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Transversality of the Rarita-Schwinger self-energy, Ward identities, and dimensional regularization

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The finite one-loop parts of the Rarita-Schwinger self-energy in the gauge $\alpha\gamma\cdot\psi+(1-\alpha)\gamma^a\delta_a^\mu\psi_\mu$ satisfy the transversality Ward identity in the new regularization scheme by dimensional reduction but not in conventional dimensional regularization. Diagonalizing and then omitting the nonpropagating Lorentz ghosts, new vertices in the non-Lorentz ghost sectors do contribute. The criteria for deriving Ward identities involving tracelessness of structure constants are shown to be equivalent to the unit Becchi-Rouet-Stora-Tyutin Jacobian provided the gauge algebra closes.

I. INTRODUCTION

The quantization of supergravity proceeds along general lines. One adds gauge-fixing terms which in turn determine the Faddeev-Popov ghost action. From this quantum action one may formally derive Ward identities by means of path-integral techniques. If one has a regularization scheme which preserves the gauge invariances and applies minimal subtractions, then the regularized theory satisfies these Ward identities.[1]

In this article we study these aspects in the example of the Rarita-Schwinger self-energy. This is the counterpart of a similar calculation of the graviton self-energy in supergravity.[2] Although mixed loops containing at the same time supersymmetry ghost C^a and general coordinate ghosts C^ν appear here, the calculation does not give information on the commutation properties of C^a and C^ν, but it only requires that C^a (and C^ν) commute (anticommute) with their antighosts. As usual in such explicit calculations, some new theoretical insights are gained. The two most important results we present below are as follows:

(i) Even though the antisymmetric part of the vierbein field can be gauged away completely, its effects remain at the quantum level. In fact, it is just because we could not satisfy some Ward identities that we found that new couplings are present. They are due to the fact that the kinetic terms involving the Lorentz antighost are nondiagonal. Diagonalizing by redefining the Lorentz ghost field, new couplings are produced in the non-Lorentz ghost sectors. Thus the lore that one can forget in quantum supergravity about the Lorentz ghosts "because they do not propagate" is incorrect.

(ii) The dimensional regularization scheme in the form usually employed[3] violates the transversality Ward identity of the Rarita-Schwinger self-energy, but a recent modification, the so-called dimensional reduction scheme,[4] preserves this Ward identity for all n. Thus, as expected for a long time, it has finally been proven that dimensional regularization and (local) supersymmetry are incompatible from the finite parts of the one-loop level onward, where the calculation becomes sensitive to the regularization method employed. The dimensional reduction scheme, keeping all spinors and vectors four-dimensional, preserves supersymmetry Ward identities. This has the important consequences that one may use the Gauss-Bonnet theorem in quantum (super) gravity, that one still can use the two-component van der Waerden formalism at the quantum level, and that propagators are n independent. Most importantly, the two-loop finiteness proof of supergravity[5] has now been completed, since there are no supersymmetry anomalies.

In order to derive Ward identities, we use the Becchi-Rouet-Stora-Tyutin (BRST) invariance of the quantum action.[6,7] This invariance can be achieved with or without closed gauge algebra, that is to say, with or without auxiliary fields.[8] However, one also needs a unit Jacobian, and this is achieved by choosing a certain measure in the path integral. In the old way of deriving Ward identities by means of nonlocal gauge parameters, one always had to make the product of Jacobian and integration measure invariant in order that the path integral be gauge independent.[9] In this case the gauge algebra must close. We show below that these two methods of deriving Ward identities are equivalent (once the gauge algebra closes).

As a byproduct, we show that in ordinary Einstein gravity the product of Jacobian and integration measure is invariant under nonlocal gauge transformations, provided one adds a measure $(\det g)^{3/2}$. Thus, it is not necessary to give arguments that the ordinary Jacobian as well as the trace of the structure constants each vanish formally.[10] Rather, it is the sum which vanishes rigorously.

In Sec. II we compare various methods for deriving Ward identities in supergravity. In Sec. III we obtain the one-loop divergences of the Rarita-Schwinger self-energy using dimensional regularization. In Sec. IV we extend these results to the finite parts, repeat the calculation using regularization by dimensional reduction, and compare the results with those of Sec. III.

II. THE WARD IDENTITY

In previous calculations[2,11] the gauge-fixing term was chosen as

$$\mathcal{L}(\text{fix}) = -\tfrac{1}{4}(\partial_\mu \sqrt{g} g^{\mu\nu})^2 + \tfrac{1}{4}\bar{\psi}\cdot\gamma \not{\partial} \gamma\cdot\psi + \lambda(e_{a\mu}\delta^\mu_b - e_{b\mu}\delta^\mu_a)^2 , \quad (1)$$

where λ is an arbitrary constant, often taken to tend to infinity. However, as we shall see, with this choice the diagram with the mixed ghost loop in which we are interested vanishes. Thus we consider a more general gauge for the local supersymmetry

$$\mathcal{L}(\text{fix sup}) = \tfrac{1}{4}[\alpha\bar{\psi}\cdot\gamma + (1-\alpha)\bar{\psi}_\mu \delta^\mu_a \gamma^a] \times \not{\partial}[\alpha\gamma\cdot\psi + (1-\alpha)\gamma^b\delta^\nu_b\psi_\nu] . \quad (2)$$

