Evolution of cosmological baryon asymmetries. II. The role of Higgs bosons

J. N. Fry, Keith A. Olive, and Michael S. Turner
Astronomy and Astrophysics Center, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637
(Received 27 May 1980)

The time evolution of the baryon asymmetry of the universe due to superheavy Higgs bosons is obtained by integrating the Boltzmann equations. The interactions included are decays, inverse decays, and annihilations of the Higgs bosons. The minimal SU(5) model is used to calculate the interactions, although our results are more generally applicable. Decays and inverse decays of these bosons damp preexisting asymmetries by $\exp(-0.3K)$ to $\exp(-2.0K)$ where $K = 2.9\times 10^{17}\alpha_H\,\mathrm{GeV}/M_H$ (α_H and M_H are the coupling strength and mass of the Higgs boson). If both C and CP are violated, then in an initially symmetrical universe a baryon asymmetry evolves and its value depends upon K and α^2/α_H (α is the gauge coupling strength). For $K \ll \alpha_H/\alpha^2$ the asymmetry produced is $kn_B/s \simeq 2\times 10^{-3}\epsilon$, and for $K \gg 1$ it is $kn_B/s \simeq 2\times 10^{-3}\epsilon(3K)^{-1.2}$ ($\epsilon/2$ = the baryon excess produced by the decay of a pair of H, $\bar{H}$ bosons). In SU(5) for $M_H \gtrsim 3\times 10^{13}$ GeV preexisting asymmetries are not damped and the observed asymmetry $kn_B/s = 10^{-9.8\pm 1.6}$ can be produced if $\epsilon \simeq 10^{-7}$. In a companion paper the role of gauge bosons is considered.

I. INTRODUCTION

A. Background

Most grand unified theories (GUT's) predict the existence of new, superheavy bosons (gauge and/or Higgs) whose interactions violate B and L conservation (B = baryon number, L = lepton number). The existence of these bosons implies that the proton has a finite lifetime ($\tau_p \simeq 10^{31\pm 1}$ yr).[1] In addition, their existence may explain the presence of baryons in the universe in the first place. It has been suggested that the out-of-equilibrium decay of these bosons in the very early universe ($t \lesssim 10^{-35}$ sec) could have produced a slight excess of baryons.[2,3] If the baryon excess to specific entropy ratio produced were $\sim 10^{-9.8}$, then this mechanism would explain the apparent absence of antimatter and the baryon to specific entropy ratio $kn_B/s = 10^{-9.8\pm 1.6}$ observed today.

In the most general GUT which breaks down to $SU(3)_c\times SU(2)_L\times U(1)$ there are three types of superheavy bosons ($M \gtrsim 10^{14}$ GeV) whose interactions violate B and L conservation,[4] and so could possibly produce a baryon excess. Classified according to $SU(2)_L$ and $SU(3)_c$ they are X, Y—an isodoublet, color triplet of vector bosons (charge $\pm\frac{4}{3}, \pm\frac{1}{3}$); X', Y'—an isodoublet, color triplet of vector bosons (charge $\pm\frac{2}{3}, \pm\frac{1}{3}$); and H—an isosinglet, color triplet of scalar bosons (charge $\pm\frac{1}{3}$). The interactions of all these bosons conserve $B-L$ (total baryon number minus total lepton number).

The superheavy gauge bosons of SU(5) are of the X and Y variety. In a companion paper[5] (hereafter referred to as FOT) we considered in detail their role in generating or damping baryon asymmetries. In an arbitrary GUT the X and Y bosons will couple to the same fermion channels but with different strength, so when the appropriate coupling strength is used the results of FOT are applicable in general to X and Y bosons. Since X' and Y' bosons couple to similar channels as those of the X and Y, we also take the results of FOT to be applicable to X' and Y' bosons (when the appropriate coupling strength is used).

