is the annihilation operator for a meson of momentum k and $\rho_S(r)$ is the scalar source density. The ground-state energy of H_{SS} is easily seen to be

$$E_{SS} = -\frac{\gamma}{2}\int \rho_S(r)\frac{e^{-m|r-r'|}}{|r-r'|}\rho_S(r')dr\,dr' \,, \tag{1.2}$$

from which follows the static Yukawa potential.

If the meson field is isovector in nature, the above simple derivation cannot be carried out. Moreover, for a vector field, a Hamiltonian that gives the desired result $+\gamma e^{-mR}/R$ does not seem to have appeared in the literature. Thus, in these cases the mesonic interaction between static sources is of some interest.

Section II of this paper discusses the appropriate Hamiltonian for a vector meson field interacting with static sources. A consistent treatment of the relativistic Hamiltonian for a Dirac particle interacting with a vector field with mass has been given earlier[3]; reduction of the Hamiltonian to the case of static sources gives

$$\begin{aligned} H_{SV} &= \int \omega(k)a^\dagger(k)a(k)dk \\ &\quad - \int [v(k)a^\dagger(k) + v^*(k)a(k)]dk \\ &\quad + \frac{\gamma}{4\pi^2}\sum_{i\neq j}\int \frac{\tilde{\rho}_i^*(k)\tilde{\rho}_j(k)}{k^2}dk \,, \\ v(k) &= \frac{gm\tilde{\rho}(k)}{k[16\pi^3\omega(k)]^{1/2}} \,, \\ \tilde{\rho}(k) &= \sum_i \tilde{\rho}_i(k) \end{aligned} \tag{1.3}$$

for the interaction with the longitudinal scalar quanta of the vector field, which are annihilated by $a(k)$. The sources are treated as distinct, with $\tilde{\rho}_i(k)$ the source distribution of source i. The interaction with the transverse part of the vector field gives spin-dependent corrections to the longitudinal field interaction that will not be considered here.

The ground-state energy of H_{SV} is easily found:

$$E_{SV}=\sum_i \Delta_i + \sum_{i<j} V_{ij}\,,$$
$$\Delta_i = \frac{\gamma m^2}{4\pi^2}\int \frac{|\tilde{\rho}_i(k)|^2}{k^2\omega^2(k)}dk\,, \tag{1.4}$$
$$V_{ij}=\gamma\int \rho_i(r)\frac{e^{-m|r-r'|}}{|r-r'|}\rho_j(r')dr\,dr'\,.$$

Note that, in contrast to the case of a scalar field, the self-energy Δ_i is finite even in the point-source limit where $\tilde{\rho}_i(k)=1$. The fact that V_{ij} has the expected Yukawa form can be regarded as a justification of the Hamiltonian of Eq. (1.3).

The generalizations of the above Hamiltonians to the case of isovector mesons interacting with n sources are clearly

$$H_{VS}=\int \omega(k)a^\dagger(k)\cdot a(k)dk - \sum_{i=1}^{n}\tau_i\cdot\int[u_i(k)a^\dagger(k)+u_i^*(k)a(k)]dk\,,$$
$$H_{VV}=\int \omega(k)a^\dagger(k)\cdot a(k)dk - \sum_{i=1}^{n}\tau_i\cdot\int[v_i(u)a^\dagger(k)+v_i^*(k)a(k)]dk + \gamma\sum_{i<j=1}^{n}\tau_i\cdot\tau_j\int\rho_i(r)\frac{1}{|r-r'|}\rho_j(r')dr\,dr'\,,$$
$$u_i(k)=\frac{g\tilde{\rho}_{Si}(k)}{[16\pi^3\omega(k)]^{1/2}}\,, \tag{1.5}$$
$$v_i(k)=\frac{gm\tilde{\rho}_i(k)}{k[16\pi^3\omega(k)]^{1/2}}\,,$$

