Time-symmetric, approximately relativistic particle interactions and radiation

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The early suggestion that radiation could be viewed as arising from a microscopic time-symmetric theory, with a derivation using statistical mechanics, has never been explicitly demonstrated. For electrodynamics we show that, for a microscopic relativistic theory of interacting particles which is described to order c^{-2}, the effects due to medium polarization (Čerenkov and Bohr formulas) are contained in the c^{-2} kinetic equation treated in other work. Thus, an explicit demonstration of radiation arising from an otherwise time-symmetric particle theory is made. The physical interpretation of real-axis singularities as Čerenkov poles leads to the above result. Comparison is made with other works, which do not appear to lead to the Čerenkov effect.

In 1910 Einstein[1] suggested that macroscopic radiation is an irreversible phenomenon derivable with statistical mechanics from a time-symmetric microscopic formulation. This theme was later elaborated by Wheeler and Feynman[2] for classical electrodynamics[3] when they derived the Lorentz-Dirac equation after making the assumption that all radiation is ultimately absorbed (absorber condition[4]) and that advanced, as well as retarded, effects occur.[5]

While the preceding references did not demonstrate explicitly the connection with statistical or statistical-mechanical arguments, there have been some attempts with stochastic methods to treat radiative effects.[6,7] However, there has been a notable lack of results relating to an explicit demonstration of the *statistical-mechanical* connection between radiation and a microscopic, time-reversible basis. Of course, in electrodynamics, more is implied here than a derivation of the macroscopic Maxwell equations from a suitable microscopic approach, since irreversibility does not *necessarily* follow.

The fundamental difficulties relating to the amalgamation of statistical mechanics and relativity still appear to hold.[8] However, progress has been made in the construction of consistent, approximately relativistic theories.[9]

In particular, a Hamiltonian particle basis to order c^{-2} exists (Darwin approximation) and, consequently, statistical mechanics to this order may be constructed. Thus, a Liouville equation exists to this order and one may proceed to determine the consequences to thermodynamics of the relativistic corrections.

Here, the question of radiation would appear not to arise, since, according to the conventional treatment, an accelerated particle moving in external fields will lose energy at a rate proportional to c^{-3}. Note that the treatment of c^{-3} effects at a microscopic level does not necessarily imply radiation losses, despite the time asymmetry[10]; in this paper we deal with an approximation for which time-reversal invariance holds at the microscopic level.

At the macroscopic level, the questions of relativistic thermodynamics involving the correctness of various transformation laws for the thermodynamic quantities remain open and are not addressed here. Under reasonable assumptions involving Poincaré invariance of the equilibrium statistical-mechanical theory,[11] it is possible to accommodate most, but not all,[12] of the contending formulations. The problems involved in attempting to find unique transformation laws do not concern us here (as in earlier work[13]) and the theory, which is approximately Lorentz invariant to the order concerned, is developed in a suitably defined rest frame for the system.

The existence of a Hamiltonian guarantees the basis for the Liouville equation[14] and the convergent kinetic equation which contains the approximately relativistic generalization of the Balescu-Lenard equation was obtained.[15] The divergences which were eliminated were the usual ones associated with the long- and short-range behavior of interacting point charges.[16] The appearance of symmetric real-axis poles in the analysis of the c^{-2} interaction contribution was initially felt[13] to be an artifact of the approximation technique. Nevertheless, they could be handled, with the net effect that to the order considered for an *equilibrium* charged system, the contribution to the thermodynamics was vanishingly small. Further comparison with the equilibrium statistical mechanics of a degenerate electron gas in which the poles were handled in the same way led to c^{-2} corrections to the correlation energy, equation of state,[17] and the exchange energy.[18] These expressions were in agreement to order c^{-2} with the quantum-field-theoretic calculations given earlier,[19] although the particle calculation to this order was carried out with no need of renormalization procedures.

It is to be shown that the poles indicated do have a physical interpretation. Indeed, it was early demonstrated in well-known papers[20] that similar, physical interpretations of real-axis poles could be made.