In this paper it is the third type of superheavy boson that we shall consider. The H bosons are Higgs bosons, and being isoscalar and spin-0 particles they couple to fermions differently than the X, X', Y, and Y' bosons. They also typically have a smaller effective coupling constant ($\alpha_H \sim 10^{-4}$–10^{-6}). It is also expected that they will be lighter than the other types of superheavy bosons, so that their impact on the baryon asymmetry may be the final (and possibly most important) one.

One might expect that the results of FOT for X and Y bosons should be qualitatively correct for H bosons when the appropriate coupling constant is used. However, there is one important difference which has to do with the usually rather weak coupling of these bosons to fermions. The key feature of the decay scenario is that when the temperature falls below the mass of the superheavy boson, these bosons remain more abundant than they would be if they were present in equilibrium numbers, because they cannot diminish rapidly enough. There are two basic ways for them to disappear—via decay into fermions or via annihilation. For $T \lesssim M$ (M = mass of superheavy boson) these rates (decay and annihilation, respectively) are

$$\Gamma_D \sim \alpha M\,, \tag{1.1a}$$

$$\Gamma_A \sim \alpha^2 (T/M)^3 M\,, \tag{1.1b}$$

where α is the coupling strength of the superheavy boson. Because of the temperature dependence and the additional factor of α, it appears that $\Gamma_A \ll \Gamma_D$, so that annihilations can be ignored. This is what has been assumed in the past. For gauge bosons this is a reasonable approximation but not necessarily so for Higgs bosons. For H bosons, the couplings in (1.1a) and (1.1b) are not equal. The α in (1.1a) should be α_H, the coupling strength of H bosons to fermions. However, since these bosons are charged and colored, they can annihilate via photons or gluons, and the α in (1.1b) should be the gauge coupling constant [$\alpha = \frac{1}{45}$ in SU(5)]. Thus for Higgs bosons (1.1) should be

$$\Gamma_D \sim \alpha_H M_H\,, \tag{1.2a}$$

$$\Gamma_A \sim \alpha^2 (T/M)^3 M_H\,. \tag{1.2b}$$

For $T \sim M_H$, the ratio Γ_A/Γ_D is $\sim \alpha^2/\alpha_H$, which is appreciable when $\alpha_H \lesssim \alpha^2$ (for $\alpha = \frac{1}{45}$, this is $\alpha_H \lesssim 5\times 10^{-4}$). Over a large range of possible Higgs couplings, annihilations can dominate decays, making their inclusion crucial.

B. Description and brief summary

To make the project of calculating the effect of Higgs bosons on the evolution of baryon asymmetries tractable we must make some simplifying assumptions. In brief they are the following: (1) The minimal SU(5) theory is used to calculate the various rates needed. (2) The C and CP violations are parametrized by $\epsilon/2$, the baryon excess produced by the decay of an $H,\bar{H}$ pair. (3) Only decays (D), inverse decays (ID), and annihilations of Higgs bosons will be considered. (4) Partial equilibrium will be assumed, i.e., B- and L-conserving interactions are assumed to be effective at maintaining thermal distributions, including the possibility of chemical potentials. (5) Maxwell-Boltzmann statistics are used. The notation and formalism is that of FOT, for more details we refer the reader there.

We find that the results depend on two parameters: $K \equiv (2.9\times 10^{17}\alpha_H\ \text{GeV}/M_H) \simeq$ (decay rate/expansion rate for $T \sim M_H$), and $\alpha^2/\alpha_H \simeq (\Gamma_A/\Gamma_D)$ at $T \sim M_H$, which is a measure of the importance of annihilations compared to decays. When C or CP is a good symmetry ($\epsilon = 0$), H bosons can only damp preexisting asymmetries. We find that this damping is independent of α^2/α_H and is given by $\sim\exp[-(0.3 \text{ to } 2)K] \simeq \exp[-(1 \text{ to } 6)\times 10^{17}\alpha_H\ \text{GeV}/M_H]$.