where $a(k)$ and the τ_i are isovector operators. The Hamiltonians H_{VS} and H_{VV} appear quite simple, but they have not been solved exactly. Instead, a numerical treatment must be used to provide information about the ground-state energies. Because H_{VS} has infinite self-energies for point sources, a cutoff function is required to guarantee finite results; a satisfactory way of providing a cutoff is available, but it does involve the introduction of an extra parameter. On the other hand, H_{VV} is finite even for point sources. Therefore, in this paper results are given for H_{VV} in the case of two identical point sources located at r_1 and r_2, so that

$$v_i(k)=\frac{gm}{k[16\pi^3\omega(k)]^{1/2}}e^{-ik\cdot r_i}\,,$$
$$H_{VV}=\int\omega(k)a^\dagger(k)\cdot a(k)dk - \sum_{i=1}^{2}\tau_i\cdot\int[v_i(k)a^\dagger(k)+v_i^*(k)a(k)]dk + \gamma\tau_1\cdot\tau_2/R\,, \tag{1.6}$$
$$R=|r_1-r_2|\,.$$

In the special case of zero-mass mesons, $v_i(k)$ vanishes and H_{VV} is simply solvable. It might seem that the ground-state energy is $\gamma\tau_1\cdot\tau_2/R$, that is, γ/R for $T=1$ and $-3\gamma/R$ for $T=0$. Not so, since the $T=1$ state will emit a meson (through the transverse interaction) and convert to $T=0$; the correct ground-state energy is $-3\gamma/R$ for both $T=0$ and $T=1$. This same conversion occurs for $m\neq 0$ whenever the energy of the $T=1$ ground state exceeds the energy of the $T=0$ ground state by an amount greater than m.

The Hamiltonian of Eq. (1.6) is symmetric under the operation of $P_{r\tau}$ which interchanges r_1 with r_2 and τ_1 with τ_2. It follows that the eigenstates of H_{VV} are either even or odd under $P_{r\tau}$. The bare two-fermion states $|t=0\rangle$ and $|t=1\rangle$ are odd and even, respectively, under $P_{r\tau}$, as are eigenstates of H containing any component of $|t=0\rangle$ or $|t=1\rangle$. For identical fermions, the $P_{r\tau}$-odd state with $T=0$ has spin $S=1$, the $P_{r\tau}$-even state with $T=1$ has $S=0$, and only these two ground-state energies are computed in this paper. For nonidentical fermions, such as particle and antiparticle, the spin is unrestricted, although the states are, of course, still eigenstates of $P_{r\tau}$.

The essential step in obtaining a ground-state energy for H_{VV} is to consider only a few modes of the meson field.[4] In particular, for two sources it seems reasonable to allow one symmetric mode and one antisymmetric mode. The annihilation operator for the mode with mode function $\phi_i(k)$ is

$$A_i=\int\phi_i^*(k)a(k)dk\,. \tag{1.7}$$

For the two modes, the annihilation operators will be denoted $A_\pm$, with the alternate notation

$$P=A_+\,,\quad M=A_-\,. \tag{1.8}$$

The mode functions $\phi_+(k)$ and $\phi_-(k)$ are symmetric and antisymmetric, respectively, under interchange of r_1 and r_2, the two source positions; they are taken to be normalized and are obviously orthogonal. In the two-mode subspace generated by $P^\dagger$ and $M^\dagger$, matrix elements of H_{VV} are equal to matrix elements of H_{MP}, where

$$H_{MP}=W_+P^\dagger\cdot P+W_-M^\dagger\cdot M-V_+\tau_+\cdot(P^\dagger+P) - V_-\tau_-\cdot(M^\dagger+M)+\gamma\tau_1\cdot\tau_2/R\,,$$
$$W_s=\int\omega(k)|\phi_s(k)|^2dk\,, \tag{1.9}$$
$$V_s=\left|\int\phi_s^*(k)[v_1(k)+s v_2(k)]dk\right|\,,$$
$$\tau_\pm=\tfrac{1}{2}(\tau_1\pm\tau_2)\,.$$

The phases of $V_\pm$ have been incorporated into P and M.

