

QED vacuum polarization in a background gravitational field and its effect on the velocity of photons

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We calculate in QED the contribution to the photon effective action from one-loop vacuum polarization on a general curved background manifold, and use it to investigate the corrections to the local propagation of photons. We find that the quantum corrections introduce tidal gravitational forces on the photons which in general alter the characteristics of propagation, so that in some cases photons travel at speeds greater than unity. The effect is nondispersive and gauge invariant. We look at a few examples, including a background Schwarzschild geometry, and we argue that although these results are controversial they do not in fact exhibit any obvious inconsistency.

I. INTRODUCTION

In QED the one-electron loop gives rise to the Euler-Heisenberg effective Lagrangian¹ for the electromagnetic field. For low frequencies this is

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & -\frac{\alpha^2}{45m^4}\left[\frac{5}{4}(F_{\mu\nu}F^{\mu\nu})^2 - \frac{7}{2}F_{\mu\nu}F_{\sigma\tau}F^{\mu\sigma}F^{\nu\tau}\right], \end{aligned} \quad (1.1)$$

where m is the electron mass. As shown by Adler,² the resulting equations of motion imply that electromagnetic waves passing through a region of intense magnetic field \vec{B} will exhibit birefringence. The velocities at which they travel are

$$\begin{aligned} v_{\parallel} &= 1 - \frac{8\alpha^2\vec{B}^2 \sin^2\theta}{45m^4}, \\ v_{\perp} &= 1 - \frac{14\alpha^2\vec{B}^2 \sin^2\theta}{45m^4} \end{aligned} \quad (1.2)$$

for polarizations respectively coplanar with and perpendicular to the plane defined by \vec{B} and the direction of propagation. Here θ is the angle between \vec{B} and the direction of propagation.

In this paper we study the analogous problem of how photon propagation in a background gravitational field is influenced by vacuum polarization. We again work in the one-loop approximation. If vacuum polarization is ignored the properties of photon propagation can be inferred from the equivalence principle, namely, that the photon travels at the speed of light in a manner independent of its polarization state, which itself remains unchanged. Vacuum polarization is an effect in which the photon exists for part of the time as a virtual e^+e^- pair. This virtual transition confers a size on the photon which is $O(\lambda_c)$, where λ_c is

the Compton wavelength of the electron. Having by this means acquired a size, the photon can be influenced in its motion by the curvature of the gravitational field. The equivalence principle is not applicable to such tidal effects.

We find, for example, that photon propagation can be polarization dependent (gravitational birefringence) and also that in certain circumstances the speed of propagation is "faster than light." This is a surprising result and we discuss carefully (1) the extent to which it implies an observable alteration of the causal structure of events in space-time and (2) whether or not such an alteration is in conflict with fundamental principles.

In Secs. II and III we set up the machinery and derive the equation for photon propagation in a gravitational field. We present two independent derivations, each of which serves to highlight a different aspect of the argument. In Sec. IV-VI we examine the consequences of our equation in different gravitational backgrounds. We present a general discussion and conclusions in Sec. VII.

II. THE EQUATION OF MOTION FOR THE PHOTON FIELD

The equation of motion for the electromagnetic field is

$$\frac{\delta W}{\delta A_{\mu}(x)} = 0, \quad (2.1)$$

where the effective action W is given by

$$W = W_0 + W_1 \quad (2.2)$$

with

$$W_0 = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu}F^{\mu\nu}, \quad (2.3)$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}.$$

The second contribution to Eq. (2.2), W_1 , incorporates the effects of virtual electron loops. It is given by

$$W_1 = \sum_{n, \text{even}} \frac{1}{n!} \int \prod_{i=1}^n d^4 x_i A_{\mu_i}(x_i) G^{\mu_1 \dots \mu_n}(x_1 \dots x_n), \quad (2.4)$$

where $G^{\mu_1 \dots \mu_n}$ is the sum over one-particle-irreducible Feynman diagrams. Since we are only concerned with the propagation of individual photons, we need only consider the contribution which is quadratic in $A_{\mu}(x)$. Gauge invariance implies that W_1 depends on $F_{\mu\nu}$ rather than A_{μ} directly.

As indicated in the Introduction, the effect of virtual electron loops is to give the photon a "size" proportional to λ_c . In order to calculate these effects we will expand W_1 in powers of $\lambda_c^2 = m^{-2}$ and retain only the lowest term.

Since λ_c is the only length scale in the theory, the coefficients in the expansion will be local functionals of A_{μ} and $g_{\mu\nu}$ with a definite total number of derivatives acting on them. More strictly, we should argue that they will be algebraic combinations of such functionals. However, this more complicated possibility would imply a disconnectedness structure which is not appropriate to the one-particle-irreducible amplitudes $G^{\mu_1 \dots \mu_n}$ we are considering when we discuss W_1 . In Sec. III the locality of the functional is verified by direct calculation.

The lowest term in the expansion for W_1 is $O(m^{-2})$ and the appropriate number of derivatives is four. There are only four independent gauge-invariant and coordinate-invariant terms satisfying these criteria.³ They may be chosen to be

$$W_1 = \frac{1}{m^2} \int d^4 x (-g)^{1/2} (a R F_{\mu\nu} F^{\mu\nu} + b R_{\mu\nu} F^{\mu\sigma} F^{\sigma\nu} + c R_{\mu\nu\sigma\tau} F^{\mu\nu} F^{\sigma\tau} + d D_{\mu} F^{\mu\nu} D_{\sigma} F^{\sigma\nu}), \quad (2.5)$$

[Our sign conventions are $g^{\mu\nu} = (+---)$ for the metric, and $R^{\mu}_{\nu\sigma\tau} = \Gamma^{\mu}_{\nu\sigma}{}_{,\tau} - \Gamma^{\mu}_{\nu\sigma,\tau} + \Gamma^{\mu}_{\lambda\sigma} \Gamma^{\lambda}_{\nu\tau} - \Gamma^{\mu}_{\lambda\tau} \Gamma^{\lambda}_{\nu\sigma}$ for the curvature tensor.] The first three terms reveal the influence of the curvature functional. The fourth survives even in flat space and represents off-mass-shell effects in the vacuum polarization. We evaluate a, b, c , and d to $O(e^2)$. In this section we calculate these numbers by comparing W_1 with results obtained in the weak-gravitational-field limit, by means of conventional Feynman diagram techniques. In Sec. III we evaluate directly the asymptotic expansion for the one-loop effective action for electrons in the presence of external gravitational and electro-



FIG. 1. Flat-space vacuum polarization amplitude.

magnetic fields.

The last coefficient d may be obtained from the flat-space vacuum polarization amplitude associated with the diagram in Fig. 1. It yields the renormalized amplitude

$$\Pi^{\mu\nu} = -(q^2 \eta^{\mu\nu} - q^{\mu} q^{\nu}) \left(1 - \frac{e^2}{60\pi^2} \frac{q^2}{m_e^2} + \dots \right). \quad (2.6)$$

The effective action W will yield the same result to $O(1/m^2)$ provided

$$d = -\frac{e^2}{120\pi^2}. \quad (2.7)$$

In fact, the precise value of d will not be important to our argument. The fact that it is $O(e^2)$ is, however, important.

