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Quantum mechanics vs local realism near the classical limit: A Bell inequality for spin s

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The quantitative quantum-mechanical analysis of the Einstein-Podolsky-Rosen experiment for correlated particles of arbitrary spin s is shown to contradict a generalized form of Bell's inequality, for suitable orientations of the detectors. As the classical ($s \to \infty$) limit is approached, the range of angles for which the contradiction arises vanishes as $1/s$.

Consider the spin-s generalization of Bohm's version of the Einstein-Podolsky-Rosen *Gedanken-experiment*[1]: two spin-s particles are flying apart in a state $|\psi\rangle$ of zero total spin. Define $|m,m'\rangle_{\hat{n},\hat{n}'}$ to be the simultaneous eigenstate of the (commuting) projections along axes $\hat{n}$ and $\hat{n}'$ of the vector spin operators $\vec{S}^{(1)}$ and $\vec{S}^{(2)}$:

$$\begin{aligned}\vec{S}^{(1)}\cdot\hat{n}\,|m,m'\rangle_{\hat{n},\hat{n}'} &= m\,|m,m'\rangle_{\hat{n},\hat{n}'}\,,\\ \vec{S}^{(2)}\cdot\hat{n}'\,|m,m'\rangle_{\hat{n},\hat{n}'} &= m'\,|m,m'\rangle_{\hat{n},\hat{n}'}\,.\end{aligned} \tag{1}$$

The spin part of the state $|\psi\rangle$ is given by[2]

$$|\phi\rangle=\frac{1}{(2s+1)^{1/2}}\sum_{m=-s}^{s}(-1)^{s-m}|m,-m\rangle_{\hat{n},\hat{n}}\,. \tag{2}$$

Being a state of zero total spin, $|\phi\rangle$ is rotationally invariant in spin space and therefore has the structure (2) whatever the direction of the axis $\hat{n}$. Putting it explicitly, the relation

$$\sum_{m=-s}^{s}(-1)^{s-m}|m,-m\rangle_{\hat{n},\hat{n}}=\sum_{m=-s}^{s}(-1)^{s-m}|m,-m\rangle_{\hat{n}',\hat{n}'} \tag{3}$$

holds as a mathematical identity, whatever the directions of $\hat{n}$ and $\hat{n}'$.

The property (3) leads immediately to Einstein, Podolsky, and Rosen's conundrum: If the spin of particle 2 is measured along any axis $\hat{n}$ and found to have the value m, then a subsequent measurement of the spin of particle 1 along the same axis $\hat{n}$ will yield the value $-m$ with probability unity. Since this is the case whatever the direction of the axis $\hat{n}$, an observer in the vicinity of one of the particles can predict with certainty the outcome of a measurement of its spin along any direction whatever, provided a second far-away observer in the vicinity of the other particle first measures the other particle's spin in that same direction.

Given these undisputed facts, and bearing in mind the spatial remoteness of particle 2 from particle 1 as well as the possibility of measuring the spin of either particle with detectors that are well localized on the scale of the interparticle separation, it is tempting to draw conclusions which the orthodox interpretation of quantum mechanics strictly forbids: namely, that associated with each particle i is a definite value $m_i(\hat{n})$ for the result of an impending measurement of the component of its spin along any axis $\hat{n}$ whatever.

Quantum mechanics tells us to resist this temptation. Given a pair of axes $\hat{n}$ and $\hat{n}'$, the numbers $m_1(\hat{n})$ and $m_1(\hat{n}')$ are the results of measurements of the observables $\vec{S}^{(1)}\cdot\hat{n}$ and $\vec{S}^{(1)}\cdot\hat{n}'$. Since these fail to commute if $\hat{n}$ is not parallel to $\hat{n}'$, it is meaningless—simply an abuse of language—to ascribe to particle 1 simultaneous values of $m_1(\hat{n})$ and $m_1(\hat{n}')$, each waiting to reveal itself should the Stern-Gerlach apparatus be aligned along $\hat{n}$ or along $\hat{n}'$.