When both C and CP are violated ($\epsilon \neq 0$), H bosons can generate a baryon excess; for $K \ll \alpha_H/\alpha^2$, the asymmetry produced is independent of K and α^2/α_H, and is $kn_B/s = 2\times 10^{-3}\epsilon$. For $K \gg 1$, the final asymmetry produced is $kn_B/s = 2\times 10^{-3}\epsilon\,(3K)^{-1.2}$ independent of α^2/α_H. In the intermediate range, $\alpha_H/\alpha^2 < K \lesssim 1$, the production does depend weakly upon α^2/α_H. For physically interesting values of α_H and M_H ($\alpha_H \sim 10^{-4}$–10^{-6} and $M_H \sim 10^{14}$ GeV), K is in this intermediate range. In the minimal SU(5) theory $\alpha_H \gtrsim 7\times 10^{-5}$; for $M_H \gtrsim 3\times 10^{13}$ GeV H bosons do not significantly damp preexisting asymmetries and $\epsilon \sim 10^{-7}$ is needed to produce the observed asymmetry, $kn_B/s = 10^{-9.8\pm 1.6}$. Unfortunately, in the minimal SU(5) model (in which there are only $\underline{5}$ and $\underline{24}$ Higgs multiplets) ϵ is less than 10^{-10} (Ref. 4). However, with the addition of one more 5-plet of Higgs bosons ϵ can be as large as 10^{-2}.

In Sec. II the assumptions that we make are reviewed in detail and the "master equations" are derived. In Sec. III the damping of initial asymmetries by H bosons is considered when C or CP is a good symmetry ($\epsilon = 0$). In Sec. IV the evolution of the baryon asymmetry when C and CP are violated ($\epsilon \neq 0$) is discussed. In Sec. V a summary of our results is presented. The details of computing the necessary matrix elements are found in the Appendix.

II. MASTER EQUATIONS

Rather than discuss kinetic theory in the Friedmann universe again here, we refer the reader to Sec. II of FOT. We will employ the same notation and conventions here; in particular $\hbar = k_B = c = 1$.

A. Assumptions

When the density of the universe is dominated by relativistic species (as it was in the early universe), the expansion rate is given by

$$H \equiv \dot{R}/R = (8\pi G\rho/3)^{1/2}\,. \tag{2.1}$$

If all particle species are present in thermal numbers then $\rho = g_*(\pi^2/30)T^4$, where g_* is the effective number of degrees of freedom of all species ($g_* \equiv$ total number of Bose degrees of freedom $+\frac{7}{8}$ of the total number of Fermi degrees of freedom). For the minimal SU(5) model at very high temperatures ($T \gtrsim 10^{16}$ GeV) $g_* \cong 160$. The comoving time coordinate (age) and temperature of the universe are related by

$$\begin{aligned} t &= 0.301 m_P T^{-2} g_*^{-1/2} \\ &= 0.154(160/g_*)^{1/2} T_P/T^2\,, \end{aligned} \tag{2.2}$$

where m_P is the Planck mass ($G^{-1/2} = 1.22\times 10^{19}$ GeV), and $T_P = 1.88\times 10^{18}$ GeV is the temperature of the universe at the Planck time (0.538×10^{-43} sec) for $g_* = 160$.

The particle content of the minimal SU(5) theory is as follows: 3 generations of quarks and leptons; 24 gauge particles—$W^\pm$, Z, γ, 8 gluons and 12 superheavy bosons (X and Y, mass $M_X = M_Y \sim 10^{15}$ GeV), and a 24 and 5 of Higgs bosons (34 degrees of freedom in total), including an isosinglet, color triplet of superheavy Higgs bosons with charge $\pm\frac{1}{3}$(H, mass $M_H \lesssim M_X$) which mediate B and L violations. In this model the allowed C and CP violations are probably too small[4] to produce $kn_B/s \simeq 10^{-9.8}$, however, enlarging the theory to permit big enough C and CP violations only changes g_* and the number of superheavy species which mediate B and L violations. The change in g_* affects the expansion rate as $g_*^{1/2}$, and the effect of each superheavy species on the baryon asymmetry can be computed.