The coefficients a, b , and c may be obtained from the coupling of a graviton to two on-mass-shell photons in the flat-space limit; that is, from the matrix element

$$\langle \gamma(q_2, \beta) | \theta^{\mu\nu}(0) | \gamma(q_1, \alpha) \rangle,$$

where $\theta^{\mu\nu}(x)$ is the energy-momentum tensor. This matrix element has been calculated by Berends and Gastmans⁴ from the diagrams in Fig. 2. We can express their result to $O(m^{-2})$ in the form

$$\langle \gamma(q_2, \beta) | \theta^{\mu\nu}(0) | \gamma(q_1, \alpha) \rangle = V_0^{\mu\nu\alpha\beta}(q_1, q_2) (1 + p^2 g_1) - P^{\alpha\beta}(q_1, q_2) (g_2 A^{\mu\nu} + g_3 B^{\mu\nu}), \quad (2.8)$$

where $V_0^{\mu\nu\alpha\beta}$ is the standard matrix element for the free electromagnetic field given by

$$V_0^{\mu\nu\alpha\beta} = \eta^{\mu\nu} (q_1 \cdot q_2 \eta^{\alpha\beta} - q_1^{\beta} q_2^{\alpha}) - q_1 \cdot q_2 (\eta^{\alpha\mu} \eta^{\beta\nu} + \eta^{\alpha\nu} \eta^{\beta\mu}) - \eta^{\alpha\beta} (q_1^{\mu} q_2^{\nu} + q_1^{\nu} q_2^{\mu}) + q_1^{\mu} q_2^{\alpha} \eta^{\nu\beta} + q_1^{\nu} q_2^{\alpha} \eta^{\mu\beta} + q_2^{\mu} q_1^{\beta} \eta^{\nu\alpha} + q_2^{\nu} q_1^{\beta} \eta^{\mu\alpha}$$

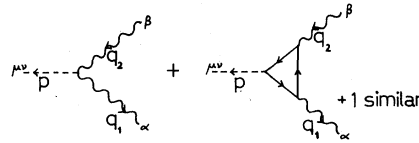


FIG. 2. Matrix elements of the energy-momentum tensor. Renormalization is effected by multiplication by $Z_3^{1/2}$ for each external photon where Z_3 is the usual flat-space photon renormalization factor.

and

$$\begin{aligned} A^{\mu\nu} &= q_1 \cdot q_2 \eta^{\mu\nu} + q_1^\mu q_1^\nu + q_2^\mu q_2^\nu, \\ B^{\mu\nu} &= q_1 \cdot q_2 \eta^{\mu\nu} - q_1^\mu q_2^\nu - q_1^\nu q_2^\mu, \\ P^{\alpha\beta} &= q_1 \cdot q_2 \eta^{\alpha\beta} - q_2^\alpha q_1^\beta. \end{aligned} \quad (2.9)$$

The form factors $\{g_i\}$ are

$$\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \frac{\alpha}{360\pi} \frac{1}{m^2} \begin{pmatrix} 11 \\ -6 \\ -2 \end{pmatrix} \left[1 + O\left(\frac{p^2}{m^2}\right) \right], \quad (2.10)$$

where α is the fine-structure constant. Using the equation

$$\begin{aligned} \langle \gamma(q_2, \beta) | \theta^{\mu\nu}(0) | \gamma(q_1, \alpha) \rangle (2\pi)^4 \delta(q_1 - q_2 - p) &= \int d^4x d^4y d^4z e^{i(q_1 \cdot x - q_2 \cdot y - p \cdot z)} \\ &\times \left(-\frac{2}{(-g)^{1/2}(z)} \frac{\delta^3 W}{\delta A_\alpha(x) \delta A_\beta(y) \delta g_{\mu\nu}(z)} \right)_{g_{\mu\nu} = \eta_{\mu\nu}}, \end{aligned} \quad (2.11)$$

we obtain the same results from W_1 provided

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\frac{\alpha}{720\pi} \begin{pmatrix} 5 \\ -26 \\ 2 \end{pmatrix}. \quad (2.12)$$

It is clear from this discussion that the first three terms on the right-hand side of Eq. (2.5) are related to the three gravitational form factors of the photon.

We are now in a position to obtain the gravitationally modified equation of motion for the electromagnetic field. From Eqs. (2.1) and (2.2) we see that it has the form

$$D_\mu F^{\mu\nu} + \frac{\delta W_1}{\delta A_\nu(x)} = 0. \quad (2.13)$$

This shows that $D_\mu F^{\mu\nu}$ is $O(e^2)$ and that, therefore, we can omit the term with coefficient d in Eq. (2.5) for W_1 since it will only influence the motion to $O(e^4)$. From Eq. (2.13) we then obtain the result

$$\begin{aligned} D_\mu F^{\mu\nu} + \frac{1}{m_e^2} D_\mu [4aR F^{\mu\nu} + 2b(R^\mu{}_\sigma F^{\sigma\nu} - R^\nu{}_\sigma F^{\sigma\mu}) \\ + 4cR^{\mu\nu}{}_{\sigma\tau} F^{\sigma\tau}] = 0, \end{aligned} \quad (2.14)$$

where a , b , and c are given by Eq. (2.12). The argument above for the omission of terms containing $D_\mu F^{\mu\nu}$ is, of course, formal. It must be supplemented by a bound on the derivatives of $F^{\mu\nu}$. A sufficient condition is that the signals comprise wavelengths $\lambda > \lambda_c$. We shall return to this point later.

We may immediately observe that, in general, the curvature is not isotropic, and so Eq. (2.12) can be expected to modify photon propagation differently in different directions. The simplest example to consider is therefore the de Sitter gravitational background for which the curvature is isotropic, i. e.,

$$R_{\mu\nu\sigma\tau} = K(g_{\mu\sigma} g_{\nu\tau} - g_{\mu\tau} g_{\nu\sigma}), \quad (2.15)$$

where K is a constant. We find from Eqs. (2.12) and (2.14) that

$$\left(1 + \frac{7\alpha K}{90\pi m^2} \right) D_\mu F^{\mu\nu} = 0, \quad (2.16)$$

and so Maxwell's equation is only altered by a trivial change of normalization. Vacuum polarization therefore does not affect the propagation of individual photons in a de Sitter background.

More interesting results are obtained from certain fields which satisfy the Einstein vacuum equation,

$$R_{\mu\nu} = 0. \quad (2.17)$$

We should emphasize that the photon is treated as a test particle, and its effect on the metric is assumed negligible. In this case we find from Eqs. (2.12) and (2.14) the result

$$D_\mu F^{\mu\nu} + \xi^2 R^{\mu\nu}{}_{\sigma\tau} D_\mu F^{\sigma\tau} = 0, \quad (2.18)$$

where

$$\xi^2 = \alpha/90\pi m_e^2. \quad (2.19)$$

We have taken advantage of Eq. (2.17) and the Bianchi identities to move the covariant derivative through the curvature tensor. For convenience we note at this point the remaining Maxwell equation

$$D_\rho F_{\mu\nu} + D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} = 0. \quad (2.20)$$

Before going on to discuss the implications of these equations for photon propagation, we consider a second derivation of the one-loop effective action W_1 .

III. ALTERNATIVE DERIVATION OF THE EFFECTIVE ACTION

The derivation of the effective action presented in Sec. II emphasized its relationship to the photon's gravitational form factors. In this section we follow a procedure developed by Schwinger⁵ and DeWitt⁶ and evaluate the one-loop effective

action in terms of the electron propagator in external gravitational and electromagnetic fields. The calculation which we give in outline below reproduces results consistent with a geometrical calculation by Gilkey.⁷ We will work in n dimensions in order to regularize the integrals which appear in the calculation.