Prior to Bell's 1964 paper[3] one could regard this doctrine as a part of quantum theology, a subject of some concern to the founders of quantum mechanics, but bearing a rather tenuous relation to quantum mechanics as practiced in its maturity. The discomfort produced by the conundrum of Einstein, Podolsky, and Rosen could then be relieved by rejecting this article of quantum theology, adopting instead a simple point of view that I shall characterize by the term "local realism."[4]

As I shall use the term here, local realism holds that one can assign a definite value to the result of an impending measurement of any component of the spin of either of the two correlated particles, whether or not that measurement is actually performed. That value may well be unknown or even inherently unknowable until the measurement is performed, but it is nevertheless assumed that functions $m_i(\hat{n})$ *exist*, defined over the entire unit sphere and taking values on the discrete set $s,\ s-1,\ldots,-s$, such that the result of measuring the spin component of particle i along a particular direction $\hat{n}$ will be the number $m_i(\hat{n})$. The functions $m_i(\hat{n})$ can vary from one run of the experiment to another, even though each run is characterized by the identical quantum state. The manner in which they vary can be regarded as the subject of an as yet unformulated hidden-variable theory, or as an intrinsically unknowable feature of the world. All we require is that the functions $m_i(\hat{n})$ exist and have some statistical distribution. The perfect correlation in the *Gedankenexperiment* in the state of zero total spin is then simply accounted for by the additional assumption that in each run of the experiment

$$m_1(\hat{n}) = -m_2(\hat{n}) \tag{4}$$

for every direction $\hat{n}$.

In 1964, however, Bell showed that in the case $s=\frac{1}{2}$ this escape from the conundrum is not only incompatible with the orthodox interpretation of quantum mechanics, but it is also inconsistent with the *quantitative numerical predictions* of quantum mechanics. There is no conceivable way to assign values to the unmeasured $m_i(\hat{n})$ that does not lead to numerical disagreement with some of the predictions of the elementary quantum mechanics of spin $\frac{1}{2}$.

Now spin $\frac{1}{2}$ is the simplest but also the least classical of the spin values. Few believers in quantum theology would doubt that a conclusion like Bell's could be derived for values of s other than $\frac{1}{2}$, but I am unaware that any such extensions have been given, and I know of at least one inconclusive attempt to do so.[5] The many generalizations and extensions of Bell's argument since 1964 have been almost entirely concerned with two-state systems or with two-dimensional subspaces of many-state systems. The focus has remained on measurements with only two possible outcomes. This imposes something of a gap between those quantum phenomena where the local realist's description is demonstrably incompatible with the predictions of quantum mechanics, and classical phenomena, measurements of which can be viewed as having a continuum of values and for which local realism is unquestionably valid. Could it therefore be that the clash between quantum mechanics and local realism is limited to the behavior of two-state systems or two-valued experiments, and that insight into the conundrum should be sought in the curious behavior of these most quantum mechanical of systems?

My purpose in this paper is to answer this question in the negative, by showing explicitly that local realism is inconsistent with the quantitative predictions of quantum mechanics not only for the spin-$\frac{1}{2}$ Einstein-Podolsky-Rosen experiment, but also for arbitrary values of the spin s right up to the very threshold of that classical ($s=\infty$) world in which it *is*, in fact, demonstrably permissible to assign definite values to all components of the spin, whether or not they are actually measured.

More specifically, I shall show that when applied to the spin-s Einstein-Podolsky-Rosen experiment, local realism contradicts at least one of a pair of predictions of the quantum theory. One prediction applies when the axes $\hat{n}$ and $\hat{n}'$ along which m_1 and m_2 are measured are at an angle of $\frac{1}{2}\pi+\theta$; the other applies when the angle is $\pi-2\theta$. Whatever the value of s there is a range of angles in the neighborhood of $\theta=0$ for which the contradiction occurs. In the limit of large s the range of angles is

$$0<\theta<A/s\,, \tag{5}$$

where $A=0.5659\ldots$.[6]