It is essential to use a flat $\not{\partial}$ and not a covariant $\not{D}$ since for the latter case the correct quantum action is still not known, except when the vierbein fields in $\not{D}$ are external fields.[12,13] For $\alpha = 1$ we find Eq. (1), while for $\alpha = 0$ one finds a gauge-fixing term bilinear in fields, which only contributes to the kinetic tensor but not to the vertices. The kinetic terms in Eq. (2) are α independent and lead to the usual Rarita-Schwinger propagator $\langle\psi_\mu(p)\bar{\psi}_\nu(p)\rangle = \frac{1}{2}\gamma_\nu \not{p}\gamma_\mu p^{-2}$ at $n = 4$. To derive the Ward identity for the Rarita-Schwinger self-energy, we consider the unweighted path integral

$$Z = \int [d\phi^i] e^{iI(\text{cl})} \delta[F_\alpha + b_\alpha - a_\alpha] \,\text{s-det}(\partial F_\alpha/\partial\xi^\beta) , \quad (3)$$

where s-det denotes the Faddeev-Popov superdeterminant,[14] ξ^β the gauge parameters, $\phi^i = \{e^m_\mu, \psi^a_\mu, S, P, A_m\}$ and $F_\alpha = \{-\partial_\mu\sqrt{g}g^{\mu\nu}, -\frac{1}{2}[\alpha\gamma\cdot\psi + (1-\alpha)\gamma^a\delta^\mu_a\psi_\mu], e_{a\mu} - e_{b\nu}\delta^b_\mu\delta^\nu_a\}$. If we can show that Z is independent of the arbitrary functions a_α and b_α, then we can multiply by "unity"

$$\int da_\alpha \exp(\tfrac{1}{2}a_\alpha\gamma^{\alpha\beta}a_\beta)(\text{s-det}\,\gamma)^{1/2} = \text{constant}, \quad (4)$$

to obtain [dropping the field-independent $(\text{s-det}\,\gamma)^{1/2}$]

$$Z = \int [d\phi^i] \exp\{i[I(\text{cl}) + \tfrac{1}{2}(F_\alpha + b_\alpha)\gamma^{\alpha\beta}(F_\beta + b_\beta)]\} \times \text{s-det}(\partial F_\alpha/\partial\xi_\beta) , \quad (5)$$

where Z is evidently still independent of b_α. In that case we have the Ward identity

$$\frac{\delta}{\delta b_\beta(y)} \frac{\delta}{\delta b_\alpha(x)} Z \Big|_{b=0} = \gamma^{\alpha\beta} + \langle\gamma^{\beta\rho}F_\rho(y)\gamma^{\alpha\sigma}F_\sigma(x)\rangle , \quad (6)$$

where $\gamma^{\alpha\beta}$ is the tree-graph contribution. We choose b_α to be nonzero only in the case of local supersymmetry. There we find the transversality condition on the Rarita-Schwinger self-energy, after dividing by $\frac{1}{4}\not{\partial}(x)$ and $\frac{1}{4}\not{\partial}(y)$

$$\langle[\alpha\gamma\cdot\psi(y) + (1-\alpha)\gamma^a\delta^\mu_a\psi_\mu(y)] \times [\alpha\bar{\psi}\cdot\gamma(x) + (1-\alpha)\bar{\psi}_\nu\delta^\nu_b\gamma^b]\rangle_{1\text{oop}} = 0 . \quad (7)$$

The tree-graph contribution is of order $\alpha = 0$ and equal to $\gamma_\mu\langle\psi_\mu\bar{\psi}_\nu\rangle\gamma_\nu = 2\not{p}^{-1}$, but the loop corrections must sum up to zero. In the next sections we will consider separately the order $\alpha = 0$, α, and α^2 terms in Eq. (7), and compare dimensional regularization with dimensional reduction for n different from 4. In this section we now turn to the question of whether Z is indeed independent of a_α.

It is well known[9] that the path integral is independent of the gauge (i.e., independent of a_α) if the integration measure times the Faddeev-Popov determinant is invariant under a particular kind of nonlocal gauge transformations. It is necessary and sufficient that

$$\delta(R^i_\alpha\xi^\alpha)/\delta\phi^i + f^\beta_{\beta\alpha}\xi^\alpha = 0 , \quad (8)$$

where ϕ^i are the classical gauge fields $(e^m_\mu, \psi^a_\mu, S, P, A_m)$ and $\delta\phi^i = R^i_\alpha(\phi)\xi^\alpha$ denotes all gauge transformations. The structure "constants" are defined by the closure of the gauge algebra

$$[\delta(\eta), \delta(\xi)]\phi^i = R^i_{\alpha,j}R^j_\beta\eta^\beta\xi^\alpha - (\xi \leftrightarrow \eta) = R^i_\gamma f^\gamma_{\alpha\beta}\eta^\beta\xi^\alpha . \quad (9)$$

The contributing parts of $\delta\phi^i$ are [8]

$$\begin{aligned}
\delta S &= \xi^\alpha\partial_\alpha S - \tfrac{1}{2}\bar{\epsilon}\gamma\cdot\psi S , \quad \text{idem}\, P \\
\delta A_m &= \xi^\alpha\partial_\alpha A_m - \tfrac{1}{2}\bar{\epsilon}\gamma\cdot\psi A_m , \\
\delta e^m_\mu &= \partial_\mu\xi^\alpha e^m_\alpha + \xi^\alpha\partial_\alpha e^m_\mu , \\
\delta\psi^a_\mu &= \partial_\mu\xi^\alpha\psi^a_\alpha + \xi^\alpha\partial_\alpha\psi^a_\mu + 2D_\mu\epsilon .
\end{aligned} \quad (10)$$

The contribution from the gauge fields to Eq.