Following standard arguments,⁸ we can write

$$\frac{\partial W_1}{\partial m} = \int d^n x (-g)^{1/2} \text{Tr} \lim_{x' \rightarrow x} S(x, x'), \quad (3.1)$$

where $S(x, x')$ is the Dirac propagator which obeys

$$(i\mathcal{D} - m)S(x, x') = i\delta(x - x')/(-g)^{1/2}(x). \quad (3.2)$$

Here \mathcal{D} is the covariant derivative including both the gravitational and electromagnetic fields:

$$\mathcal{D} = \gamma^a e_a^\mu(x) (\partial_\mu + \frac{1}{2} \omega_{\mu bc} \sigma^{bc} + ieA_\mu), \quad (3.3)$$

where

$$\sigma^{bc} = \frac{1}{4}[\gamma^b, \gamma^c], \quad (3.4)$$

$$\omega_{\mu bc} = e_{b\nu} \partial_\mu e_\nu^c + \Gamma_{\lambda\mu}^\nu e_{b\nu} e_\lambda^c,$$

and $e_a^\mu(x)$ is an appropriate vierbein field.

Following DeWitt,⁶ we set

$$S(x, x') = (i\mathcal{D} + m)G(x, x'), \quad (3.5)$$

where the bispinor $G(x, x')$ satisfies

$$(D^2 + ie\sigma^{\mu\nu} F_{\mu\nu} - \frac{1}{4}R + m^2)G(x, x') = -i\delta(x - x')/(-g)^{1/2}. \quad (3.6)$$

We can express G in the proper-time representation⁹

$$G(x, x') = \frac{i}{(4\pi i)^{n/2}} \int_0^\infty dt t^{-n/2} F(x, x', t) \times \exp\left(-\frac{i\sigma}{2t} - im^2 t\right). \quad (3.7)$$

Here $\sigma = \sigma(x, x')$ is the standard world function which satisfies¹⁰

$$\sigma_{,\mu} \sigma^{,\mu} = 2\sigma, \quad (3.8)$$

$$\sigma(x, x) = 0.$$

The bispinor $F(x, x', t)$ is a regular function of t at $t=0$ with the property

$$F(x, x, 0) = 1. \quad (3.9)$$

If we expand

$$F(x, x', t) = \sum_{r=0}^\infty (it)^r f_r(x, x'), \quad (3.10)$$

then Eq. (3.7) implies

$$\frac{1}{2}(\sigma_{,\lambda}{}^\lambda - n)f_r + r f_r + \sigma_{,\lambda} f_{r,\lambda} + f_{r-1,\lambda}{}^\lambda + (ie\sigma^{\mu\nu} F_{\mu\nu} - \frac{1}{4}R)f_{r-1} = 0, \quad (3.11)$$

with $f_{-1} = 0$. Taking into account the boundary condition Eq. (3.9), Eq. (3.11) has for the case $r=0$ the solution

$$f_0(x, x') = \Delta^{1/2}(x, x')I(x, x'), \quad (3.12)$$

where

$$\Delta(x, x') = \frac{-\det(-\sigma_{,\mu}{}^{\mu'}(x, x'))}{[g(x)g(x')]^{1/2}}$$

and $I(x, x')$ is the parallel-displacement matrix for spinors. Note that Δ satisfies

$$\frac{1}{2}(\sigma_{,\lambda}{}^\lambda - n)\Delta^{1/2} - \sigma_{,\lambda}(\Delta^{1/2})^{,\lambda} = 0, \quad (3.13)$$

$$\Delta(x, x) = 1.$$

In order to calculate W_1 from Eq. (3.1), we require the coincidence limits $f_r(x, x)$. These may be calculated in the standard way by repeated differentiation of Eqs. (3.8), (3.11), and (3.13). Some relevant details are given in Table I and in the Appendix. The particular results we need are

$$\text{Tr} f_2(x, x) \cong -\frac{2}{3} 2^{n/2} e^2 \left(-\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}\right), \quad (3.14a)$$

$$\text{Tr} f_3(x, x) \cong \frac{e^2}{360} 2^{n/2} (5R F_{\alpha\beta} F^{\alpha\beta} - 26R^{\alpha\beta} F_{\alpha\gamma} F_\beta{}^\gamma + 2R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta} + 24F^{\alpha\beta}{}_{,\beta} F_{\alpha\gamma}{}^{,\gamma}). \quad (3.14b)$$

Here $2^{n/2}$ is the dimension of the Dirac γ matrices.¹¹ We have included on the right-hand side of Eqs. (3.14a) and (3.14b) only those terms which are at most linear in the curvature. These results are consistent with those of Gilkey,⁷ who considered the more general case of a non-Abelian gauge field and retained higher orders in the curvature.

From Eqs. (3.1), (3.5), and (3.7) we conclude that

$$W_1 = \int dm \int d^n x (-g)^{1/2} \text{Tr} \lim_{x' \rightarrow x} [(i\mathcal{D} + m)G(x, x')] + \text{const.} \quad (3.15)$$

It is not hard to verify that the coincidence limits of $f_r(x, x')$ and its derivatives involve combinations of even numbers of γ matrices. The trace in the integrand of Eq. (3.15) then eliminates the contribution from \mathcal{D} . On performing the m integration and using the integral representation for G and the expression for F , we find that

$$W_1 = \sum_{r=0}^\infty \left(\frac{1}{(4\pi)^{n/2}} \frac{\Gamma(r+1-n/2)}{(n-2r)} m^{n-2r} \times \int d^n x (-g)^{1/2} \text{Tr} f_r(x, x) \right). \quad (3.16)$$

Since we are concerned with photon propagation,

TABLE I. Some coincidence limits.

Bitensor (x, x')	Coincidence limit $x' \rightarrow x$
σ	0
σ, μ	0
$\sigma, \mu\nu$	$g_{\mu\nu}$
$\sigma, \lambda\mu\nu$	0
$\sigma, \kappa\lambda\mu\nu$	$\frac{1}{3}(R_{\kappa\mu\nu\lambda} + R_{\kappa\nu\mu\lambda})$
$\Delta^{1/2}$	1
$\Delta^{1/2}, \mu$	0
$\Delta^{1/2}, \mu\nu$	$\frac{1}{6}R_{\mu\nu}$
I	1
I, μ	0
$I, \mu\nu$	$\frac{1}{2}W_{\mu\nu}$
$I, \lambda\mu\nu$	$\frac{1}{3}(W_{\lambda\mu, \nu} + W_{\lambda\nu, \mu})$
$I, \kappa\lambda\mu\nu$	$\frac{1}{4}(W_{\kappa\lambda}W_{\mu\nu} + W_{\kappa\mu}W_{\lambda\nu} + W_{\kappa\nu}W_{\lambda\mu}$ $+ W_{\kappa\lambda, \mu\nu} + W_{\kappa\mu, \lambda\nu} + W_{\kappa\nu, \lambda\mu})$ $-\frac{1}{12}[(R_{\lambda\mu\nu}^{\alpha} + R_{\nu\mu\lambda}^{\alpha})W_{\alpha\kappa} + (R_{\kappa\nu\mu}^{\alpha} + R_{\mu\nu\kappa}^{\alpha})W_{\alpha\lambda}$ $+ (R_{\kappa\nu\lambda}^{\alpha} + R_{\lambda\nu\kappa}^{\alpha})W_{\alpha\mu} + (R_{\kappa\mu\lambda}^{\alpha} + R_{\lambda\mu\kappa}^{\alpha})W_{\alpha\nu}]$ $-R^{\alpha\beta\gamma\delta}W_{\alpha\beta}W_{\gamma\delta} + 3R^{\alpha\beta}W_{\alpha\gamma}W_{\beta\gamma} + \frac{2}{3}W^{\alpha\beta},_{\beta}W_{\alpha\gamma},^{\gamma}$ $+ 4W_{\alpha\beta}W^{\alpha\gamma},^{\beta} + \frac{4}{3}W^{\alpha\beta},^{\gamma}W_{\alpha\beta},^{\gamma} - \frac{2}{3}R^{\alpha\beta}W_{\alpha\gamma},^{\gamma}_{\beta}$
$I, \kappa^{\lambda} \lambda^{\mu}$	

we shall ignore the contributions to W_1 from f_0, f_1 , and the unlisted curvature terms in Eqs. (3.14a) and (3.14b), which only influence the vacuum value of W_1 . It is easily verified that the contribution from Eq. (3.14a) is removed by the standard photon wave-function renormalization:

$$\delta Z_3 \cong \frac{8}{3} \frac{1}{(n-4)} \frac{e^2}{(4\pi)^2}. \quad (3.17)$$

The third term is finite at $n=4$ and yields

$$W_1 \approx \frac{1}{(4\pi)^2} \frac{1}{m^2} \int d^4x (-g)^{1/2} [-\frac{1}{2} \text{Tr} f_3(x, x)]. \quad (3.18)$$

By referring to Eq. (3.14b) we see that this is identical to the result of Sec. II.