The argument is based on the trivial inequality

$$s\,|m_1(\hat{a})+m_1(\hat{b})| \geq -m_1(\hat{a})m_1(\hat{c}) - m_1(\hat{b})m_1(\hat{c})\,, \tag{6}$$

which the variables $m_1(\hat{n})$ of local realism must obey for any three directions $\hat{a}$, $\hat{b}$, and $\hat{c}$. By using the identity (4) and averaging over many runs of the experiment we arrive at the inequality

$$s\langle |m_1(\hat{a})-m_2(\hat{b})|\rangle_{\rm av} \geq \langle m_1(\hat{a})m_2(\hat{c})\rangle_{\rm av} + \langle m_1(\hat{b})m_2(\hat{c})\rangle_{\rm av}\,, \tag{7}$$

which any theory accepting local realism must obey.

Each term in (7), however, is a mean of data obtained by measurements of two commuting observables (a component of the spin of particle 1 and another component of the spin of particle 2). Each term is therefore a meaningful quantity within the framework of ordinary quantum mechanics and has a well-defined theoretical value which is not difficult to compute when the two spin-s particles are in a spin state $|\phi\rangle$ of zero total spin given by Eq. (2).

We can therefore test the predictions of quantum mechnics against the inequality (7). An elementary calculation[7] gives

$$\begin{aligned}\langle m_1(\hat{n})m_2(\hat{n}')\rangle_{\rm av} &= \langle\phi\,|\,\vec{S}^{(1)}\cdot\hat{n}\,\vec{S}^{(2)}\cdot\hat{n}'\,|\,\phi\rangle \\ &= -\tfrac{1}{3}s(s+1)\hat{n}\cdot\hat{n}'\,.\end{aligned} \tag{8}$$

The other type of average has a somewhat more complicated structure

$$\langle |m_1(\hat{n})-m_2(\hat{n}')|\rangle_{\rm av} = \sum_{m,m'} |m-m'|P(m,m',\alpha)\,, \tag{9}$$

where $P(m,m',\alpha)$ is the probability of spin measurements of particles 1 and 2 yielding the values m and m' when the angle between the axes $\hat{n}$ and $\hat{n}'$ is α. The probability P is given by

$$
\begin{aligned}
P(m,m',\alpha) &= |\langle\phi\,|m,m'\rangle_{\hat n,\hat n'}|^2\\
&= \frac{1}{2s+1}\left|\sum_{m''}(-1)^{s-m''}\right.\\
&\qquad\left.\times\,{}_{\hat n',\hat n'}\langle m'',-m''\,|m,m'\rangle_{\hat n,\hat n'}\right|^2\\
&= \frac{1}{2s+1}\,|{}_{\hat n',\hat n'}\langle -m',m'\,|m,m'\rangle_{\hat n,\hat n'}|^2\\
&= \frac{1}{2s+1}\,|{}_{\hat n'}\langle -m'\,|m\rangle_{\hat n}|^2. \qquad (10)
\end{aligned}
$$

The quantity on the last line of (10) is nothing but the modulus squared of a component of the rotation matrix $d_{m,-m'}(\alpha)$, and P can therefore be written in the form

$$
\begin{aligned}
P(m,m',\alpha) &= \frac{1}{2s+1}\,|\langle m\,|e^{i\alpha S_y}|-m'\rangle|^2\\
&= \frac{1}{2s+1}\,|\langle m\,|e^{i(\alpha-\pi)S_y}|m'\rangle|^2 \qquad (11)
\end{aligned}
$$

[where the y axis is perpendicular to the plane of the axes $\hat n$ and $\hat n'$, and the quantization axis z with respect to which both m and m' are defined in (11) is taken to lie along $\hat n$].

We now consider the case in which $\hat a$, $\hat b$, and $\hat c$ are three coplanar vectors, with $\hat a$ and $\hat b$ making the same angle $\frac{1}{2}\pi+\theta$ with $\hat c$, and the angle $\pi-2\theta$ with each other (Fig. 1). We use (9) and (11) to give the quantum-mechanical evaluation of the left side of the local realist's inequality (7), and (8) to evaluate the right side. The result is

$$
\begin{aligned}
f_s(\theta) &\equiv \frac{1}{2s+1}\sum_{m,m'}|m-m'|\;|\langle m\,|e^{-2i\theta S_y}|m'\rangle|^2\\
&\geq \tfrac{2}{3}(s+1)\sin\theta. \qquad (12)
\end{aligned}
$$

The validity of (12) for all angles θ is a necessary condition for the quantum-mechanical predictions for the spin-s Einstein-Podolsky-Rosen experiment to be consistent with the assumptions of local realism.