Rather than follow the evolution of all the particle species and consider all the interactions, we will follow only the fermions and the superheavy H bosons and will consider only interactions which involve the H bosons. We will assume that all other species are present in thermal numbers.

We will also assume partial equilibrium, that is, B- and L-conserving interactions are completely effective (rates $\Gamma >$ expansion rate H), so that all particle distributions are thermal with possible nonzero chemical potentials. This assumption is justifiable for two reasons: (i) one expects B- and L-conserving interactions to be effective for $T \lesssim 3 \times 10^{15}$ GeV (Refs. 5 and 6) and the temperatures of interest ($T \lesssim M_H \lesssim M_X \sim 10^{15}$ GeV) are probably less than 10^{15} GeV; and (ii) as we found in FOT, the final results are insensitive to this assumption.

Since no species is expected to be degenerate we will use Maxwell-Boltzmann statistics rather than Fermi-Dirac and Bose-Einstein which were used in FOT. With the assumption of partial equilibrium particle phase-space distributions can be written as

$$N_F(p,t) = F(t)\exp(-p/T)\,,$$
$$N_H(p,t) = H(t)a(T)^{-1}\exp[-(p^2+M_H^2)^{1/2}/T]\,, \qquad (2.3)$$

where $F = U$, D, L or ν, $p = |\vec{p}|$ and $a(T) = 0.5(M/T)^2K_2(M/T)$ (K_2 is a modified Bessel function of the second kind[7]). U, D, L, ν, and H refer to all uplike quarks (total degeneracy $g_U = 18 = 2$ spins$\times 3$ colors$\times 3$ generations), all downlike quarks ($g_D = 18$), all electronlike leptons ($g_L = 6$), all neutrinos ($g_\nu = 3$), and H bosons ($g_H = 3 = 3$ colors), respectively. The corresponding antiparticle distributions are denoted by $\overline{U}$, $\overline{D}$, $\overline{L}$, $\overline{\nu}$, and $\overline{H}$.

The number density of species i ($i = U$, D, L, ν, or H) per proper volume is

$$n_i = \int N_i(p,t)p_i^0 d\Pi_i = I(t)(g_i/\pi^2)T^3\,, \qquad (2.4)$$

where $d\Pi_i = (2\pi)^{-3}g_i\, d^3p_i/p_i^0$. The quantity $a(T)$ was chosen so that (2.4) also holds for H bosons, that is, $H(t)$ reflects the abundance of H bosons relative to their abundance if they were very relativistic (similar to the other species).

The following quantities turn out to be more useful: $U_\pm = U \pm \overline{U}$, $D_\pm = D \pm \overline{D}$, $L_\pm = L \pm \overline{L}$, $\nu_\pm = \nu \pm \overline{\nu}$, and $H_\pm = H \pm \overline{H}$. The assumption of partial equilibrium means that $U_+ = D_+ = L_+ = \nu_+ = 2$. However, we do not intend to extend the condition of partial equilibrium to the H bosons also. If H_+ were equal to $2a$ (the value it would have in equilibrium), there would be no baryon generation. We will follow the evolution of H_+ due to interactions (B and L conserving and nonconserving). For the H bosons the condition of partial equilibrium only means that those H bosons present are thermally distributed in energy (2.3).

B. Interactions

1. Decays and inverse decays

With the assumptions discussed above there are six quantities which evolve with time—U_-, D_-, L_-, ν_-, H_-, and H_+. The evolution of the $-$ quantities only involves interactions which do not conserve B and L. The only B- and L-violating interactions which we include are decays (D) and inverse decays (ID) of the H boson. The H boson has three decay channels $H \to \overline{U}\overline{D}, UL, D\nu$.

Baryon-nonconserving processes mediated by H bosons (e.g., $\overline{U}\overline{D} \to UL$) are not considered; these processes are $O(\alpha_H^2)$ while D and ID are $O(\alpha_H)$, and since $\alpha_H \sim 10^{-4} - 10^{-6}$ they are much less important (in FOT we found that for $\alpha \lesssim 10^{-2}$ they were negligible). In addition we are ignoring B- and L-violating processes due to other superheavy boson species. However, these processes are only effective ($\Gamma > H$) for $T \sim M$ (M = mass of superheavy boson), so we assume for now that we can treat the effect of each superheavy boson sequentially. The details of this procedure will be discussed in a third paper in this series.