In ignoring the vacuum contribution to W_1 we do not mean to imply that it does not represent a problem for quantum field theory in a gravitational background. On the contrary, it is very much bound up with the problems of calculating the back reaction of the quantum field theory on the metric. However, if it is a valid approximation (as we presume) to take the space-time manifold as given, then the problems of the vacuum functional can be separated from those of calculating the renormalized Green's functions for quantum

fields propagating in that space-time.

A final point concerns the use of the asymptotic expansion for $G(x, x')$ implied by Eqs. (3.7) and (3.10). Of course, this refers only to the singular part of $G(x, x')$. It adequately represents all local effects due to the curvature. However, it is insensitive to the effects of boundary conditions on the electron field. We shall make some brief comments on this point in Sec. VII.

IV. GRAVITATIONAL-WAVE BACKGROUND

In order to derive the equation for the characteristics of photon propagation we make the simplest geometrical-optics plane-wave approximation.¹² We should emphasize that although this is an approximation for the actual solution of the field equation for the photon, it exactly determines the characteristics of propagation and hence the causal structure of the solution. We also carry through the derivation in a gauge-invariant manner to demonstrate that the results do not depend in any way upon a choice of gauge, although the same results can in fact be obtained with a little less effort when working in the Lorentz gauge.

We set

$$F_{\mu\nu} = f_{\mu\nu} e^{i\theta} \quad (4.1)$$

and regard $f_{\mu\nu}$ as slowly varying in comparison with θ . If we now put $k_\mu = D_\mu \theta$ and ignore all other derivatives, we obtain from Eq. (2.17) (we are interested in the Ricci-flat case)

$$k_\mu f^{\mu\nu} + \xi^2 R^{\mu\nu}{}_{\sigma\tau} k_\mu f^{\sigma\tau} = 0, \quad (4.2)$$

and from Eq. (2.20)

$$k_\rho f_{\mu\nu} + k_\mu f_{\nu\rho} + k_\nu f_{\rho\mu} = 0. \quad (4.3)$$

Combining these two equations we find

$$k^2 f^{\rho\nu} - 2\xi^2 k^\rho R^{\nu\mu}{}_{\sigma\tau} k_\mu f^{\sigma\tau} = 0. \quad (4.4)$$

This is a homogeneous equation for $f^{\sigma\tau}$, and in order that it have nontrivial solutions k must obey a determinantal condition. From Eq. (4.3) it follows that

$$f_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu \quad (4.5)$$

for some vector a_ν . Hence, for given k^μ , $f_{\mu\nu}$ has three independent components (two polarizations and one magnitude).

It is illuminating to see how the above analysis works in specific cases. Consider a background which is a plane gravitational wave traveling in the 3 direction. In the transverse traceless gauge the metric is

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (4.6)$$

where $h_{\mu\nu}$ vanishes except for the components h_{ij} ($i, j = 1, 2$) and the matrix $\{h_{ij}\}$ is symmetric and traceless.¹² It is sufficient for the purposes of illustration to consider the case

$$h_{11} = -h_{22} = b \equiv B \cos \omega(x^0 - x^3), \quad (4.7)$$

$$h_{12} = h_{21} = 0.$$

The components of the curvature tensor are zero apart from those given by

$$R_{aibj} = -\frac{1}{2} \partial_a \partial_b h_{ij} = (-1)^{a+b} \frac{1}{2} \omega^2 h_{ij}, \quad (4.8)$$

where $a, b = 0, 3$ and $i, j = 1, 2$. On introducing the orthonormal tetrad $\{e_0, e_1, e_2, e_3\}$ and the bivectors

$$\Omega_i^{\mu\nu} = e_+^\mu e_i^\nu - e_i^\mu e_+^\nu, \quad i = 1, 2, 3 \quad (4.9)$$

where $e_+^\mu = \frac{1}{2}(e_0^\mu + e_3^\mu)$, we can write

$$R^{\mu\nu\sigma\tau} = 2\omega^2 b (\Omega_1^{\mu\nu} \Omega_1^{\sigma\tau} - \Omega_2^{\mu\nu} \Omega_2^{\sigma\tau}) \quad (4.10)$$

to within the weak-field approximation.

Now set

$$p_i^\nu = k_\mu \Omega_i^{\mu\nu} = k_+ e_i^\nu - k_i e_+^\nu, \quad (4.11)$$

$$\Lambda_i^{\mu\nu} = k^\mu p_i^\nu - k^\nu p_i^\mu.$$

Equation (4.4) becomes

$$k^2 f^{\rho\nu} + 2\xi^2 \omega^2 b (\Lambda_1^{\rho\nu} \Omega_1^{\sigma\tau} - \Lambda_2^{\rho\nu} \Omega_2^{\sigma\tau}) f_{\sigma\tau} = 0. \quad (4.12)$$

We can express Eq. (4.12) in terms of three independent components of $f_{\mu\nu}$. We write

$$f_i = \Omega_i^{\sigma\tau} f_{\sigma\tau}, \quad i = 1, 2, 3 \quad (4.13)$$

and note that

$$\Omega_i^{\sigma\tau} \Lambda_{j\sigma\tau} = 2p_i \cdot p_j. \quad (4.14)$$

Equation (4.12) then becomes

$$(k^2 + 4\xi^2 \omega^2 b p_1^2) f_1 = 0,$$

$$(k^2 - 4\xi^2 \omega^2 b p_2^2) f_2 = 0, \quad (4.15)$$

$$4\xi^2 \omega^2 b (p_1 \cdot p_3 f_1 - p_2 \cdot p_3 f_2) + k^2 f_3 = 0.$$

The determinantal condition on k is therefore

$$k^2 (k^2 + 4\xi^2 \omega^2 b p_1^2) (k^2 - 4\xi^2 \omega^2 b p_2^2) = 0. \quad (4.16)$$

The root $k^2 = 0$ implies that either $k_+ = 0$ or $f_1 = f_2 = 0$. Using Eq. (4.5) we see that the latter implies

$$p_1 \cdot a = p_2 \cdot a = 0. \quad (4.17)$$

From Eq. (4.2) we find then

$$k^2 a^\nu - k^\nu k \cdot a + 4\xi^2 \omega^2 b (p_1^\nu p_1 \cdot a - p_2^\nu p_2 \cdot a) = 0, \quad (4.18)$$

and, therefore, that $k \cdot a$ vanishes. There are now two possibilities:

(i) If e_+^μ , k^μ , p_1^μ , and p_2^μ are linearly dependent it is easy to deduce that k^μ is parallel to e_+^μ (given the above conditions), and so $k_+ = 0$.

(ii) Otherwise, express a^μ as a linear combination of these four vectors. Contraction with k^μ , p_1^μ , and p_2^μ then shows that

$$a_\mu = \lambda k_\mu \quad (4.19)$$

for some λ , and $f_{\mu\nu}$ vanishes. The root $k^2 = 0$ therefore corresponds either to $f_{\mu\nu} = 0$ or to the special case of photons propagating parallel to the gravitational wave.