The function $f_s(\theta)$ appearing in (12) can be evaluated without excessive effort for small values of the spin. Letting $x=\sin\theta$, one can show that the inequality (12) assumes the forms

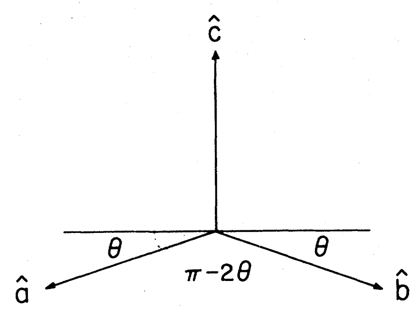

FIG. 1. The axes $\hat a$, $\hat b$, and $\hat c$. Local realism is inconsistent with the combined predictions of quantum mechanics for two Stern-Gerlach experiments. In one the axes of the detectors are a and c; in the other, a and b. The inconsistency occurs for the range of angles given in Eq. (5).

$$
\begin{aligned}
&f_{1/2}(\theta)=x^2\geq x \quad (\text{spin } \tfrac{1}{2}),\\
&f_1(\theta)=\frac{8x^2-4x^4}{3}\geq \tfrac{4}{3}x \quad (\text{spin } 1),\\
&f_{3/2}(\theta)=5x^2-6x^4+3x^6\geq \tfrac{5}{3}x \quad (\text{spin } \tfrac{3}{2}),\\
&f_2(\theta)=\frac{40x^2-84x^4+96x^6-40x^8}{5}\geq 2x \quad (\text{spin } 2).
\end{aligned} \qquad (13)
$$

The spectacular (and atypical) behavior of spin $\frac{1}{2}$ is evident: the inequality is violated for any angle θ whatsoever. More generally, the violation occurs for angles $0<\theta<\theta_s$, where

$$
\begin{aligned}
&\theta_{1/2}=\pi/2=90^\circ,\\
&\theta_1=0.6624=38.17^\circ,\\
&\theta_{3/2}=0.4203=24.08^\circ,\\
&\theta_2=0.3068=17.58^\circ.
\end{aligned} \qquad (14)
$$

A simple lower bound for θ_s can be derived by noting that the size of $f_s(\theta)$ can only increase if the integer $m-m'$ is replaced by its square in the expression (12). The inequality (12) will therefore certainly fail if

$$
\tfrac{2}{3}(s+1)\sin\theta>\frac{1}{2s+1}\sum_{mm'}(m-m')^2\,|\langle m\,|e^{-2i\theta S_y}|m'\rangle|^2. \qquad (15)
$$

The right side of (15) can be evaluated as follows:

$$
\begin{aligned}
\frac{1}{2s+1}\sum_{mm'}(m-m')^2|\langle m\,|e^{-2i\theta S_y}|m'\rangle|^2 &= -\frac{1}{2s+1}\sum_{mm'}\langle m\,|[S_z,e^{-2i\theta S_y}]\,|m'\rangle\langle m'\,|[S_z,e^{2i\theta S_y}]\,|m\rangle\\
&= \frac{2}{2s+1}\,\mathrm{tr}(S_z^{\,2}-e^{-2i\theta S_y}S_z e^{2i\theta S_y}S_z)\\
&= \frac{2}{2s+1}\,\mathrm{tr}[S_z^{\,2}-(S_z\cos2\theta+S_x\sin2\theta)S_z]\\
&= \frac{2(1-\cos2\theta)}{2s+1}\,\mathrm{tr}S_z^{\,2}=\frac{4\sin^2\theta}{3}\,s(s+1). \qquad (16)
\end{aligned}
$$