(8) is

$$\delta(R^i_\alpha \xi^\alpha)/\delta\phi^i = \partial_\alpha \xi^\alpha(-1-1-4+4-16-4+16) + \bar\psi\cdot\gamma\epsilon(\tfrac{1}{2}+\tfrac{1}{2}+2+0-3)\,. \quad (11)$$

Note that the Rarita-Schwinger contributions in the supertrace[14] acquire an extra minus sign. All derivatives are right derivatives.[15] Thus, from

$$\delta\psi^a_\mu = 2(D_\mu\epsilon)^a = 2\partial_\mu\epsilon^a + (\sigma^{mn}\epsilon)^a[\omega_{\mu mn}(e) + \tfrac{1}{2}\bar\psi_\mu\gamma_m\psi_n + \tfrac{1}{4}\bar\psi_m\gamma_\mu\psi_n] \quad (12)$$

one finds indeed the contribution

$$-(\sigma^{mn}\epsilon)^a(\tfrac{1}{2}\bar\psi_\mu\gamma_m\delta^\mu_n - \tfrac{1}{2}\bar\psi_n\gamma_m\delta^\mu_\mu + \tfrac{1}{2}\bar\psi_m\gamma_\mu\delta^\mu_n)^a = -3\bar\psi\cdot\gamma\epsilon\,. \quad (13)$$

The contribution from the trace of the structure constants follows from the commutators of the three gauge invariances[8] which are general coordinate transformations (g), local Lorentz rotation (l), and local supersymmetry transformations (s). To give an example, from

$$[\delta_g(\xi^\alpha), \delta_l(\lambda^{mn})] = \delta_l(-\xi^\alpha\partial_\alpha\lambda^{mn})$$

one gets the contribution $6\,\partial_\alpha\xi^\alpha$. In general, one takes the right derivative with respect to ξ^α of the composite parameter $f^\alpha_{\beta\gamma}\eta^\gamma\xi^\beta$. In this way one finds for the g, l, and s contributions, respectively,

$$f^\beta_{\beta\alpha}\xi^\alpha = (5+6-4)(\partial_\alpha\xi^\alpha) + (0+0-1)(\bar\psi\cdot\gamma\epsilon)\,. \quad (14)$$

Again a trace over fermionic indices acquires an extra minus sign, so that from

$$[\delta_s(\epsilon_1), \delta_s(\epsilon_2)] = \delta_s(\psi_\mu\bar\epsilon_1\gamma^\mu\epsilon_2) + \text{irrelevant } l \text{ and } g \text{ terms}, \quad (15)$$

one finds indeed $-\bar\psi\cdot\gamma\epsilon$.

Thus the sum of Eqs. (11) and (14) does not vanish. However, one can add a measure $(\det e)^{-1}$, in order to make the $\partial_\alpha\xi^\alpha$ terms cancel. *The crucial test is then whether the $\bar\psi\cdot\gamma\epsilon$ terms also cancel with this same measure.* This is indeed the case. The path integral thus reads

$$Z = \int [de^m_\mu d\psi^a_\mu dS\, dP\, dA_m (\det e)^{-1}\Delta_{\rm FP}]\delta(F_\alpha - a_\alpha)e^{iI(\rm cl)}, \quad (16)$$

where F_α are the gauges chosen. Z is independent of a_α since under the nonlocal gauge transformation $\delta\phi^i = R^i_\alpha(F_{\alpha,j}R^j_\beta)^{-1}\lambda_\beta(x)$ the factor within square brackets is invariant, its variation being proportional to Eq. (11) plus Eq. (14), plus the variation of $(\det e)^{-1}$

$$(\partial_\alpha\xi^\alpha)(-6+7-1) + \bar\psi\cdot\gamma\epsilon(0-1+1) = 0\,. \quad (17)$$

In none of the results above do the Lorentz transformation parameters occur, since the Lorentz generators $(\sigma_{mn})^{ab}$ and $(\delta^a_m\delta^b_n - \delta^b_m\delta^a_n)$ have no diagonal elements.

We stress that it is only the sum in Eq. (8) plus the measure which is invariant, not each term separately. For Einstein gravity, Capper and Ramon Medrano[10] argued that each term separately is formally zero. However, the sum is rigorously zero if one adds again a measure $(\det g)^{3/2}$. Using dimensional regularization or regularization by dimensional reduction, these measures do not contribute, and anyway they are absorbed by the renormalization constants in renormalizable theories.

The auxiliary fields S, P, and A_m appear in the action as $-\frac{1}{3}\det e(S^2+P^2-A_m{}^2)$ and can be integrated away. (This remains true in the quantum action, since the terms which are bilinear in ghosts but linear in S,P,A_m are added). The measure then becomes $(\det e)^{-4}$, in agreement with the results obtained by requiring BRST invariance of simple supergravity with open algebras.[13]

This brings us to the last point to be discussed in this section, the relation between Slavnov-Taylor invariance and BRST invariance of the path integral. It is well known that Ward identities such as Eq. (7) can also be derived from BRST invariance by making a BRST variation of $\langle C^{*\beta}(x)F_\alpha(y)\rangle$. However, one must also show that in the case of BRST transformations the Jacobian equals unity and this was shown in Ref. 7. Indeed, the antighost fields do not contribute, since they rotate into the gauge-fixing terms while the ghost-field transformation rules were first found in Ref. 6:

$$\delta C^\nu = (-C^\lambda\partial_\lambda C^\nu + \hat C\gamma^\nu C)\Lambda\,,$$
$$\delta C_{ab} = (-C^\lambda\partial_\lambda C_{ab} - C^m_a C_{mb} + \omega, S, P, A_m \text{ terms})\Lambda\,, \quad (18)$$
$$\delta C^a = [C^\lambda\partial_\lambda C^a - \tfrac{1}{2}\psi^a_\mu\hat C\gamma^\mu C + \tfrac{1}{2}C_{mn}(\sigma^{mn}C)^a]\Lambda\,.$$