The second root is

$$k^2 - 4\xi^2 \omega^2 b k_+^2 = 0. \quad (4.20)$$

If $k_+ = 0$ then also $k^2 = 0$ and again we have k^μ parallel to e_+^μ . Otherwise we have

$$f_2 = 0, \quad (4.21)$$

$$k_1 f_1 + 2k_+ f_3 = 0.$$

When $k_1 = k_2 = 0$ these conditions are satisfied by a photon polarized in the 1 direction. The characteristic equation, Eq. (4.20), becomes

$$2k_+ [(1 - \xi^2 \omega^2 b) k_0 - (1 + \xi^2 \omega^2 b) k_3] = 0. \quad (4.22)$$

The solution $k_+ = 0$ again corresponds to a photon propagating parallel to the gravitational wave with speed unity. The other solution is a photon traveling antiparallel to the gravitational wave, and to $O(e^2)$ it has velocity

$$k_0/k_3 = 1 + 2\xi^2\omega^2b. \quad (4.23)$$

The speed of travel therefore is greater than unity in those regions where b is positive and less than unity in those regions where b is negative. The other polarization behaves in a complementary fashion so at least one type of photon has a speed greater than unity at each point in space. It must be emphasized that the propagation is nondispersive—the phase and group velocities are the same. In other words, although the light cone has been modified [Eq. (4.20)], it nevertheless remains a cone, and depending on the photon polarization the cone may lie outside the original light cone except for certain special directions of propagation.

We will now turn to the Schwarzschild metric, being of particular physical interest, where we will find similar results occurring.

V. SCHWARZSCHILD BACKGROUND

In standard coordinates the Schwarzschild metric is¹²

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (5.1)$$

If we introduce the orthonormal tetrad $\{e_t, e_r, e_\theta, e_\phi\}$ and the bivectors

$$\begin{aligned} U^{\alpha\beta} &= e_t^\alpha e_r^\beta - e_t^\beta e_r^\alpha, \\ V^{\alpha\beta} &= e_\theta^\alpha e_\phi^\beta - e_\theta^\beta e_\phi^\alpha \end{aligned} \quad (5.2)$$

we can express the curvature tensor as

$$R^{\mu\nu\sigma\tau} = \frac{MG}{r^3} [g^{\mu\sigma} g^{\nu\tau} - g^{\nu\sigma} g^{\mu\tau} + 3(U^{\mu\nu} U^{\sigma\tau} - V^{\mu\nu} V^{\sigma\tau})]. \quad (5.3)$$

Equation (4.4) becomes, in this case,

$$\left(1 + \frac{2MG}{r^3} \xi^2\right) k^2 f^{\rho\nu} + \frac{6MG}{r^3} \xi^2 (k^\rho V^{\nu\mu} V^{\sigma\tau} - k^\rho U^{\nu\mu} U^{\sigma\tau}) k_\mu f_{\sigma\tau} = 0. \quad (5.4)$$

The important components of $f_{\mu\nu}$ are

$$\begin{aligned} g &= U^{\mu\nu} f_{\mu\nu}, \\ h &= V^{\mu\nu} f_{\mu\nu}. \end{aligned} \quad (5.5)$$

A third component may be chosen arbitrarily to be

$$f = (e_t^\mu e_\theta^\nu - e_t^\nu e_\theta^\mu) f_{\mu\nu}. \quad (5.6)$$

If we introduce the vectors

$$\begin{aligned} l^\nu &= k_\mu U^{\mu\nu}, \\ m^\nu &= k_\mu V^{\mu\nu}, \end{aligned} \quad (5.7)$$

then Eq. (5.4) has the form

$$k^2 f^{\rho\nu} + \frac{1}{2} \epsilon [(k^\rho l^\nu - k^\nu l^\rho)g - (k^\rho m^\nu - k^\nu m^\rho)h] = 0, \quad (5.8)$$

where

$$\epsilon = (6MG \xi^2 / r^3) / \left(1 + \frac{2MG \xi^2}{r^3}\right) \approx \frac{6MG \xi^2}{r^3}. \quad (5.9)$$

It follows from Eq. (5.8) that

$$\begin{aligned} (k^2 + \epsilon l^2)g &= 0, \\ (k^2 - \epsilon m^2)h &= 0, \\ \epsilon(k_\theta k_r g - k_t k_\phi h) + k^2 f &= 0. \end{aligned} \quad (5.10)$$

The determinantal condition is

$$k^2(k^2 + \epsilon l^2)(k^2 - \epsilon m^2) = 0. \quad (5.11)$$

As in Sec. IV, the root $k^2 = 0$ may be ignored for general directions of propagation.

The second root yields the modified light cone

$$(1 - \epsilon)(k_t^2 - k_r^2) - k_\theta^2 - k_\phi^2 = 0. \quad (5.12)$$

It is easily checked that the corresponding solution is (for some λ)

$$f_{\mu\nu} = \lambda(k_\mu l_\nu - k_\nu l_\mu). \quad (5.13)$$

The third root yields

$$k_t^2 - k_r^2 - (1 - \epsilon)(k_\theta^2 + k_\phi^2) = 0 \quad (5.14)$$

and

$$f_{\mu\nu} = \lambda(k_\mu m_\nu - k_\nu m_\mu). \quad (5.15)$$

For transverse photon motion, say $k_r = k_\theta = 0$, Eq. (5.12) implies to $O(\epsilon^2)$ that

$$|k_t/k_\phi| = 1 + \frac{1}{2}\epsilon, \quad (5.16)$$

and Eq. (5.13) yields

$$f_{\mu\nu} \propto [k_\mu (e_r)_\nu - k_\nu (e_r)_\mu]. \quad (5.17)$$

That is, the photon with radial polarization travels with a velocity greater than unity. Equation (5.14) implies that

$$|k_t/k_\phi| = 1 - \frac{1}{2}\epsilon, \quad (5.18)$$

and Eq. (5.15) yields

$$f_{\mu\nu} \propto [k_\mu (e_\theta)_\nu - k_\nu (e_\theta)_\mu]. \quad (5.19)$$

Hence the photon with transverse polarization travels with a velocity less than unity. Since for a radially directed photon $l^2 = -k^2$ and $m^2 = 0$, it is obvious from Eqs. (5.12) and (5.14) that such a photon will travel with unit velocity whatever its polarization.

We have thus demonstrated gravitational birefringence in the plane-wave and Schwarzschild metrics. In order to lend some reality to the effect, we calculate the polarization dependence of the bending of light in a Schwarzschild metric.

For a null ray in a metric

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (5.20)$$

the angular deflection $\Delta\phi$ is given by¹³

$$\pi + \Delta\phi = 2 \int_{r_0}^{\infty} \frac{dr}{r} \left[\frac{A(r)}{r^2 \frac{B(r_0)}{B(r)} - 1} \right]^{1/2}, \quad (5.21)$$

where r_0 is the minimum value of r on the orbit.

For the Schwarzschild metric itself this yields the Einstein deflection

$$\Delta\phi = 4MG/r_0. \quad (5.22)$$

Since the determination of the deflection of the classical trajectory from the relation (5.12) is reduced to a problem in differential geometry, the simplest procedure is to take the wave vector as a null vector in an effective metric with

$$A(r) = \left(1 - \frac{2MG}{r}\right)^{-1} (1 + \epsilon), \quad (5.23)$$

$$B(r) = \left(1 - \frac{2MG}{r}\right) (1 + \epsilon).$$

Relative to the Schwarzschild metric, we have to lowest order in G and ϵ^2 ,

$$\delta A(r) = \delta B(r) = \epsilon = (6MG/r^3)\xi^2. \quad (5.24)$$

Now under a perturbation $\delta A, \delta B$ of the metric, the deflection is changed by an amount

$$\delta\Delta\phi \cong \int_{r_0}^{\infty} \frac{dr}{r} \left(\frac{\delta A}{(r^2/r_0^2 - 1)^{1/2}} - \frac{r^2}{r_0^2} \frac{\delta B(r_0) - \delta B(r)}{(r^2/r_0^2 - 1)^{3/2}} \right). \quad (5.25)$$

If we insert the values for δA and δB in Eq. (5.24), we find then

$$\delta\Delta\phi = \frac{-\alpha}{45\pi} \left(\frac{\lambda_c}{r_0}\right)^2 \left(\frac{4MG}{r_0}\right). \quad (5.26)$$

(It is also straightforward to verify that the same correction is obtained by direct calculation in the usual Schwarzschild metric.)