The ground-state energy E_{MP} of H_{MP} depends on the constants W, V, and γ/R. Therefore, a simple variation shows that ϕ has the form

$$\phi_s(k) = c_s \frac{v_1(k) + s v_2(k)}{\omega(k) + b_s}\,, \tag{1.10}$$

where c_s is a normalization constant

$$c_s^{-2} = \int \frac{|v_1(k) + s v_2(k)|^2}{[\omega(k) + b_s]^2} dk \tag{1.11}$$

and b_s are real parameters. Hence, the ground-state energy of the system in the two-mode approximation is

$$E(R) = \min_{b_s} E_{MP}(b_s, R)\,. \tag{1.12}$$

The potential energy is $V(R)$,

$$V(R) = E(R) - E(\infty)\,. \tag{1.13}$$

Section III gives details of the method of obtaining ground-state energies of H_{MP}.

Figure 1 shows $V(R)$ for both $T=0$ and $T=1$ with $\gamma=1$. Also shown are the second-order perturbation values $V^{(2)}(R)$; these are $\gamma e^{-mr}/R$ and $-3\gamma e^{-mR}/R$ for $T=1$ and $T=0$, respectively.

The variation with γ is shown in Figs. 2 and 3. In these figures, $V(R)/\gamma$ is plotted for $\gamma=\frac{1}{4}$, 1, and 4, together with $V^{(2)}(R)/\gamma$. Figure 2 is plotted for $T=0$ and Fig. 3 gives the $T=1$ potentials.

In every case, $V(R)$ is more attractive than $V^{(2)}(R)$. The function $V(R)/\gamma$ is monotonic in γ. For $T=0$, $V(R)$ is attractive, as is $V^{(2)}(R)$, but for $T=1$, $V(R)$ is only repulsive for weak coupling and/or distances greater than about γ/m; for short distances and/or strong coupling $V(R)$ is attractive, opposite in sign to $V^{(2)}(R)$.

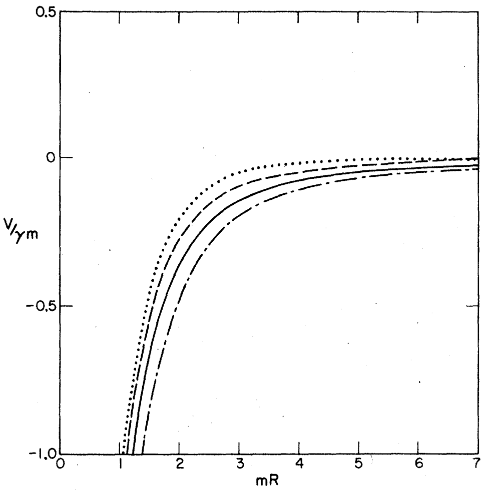

FIG. 2. The $T=0$ potentials for various values of γ. The dotted curve is the one-meson-exchange potential; the dashed, solid, and dash-dotted curves are for $\gamma=1/4$, 1, and 4, respectively.

For the case of an isovector scalar exchange, the corresponding results are conjectured to be that for $T=1$, $V(R)$ is somewhat more attractive than $V^{(2)}(R)$. For $T=0$, $V(R)$ is repulsive for weak coupling and/or large separations; for short distances and/or strong coupling, $V(R)$ is attractive, opposite in sign to $V^{(2)}(R)$.

Thus, there is a qualitative difference in the static potentials between isoscalar fields that couple through the Abelian current operators 1 and isovector fields that couple through the non-Abel-

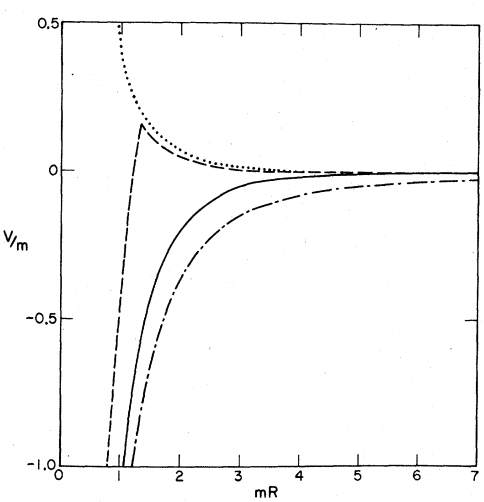

FIG. 1. Static meson potentials $V(R)/m$ for $\gamma=1$. The dashed and dash-dotted curves are for $T=1$ and $T=0$, respectively. The dotted and solid curves are the one-meson-exchange potentials for $T=1$ and $T=0$, respectively.