Hence, the inequality (12) will fail for[8]

$$0<\sin\theta<1/2s. \tag{17}$$

As the classical ($s\to\infty$) limit is approached, the contradiction between quantum mechanics and local realism occurs in an angular range of order $1/s$. We can easily verify that in the classical limit itself the contradiction has vanished. In this limit the quantities appearing in the original inequality (6) are simply the components along axes $\hat{a}$, $\hat{b}$, and $\hat{c}$ of the angular momentum vector of a classical gyroscope. This angular momentum has magnitude s and is randomly directed. The left side of (6) is just

$$|\vec{S}\cdot(\hat{a}+\hat{b})| = |S_z|\ |\hat{a}+\hat{b}| = 2|S_z|\sin\theta \tag{18}$$

(where the z axis is taken along the direction of $a+b$). The average of S_z over orientations of the gyroscope is $\frac{1}{2}s$, and the corresponding average of $m_1(\hat{a})m_1(\hat{c})$ or $m_1(\hat{b})m_1(\hat{c})$ is $s^2(\sin\theta)/3$. Consequently, in the classical limit the inequality reduces to the unexceptionable requirement that $s(\sin\theta)\geq(\frac{2}{3})s(\sin\theta)$.

From the perspective of the classical limit the failure of (12) to hold at small angles is not because the right side is too large but because the left side grows too slowly (quadratically in θ for small enough θ). This quadratic growth, which becomes linear in the classical limit, is due to the discreteness of the sum in (12)—i.e., to the quantization of the allowed values of any spin component. The closest a spin can come to being classical for fixed s is in a state of maximum alignment in which case the spread in the components of the spin perpendicular to the axis along which $m=s$ is of order $s^{1/2}$. The attempt to represent the spin by a classical vector is thus fuzzy over an angular spread of order $s^{-1/2}$. When the axes deviate from perfect perpendicularity by a still smaller angle of order s^{-1}, we are simply failing to distinguish the misaligned configuration from the perpendicular one by enough to allow the linear θ dependence characteristic of local realism (and exact classical behavior) to set in. Why the inequality should fail to hold for θ of order s^{-1} rather than $s^{-1/2}$ is not clear. It would be interesting to know whether this is an indication that a still stronger version of Bell's inequality remains to be found, or whether this is a genuine manifestation of some intrinsic aspect of the transition from quantum to classical behavior.

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APPENDIX A

We note here that a more straightforward attempt to generalize Bell's inequality to spin s is inconclusive. We use the form of the argument given by Friedberg.[9] A lower bound can be derived for the quantity $m_1(\hat{a})m_1(\hat{b})+m_1(\hat{b})m_1(\hat{c})+m_1(\hat{c})m_1(\hat{a})$ by noting that to make it negative requires at least one of the m_1's to be negative, but not all three. Since two negative m_1's lead to the same lower bound as one negative one (by reversing the sign of all three), it suffices to consider the case in which $m_1(\hat{c})$ is negative and the other two are positive. One then has

$$\begin{aligned} m_1(\hat{a})m_1(\hat{b})+m_1(\hat{b})m_1(\hat{c})+m_1(\hat{c})m_1(\hat{a}) &= -|m_1(\hat{c})|[m_1(\hat{a})+m_1(\hat{b})]+m_1(\hat{a})m_1(\hat{b}) \\ &\geq -s[m_1(\hat{a})+m_1(\hat{b})]+m_1(\hat{a})m_1(\hat{b}) \\ &= [s-m_1(\hat{a})][s-m_1(\hat{b})]-s^2 \\ &\geq -s^2. \end{aligned} \tag{A1}$$

Taking averages, using the local realist's identity (4), and evaluating the resulting mean values with the result (8) of quantum mechanics, we find the inequality

$$\begin{aligned} s &\geq -\tfrac{1}{3}(s+1)(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}) \\ &= \tfrac{1}{3}(s+1)[\tfrac{3}{2}-\tfrac{1}{2}(\hat{a}+\hat{b}+\hat{c})^2]. \end{aligned} \tag{A2}$$

The right side of (A2) is largest when $\hat{a}+\hat{b}+\hat{c}=0$. In this case (A2) reduces to the simple condition $s\geq 1$. Thus spin $\frac{1}{2}$ fails the test of local realism, spin 1 just passes, and the higher spins get by comfortably.