The deflection of the photons polarized transversely to the orbit is equal and opposite. The two beams acquire a separation angle

$$\delta\phi = \frac{2\alpha}{45\pi} \left(\frac{\lambda_c}{r_0}\right)^2 \left(\frac{4MG}{r_0}\right). \quad (5.27)$$

If we use solar parameters in the above formula then we find

$$\delta\phi \cong 3 \times 10^{-47} \Delta\phi. \quad (5.28)$$

Clearly this is immeasurably small. A substantial effect might, however, be observed near a small black hole whose Schwarzschild radius is

more nearly comparable to λ_c .

The result (5.27) appears to be rather different from the related result of Berends and Gastmans,⁴ and this deserves some comment. Berends and Gastmans in effect calculated the matrix element of $(W_0 + W_1)$ between asymptotic photon states with a massive scalar field as the gravitational source, and obtained $O(\alpha)$ corrections to the helicity-flip and helicity-nonflip amplitudes. Now one might at first suppose that the helicity-nonflip differential cross section could be related to a classical trajectory with impact parameter b via the relation

$$b db = - \frac{d\sigma}{d\Omega} \sin\theta d\theta. \quad (5.29)$$

For the $O(1)$ term this reproduces Eq. (5.22), but for the $O(\alpha)$ correction it gives the dispersive result

$$\delta\phi = \frac{44\alpha}{45\pi} \left(\frac{\lambda_c}{r_0}\right)^2 (EMG)^2 \ln \frac{2MG}{r_0} \left(\frac{4MG}{r_0}\right), \quad (5.30)$$

where E is the photon energy; the corresponding classical quantity is the average of the equal and opposite deflections (5.26) for the two polarizations, namely, zero. The reason for this apparent contradiction is that Eq. (5.29) is a highly artificial way of relating the classical and quantum pictures, and, in fact, it only works for a Coulomb-type potential. As a simple example, consider nonrelativistic scattering by a potential:

$$V(r) \propto r^{-n}. \quad (5.31)$$

Then for small-angle scattering the classical trajectory gives a deflection:

$$\delta\phi \propto b^{-n}, \quad (5.32)$$

whereas the quantum approach via the Born approximation and Eq. (5.29) gives

$$\delta\phi \propto b^{1/(n-2)} \quad (1 \leq n < 2). \quad (5.33)$$

These only agree for $n=1$. Further, when the tail of the potential falls sufficiently quickly at large r (but does not vanish), the total quantum cross section will be finite, leading to a finite upper bound on b from Eq. (5.29), whereas classically it is clear that there will always be some deflection for any b . We therefore expect the two methods to agree for the leading contribution to the photon deflection, but not for the $O(\alpha)$ correction, which is tidal in nature and behaves like an r^{-3} potential.

In order to determine which of the calculations is appropriate to our discussion we must examine the situation more closely. The quantum calcula-

tion assumes initial and final photon states which are asymptotic pure-momentum states; it is implicit in this case that wave packets have a size which is large compared with the impact parameter. Physically speaking, it corresponds to viewing the deflection of light at a sufficiently great range from the Sun so that we see the diffraction pattern as described by $d\sigma/d\Omega$. (Note that this is the diffraction pattern caused by the curvature of space and has nothing to do with the diffraction pattern as an edge effect on an opaque Sun, which we are ignoring.)

If, on the other hand, we are quite close to the Sun—on the Earth's orbit—then we assume that it is a reasonable approximation to regard the photon as a wave packet whose dimensions are small compared to the impact parameter and whose trajectory lies along a fairly localized path in the gravitational field. (A corresponding analogy in ordinary particle physics is the difference between the elastic scattering of two particles in an accelerator, and the deflection of a beam of electrons in a cathode-ray tube.) We therefore maintain that the result (5.27) is the appropriate one for the Earth-Sun system, though there may be additional difficulties arising from constraints on frequencies, which we discuss in Sec. VII.

In support of the consistency of this view, we notice that the helicity-nonflip amplitude leading to the nonzero correction (5.30) depends only on the contribution

$$V_0^{\mu\nu\alpha\beta}(q_1, q_2)(1 + p^2 g_1) \quad (5.34)$$

from Eq. (2.8), which in turn depends only on the form factor $g_1(p^2)$, and this form factor can be identified with a linear combination of the two terms $R_{\mu\nu}F^{\mu\alpha}F^\nu{}_\alpha$ and $RF^{\alpha\beta}F_{\alpha\beta}$ in the effective action W_1 . Thus the correction to $d\sigma/d\Omega$ for unpolarized photons comes only from the interaction with the region of nonvanishing $T^{\mu\nu}$, i. e., the gravitational source. So if we assume that the photon wave packet can consistently be constructed so as to "miss" the source when observing close to the Sun, then we would expect a null result for Eq. (5.30). The interaction with the source must of course be included in the calculation of the diffraction pattern seen at large distances, and the calculation of $d\sigma/d\Omega$ by Berends and Gastmans is then appropriate. The diffraction pattern can also be expected to depend on the ratio of the wavelength of light to the length scale of the source, c. f., the dispersive factor $(EMG)^2$ in Eq. (5.30). The leading-order term is nondispersive because the Coulomb-type potential is the only one which does not contain an intrinsic length scale.

VI. ROBERTSON-WALKER BACKGROUND AND FRIEDMANN COSMOLOGY

In standard form the Robertson-Walker metric is given by¹²

$$ds^2 = dt^2 - R^2(t) \left(\frac{d\chi^2}{1 - K\chi^2} + \chi^2(d\theta^2 + \sin^2\theta d\phi^2) \right). \quad (6.1)$$

The constant K can take the values -1 , 0 , and $+1$ corresponding to open, spatially flat, and closed universes. The coordinate t gives the proper time of a comoving observer, and $R(t)$ is a scale factor for the universe. [In the case of a closed universe $K=+1$, $R(t)$ is just the radius of the hypersphere.] The Riemann curvature tensor can be expressed in the form

$$R_{\alpha\beta\gamma\delta} = \left(\frac{\dot{R}^2 + K}{R^2} - \frac{\ddot{R}}{R} \right) (u_\alpha u_\gamma g_{\beta\delta} - u_\beta u_\gamma g_{\alpha\delta} + u_\beta u_\delta g_{\alpha\gamma} - u_\alpha u_\delta g_{\beta\gamma}) - \left(\frac{\dot{R}^2 + K}{R^2} \right) (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}), \quad (6.2)$$

where $u^\mu = (e_t)^\mu = (1, 0, 0, 0)$ in a comoving orthonormal frame.