The caret denotes a Majorana bar $C^a = \mathfrak{C}^{ab}\hat C^b$. One finds a contribution to the Jacobian

$$(5+6-4)(\partial_\lambda C^\lambda\Lambda) - (\bar\psi\cdot\gamma C)\Lambda\,. \quad (19)$$

Clearly, if one adds these terms to Eq. (11) where one replaces ξ^α, λ^{mn}, and ϵ^a by $C^\alpha\Lambda$, $C^{mn}\Lambda$, and $C^a\Lambda$, respectively, then the sum cancels and the BRST Jacobian equals unity under *all three* local symmetries provided one has the *same* measure $(\det e)^{-1}$ in each case.

In fact, one can understand why the BRST Jacobian equals unity with the same measure as was needed for Slavnov-Taylor invariance. The ghost contribution to the Jacobian is given by

$$\delta(\delta^{\rm BRST}C^\alpha)/\delta C^\alpha = f^\alpha_{\alpha\beta}C^\beta\Lambda \quad (20)$$

and this is precisely the same as the term with the structure constant in Eq. (8). The sign here is + rather than −, since ghosts have opposite statistics to the parameters to which they belong, so that right differentiating $\delta C^A = -\frac{1}{2} f^A_{KL} C^L \Lambda C^K$ acquires an extra minus sign relative to right differentiating $f^\alpha_{\beta\gamma}\eta^\gamma\xi^\beta$. Thus we can deduce from the preceding results for Slavnov-Taylor invariance that also for BRST invariance the measure is $(\det e)^{-1}$.

In Ref. 7 the authors avoided the measure $(\det e)^{-1}$ by taking A_μ rather than A_m as an independent field. One can find relations to other choices by using the following simple theorem:

$$\delta(A^\mu)'/\delta A^\mu = (\delta A'_m/\delta A_m) + \delta(e^{m\mu})'/\delta e^{m\mu}, \quad (21)$$

where the prime denotes an infinitesimal transformation. Note that the last term is equal to $\delta(\det e)'$. Similarly, one could have taken ψ^m rather than ψ_μ, while any power of $(\det e)$ in the measure can be counterbalanced by multiplying fields by powers of $(\det e)$.

III. DIMENSIONAL REGULARIZATION OF THE TRANSVERSALITY WARD IDENTITY

The supergravity action[16,17] consists of the Einstein-Hilbert and the Rarita-Schwinger actions as well as a complicated four-fermion interaction whose explicit form is not needed for our present one-loop calculation,

$$\mathcal{L}^{SG} = -\frac{1}{2\kappa^2} eR(e) - \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma + \text{``}\psi^4\text{''}. \quad (22)$$

Here $e = \det e_{a\mu}$ with $e_{a\mu}$ the spin-2 vierbein field (the graviton), ψ_μ is the spin-$\frac{3}{2}$ Majorana field, and D_ρ is the gravitationally covariant derivative. We work in the second-order formalism[16] without auxiliary fields. The elimination of the auxiliary fields leads to a four-ghost coupling[8,18] but this does not contribute to our one-loop calculation. We use the positive Pauli metric $[g_{\mu\nu} = g^{\mu\nu} = \delta^\mu_\nu = (++++)$ in flat space with $\mu,\nu = 1,2,3,4]$, $\gamma_5^2 = \gamma_a^2 = 1$ $(a = 1,2,3,4)$ and Greek (Latin) indices denote world (local Lorentz) tensors. From now on we put $\kappa = 1$, and we define $c_{a\mu} = e_{a\mu} - \delta_{a\mu}$ as the quantum field variable of the graviton.

The nonghost vertex of two Rarita-Schwinger fields and one graviton from the Rarita-Schwinger action is given by $\mathcal{L}_{ng} = \mathcal{L}_I + \mathcal{L}_{II}$ where

$$\mathcal{L}_I = -\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_5\gamma_a\partial_\rho\psi_\sigma c_{a\nu},$$
$$\mathcal{L}_{II} = -\tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu abc}\bar\psi_\mu\gamma_c\psi_\sigma\partial_b c_{\rho a}. \quad (23)$$

The most useful gauge choices are

$$F_\alpha = \{-\partial_\mu\sqrt{g}g^{\mu\nu},\ e_{a\mu} - e_{\mu a},\ -\tfrac{1}{2}[\alpha\gamma\cdot\psi + (1-\alpha)\bar\gamma\cdot\psi]\}, \quad (24)$$

where $\bar\gamma^\mu = \gamma^a\delta^\mu_a$ is field independent and α is an arbitrary constant. The gauge-fixing terms can be chosen to be

$$\mathcal{L}_{fix} = -\tfrac{1}{4}(\partial_\mu\sqrt{g}g^{\mu\nu})^2 + \lambda(e_{a\mu} - e_{\mu a})^2 + \tfrac{1}{4}[\alpha\bar\psi\cdot\gamma + (1-\alpha)\bar\psi\cdot\bar\gamma]\bar{\not\partial}[\alpha\gamma\cdot\psi + (1-\alpha)\bar\gamma\cdot\psi]. \quad (25)$$