2. Annihilations

The evolution of H_+ involves all interactions which allow the number of H bosons to change. The interactions which we consider are D, ID, and annihilations. The seven annihilation processes which we include are (i) $H + \overline{H} \rightleftharpoons U + \overline{U}$, (ii) $H + \overline{H} \rightleftharpoons D + \overline{D}$, (iii) $H + \overline{H} \rightleftharpoons L + \overline{L}$, (iv) $H + \overline{H} \rightleftharpoons \nu + \overline{\nu}$, (v) $H + \overline{H} \rightleftharpoons G + G$, (vi) $H + \overline{H} \rightleftharpoons \gamma + \gamma$, and (vii) $H + \overline{H} \rightleftharpoons \gamma + G$ (G = gluon). All seven of these processes are of order α^2 (α = gauge coupling constant, which we assume to be $\simeq \frac{1}{45}$).

The reason for including these processes here and not in FOT is simple; for $T \lesssim M_H$ the decay rate $\Gamma_D \sim \alpha_H M_H$, while the annihilation rate $\Gamma_A \sim \alpha^2 M_H (T/M_H)^3$. For $\alpha_H \lesssim \alpha^2 \approx 10^{-3}$, Γ_A is $\gtrsim \Gamma_D$. Since it is the nonequilibrium abundance of H bosons which permits baryon generation, the effect of annihilations could be crucial.

We have ignored the process $H \rightleftarrows hY$, where h = a light (at these energies massless) Weinberg-Salam Higgs boson, which is of order α. If, as expected, M_H is less than M_Y, then this process is kinematically forbidden, and the process $H \to h + 2$ fermions mediated by a virtual Y is of order α^2 and is also suppressed by a Y propagator.

On the other hand, if $M_H > M_Y$, then the process $H \to hY$ would be very important since $H \to hY$, $Y \to 2$ fermions is a two-step decay process of order α. However, in this case, the H boson effectively behaves like a Y boson (interactions have the same rates, etc.) and the results of FOT for gauge bosons are applicable.

Finally, we have also ignored the contact interactions involving two H bosons and two other particles, largely because the Yukawa couplings for these processes are unspecified even in the minimal SU(5) model. Even so, they are not expected to be important.

3. *C and CP violation*

In order for a baryon excess to evolve both C and CP must be violated in the decays of the H bosons. As in FOT we will parametrize these violations by ϵ, where $\epsilon/2$ = net baryon excess produced by the decay of a pair of $H, \bar{H}$ bosons. The details of how this is done in a manner consistent with unitarity are contained in Sec. III of FOT.

The computation of all the required matrix elements for D, ID, and annihilations is done in the Appendix. In computing these quantities we assume that C or CP is conserved, the requisite C and CP violations being added as discussed above.

C. The Boltzmann equations

Given the assumptions discussed in Subsection A the Boltzmann equations reduce to a set of coupled ordinary differential equations for U, D, L, ν, H, and their barred counterparts or $U_\pm$, $D_\pm$, $L_\pm$, $\nu_\pm$, and $H_\pm$. A typical equation is (see Secs. II and III of FOT for more details)

$$\dot{I}(t) = (g_i T^3/\pi^2)^{-1} \sum_{\substack{j,k,\cdots \\ l,m,\cdots}} \Big[L(t) M(t) \cdots \int \cdots \int W(l, m, \ldots \to i,j,k,\ldots) \exp(-E_T/T)\, d\Pi_i\, d\Pi_j\, d\Pi_k \cdots d\Pi_l\, d\Pi_m \cdots$$
$$- I(t) J(t) K(t) \cdots \int \cdots \int W(i,j,k \ldots \to l, m, \ldots)$$
$$\times \exp(-E_T/T)\, d\Pi_i\, d\Pi_j\, d\Pi_k \cdots d\Pi_l\, d\Pi_m \cdots \Big], \tag{2.5}$$