The argument in the text uses a local realist's inequality which is, if anything, simpler than (A1). However, another quantum-mechanical mean value [Eq. (9)] enters into the evaluation of the inequality, which is rather more intricate than the simple bilinear expression (8), which is all that is needed to arrive at (A2).

APPENDIX B

We show below that if θ is taken to be of the form $\theta=a/s$, for fixed a, then the limit of the inequality (12) as $s\to\infty$ is

$$\lim_{s\to\infty} f_s(a/s) = \int_{-\infty}^{\infty}\frac{dx}{2\pi x^2}\left[1-\frac{\sin(4a\sin x)}{4a\sin x}\right] \geq \tfrac{2}{3}a. \tag{B1}$$

This inequality fails to hold when a is less than 0.5659..., which is remarkably close to the crude estimate for the crossover, $a<0.5$, given by the large-s limit of (17).

To establish (B1) note first that a series of steps analogous to those taken in (16) establishes that if $|m-m'|$ is replaced by $(m-m')$ in the form (12) for $f_s(\theta)$, then the resulting expression vanishes. Consequently, one can rewrite $f_s(\theta)$ in the form

$$f_s(\theta)=\frac{2}{2s+1}\sum_{m,m'}(m-m') \times \eta(m-m')\,|\langle m|e^{-2i\theta S_y}|m'\rangle|^2, \tag{B2}$$

where the η function vanishes for negative values of its argument and is unity for positive values. We use the integral representation

$$\eta(x)=\int\frac{d\zeta}{2\pi i}\,e^{i\zeta x}/\zeta\,, \tag{B3}$$

where the contour is along the real axis except for an infinitesimal dip below the pole at $\zeta=0$. Substituting this into (B2), we find

$$\begin{aligned} f_s(\theta)&=\frac{2}{2s+1}\int\frac{d\zeta}{2\pi i\zeta} \times\sum_{m,m'}\frac{1}{i}\,\frac{\partial}{\partial\zeta}\,e^{i\zeta(m-m')}\,|\langle m|e^{-2i\theta S_y}|m'\rangle|^2\\ &=-\frac{2}{2s+1}\int\frac{d\zeta}{2\pi\zeta}\,\frac{\partial}{\partial\zeta}\,\mathrm{tr}[e^{i\zeta S_z}e^{-2i\theta S_y}e^{-i\zeta S_z}e^{2i\theta S_y}]\\ &=-\frac{2}{2s+1}\int\frac{d\zeta}{2\pi\zeta}\,\frac{\partial}{\partial\zeta}\,\mathrm{tr}[e^{-2i\theta(S_y\cos\zeta+S_x\sin\zeta)}e^{2i\theta S_y}]\,. \end{aligned} \tag{B4}$$

Now

$$e^{-2i\theta(S_y\cos\zeta+S_x\sin\zeta)}e^{2i\theta S_y}=e^{i\beta\hat{n}\cdot\vec{S}}\,, \tag{B5}$$

where the direction of the axis $\hat{n}$ is immaterial, and the angle β is that of a single roation equivalent to a rotation through 2θ about $\hat{y}$, followed by a rotation through -2θ about $\hat{y}\cos\zeta+\hat{x}\sin\zeta$:

$$\cos\tfrac{1}{2}\beta=\cos^2\theta+\cos\zeta\,\sin^2\theta\,. \tag{B6}$$

Using the form (B5) we can immediately evaluate the trace, to find

$$f_s(\theta)=-\frac{2}{2s+1}\int\frac{d\zeta}{2\pi\zeta}\,\frac{\partial}{\partial\zeta}\left[\frac{\sin(s+\frac{1}{2})\beta}{\sin\frac{1}{2}\beta}\right]. \tag{B7}$$

When $\theta=a/s$, we find from (B6) and (B7) that

$$\lim_{s\to\infty}f_s(a/s)=-\frac{1}{2a}\int\frac{d\zeta}{2\pi\zeta}\,\frac{\partial}{\partial\zeta}\left[\frac{\sin(4a\sin\frac{1}{2}\zeta)}{\sin\frac{1}{2}\zeta}\right]. \tag{B8}$$

An integration by parts then yields the right side of (B1) (after a change of variables to $x=\zeta/2$). (There is no longer a pole at $x=0$ so the integration can be taken entirely along the real axis.)