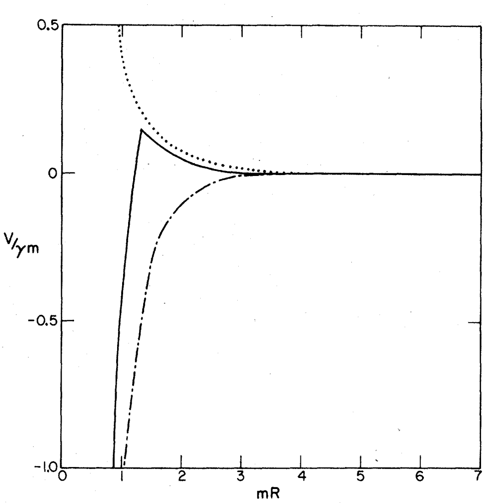

FIG. 3. The $T=1$ potentials for various values of γ. The dotted curve shows both the one-meson-exchange potential and the potential for $\gamma=1/4$. The solid and dashed curves show the potentials for $\gamma=1$ and $\gamma=4$, respectively.

ian current operators τ. In the Abelian case, the static potential is identical to the second-order perturbation potential or one-meson-exchange potential; in the non-Abelian case, the static potential need not even have the sign of the one-meson-exchange potential.

The computation of quark-quark and quark-antiquark potentials in a static limit of quantum chromodynamics (QCD) is a considerably more complicated problem than the interactions treated above.[5] However, several points discussed in the present work may be applicable to the QCD potentials. The static limit of the vector-meson interaction discussed in Sec. II seems potentially useful, and the use of both a symmetric and an antisymmetric mode may be a practical extension of Tomonaga's original ideas. The instability of the $T=1$ state discussed after Eq. (1.16) and evident in Figs. 1 and 3 will also need to be taken into account.

II. HAMILTONIAN FOR VECTOR MESONS

The old form for the interaction Hamiltonian density in the case of a longitudinal vector field $\vec{V}_L$ interacting with a Dirac field ψ is

$$\mathcal{H}_{I,1}(\psi,\vec{\pi}_L,\vec{V}_L) = -g\vec{V}_L\cdot :\psi^\dagger\vec{\alpha}\psi: + \frac{g}{m^2}:\psi^\dagger\psi:\nabla\cdot\vec{\pi}_L + \frac{g^2}{2m^2}:\psi^\dagger\psi\psi^\dagger\psi: . \quad (2.1)$$

In this form, the field ψ depends on the gauge. In Ref. 3 it was shown that a gauge-invariant Dirac field can be defined by

$$\chi(\vec{r}) = \exp\left(-\frac{ig}{\nabla^2}\nabla\cdot\vec{V}_L(\vec{r})\right)\psi(\vec{r}) = \exp\left(\frac{ig}{4\pi}\int\frac{1}{|\vec{r}-\vec{s}|}\nabla\cdot\vec{V}_L(\vec{s})d\vec{s}\right)\psi(\vec{r}) . \quad (2.2)$$