Here $\bar{\not\partial}$ is field independent.[12] We let $\lambda \to \infty$ so that only the symmetric part of the vierbein field remains. There is a vertex $\alpha\mathcal{L}_{fix}$ of order α from the gauge-fixing term where

$$\mathcal{L}_{fix} = -\tfrac{1}{2}\bar\psi_\mu\gamma_a\bar\gamma_\rho\bar\gamma_\nu\partial_\rho\psi_\nu c_{\mu a}. \quad (26)$$

We list here all the propagators:

$$\begin{aligned} P^{(2)}_{\mu\nu,\rho\sigma} &= -i(2p^2)^{-1}(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\rho\sigma}),\\ P^{(3/2)}_{\mu\nu} &= (2p^2)^{-1}\gamma_\nu\not p\gamma_\mu,\\ P^{(1)}_{\mu\nu} &= -i(p^2)^{-1}\delta_{\mu\nu},\\ P^{(1/2)} &= -\not p^{-1}. \end{aligned} \quad (27)$$

The ghost terms are obtained by varying the gauge functions and we exhibit here only those terms relevant to our calculation:

$$\begin{aligned}\mathcal{L}_{gh} = {}& \partial_\mu C^*_\nu[-\partial^\mu C^\nu + (\bar\psi^\mu\gamma^\nu + \bar\psi^\nu\gamma^\mu - \bar\psi\cdot\gamma\delta^{\mu\nu})C] + C^{*ab}(2C_{ab} + \partial_b C_a - \partial_a C_b)\\ & -\tfrac{1}{2}(1-\alpha)\bar C[\partial_\alpha(\bar\gamma\cdot\psi)C^\alpha + \bar\gamma^\mu\psi_\alpha\partial_\mu C^\alpha + \tfrac{1}{2}\bar\gamma^\mu\sigma_{ab}\psi_\mu C^{ab} + 2\bar\gamma^\mu D_\mu C]\\ & -\frac{\alpha}{2}\bar C[\partial_\alpha(\gamma\cdot\psi)C^\alpha + \tfrac{1}{2}\gamma^\mu\sigma_{ab}\psi_\mu C^{ab} + \tfrac{1}{2}\gamma_a\psi_b C^{ab} + 2\gamma^\mu D_\mu C].\end{aligned} \quad (28)$$

We note that the kinetic term in the Lorentz sector is $2C^{*ab}[C_{ab} + \frac{1}{2}(\partial_b C_a - \partial_a C_b)]$. It is convenient to redefine $C'_{ab} = C_{ab} + \frac{1}{2}(\partial_b C_a - \partial_a C_b)$ so that the kinetic term for the Lorentz ghosts becomes $2C^{*ab}C'_{ab}$ which are then nonpropagating. The ghost vertex with one Rarita-Schwinger field, one supersymmetry ghost, and one general coordinate ghost can be cast in the following form:

$$\mathcal{L}_{gh} = \mathcal{L}_a + \mathcal{L}_b + \mathcal{L}_c + \mathcal{L}_d - \alpha\mathcal{L}_c + \alpha\mathcal{L}_e, \quad (29)$$

where $\mathcal{L}_d$ contains the vertices induced by dia-

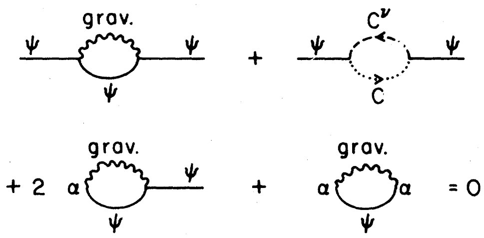


FIG. 1. The four classes of graphs contributing to the Ward identity. Note that each class contains graphs which may depend on α.

gonalizing the Lorentz ghost:

$$\begin{aligned}
\mathcal{L}_a &= \partial_\mu C_\nu^*(\bar{\psi}_\mu \bar{\gamma}_\nu + \bar{\psi}_\nu \gamma_\mu - \bar{\psi}\cdot\bar{\gamma}\delta_{\mu\nu})C\,,\\
\mathcal{L}_b &= -\tfrac{1}{2}\bar{C}[\partial_\alpha(\bar{\gamma}\cdot\psi)]C^\alpha\,,\\
\mathcal{L}_c &= -\tfrac{1}{2}\bar{C}\bar{\gamma}^\mu\psi_\alpha\partial_\mu C^\alpha\,,\\
\mathcal{L}_d &= \tfrac{1}{4}\bar{C}\bar{\gamma}^\mu\sigma_{ab}\psi_\mu\partial_b C^\nu\delta_\nu^a\,,\\
\mathcal{L}_e &= \tfrac{1}{4}\bar{C}\gamma_a\psi_\mu(\partial_b C^\nu\delta_\nu^a - \partial_a C^\nu\delta_\nu^b)\,.
\end{aligned} \tag{30}$$

Thus the effects of the Lorentz ghosts are present in the effective quantum action, even though the Lorentz ghosts themselves can be dropped. At this point we have complete Feynman rules. We can proceed to calculate the contributions to the Ward identity. The Ward identity, which we derived in Sec. II, is given in Fig. 1. Now this should hold separately for different powers of α.

The results of our calculations of the pole terms to order $\alpha = 0$, α, and α^2 are given in Figs. 2, –4, respectively. We draw the reader's attention to the fact that all graphs with ghosts are of mixed type, namely, they contain one supersymmetry and one general coordinate ghost. We have not added an extra minus sign for these loops, and note that the Ward identities are satisfied. The sum of all ghost contributions vanishes for $\alpha = 1$ and this was the reason that we chose instead of $\gamma\cdot\psi$ as gauge-fixing term $\alpha\gamma\cdot\psi + (1-\alpha)\bar{\gamma}\cdot\psi$. In computing these diagrams one needs Wick contractions, for example, $\langle C^{*\nu}C^\mu\rangle$. These lead to extra signs: $\langle C^{*\nu}C^\mu\rangle = -\langle C^\mu C^{*\nu}\rangle$ but $\langle C^{*a}C^b\rangle = +\langle C^b C^{*a}\rangle$. However, these are diagonal signs and have nothing to do with $C^a C^\nu = \pm C^\nu C^a$. We always need to interchange C^a and C^ν either twice or not at all in a given diagram (this is well-known in Feynman diagrams and the reader may easily check this). Thus one obtains the same result whether C^a and C^ν commute or anticommute.