It has been demonstrated[9] that the most general form for a two-body, approximately relativistic Lagrangian following from a variational principle for interacting point particles, contains the Darwin approximation as a special case:

$$L = L_0 - V + I_{\mathrm{PN}} \equiv L_0 + L_1 , \tag{1}$$

where L_0 is the total free-particle Lagrangian to the order concerned, V is the total Newtonian potential, and I_{PN} is the post-Newtonian term

$$I_{\mathrm{PN}} = \frac{1}{2c^2} \sum g_i g_j \left[\vec{\mathrm{v}}_i \cdot \vec{\mathrm{v}}_j V_{ij} - (\vec{\mathrm{v}}_i \cdot \vec{\mathrm{R}}_j)(\vec{\mathrm{v}}_j \cdot \vec{\mathrm{R}}_j) \frac{1}{R_{ij}} \frac{dV_{ij}}{dR_{ij}} \right]. \tag{2}$$

The Hamiltonian which follows for the above Lagrangian in this case is

$$H = H_0 + V + I'_{\mathrm{PN}} \equiv H_0 + H_1 , \tag{3}$$

where H_0 is the total free-particle Hamiltonian and

$$I'_{\mathrm{PN}} = -\frac{1}{2c^2} \sum_{i<j} g_i g_j \left[\frac{\vec{\mathrm{P}}_i \cdot \vec{\mathrm{P}}_j}{m_i m_j} V_{ij} - \frac{(\vec{\mathrm{P}}_j \cdot \vec{\mathrm{R}}_{ij})(\vec{\mathrm{P}}_j \cdot \vec{\mathrm{R}}_{ij})}{m_i m_j} \frac{1}{R_{ij}} \frac{dV_{ij}}{dR_{ij}} \right], \tag{4}$$

where $\vec{\mathrm{P}}$ is the canonical momentum, considered to order c^{-2}. The Hamiltonian form is also a particular case of those general cases to order c^{-2}, which follow from consideration of the approximate generators of the Poincaré group.[9]

It is to be stressed that in using the Hamiltonian (3), or the Lagrangian (1), then even if one takes the viewpoint of the slow-velocity expansion of an exact field theory, the situation is correct only to order c^{-2}.[21] Thus, to argue[22,23] that (1) is correct to this order and then to incorporate terms of order c^{-3} and higher *ab initio* in a Hamiltonian (including many-body terms) cannot be correct. In writing the Lagrangian (1), terms of order c^{-3} have already been disregarded.[24]

The treatment of retardation (or advanced effect) is not involved in the c^{-2} approximation.[25] Thus, there is a fundamental inconsistency in treating particles in such a way that brings in c^{-3} effects[22] (same order effects as are associated with radiation damping). Another problem relates to treating particles as moving in straight-line trajectories (while neglecting radiation). With regard to the magnetic interactions, it may be seen that such an approximation is equivalent to assuming that the particles have independent trajectories. Inserting the independent-particle approximation

$$F_{\mathrm{N}}(t) = \prod_{i=1}^{N} f(\vec{\mathrm{R}}_i, \vec{\mathrm{P}}_i, t) \tag{5}$$

into the Liouville equation

$$\frac{\partial F_N(t)}{\partial t} = i L_N F_N(t) , \tag{6}$$

where L_N is the Liouville operator,[15] and projecting out the single-particle distribution f we get, using (5),

$$\frac{\partial f(\vec{\mathrm{R}}, \vec{\mathrm{P}}, t)}{\partial t} = \frac{\partial H_0^{(1)}}{\partial \vec{\mathrm{R}}} \cdot \frac{\partial f}{\partial \vec{\mathrm{P}}} - \frac{\partial H_0^{(1)}}{\partial \vec{\mathrm{P}}} \cdot \frac{\partial f}{\partial \vec{\mathrm{R}}} + (N-1) \int d\vec{\mathrm{R}}' d\vec{\mathrm{P}}' \left(\frac{\partial H_1^{(2)}}{\partial \vec{\mathrm{R}}} \cdot \frac{\partial}{\partial \vec{\mathrm{P}}} - \frac{\partial H_1^{(2)}}{\partial \vec{\mathrm{P}}} \cdot \frac{\partial}{\partial \vec{\mathrm{R}}} \right) \times f(\vec{\mathrm{R}}, \vec{\mathrm{P}}, t) f(\vec{\mathrm{R}}', \vec{\mathrm{P}}', T) , \tag{7}$$

where $H_0 = \sum H_0^{(1)}$ and $H_1 = \sum H_1^{(2)}$. Equation (7) is the linearized approximation; it is seen that if $f = f(|\vec{\mathrm{R}}|, |\vec{\mathrm{P}}|, t)$, then there is no contribution from $I'^{(2)}_{\mathrm{PN}}$ on the right-hand side after integration. This is so in the important case of spherical symmetry in phase space (a spatial homogeneous system being a special case).