When the necessary change in $\vec{\pi}_L$ is made to make the transformation canonical, namely,

$$\vec{\Pi}_L(\vec{r}) = \vec{\pi}_L(\vec{r}) + \frac{g}{\nabla^2}\vec{\nabla}[\psi^\dagger(\vec{r})\psi(\vec{r})] , \quad (2.3)$$

then the interaction Hamiltonian density is

$$\mathcal{H}_{I,2}(\chi,\vec{\Pi}_L,\vec{V}_L) = g:\chi^\dagger\chi:\frac{1}{\nabla^2}\nabla\cdot\Pi_L - \frac{g^2}{2}:\chi^\dagger\chi\frac{1}{\nabla^2}\chi^\dagger\chi: \quad (2.4)$$

and the interaction Hamiltonian can be written

$$H_{I,2} = -gm\int\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{k[16\pi^3\omega(k)]^{1/2}}:\chi^\dagger(\vec{r})\chi(\vec{r}):[b(\vec{k})+b^\dagger(-\vec{k})]d\vec{k}\,d\vec{r} + \frac{g^2}{8\pi}\int\frac{1}{|\vec{r}-\vec{s}|}:\chi^\dagger(\vec{r})\chi(\vec{r})\chi^\dagger(\vec{s})\chi(\vec{s}):d\vec{r}\,d\vec{s} . \quad (2.5)$$

When the density $\chi^\dagger(\vec{r})\chi(\vec{r})$ is replaced by a static source density $\rho(\vec{r})$, the Hamiltonian of Eq. (1.3) is obtained; the restriction $i\neq j$ in Eq. (1.3) comes from the normal product in Eq. (2.5).

For particle-antiparticle interactions, the sign of the source density $\tilde{\rho}(k)$ must be changed for the antiparticles because of the normal ordering. It follows that for isoscalar-meson interaction, particles and antiparticles couple oppositely. For particle-antiparticle interactions there is also a contact term that can be dropped for nonoverlapping static sources.

The isovector interaction corresponding to Eq. (2.5) has $:\chi^\dagger\chi:$ replace by $:\chi^\dagger\tau\chi:$ in Eq. (2.5). As is shown in Ref. 2, the antiparticle part of $:\chi^\dagger\tau\chi:$ is $\psi_A^\dagger\tau\psi_A$, so that particle and antiparticle static sources have the same static isovector-vector-meson interaction. Again the contact term is ignored for nonoverlapping sources.

III. APPROXIMATE EIGENSTATES OF H_{MP}

Consider first the single-mode subspaces generated by $M^\dagger$ or $P^\dagger$. In the M-mode subspace, H_{MP} is equivalent to H_M; in the P-mode subspace to H_P,

$$H_M = W_-^\dagger M^\dagger\cdot M - V_-\tau_-\cdot(M^\dagger+M) + \gamma\tau_1\cdot\tau_2/R ,$$
$$H_P = W_+P^\dagger\cdot P - V_+\tau_+\cdot(P^\dagger+P) + \gamma\tau_1\cdot\tau_2/R . \quad (3.1)$$

Let t be the fermion isospin; states with $t=1$ and 0 are symmetric and antisymmetric, respectively, under fermion isospin exchange. It follows that τ_+ and τ_- are diagonal and off diagonal, respectively, in fermion isospin t. Moreover, τ_+ annihilates states with $t=0$. Thus, the P-mode Hamiltonian H_P is diagonal in t,

$$H_{P,t=0} = W_+P^\dagger\cdot P - 3\gamma/R ,$$
$$H_{P,t=1} = W_+P^\dagger\cdot P - \nabla_+\tau_+\cdot(P^\dagger+P) + \gamma/R , \quad (3.2)$$

and it follows directly that the lowest eigenvalue of H_P in $T=0$ states is

$$E_{0,P} = -3\gamma/R . \quad (3.3)$$

On the other hand, the M-mode Hamiltonian H_M has parts that are diagonal in t and parts that are off diagonal. For $T=0$, the M states are relatively simple because

$$(\tau_-\cdot M^\dagger)^2|t=0\rangle = (\tau_1\cdot M^\dagger)^2|t=0\rangle = M^\dagger\cdot M^\dagger|t=0\rangle ,$$
$$\tau_+\cdot M^\dagger|t=0\rangle = 0 , \quad (3.4)$$
$$\tau_+\cdot M^\dagger\tau_-\cdot M^\dagger = \tau_-\cdot M^\dagger\tau_+\cdot M^\dagger = 0 ,$$

so that the general form of an M-mode state with $T=0$ is

$$\sum_0^\infty a_n(\tau_-\cdot M^\dagger)^n|t=0\rangle=\sum_0^\infty a_n|M,n,T=0\rangle\,. \tag{3.5}$$