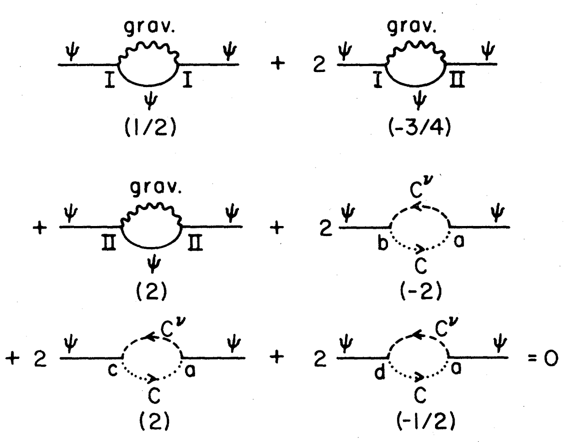


FIG. 2. Order-α^0 contributions to the Ward identity for the one-loop divergent parts. The numbers in brackets are the contributions to the Ward identity except for an overall factor $\pi^2 \not{p}(4-n)^{-1}$ and the explicit factors 2 indicate that both orders of the vertices have been added together.

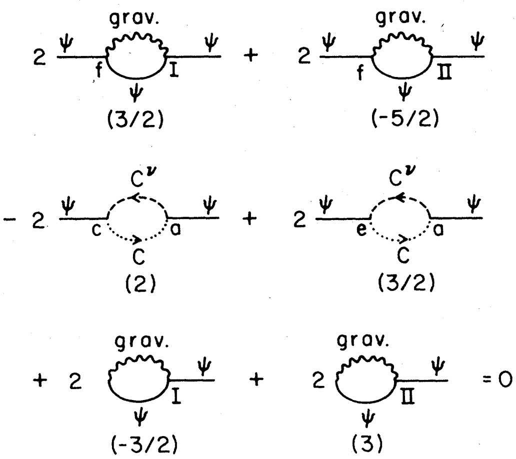


FIG. 3. Order-α contributions to the Ward identity for the one-loop divergent parts.

IV. DIMENSIONAL REGULARIZATION OF THE TRANSVERSALITY WARD IDENTITY BY DIMENSIONAL REDUCTION FOR GENERAL n

In Sec. III transversality of the Rarita-Schwinger self-energy was verified for the divergent part at the one-loop level. We now extend this to include finite terms. The calculation then becomes sensitive to the regularization method employed.

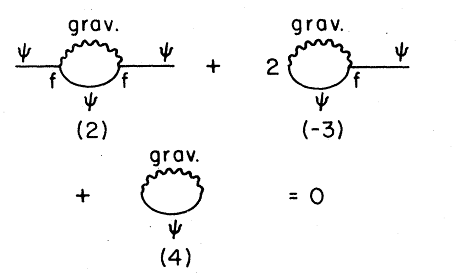


FIG. 4. Order-α^2 contributions to the Ward identity for the one-loop divergent parts.

For global supersymmetry it has been shown[4] that a modified form of dimensional regularization, which we call dimensional reduction, preserves supersymmetric Ward identities. The technique consists essentially of continuing coordinates and momenta to n dimensions ($n<4$) while retaining the four-dimensional nature of all other vector indices; thereby the equality of Bose and Fermi degrees of freedom characteristic of supersymmetric theories is preserved. By contrast, in conventional dimensional regularization all tensors are continued to n dimensions and it was shown[19] for a variety of globally supersymmetric theories that the supersymmetric Ward identities are violated by finite terms at the one-loop level. Our purpose here is to investigate whether the method of dimensional reduction is also applicable to locally supersymmetric theories. We also consider the result of using conventional dimensional regularization.

In both cases it is convenient to relate all the Feynman integrals which arise to a single standard one given by

$$I(p^2)=i\int\frac{d^nk}{k^2(p-k)^2}\,.$$

The following relations are useful:

$$i\int\frac{k_\mu d^nk}{k^2(p-k)^2}=\tfrac{1}{2}Ip_\mu\,,$$

$$i\int\frac{k_\mu k_\nu d^nk}{k^2(p-k)^2}=\frac{1}{4(n-1)}I(np_\mu p_\nu-p^2\delta_{\mu\nu})\,, \tag{31}$$

$$i\int\frac{k_\mu k_\nu k_\lambda d^nk}{k^2(p-k)^2}=\frac{1}{8(n-1)}I[(n+2)p_\mu p_\nu p_\lambda-\delta_{\mu\nu}p_\lambda-\delta_{\mu\lambda}p_\nu-\delta_{\lambda\nu}p_\mu]\,.$$

Note that the Kronecker δ's in the above expressions are n dimensional. It is frequently convenient for algebraic purposes to apply these relations at an intermediate stage in the calculation of a particular diagram. It is then important to distinguish (in dimensional reduction) between the Kronecker δ's thereby arising, and the four-dimensional ones from the Feynman rules.