To assess more carefully the role of the c^{-2} term, we turn to the long-range part of the kinetic equation (the c^{-2} generalization of the Balescu-Lenard equation) as derived by using cluster methods from the Liouville and Master equations.[15] For a spherical symmetric distribution $\phi(\vec{\mathrm{P}}_1)$

$$\frac{\partial \phi(\vec{\mathrm{P}}_1)}{\partial t} = 16\pi^3 e^4 C \int d\vec{\mathrm{P}}_2 \int d\vec{\mathrm{l}}\, \vec{\mathrm{l}} \cdot \frac{\partial}{\partial \vec{\mathrm{P}}_1} \frac{\delta(\vec{\mathrm{l}} \cdot \vec{\mathrm{g}}_{12})}{l^4} \left(\frac{1}{|\epsilon|^2} + \frac{P_{1\perp}{}^2 P_{2\perp}{}^2}{2(mc)^4 |\eta|^2} \right) \vec{\mathrm{l}} \cdot \left(\frac{\partial}{\partial \vec{\mathrm{P}}_1} - \frac{\partial}{\partial \vec{\mathrm{P}}_2} \right) \phi_1(\vec{\mathrm{P}}_1) \phi_1(\vec{\mathrm{P}}_2) , \tag{8}$$

where

$$\epsilon \equiv 1 + (4\pi i e^2 C / l^2) \int d\vec{\mathrm{P}}_3 \delta_-(\vec{\mathrm{l}} \cdot \vec{\mathrm{g}}_{13}) \vec{\mathrm{l}} \cdot \frac{\partial \phi_1(\vec{\mathrm{P}}_3)}{\partial \vec{\mathrm{P}}_3} \tag{9}$$

and

$$\eta = 1 - (4\pi i e^2 C/l^2) \int d\vec{P}_3 \delta_-(\vec{l}\cdot\vec{g}_{13}) \frac{P_{3\perp}^2}{2(mc)^2} \vec{l} \cdot \frac{\partial \phi_1(\vec{P}_3)}{\partial \vec{P}_3}. \tag{10}$$

The right-hand side of (8) may be cast in the form

$$-\frac{\partial}{\partial \vec{P}_1} \cdot \left[\vec{F}\phi(\vec{P}_1) - \vec{D} \cdot \frac{\partial \phi(\vec{P}_1)}{\partial \vec{P}_1} \right], \tag{11}$$

where

$$\vec{F} = 16\pi^3 e^4 C \int d\vec{P}_2 \int d\vec{l}\, \vec{l} \frac{\delta(\vec{l}\cdot\vec{g}_{12})}{l^4} \left[\frac{1}{|\epsilon|^2} + \frac{P_{1\perp}^2 P_{2\perp}^2}{2(mc)^4 |\eta|^2} \right] \vec{l} \cdot \frac{\partial \phi(\vec{P}_2)}{\partial \vec{P}_2} \tag{12}$$

and

$$\vec{D} = 16\pi^3 e^4 C \int d\vec{P}_2 \int d\vec{l}\, \vec{l}\, \vec{l} \frac{\delta(\vec{l}\cdot\vec{g}_{12})}{4} \left[\frac{1}{|\epsilon|^2} + \frac{P_{1\perp}^2 P_{2\perp}^2}{2(mc)^4 |\eta|^2} \right] \phi(\vec{P}_2). \tag{13}$$