As before, we now take $F_{\mu\nu} = f_{\mu\nu} e^{i\theta(x)}$ with $k_\mu \equiv \theta_{,\mu}$, and consider only the derivatives on the phase factor. The argument is identical to the argument of the previous sections, except that we now have contributions from $R_{\alpha\beta}$ and R as well in Eq. (2.14); so we will only quote the result. For physical polarizations we find

$$k^2 = 11\xi^2 \left(\frac{\dot{R}^2 + K}{R^2} - \frac{\ddot{R}}{R} \right) (k \cdot u)^2, \quad (6.3)$$

where ξ^2 is given by Eq. (2.19). The velocity of the photons in an orthonormal frame is therefore

$$\left| \frac{k^0}{|\mathbf{k}|} \right| \approx 1 + \frac{1}{2}\xi^2 \left(\frac{\dot{R}^2 + K}{R^2} - \frac{\ddot{R}}{R} \right). \quad (6.4)$$

This velocity is the same in all directions and for all polarizations, which is to be expected because the Robertson-Walker metric describes a general isotropic homogeneous space. If we now follow the standard Friedmann cosmological model, we assume that space-time is filled with a homogeneous isotropic fluid, with energy density ρ and isotropic pressure p , so that the energy-momentum tensor $T^{\mu\nu}$ is given by

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu - pg^{\mu\nu}. \quad (6.5)$$

The Einstein field equations then give

$$3 \left(\frac{\dot{R}^2 + K}{R^2} \right) = 8\pi G\rho, \\ - \left(2 \frac{\ddot{R}}{R} + \frac{\dot{R}^2 + K}{R^2} \right) = 8\pi Gp, \quad (6.6)$$

from which one derives

$$\frac{\ddot{R}}{R} = -\frac{4}{3}\pi G(\rho + 3p), \quad (6.7)$$

and so

$$\left| \frac{k^0}{|\mathbf{k}|} \right| = 1 + 22\xi^2\pi G(\rho + p). \quad (6.8)$$

The photon velocity is therefore clearly greater than one for all physically reasonable pressures. Further, we can make the usual assumption that the early universe is radiation dominated with

$$p = \frac{1}{3}\rho. \quad (6.9)$$

(This is believed to be valid before about 10^{12} sec.)

The first law of thermodynamics gives

$$\rho R^4 = \text{constant} \quad (6.10)$$

for radiation. It then follows that the Universe had a singularity at a finite proper time in the past, with

$$P(t) \propto t^{1/2}, \quad (6.11)$$

the value of the constant K becoming unimportant as $t \rightarrow 0$. Equation (6.4) finally gives

$$\left| \frac{k^0}{|\mathbf{k}|} \right| \approx 1 + \frac{11}{4}\xi^2/t^2 \quad (6.12)$$

and the velocity of photons increases like t^{-2} as we go back in time towards the big-bang singularity. Of course, we do not believe Eq. (6.12) once the $O(e^2)$ correction becomes really large. This only happens at $t \approx 10^{-23}$ sec.

VII. GENERAL DISCUSSION AND CONCLUSIONS

So far we have discussed in detail the propagation of photons in some specific gravitational fields and have found that the effect of vacuum polarization in these cases may be to increase the velocity of photons beyond the flat-space value. Such a controversial result clearly requires very careful consideration.

There are some obvious constraints under which the result was obtained. As indicated in Sec. II, the derivation is applicable only to wavelengths λ which satisfy

$$\lambda > \lambda_c. \quad (7.1)$$

In obtaining the characteristics of the modified wave equation [Eq. (2.14)] we are implicitly considering wavelengths such that

$$\lambda \ll L, \quad (7.2)$$

where L^{-2} is a typical curvature size. It follows that to apply the velocity analysis [as opposed to deriving Eq. (2.14) itself] we require

$$L \gg \lambda_c. \quad (7.3)$$

Since the velocity shift δv is given by

$$\delta v = O(\alpha\lambda_c^2/L^2), \quad (7.4)$$

it is necessarily small, though not zero.

The important point to establish is whether or not this velocity shift implies an alteration in the causality structure of events. Obviously the answer depends on the wave-front velocity and this in turn depends on the velocity of the very-short-wavelength signals for which

$$\lambda_c > \lambda \approx 0. \quad (7.5)$$

Since our derivation was for signals which do not satisfy this condition, it would seem that we cannot draw an immediate conclusion. However, the effect of vacuum polarization is to make space act as a dispersive medium. In that case the long-wavelength (low-frequency) velocity is related to the short-wavelength (high-frequency) velocity by absorption in such a way that the latter velocity is always greater than the former. This is usually stated in terms of the frequency-dependent refractive index $n(\omega)$, where from dispersion relations we have

$$n(\infty) = n(0) - \frac{2}{\pi} \int_0^\infty \frac{d\omega}{\omega} \text{Im}n(\omega). \quad (7.6)$$

Since absorption measured by $\text{Im}n(\omega)$ is never negative, it follows that $n(\infty) \leq n(0)$, which implies the above conclusion. If we are justified in applying these ideas in the curved-space context, then we can conclude that our calculation of the long-wavelength velocity is a lower bound on the wave-front velocity. It follows then that the causal relationships of events are no longer constrained by the null cone of the background metric but by an effective light cone which may lie outside the former cone. It does not, of course, make sense to consider renormalizing the "speed of light" constant c because the correction depends on several variables—the local curvature, the direction of polarization, and the direction of propagation.

The usual objection to faster-than-light transmission of information is that one can set up a paradox involving backwards travel in time. If an observer at event A sends a signal to an observer at event B by a spacelike vector, then there can be another observer C with an ordinary timelike motion who sees B happen before A . Relativistic invariance of the laws of physics ensures that if a spacelike velocity with travel backwards in time is possible from A to B (as seen by C), then B can send a signal back to the world line of A which arrives before A emitted its first signal, and this is clearly unreasonable. However, the key to

this problem is the very assumption of relativistic invariance of the laws governing the motion. The crucial point about the faster-than-light motion calculation in this paper is that it depends on the frame of reference. The change in the velocity is a strictly tidal effect, and different observers will see different tidal effects according to their motion—determined, of course, by the usual Lorentz transformations on the Riemann curvature tensor. In the particular frames we chose we found that the spacelike photon velocities corresponded to motion forward in time only, so in that frame it would be impossible to set up a paradox in the manner described, and, therefore, it would be impossible in any frame. Although it would be possible to find an observer with timelike motion who saw the light signal propagating causally in one direction backwards in time, the return signal in the other direction could only go forward in time in such a way as to arrive at the source *after* the emission of the original signal.

Arising out of this is the very important point that as soon as quantum effects are introduced on a curved background manifold, it seems to be impossible to divorce the discussion of the causal structure of the manifold from the choice of frames of reference, in the sense that the curvature effects distinguish between frames.

This brings us to the question of compatibility with the equivalence principle, which is at the foundation of general relativity. We believe that there is no contradiction here either, since the equivalence principle amounts to the statement that each point of the manifold has a Minkowski tangent space. It specifically determines the behavior of a particle when all curvature effects are ignored, whereas the corrections to photon propagation calculated here are tidal in nature. Nevertheless it may seem strange that the tidal effects should alter the causality structure of the manifold, and so we offer a possible heuristic explanation. One might well expect that tidal forces acting on a large “classical” body would not alter the light cone because the classical body can be regarded as a set of pointlike elements bound by forces which themselves respect the light cone. But for a “large” photon the binding forces are genuinely quantum in nature, being one-loop effects, and are represented by a sum over possible momenta of virtual electrons. The difference here is that the virtual electrons can have spacelike momenta. This is, of course, a very loose argument, but it indicates that a little care is needed in extending classical concepts about the effects of tidal forces to the quantum level.

We may go further and conjecture that in general the equivalence principle may only be relevant to

the “classical limit” of interacting quantum theories, because if there is a mass scale in the theory—and here, it is the electron mass—then as soon as Planck’s constant is introduced, the elementary particles in the theory acquire in their description a length scale corresponding to the Compton wavelength of that mass, and we would in general expect to find tidal forces acting on those elementary particles. Expressed another way, at the quantum level there may be no such thing as an observable elementary particle in the sense required by the equivalence principle.