For illustrative purposes we consider the evaluation of the first diagram in Fig. 6. For the case of dimensional reduction, the Feynman rules are exactly as given in the previous section, and we obtain

$$\frac{i}{2p^2}\int\frac{k_{\hat\rho}k_{\hat\tau}\,d^nk}{k^2(p-k)^2}\times p_{\hat\mu}\epsilon^{\mu\nu\rho\sigma}(\delta_{a\alpha}\delta_{\nu d}+\delta_{ad}\delta_{\nu\alpha}-\delta_{a\nu}\delta_{\alpha d})\gamma_5\gamma_a\gamma_\alpha\gamma_\tau\gamma_\sigma\gamma_d=\frac{3}{4(n-1)}I\epsilon^{\mu\nu\rho\sigma}p_{\hat\mu}\gamma_5\gamma_\sigma\gamma_{\hat\rho}\gamma_\nu=\tfrac{3}{2}I\not{p} \tag{32}$$

(where careted indices are n dimensional, uncareted four dimensional).

If we attempt instead to apply conventional dimensional regularization, a number of complications arise. Both graviton and Rarita-Schwinger propagators become n dependent:

$$P^{(2)}_{\mu\nu,\rho\sigma}=\frac{-i}{2p^2}\left(\delta_{\mu\rho}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\rho}-\frac{2}{n-2}\delta_{\mu\nu}\delta_{\rho\sigma}\right),$$

$$P^{(3/2)}_{\mu\nu}=\frac{1}{(n-2)p^2}\left(\gamma_\nu\not{p}\gamma_\mu+(4-n)\delta_{\mu\nu}\not{p}+2(n-4)\frac{p_\mu p_\nu\not{p}}{p^2}\right)\,. \tag{33}$$

The derivation of the Rarita-Schwinger propagator using general n spin-$\frac{3}{2}$ projection operators is sketched in Appendix A, where some relations useful in evaluating the diagrams are also noted. The treatment of the antisymmetric Levi-Civita tensor also requires explanation, since it has no obvious generalization to n dimensions. However, in the calculations it can always be replaced by an expression which can be generalized by, for example, the relation

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\sigma=\{\sigma_{\nu\rho},\gamma_\mu\}\,. \tag{34}$$

Of course the uniqueness of this procedure is questionable, which serves to illustrate the unsatisfactory nature of the conventional technique for supersymmetric theories.

For the same diagram as considered above, we now find the following expression:

$$i\int\frac{d^nk\,k_\rho p_\mu}{k^2(p+k)^2}\epsilon^{\mu\nu\rho\sigma}\gamma_5\left[\gamma_\alpha P^{(3/2)}_{\sigma\alpha}(k)\gamma_\nu+\gamma_d P^{(3/2)}_{\sigma\nu}(k)\gamma_d-\frac{2}{n-2}\gamma_\nu P^{(3/2)}_{\sigma d}(k)\gamma_d\right]=\frac{n+2}{4(n-1)(n-2)}I\epsilon_{\mu\nu\rho\sigma}p_\mu\gamma_5\gamma_\sigma\gamma_\rho\gamma_\nu=\frac{n+2}{4}I\not{p} \tag{35}$$

(all indices n dimensional).

Results for both cases are given in Figs. 5,–7 for the α^0, α, and α^2 terms, respectively. We see that with dimensional reduction the Ward identity is identically satisfied irrespective of the value of n, while, as anticipated, dimensional regularization works only at $n=4$ for the α^0 terms, although remarkably it too satisfies the identity for the α and α^2 terms. We conclude that the

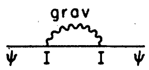
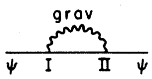
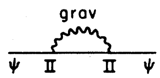
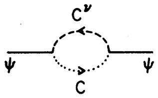
Diagram	Coefficient of $I(p^2)\not p$ Dimensional Reduction	Coefficient of $I(p^2)\not p$ Dimensional Regularization
	$\frac{1}{4}$	$\frac{n(n^2-6n+10)(n-3)}{16(n-2)}$
	$-\frac{3}{4}$	$-\frac{(n+2)}{8}$
	1	$\frac{n^2+n-4}{16}$
	-2	-2
	2	2
	$-\frac{1}{2}$	$-\frac{1}{2}$
TOTAL	0	$\frac{(n-4)(n^3-4n^2+9n-8)}{16(n-2)}$

FIG. 5. Order-α^0 contributions to the Ward identity for the one-loop finite parts.

dimensional reduction technique gives satisfactory results for supergravity.

Note added. In this paper we have discussed a particular Ward identity, but anomalies were not considered. Recently, however, some papers have appeared in which the role of anomalies in dimensional regularization by dimensional reduction has been discussed. We refer to H. Nicolai and P. Townsend, Phys. Lett. 93B 111 (1980); proceedings of the Erice conference on the unification of the fundamental interactions, edited by S. Ferrara, J. Ellis, and P van Nieuwenhuizen (unpublished), and P. Majumdar, E. Poggio, and H. Schnitzer, Phys. Rev. D 21, 2203 (1980).

In a recent paper W. Siegel [Phys. Lett. 94B, 37 (1980)] has argued that his technique of regularization by dimensional reduction is mathematically inconsistent. It seems to us, however, that the inconsistencies found by him are in fact nothing more than the usual ambiguities associated with the axial anomaly. More work is certainly needed to clarify this point further.