According to (12), a particle of momentum $\vec{P}_1$ moving through the medium will lose energy at the rate $\vec{P}_1 \cdot \vec{F}/m$ or

$$16\pi^3 e^4 C \int d\omega\, d^2 P_{2\perp} \int d\vec{l}\; \delta\left(\omega - \frac{\vec{l}\cdot\vec{P}_1}{m}\right) \frac{(\vec{l}\cdot\vec{P}_1)}{l^4} \left[\frac{1}{\epsilon_2} \mathrm{Re}\frac{i}{\epsilon} - \frac{P_{1\perp}^2 P_{2\perp}^2}{2(mc)^4 \eta_2} \mathrm{Re}\frac{i}{\eta} \right] \frac{\partial \phi(\vec{P}_2)}{\partial P_{2z}}, \tag{14}$$

where we have defined $\omega = \vec{l}\cdot\vec{P}_2/m$, $\epsilon = \epsilon_1 + i\epsilon_2$, and $\eta = \eta_1 + i\eta_2$. Also, $\delta(\vec{l}\cdot\vec{g}_{12}) \sim \delta(\omega - \vec{l}\cdot\vec{P}_1/m)$ and we have rewritten the form of $|\epsilon|^{-2}$.

For simplicity, we may rewrite the forms ϵ and η for the Maxwell-Boltzmann distribution (the Jüttner distribution leads to more complicated forms; the qualitative results should, however, be the same, since the general terms of order higher than c^{-2} are involved)

$$\epsilon = 1 + (1 - \omega I)/l^2 r_D^2, \tag{15}$$

$$\eta = 1 - (1 - \omega I)/l^2 r_R^2, \tag{16}$$

where

$$I = \frac{P}{2} \int d\vec{P} \frac{1}{\omega - (\vec{l}\cdot\vec{P}/m)} \vec{P}_\perp \cdot \frac{\partial \phi_1(\vec{P})}{\partial \vec{P}} + i\frac{\pi}{2} \int d\vec{P}\, \delta\left(\omega - \frac{\vec{l}\cdot\vec{P}}{m}\right) \vec{P}_\perp \cdot \frac{\partial \phi_1(\vec{P})}{\partial \vec{P}}. \tag{17}$$

The forms (15) and (16) become explicitly

$$\epsilon = 1 + \frac{\omega_0^2 \beta m}{l^2} \left[1 - \frac{\omega(2\beta m)^{1/2}}{l} \psi\left(\frac{\omega(2\beta m)^{1/2}}{l} \right) \right] - \frac{i\omega}{l} \left(\frac{\omega_0}{l} \right)^2 (\beta m)^{3/2} \left(\frac{\pi}{2} \right)^2 e^{-\beta m (\omega/l)^2/2} \tag{18}$$

and

$$\eta = 1 - \left(\frac{\omega_0}{lc} \right)^2 \left[1 - \frac{\omega(2\beta m)^{1/2}}{l} \psi\left(\frac{\omega(2\beta m)^{1/2}}{l} \right) \right] + \frac{i\omega}{l} \left(\frac{\omega_0}{lc} \right)^2 \left(\frac{\beta m}{2\pi} \right)^{1/2} e^{-\beta m (\omega/l)^2/2}. \tag{19}$$

Using (18), we examine the first term in (14). This becomes

$$4\pi e^2 \mathrm{Re}\, i \int d\omega\, \omega \int d\vec{l}\, \frac{\delta(\omega - \vec{l}\cdot\vec{P}_1/m)}{l^2} \frac{\exp\{-(\beta/2ml^2)[\omega^2 - (\vec{l}\cdot\vec{P}/m)^2]\}}{\epsilon}. \tag{20}$$

Making use of the relation [assuming an extremely small imaginary part in (18)]

$$\mathrm{Re}\frac{i}{\epsilon} = \pi\delta(\epsilon) \simeq \pi\delta\left(1 - \frac{\omega_0^2}{\omega^2}\right), \tag{21}$$

where we have taken the large-wavelength (small-l) form of (18) [recall that (8) takes long-range collective effects into account]; then (20) is ($\mu \equiv \cos\theta$)

$$\frac{8\pi^3 e^2 m}{\omega_0^2 P_1} \int d\omega\, \omega^4 \delta(\omega - \omega_0) \int \frac{dl}{l} d\mu\, \delta(\mu - m\omega/lP_1) = \frac{e^2 \omega_0^2}{v_1} \ln\left(\frac{l_{\max}}{l_{\min}} \right). \tag{22}$$

For a suitable choice of the integration limits, we may obtain the Bohr scattering losses (longitudinal) due to polarization of the medium.