The basis states $|M,n,T=0\rangle$ can be normalized and used to evaluate the matrix of H_M, with the result

$$(H_M)=\begin{pmatrix} -3\Gamma & -3^{1/2}V_- & & & & \\ -3^{1/2}V_- & W_-+\Gamma & -2^{1/2}V_- & & & \\ & -2^{1/2}V_- & 2W_--3\Gamma & -5^{1/2}V_- & & \\ & & -5^{1/2}V_- & 3W_-+\Gamma & -4^{1/2}V_- & \\ & & & -4^{1/2}V_- & 4W_--3\Gamma & -7^{1/2}V_- \\ & & & & -7^{1/2}V_- & \text{etc.,} \end{pmatrix}. \tag{3.6}$$

This matrix can be diagonalized by brute force on a computer, but it suffers the same problems that arise in treating the corresponding famous matrix for isoscalar scalar mesons interacting with a simple source

$$H_A=WA^\dagger\cdot A-V(A^\dagger+A)\,, \tag{3.7}$$

$$(H_A)=\begin{pmatrix} 0 & -V & 0 & 0 & 0 \\ -V & W & -2^{1/2}V & 0 & 0 \\ 0 & -2^{1/2}V & 2W & -3^{1/2}V & 0 \\ 0 & 0 & -3^{1/2}V & 3W & -4^{1/2}V \\ 0 & 0 & 0 & -4^{1/2}V & \text{etc.,} \end{pmatrix}. \tag{3.8}$$

Here the eigenvector has components

$$v_n=(V/W)^n/(n!)^{1/2}\,, \tag{3.9}$$

so that for $V/W>2$ or 3, very large matrices must be used to get reasonable accuracy in the lowest eigenvalue. In the case of H_A of Eq. (3.7) the exact ground state is the simple coherent state

$$|y\rangle_A\equiv e^{yA^\dagger}|\Omega\rangle\,,\quad y=V/W\,, \tag{3.10}$$

where $|\Omega\rangle$ is the meson vacuum. For any y, the coherent state $|y\rangle$ satisfies

$$A\,|y\rangle_A=y\,|y\rangle_A\,. \tag{3.11}$$

In the case of H_M, the convergence difficulties for large values of V/W can be circumvented by using a coherent state as a starting point. Various coherent states can be imagined; as is discussed in Ref. 6, the useful choice is the special coherent state $|y\rangle_M$ that satsifies

$$\tau_-\cdot M\,|y\rangle_M=y\,|y\rangle_M\,. \tag{3.12}$$

This condition is the isovector analog of Eq. (3.11).

For $T=0$, the state $|y\rangle_M$ is given by

$$|y,T=0\rangle_M=D_{0,M}(y\tau_-\cdot M^\dagger)\,|t=0\rangle\,,\qquad D_{0,M}(x)=\frac{e^x}{x}-\frac{\sinh x}{x^2}\,. \tag{3.13}$$

[Remarkably, $D_{0,M}(x)$ is identical to the function that gives the special coherent state for an isovector field interacting with a single fermion source.[6]] The special coherent state $|y,T=0\rangle_M$ has been chosen so that $D_{0,M}(0)=1$; for $y\to 0$ it reduces to the bare state $|t=0\rangle$. Its normalization then follows from Eq. (3.12):

$$_M\langle y,T=0\,|y,T=0\rangle_M=D_{0,M}(y^2)\,. \tag{3.14}$$

In contrast to the state $|y\rangle_A$ of Eq. (3.11), there is not a value of y for which $|y,T=0\rangle_M$ is an eigenvector of H_M. However, the value of y that minimizes

$$F_{0,M}(y)=\frac{_M\langle y,T=0\,|H_M|\,y,T=0\rangle_M}{_M\langle y,T=0\,|\,y,T=0\rangle_M} \tag{3.15}$$

gives the variationally best special coherent state for H_M. Moreover, it is possible to evaluate the matrix of H_M in the basis constructed on $|y,T=0\rangle_M$; it turns out that only a few (up to four) basis vectors are needed for six or eight decimal places of accuracy in the ground-state eigenvalue of H_M for any value of γ.