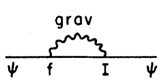
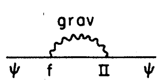
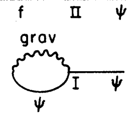
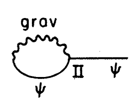
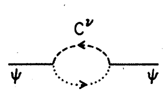
Diagram	Coefficient of $I(p^2)\not p$ Dimensional Reduction	Coefficient of $I(p^2)\not p$ Dimensional Regularization
	$\frac{3}{2}$	$\frac{n+2}{4}$
	$-\frac{5}{2}$	$-\frac{(n^2+3n-18)}{4}$
	$-\frac{3}{2}$	$-\frac{(n+2)}{4}$
	3	$\frac{n^2+2n-12}{4}$
	-2	-2
	$\frac{3}{2}$	$\frac{n+2}{4}$
TOTAL	0	0

FIG. 6. Order-α contributions to the Ward identity for the one-loop finite parts.

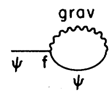
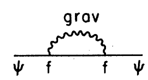
Diagram	Coefficient of $I(p^2)\not p$ Dimensional Reduction	Coefficient of $I(p^2)\not p$ Dimensional Regularization
	-2	$-\frac{(n^3-5n^2+12n-16)}{4(n-2)}$
	3	$\frac{n^3-6n^2+16n-20}{2(n-2)}$
	-1	$-\frac{(n^3-7n^2+20n-24)}{4(n-2)}$
TOTAL	0	0

FIG. 7. Order-α^2 contributions to the Ward identity for the one-loop finite parts.

APPENDIX A

In the class of gauges used, the Rarita-Schwinger propagator is given by the inverse of the expression[2]

$$\tfrac{1}{2}\gamma_\mu \not p \gamma_\nu . \tag{A1}$$

The inverse of (A1) was found using the following set of spin-$\frac{3}{2}$ projection operators:

$$\begin{aligned} &P^{1/2,ss}_{\mu\nu} = \frac{1}{n-1}\hat\gamma_\mu\hat\gamma_\nu , \quad P^{1/2,tt}_{\mu\nu} = \omega_\mu\omega_\nu , \\ &P^{1/2,st}_{\mu\nu} = \frac{1}{(n-1)^{1/2}}\hat\gamma_\mu\omega_\nu , \quad P^{1/2,ts}_{\mu\nu} = \omega_\mu\hat\gamma_\nu , \\ &P^{3/2}_{\mu\nu} = \delta_{\mu\nu} - P^{1/2,ss}_{\mu\nu} - P^{1/2,tt}_{\mu\nu} , \end{aligned} \tag{A2}$$

where

$$\hat\gamma_\mu = \gamma_\mu - \frac{k_\mu \not k}{k^2} , \quad \omega_\mu = \frac{k_\mu \not k}{k^2} .$$

These obey the orthogonality condition

$$\sum_\nu P^{J,ij}_{\mu\nu} P^{J',kl}_{\nu\sigma} = \delta^{JJ'}\delta^{jk}P^{J,il}_{\mu\sigma} . \tag{A3}$$

Then, writing

$$\tfrac{1}{2}\gamma_\nu \not{p} \gamma_\mu = \sum_{J,i,j} a^J_{ij} P^{J,ij}_{\mu\nu} \not{p} \qquad \text{(A4)}$$

we find

$$a^J_{ij} P^{J,ij}_{\mu\nu} \not{p} = P^{J,ii}_{\mu\rho} \tfrac{1}{2}\gamma_\sigma \not{p} \gamma_\rho P^{J,jj}_{\sigma\nu}. \qquad \text{(A5)}$$

Then by simple algebra one finds

$$a^{1/2}_{tt} = \tfrac{1}{2}, \quad a^{1/2}_{st} = -a^{1/2}_{ts} = \frac{(n-1)^{1/2}}{2},$$
$$a^{1/2}_{ss} = \frac{(n-3)}{2}, \quad a^{3/2} = -1 \qquad \text{(A6)}$$

and hence that the Rarita-Schwinger propagator is

$$P^{(3/2)}_{\mu\nu} = \frac{1}{p^2(n-2)}\left[\gamma_\nu \not{p} \gamma_\mu + (4-n)\delta_{\mu\nu}\not{p} + 2(n-4)\frac{p_\mu p_\nu \not{p}}{p^2}\right]. \qquad \text{(A7)}$$

The following identities are useful:

$$\gamma_\mu P^{(3/2)}_{\mu\nu} = P^{(3/2)}_{\nu\mu}\gamma_\mu = \frac{2p_\nu}{p^2},$$
$$P^{(3/2)}_{\mu\nu}\gamma_\mu = \frac{2\gamma_\nu \not{p}}{p^2},$$
$$\gamma_\nu P^{(3/2)}_{\mu\nu} = \frac{2\not{p}\gamma_\mu}{p^2},$$
$$P^{(3/2)}_{\mu\mu} = \frac{(3-n)\not{p}}{p^2}. \qquad \text{(A8)}$$

Some formulas for γ matrix algebra in n dimensions used in Sec. IV are

$$\gamma_\lambda \gamma_\beta \not{p} \gamma_\lambda \gamma_\beta = [2n - (n-2)^2]\not{p},$$
$$\gamma_\lambda \gamma_\rho \gamma_\sigma \gamma_\lambda = (n-2)\gamma_\rho\gamma_\sigma + 2\gamma_\sigma\gamma_\rho,$$
$$\gamma_\lambda \gamma_\rho \gamma_\sigma \gamma_\tau \gamma_\lambda = (n-6)\gamma_\tau\gamma_\sigma\gamma_\rho + 2(4-n)(\delta_{\rho\sigma}\gamma_\tau + \delta_{\sigma\tau}\gamma_\rho - \delta_{\rho\tau}\gamma_\sigma). \qquad \text{(A9)}$$

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