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[1]A. Einstein *et al.*, Phys. Rev. 47, 777 (1935); D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, New Jersey, 1951), pp. 614–619.

[2]An elementary proof that (2) gives a state of zero total spin follows from demonstrating that it is an eigenstate of any component of the total spin operator $\vec{S}$ with zero eigenvalue. With the z axis taken to be along $\hat{n}$ this is immediately evident for $S_z=S_z^{(1)}+S_z^{(2)}$. It follows for the two other linearly independent components $S_\pm=S_\pm^{(1)}+S_\pm^{(2)}$ directly from the elementary properties of the spin raising and lowering operators.

[3]J. S. Bell, Physics (N. Y.) 1, 195 (1964).

[4]Since the appearance of Bell's 1964 paper (Ref. 3), his argument has been extended to cases in which the observed correlations between measurements of the two spins along parallel axes are not perfect. A review of such developments has been given by J. F. Clauser and A. Shimony, Rep. Prog. Phys. 41, 1991 (1978). These extensions employ a rather weaker notion of local realism than I am using here. In such contexts my version of local realism is sometimes called "determinism," even though the $m_i(\hat{n})$ are stochastic variables whose statistics can be unknown or even unknowable. In this paper I do not attempt such extensions for my generalization of Bell's argument to the spin-s case. My point of view is that of the 1964 paper: the Einstein-Podolsky-Rosen experiment is regarded as a *Gedankenexperiment* and local realism is a theoretical construct to account for the remarkable perfect correlations predicted by the quantum theory. The point is to show that the quantum *theory*, which suggests local realism in the strong sense of the term, is nevertheless *numerically* inconsistent with the theoretical premises of such local realism. I believe that in this paper I am using the term local realism in a sense similar to that of B. d'Espagnat, Sci. Am. 241 (5), 158 (1979).

[5]A. Baracca *et al.*, Int. J. Theor. Phys. 15, 473 (1976). Another straightforward attempt at a generalization that fails to work for $s\geqslant 1$ is described below in Appendix A.

[6]I demonstrate the result in the text for a range of angles (5) given by $A=0.5$. A considerably more elaborate argument (Appendix B) shows that the asymptotically

exact range for which the inequality (12) fails is given by $A=0.5659\ldots$.

[7]Because the state $|\phi\rangle$ is isotropic in spin space, the real tensor $\langle\phi|\frac{1}{2}(S_\mu^1 S_\nu^2 + S_\nu^2 S_\mu^1)|\phi\rangle$ must be proportional to the unit tensor $\delta_{\mu\nu}$. Taking the trace of both quantities shows that the proportionality constant must be

$$\frac{1}{6}\langle\phi|\vec{S}^1\cdot\vec{S}^2+\vec{S}^2\cdot\vec{S}^1|\phi\rangle=\frac{1}{6}\langle\phi|(\vec{S}^1+\vec{S}^2)^2-(\vec{S}^1)^2-(\vec{S}^2)^2|\phi\rangle .$$

Since $|\phi\rangle$ is a state of zero total spin, the expectation value of $(\vec{S}^1+\vec{S}^2)^2$ vanishes, and since the particles have spin s, $(\vec{S}^i)^2=s(s+1)$, $i=1, 2$. Therefore,

$$\langle\phi|\tfrac{1}{2}(\vec{S}_\mu^1 S_\nu^2+S_\nu^2 S_\mu^1)|\phi\rangle=-\tfrac{1}{3}s(s+1)\delta_{\mu\nu},$$

from which Eq. (8) follows.

[8]This estimate is quite close to the asymptotically, exact result derived in Appendix B.

[9]R. M. Friedberg, as described in M. Jammer, *The Philosophy of Quantum Mechanics* (Wiley, New York, 1974), pp. 244–247.