The second part of (14), using (19), is

$$\frac{2\pi e^2}{(mc)^2}\,\mathrm{Re}\,i\int\frac{d\omega}{\omega}\int\frac{d\vec{l}}{l^2}(\vec{l}\cdot\vec{P}_1)\left[P_1{}^2-\left(\frac{m\omega}{l}\right)^2\right]\delta(\omega-\vec{l}\cdot\vec{P}_1/m)\,\frac{\exp\{-(\beta/2ml)[\omega^2-(\vec{l}\cdot\vec{P}/m)^2]\}}{\eta}\,. \tag{23}$$

Similarly, when one examines the small-wavelength behavior,

$$\mathrm{Re}\frac{i}{\eta}=\pi\delta(\eta)\simeq\pi\delta\left[1-\left(\frac{\omega_0}{lc}\right)^2\right], \tag{24}$$

and (23) becomes

$$\frac{8\pi^3e^2}{mc^2}P_1\int d\omega\,\omega\int dl\,d\mu(1-\mu^2)\delta\left(\mu-\frac{m\omega}{lP_1}\right)\delta\left(l-\frac{\omega_0}{c}\right)\frac{l^2c^2}{\omega_0{}^2}=\frac{e^2P_1}{mc^2}\int d\omega\,\omega(1-\cos^2\alpha)\,, \tag{25}$$

where

$$\cos\alpha=m\omega c/\omega_0P_1\,. \tag{26}$$

Equation (25) is the expression for Čerenkov radiation loss. It results from the transverse interaction of the particle with the medium and the condition for radiation for a subluminal particle is $\omega c/\omega_0 v<1$ or $\omega<\omega_0$.[26]

It is important to note that the Čerenkov contribution is of order c^{-2} and, so, it falls within the bounds of the previously noted restriction. Thus, although the c^{-2} Hamiltonian was time symmetric, the statistical-mechanical treatment has resulted in radiation, an effect associated with irreversibility.

The partial (long-range) kinetic equation (8) involving the Darwin approximation[15] was also later investigated by others.[27] The latter references include effects which go beyond the c^{-2} approximation of (3) in arriving at an effective Hamiltonian or they pursue a coarse-graining procedure. One of the results of these procedures (which, we have already pointed out, effectively add higher orders to the Hamiltonian while denying the same higher order in the Lagrangian[24]) is that for low $\omega\equiv\vec{l}\cdot\vec{P}/m$ we have $\omega_0{}^2=-l^2c^2$. This forbids the propagation of low-ω Čerenkov radiation, whereas the result which obtains here, with $\omega_0{}^2=l^2c^2$, allows it. In earlier work,[25] we have also pointed out the possibility of nondamped waves in the charged system.

The behavior indicated by the presence of real-axis poles, thus, is physically significant. Of course, if the system is large enough, the absorption will ultimately take place (recall the Wheeler-Feynman condition of complete absorption[2]). The statistical-mechanical calculation implies that the thermodynamic limit has been taken. Thus, overall energy conservation for an infinite system is implied, although in finite, but large, plasmas it is impossible to eliminate the Čerenkov radiation; however, the condition $\vec{l}\cdot\vec{v}<\omega_0$ for the radiation clearly brings in the density effect through the plasma frequency, ω_0, and nonconservation in such a case is tied to this effect.

To see further the connection with radiation, we observe that the pair distribution function in equilibrium leads to an effective interaction[21]:

$$-\vec{\mu}_1\vec{\mu}_2:\frac{\overleftrightarrow{\pi}-3\hat{r}\hat{r}}{r^3}-\vec{\mu}_1\vec{\mu}_2:\left[(\overleftrightarrow{\pi}-\hat{r}\hat{r})\frac{\cos(r/r_R)}{r_R{}^2r}-(\overleftrightarrow{\pi}-3\hat{r}\hat{r})\left(\frac{\sin(r/r_R)}{r_Rr^2}+\frac{\cos(r/r_R)}{r^3}\right)\right]=-\vec{\mu}_1\cdot\vec{E}_{12}\,, \tag{27}$$