where $i,j,k,l,m,\ldots$ are U, D, L, ν, H or their barred counterparts (whenever H or $\bar{H}$ appear there should also be a factor of a^{-1}), $E_T = E_i + E_j + E_k + \cdots = E_l + E_m + \cdots$, T is the universal temperature and $W = 2^{-n}(m!)^{-1} |\mathfrak{M}|^2 (2\pi)^4 \delta^4(p_i + p_j + p_k + \cdots - p_l - p_m - \cdots)$. The matrix element $|\mathfrak{M}|^2$ is calculated according to the conventions of Bjorken and Drell,[8] n is the total number of incoming or outgoing particles and there is a factor of $(m!)^{-1}$ for each set of m *incoming or outgoing* identical particles.

The set of Eqs. (2.5) for our system is

$$U'_-/(zK) = -\gamma H_-/2 - a\gamma(UD)_-/2 - a\gamma(UL)_-/4 + 3\epsilon\gamma(H_+ - 2a)/4, \tag{2.6a}$$

$$D'_-/(zK) = (-\gamma + \tilde{\gamma}/2) H_- - a\gamma(UD)_-/2 - a\tilde{\gamma}(D\nu)_-/4 + 3\epsilon\gamma(H_+ - 2a)/8, \tag{2.6b}$$

$$L'_-/(zK) = 3\gamma H_-/2 - 3a\gamma(UL)_-/4 + 9\epsilon\gamma(H_+ - 2a)/8, \tag{2.6c}$$

$$\nu'_-/(zK) = 3\tilde{\gamma} H_- - 3a\tilde{\gamma}(D\nu)_-/2, \tag{2.6d}$$

$$H'_-/(zK) = -(3\gamma + 3\tilde{\gamma}) H_- - 3a\gamma(UD)_- + 3a\gamma(UL)_-/2 + 3a\tilde{\gamma}(D\nu)_-/2, \tag{2.6e}$$

$$H'_+/(zK) = -(9\gamma + 3\tilde{\gamma})(H_+ - 2a) - \gamma_A (H_+^2 - 4a^2), \tag{2.6f}$$

where $z = M_H/T$, the prime denotes d/dz, and $K = (2.9 \times 10^{17}\ \text{GeV}\, \alpha_H/M_H)(160/g_*)^{1/2}$. The dimensionless quantities γ, $\tilde{\gamma}$, and γ_A are related to the decay, inverse decay, and annihilation rates, $\Gamma_D \sim \gamma \alpha_H M_H$ or $\tilde{\gamma} \alpha_H M_H$, $\Gamma_{\rm ID} \sim a\gamma\alpha_H M_H$ or $a\tilde{\gamma}\alpha_H M_H$, $\Gamma_A \sim \gamma_A \alpha_H M_H$, and are given by

$$\gamma = \tfrac{1}{8}(M_H/T)a^{-1}\int_{M/T}^{\infty}[U^2-(M/T)^2]^{1/2}e^{-U}dU\,, \tag{2.7a}$$

$$\tilde{\gamma} = \gamma(\alpha_H'/\alpha_H)\,, \tag{2.7b}$$

$$\gamma_A = \tfrac{1}{3}(\alpha_H M_H)^{-1}(\pi^2/a^2T^3)\sum_{i=1}^{7}\int\int\int\int W_i(H\overline{H}\to 12)\exp[-(E_1+E_2)/T]d\Pi_H\, d\Pi_{\overline{H}}\, d\Pi_1\, d\Pi_2\,, \tag{2.7c}$$

$$\gamma_A \approx 1.34(\alpha^2/\alpha_H)(T/M_H)^4/[1+6(T/M_H)^3]+1.22(\alpha^2/\alpha_H)(T/M_H)^3/[1+3.41(T/M_H)^2]\,, \tag{2.7d}$$

where (2.7d) is obtained from (2.7c) by smoothly interpolating between the high- and low-temperature limits of (2.7c). The W_i in (2.7c) refer to the seven annihilation reactions discussed earlier.