Since $F_{0,M}(y)$ is less than $E_{0,P}$ of Eq. (3.3), it is natural to use the state $|y,T=0\rangle_M$ as a starting point for diagonalizing H_{MP}. For this purpose, eigenstates of t are useful:

$$|y,T=0,t=0\rangle=\tfrac{1}{2}(\,|y,T=0\rangle_M+|-y,T=0\rangle_M)\,,$$
$$|y,T=0,t=1\rangle=\tfrac{1}{2}(\,|y,T=0\rangle_M-|-y,T=0\rangle_M)\,. \tag{3.16}$$

Then the set of single-excitation states is spanned by the basis vectors

$$|y,0,0\rangle\,,\quad |y,0,1\rangle\,,\quad \tau_+\cdot P^\dagger|y,0,1\rangle\,,\qquad \tau_-\cdot M^\dagger|y,0,0\rangle\,,\quad \tau_-\cdot M^\dagger|y,0,1\rangle\,, \tag{3.17}$$

where it can be shown that $M^\dagger \cdot M|y,0,t\rangle$ can be written as a linear combination of the basis vectors. The $T=0$ energy shown in Sec. I is obtained by diagonalizing H_{MP} in the subspace spanned by the basis vectors of (3.17), then minimizing with respect to y and b_s. For $T=0$, the ground-state eigenvalues of H_{PM} computed by this procedure have an accuracy of better than 1%. However, since $V(R)$ is a difference of $E(R)$, it can be substantially less accurate for large values of R.

For $T=1$, the procedure is similar. In this case, the special coherent state $|z,T=1\rangle_P$ defined by

$$\tau_+ \cdot P|z\rangle_P = z|z\rangle_P \qquad (3.18)$$

gives a lower expectation of H_{MP} than the special coherent state $|y,T=1\rangle_M$. The state $|z,T=1\rangle_P$ has $t=1$. The subspace used for $E(R)$ is spanned by the vectors

$$|z\rangle\,,\quad \tau_+ \cdot P^\dagger|z\rangle\,,\quad P^\dagger \cdot P|z\rangle\,,\quad \tau_- \cdot M^\dagger|z\rangle\,,\quad (\tau_- \cdot M^\dagger)^2|z\rangle\,, \qquad (3.19)$$

where one double-excitation state is included because it was found to give a 5% correction to E for $\gamma=1$. No other double-excitation state was found to give a correction to E of as much as 1%.

ACKNOWLEDGMENT

This work was supported by the U. S. Department of Energy.

[1]W. D. Brown, R. D. Puff, and L. Wilets, Phys. Rev. C 2, 331 (1970); M. Bolsterli, Ann. Phys. (N.Y.) 62, 569 (1971); R. L. Bowers, J. A. Campbell, and R. L. Zimmerman, Phys. Rev. D 7, 2278 (1973); 7, 2289 (1973); J. D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974); M. Bolsterli, Phys. Rev. D 11, 312 (1975); 11, 784 (1975); L. Wilets, R. D. Puff, D. Chiang, and W. T. Nutt, Phys. Rev. C 14, 2269 (1976); S. A. Chin, Ann. Phys. (N.Y.) 108, 301 (1977).

[2]M. Bolsterli, in *Advances in Nuclear Science*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1979), Vol. XI, p. 367 and references therein.

[3]M. Bolsterli, Phys. Rev. D 14, 945 (1976).

[4]J. Tomonaga, Progr. Theor. Phys. (Japan) 2, 6 (1947).

[5]R. Giles and L. McLerran, Phys. Rev. D 19, 3732 (1979).

[6]M. Bolsterli, Phys. Rev. D 16, 1739 (1977).