In terms of the usual quantum-mechanical formulation of field theory, the results imply that the commutativity properties of interacting and free quantum fields differ. There will be a region outside the original null cone but inside the effective light cone where the interacting fields will not commute. Since perturbation-theory calculations are expressed in terms of free fields, one might worry that the incompatibility of the two sets of commutation relations reflects an inconsistency in the formulation of the theory. However, to set up the perturbation theory it is only necessary to have a set of spacelike slices of the manifold, on each member of which the fields at different points commute (or anticommute). Our choice of spacelike slices is not restricted very much [only to $O(\hbar)$] by the modification of the “causality cone” which is implied by our result. On the other hand, it would appear to raise doubts about the adequacy of perturbation theory based on the original null cone. Although this point requires further analysis, we could argue that there is no immediate problem in relation to micro-causality.

Even while accepting that no general principle is violated by the result, one might still find it unattractive and question the technical aspects of the derivation. Firstly, we note that the whole calculation was formulated in a gauge-invariant fashion throughout, and in a coordinate-invariant fashion up to the point where a frame was chosen to express the curvature tensor in a simple form, so there can be no query against it on these grounds. Secondly, the two expansions used, namely, that in the fine-structure constant and that in the Compton wavelength of the electron, seem to be innocuous. The former is used in a perfectly conventional way. The latter amounts to no more than the power-series expansion of the photon’s gravitational form factors at a typical momentum transfer $p^2 \approx L^{-2}$, which is determined by the length scale of the curvature and its derivatives. From Eq. (7.3) we have that $p^2 \ll m^2$. Hence, given the analyticity properties of these form factors, we can expect the expansion to be

convergent.

From numerical estimates in the text it is evident that the effect certainly has no observational consequences in the gravitational fields appropriate to present-day astrophysics and cosmology. Equally, it may be of great importance in, say, the very early universe. For example results such as (6.12) could have consequences for the isotropy of the 2.7 °K microwave background.¹⁴ However, we feel that the main significance of the result is theoretical and does not depend on its observability.

Nevertheless, from one point of view the weakness of the effect does create a problem of principle in relation to observability. It would be quite reasonable to take the attitude that since the derivation was applicable to signals satisfying the wavelength restriction Eq. (7.1), one should seek changes in causal structure consistently in terms of such signals. We will examine two cases in detail.

In the Schwarzschild metric the angular split of the photon beam was [see Eq. (5.27)]

$$\delta\phi = O\left(\alpha\left(\frac{\lambda_c}{r_0}\right)^2 \Delta\phi\right). \quad (7.7)$$

If we wish to resolve this angle at wavelengths $\lambda \approx \lambda_c$ then the lateral dimension d of the beam must satisfy

$$\frac{\lambda_c}{d} \ll \delta\phi. \quad (7.8)$$

However, this implies that

$$\frac{r_0}{d} \ll O\left(\alpha\left(\frac{\lambda_c}{r_0}\right)\Delta\phi\right), \quad (7.9)$$

from which it follows that the width of the beam d must be immensely greater than the radius r_0 at which we are trying to measure the split. This is a hopeless situation for the observation of the split which requires wavelengths much shorter than λ_c for its resolution.

The second case concerns the position discrepancy of a signal δs , which is induced by the velocity shift δv in the Friedmann-Robertson-Walker (FRW) model considered in Sec. VI. We consider a time interval $\lambda_c \ll t_0 \leq t \leq t_1$ during which the redshift caused by the expansion does not seriously alter the frequency content of the signal. In that case we have

$$\delta s < \int_{t_0}^{\infty} \delta v dt. \quad (7.10)$$

Now [see Eq. (6.12)]

$$\delta v = O\left(\alpha\frac{\lambda_c^2}{t^2}\right). \quad (7.11)$$

Therefore,

$$\delta s = O(\alpha\lambda_c(\lambda_c/t_0)). \quad (7.12)$$

This implies $\delta s \ll \lambda_c$, a discrepancy which cannot be resolved with long-wavelength signals alone.

Roughly speaking, what is happening here and in a number of other cases is the following. In a situation where the curvature length scale is L , the length of time available for examining signals of long wavelength is also L . From Eq. (7.4), we see that the position discrepancy is

$$\delta s \approx L\delta v = O\left(\alpha\lambda_c\left(\frac{\lambda_c}{L}\right)\right) \ll \lambda_c. \quad (7.13)$$

This argument tends to suggest that the alteration in causality cannot easily be observed purely in terms of long-wavelength signals. In order to get a significant effect for these wavelengths it would seem necessary to arrange for the photon to retrace its path (e.g., with the use of mirrors) a number of times which is large compared with L/λ_c , and it is not entirely clear that it would be possible to introduce such a theoretical artifice without introducing other theoretical complications at the same time. A possible position would then be to say that there is no problem of causality at all. However, we feel that this does not give proper weight to the idea discussed above that the absorptive processes will raise the wave-front velocity above the low-frequency velocity. In the end, however, a resolution of this question depends on a direct examination of short-wavelength signals. Within the framework of quantum field theory this is tantamount to examining the operator-product expansion for $j_\mu(x)j_\nu(y)$, where $j_\mu(x)$ is the electric current in a curved-space background. This is in any case a problem of intrinsic interest, which we feel our discussion has increased.

Our calculation of the electron Green's function took no account of the effect of any boundary conditions that might have to be imposed. We would expect any such effects to depend on the size of the manifold rather than on the local curvature, and to be rather different in nature, since the local curvature can be varied continuously in a region of the manifold without altering the boundary conditions. Ford¹⁵ has calculated in QED the effect of boundary conditions for spinors on the propagation of photons in a periodic flat spacetime, and he also found corrections to the photon velocity. There, however, the corrections are dispersive and the characteristics of propagation remain on the original light cone, which is a very different sort of behavior from the one we have calculated in this paper.

In conclusion, we have calculated the one-loop

electron contributions in QED to the photon effective action on a general curved-background manifold, and we have found that the tidal effects so generated can result in photons propagating faster than light. They are best described by saying that the effective light cone produced lies outside the null cone of the metric in some places. We have argued that there is no logical or experimental inconsistency in these results. It seems likely that either perturbative QED is inadequate as a theory when extended to a general-relativistic background, or that photons indeed can travel "faster than light". In view of the enormous success of perturbative QED as a theory, this latter possibility should be considered quite seriously. In either case the implications for a satisfactory quantum theory of gravitation would seem to be rather far-reaching.

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APPENDIX: DETAILS IN THE CALCULATION OF W_1

The natural "field" in the calculation of W_1 in Sec. III is the commutator of two covariant derivatives on a charged spinor:

$$\psi_{,\mu\nu} - \psi_{,\nu\mu} \equiv W_{\mu\nu}\psi, \quad (\text{A1})$$

where

$$W_{\mu\nu} = -\frac{1}{2}\sigma^{\alpha\beta}R_{\alpha\beta\mu\nu} - ieF_{\mu\nu}. \quad (\text{A2})$$

In terms of this we get the coincidence limits listed in Table I for the covariant derivatives of charged and uncharged quantities. Note that all derivatives are with respect to x , and that terms of second order or higher in the Riemann curvature have been dropped.

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⁵J. Schwinger, *Phys. Rev.* **82**, 664 (1951).

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⁹For the proper-time representation adapted for dimen-

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¹⁰Note that throughout this section the derivatives indicated are all fully covariant with respect to the gravitational and electromagnetic fields, as appropriate for uncharged (scalar, vector) or charged (spinor) quantities.

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¹²See, for example, C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

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¹⁴This possibility was suggested to us by G. W. Gibbons.

¹⁵L. H. Ford, King's College report (unpublished).