From Eq. (2.6f) we see that the equilibrium abundance of H_+ is $2a$ (obtained by setting $H_+'=0$). For $T \gg M_H$, $2a \approx 2$ and for $T \ll M_H$, $2a \approx (\pi/2)^{1/2} \times (M_H/T)^{3/2}\exp(-M_H/T)$. When H_+ assumes its equilibrium value the asymmetry producing terms in (2.6a)–(2.6c) vanish as required by unitarity and CPT invariance.

In the minimal SU(5) model α_H is uniquely determined because the H bosons reside in a 5-plet with the Weinberg-Salam Higgs boson which generates fermion masses [as well as breaking down the SU(2)× U(1) symmetry]. In this case there are two couplings to each generation, $\lambda_\alpha = g(M_{D\alpha}/2M_W)$ and $\xi_\alpha = g(M_{U\alpha}/2M_W)$, where $g^2=4\pi\alpha$, M_W is the W mass, $M_{U\alpha}$ and $M_{D\alpha}$ are the current-algebra masses of the uplike and downlike fermions, and $\alpha = 1,2,3$ specifies the generation. Since first- and second-generation masses are small ($m_u \sim 5$ MeV, $m_d \sim 8$ MeV, $m_s \sim 150$ MeV, $m_c \sim 1$ GeV) compared to third-generation masses ($m_b \sim 5$ GeV, $m_t \gtrsim 15$ GeV), the H bosons primarily couple to the third generation. Since we treat generation as another degenerate degree of freedom the two generation-averaged coupling constants are $\alpha_\lambda = (\alpha/12)(m_b/M_W)^2$ and $\alpha_\xi = (\alpha/12)(m_t/M_W)^2$. In terms of these what we call the coupling constants are

$$\alpha_H = \alpha_\xi + \alpha_\lambda = (\alpha/12)(m_b^2+m_t^2)/M_W^2 \gtrsim 7\times 10^{-5}$$

and

$$\alpha_H' = (\alpha/12)(m_b^2/M_W^2) \simeq 7\times 10^{-6}\,.$$

In an arbitrary GUT, λ and ξ are not necessarily constrained in this way, and so there is greater lattitude in the possible values of α_H, α_H'. The range often quoted is $\alpha_H, \alpha_H' \sim 10^{-4}$–$10^{-6}$.

D. Numerical integration

We used a third-order Runge-Kutta scheme to integrate Eqs. (2.6a)–(2.6f). The integration began at the Planck time, $z = M_H/T_P$, with $H_+ = 2a(T_P) \approx 2$ (equilibrium value) and $H_- = 0$. U_+, D_+, L_+, and ν_+ are always 2. The initial fermion asymmetries which were used are discussed in the text. More details on the numerical intergration are given in FOT Sec. III.

III. C OR CP CONSERVED ($\epsilon = 0$)

If the interactions of the H bosons conserve C or CP, then $\epsilon = 0$ and the Higgs bosons can only damp preexisting asymmetries. Since annihilations conserve both B and L, the master equations for the − quantities (2.6a)–(2.6e) do not directly depend upon α^2/α_H (the relative effectiveness of annihilations). They indirectly depend upon α^2/α_H since the asymmetry-producing terms are proportional to $\epsilon(H_+ - 2a)$ and the evolution of H_+ depends upon annihilations and α^2/α_H, but when $\epsilon = 0$ this coupling does not exist, and the − equations do not depend upon annihilations at all. Thus any damping is independent of α^2/α_H.

Since we have not considered any baryon-non-conserving processes of order $\geq O(\alpha_H^2)$, e.g., fermion collisions, fermion-Higgs collisions (Compton-type) or Higgs-Higgs to fermion-fermion collisions (annihilationlike), the damping which occurs is due solely to the two-step process ID and D: for example, $UD \to \overline{H}$, $\overline{H} \to \overline{U}\overline{L}$ ($\Delta B = -1$, $\Delta L = -1$).