where $\vec{\mu}\equiv er_R\vec{P}/mc$ and $r_R=c/\omega_0$; this has the form of a dipole-dipole interaction; the leading term was first obtained by Trubnikov.[27] However, the oscillating terms were replaced by other nonoscillating ones. Moreover, the $\vec{E}_{12}$ defined in (27) is a macroscopic Maxwell solution. Of course, in the limit as $v\to 0$, the Darwin approximation (Lagrangian or Hamiltonian) emerges. The nonoscillatory results, following from the screening of magnetic interaction, as noted earlier in this paper, cannot give results which explain the physically observed c^{-2} Čerenkov effects. The dipole radiation forms [bracketed terms in (27)] gives, for $r\gg r_R$ and $l_0=r_R{}^{-1}=\omega_0/c$,

$$\vec{E}_r=[\vec{\mu}-(\vec{\mu}\cdot\hat{r})\hat{r}]l_0{}^2\cos l_0r/r\,. \tag{28}$$

Defining $\vec{B}_r=\hat{r}\times\vec{E}$ then leads to

$$\vec{B}_r=(\hat{r}\times\vec{\mu})l_0{}^2\cos l_0r/r\,. \tag{29}$$

This dependence of $\vec{E}$ and $\vec{B}$ is associated with radiation and (without taking into account a radiation absorption mechanism) clearly gives rise to a net efflux of energy for finite boundaries; it follows, also, that the long range can be expected to lead to infinities in the limit of an infinite system,[28] unless special boundary conditions (absorber conditions) are brought into play. Defining the complex Poynting vector

$$\vec{S}=\frac{c}{8\pi}\vec{E}\times\vec{B}^*, \tag{30}$$

we get from (28) and (29)

$$\vec{S}=\frac{c\hat{r}l_0^4\mu^2}{8\pi r^2}\sin^2\theta=\frac{\hat{r}v^2e^2l_0^2}{8\pi c r^2}\sin^2\theta. \tag{31}$$

The angle θ lies between the direction of the particle velocity $\vec{v}$ (associated with dipole moment $\vec{\mu}$) and the direction of $\hat{l}$. The energy per unit time per solid angle is then[29]

$$\frac{dP}{d\Omega}=\frac{v^2e^2l_0^2}{8\pi c}\sin^2\theta. \tag{32}$$

The far fields $\vec{E}_r$ and $\vec{B}_r$ in (28) and (29) satisfy the equation

$$\nabla^2\begin{bmatrix}\vec{E}_r\\ \vec{B}_r\end{bmatrix}+\left(\frac{\omega_0}{c}\right)^2\begin{bmatrix}\vec{E}_r\\ \vec{B}_r\end{bmatrix}=0. \tag{33}$$

This again implies that $l^2c^2=\omega_0^2$, and that (33) are the wave equations for which $(\omega_0/c)^2=(\omega n/c^2)$, so that $n=\omega_0/\omega$. This also follows from the Čerenkov relation (26). It has been argued[30] that the Darwin approximation should be good for plasma simulation in the region $n^2\gg 1$. This corresponds to $\epsilon_T=n^2\equiv l^2c^2/\omega^2$. Thus, once again, we see $l^2c^2/\omega^2=\omega_0^2/\omega^2$, or $\omega_0^2=l^2c^2$, for low ω.

For infinite systems one may even employ a physical, exponential cutoff, when absorption is taken into account, due to the random thermal motion and Doppler broadening.[31] This is of the order of $(mc^2/kT)^{1/2}r_R$: For higher temperatures, such a cutoff would be well within reasonable system scale parameters. Absorption of radiation at distances of the order of $r_R=c/\omega_0$ could lead to the behavior (vortices) suggested earlier.[32]

We have shown that a time-symmetric Hamiltonian (order c^{-2}) in a statistical-mechanical calculation can lead to radiation effects of the same order (Čerenkov radiation). The physical interpretation of the presence of real-axis poles is, thus, given. Treatments which require screening of magnetic interactions cannot lead to these effects, since they lead to the dispersion relation $\omega_0^2=-l^2c^2$ at low ω. Arguments, which are based upon the assumption that the Darwin Lagrangian may be used to order c^{-2}, but that the Hamiltonian appropriate to the analysis must include *ab initio* orders of c^{-3} and higher, are not carried out consistently.

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