A. Initial $B-L \neq 0$

Because the − equations are linear and homogeneous, all results simply scale with the initial asymmetry. If the initial asymmetry has $B-L \neq 0$, then both B and L can never be reduced to zero since the interactions of the H exactly conserve $B-L$. As we discussed in FOT (Sec. IV) when the initial asymmetry has $B-L \neq 0$, the actions of the H bosons can at most redistribute the initial B and L asymmetries (if $K \gtrsim 1$), with the final asymmetries being of the same order of magnitude as the initial $B-L$ asymmetry.

B. Initial $B-L=0$

If the initial asymmetry has $B-L=0$, then no such restrictions apply (i.e., both B and L can be reduced to zero); this is the class of initial asymmetries we will now discuss in detail. In Fig. 1 the time evolution of the baryon asymmetry is shown for $K=0.2$ and 10 and for $\alpha'_H/\alpha_H=0.1$ and 1.0. Most of the damping occurs near $T\sim M_H$. This is because for $T\gg M_H$, both decays and inverse decays are suppressed by the time dilation ($\Gamma_D\sim\Gamma_{ID}$) $\sim\alpha_H M_H^2/T$); for $T\ll M_H$, inverse decays are blocked since typical fermion pairs ($E\sim T\ll M_H$) are not energetic enough to produce an H boson. As $K(=2.9\times10^{17}\alpha_H/M_H\simeq\Gamma_D/H$ for $T\simeq M_H$) increases, the damping increases markedly.

Figure 2 shows the damping factor $(kn_B/s)_f/(kn_B/s)_i$, as a function of K for $\alpha'_H=0.1\alpha_H$ and $\alpha'_H=\alpha_H$. Near $K\simeq1$ the slope changes; this is easily understood. If one assumes that the rates are rapid enough so that $H'_-\approx0$ (reasonable for $K\gtrsim1$), then the equations for U_-, D_-, L_-, and ν_- form a set of four coupled, linear ordinary differential equations. They are most easily analyzed in terms of their eigenvalues and eigenmodes,

$$A'_j=-(zKa\gamma)\lambda_jA_j\ ,$$
$$A_j(z)=A_j(0)\exp\left[-\lambda_jK\int_0^z z'a\gamma\,dz'\right]\ , \tag{3.1}$$

where the A_j are linear combinations of U_-, D_-, L_-, and ν_-, and $j=1$–4. Two of the eigenvalues, λ_1 and λ_2, are zero corresponding to the conserved quantitites $B-L$ and Q ($\equiv$total charge).

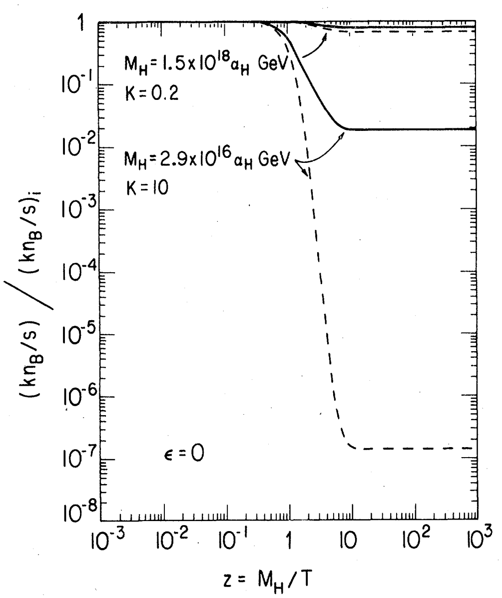

FIG. 1. The time evolution ($z\sim t^{1/2}$) of an initial asymmetry with $B-L=0$ is shown for $\epsilon=0$ (C or CP conserved). Results are presented for $K\equiv2.9\times10^{17}\alpha_H/M_H=0.2$ and $K=10$; the solid curves are for $\alpha'_H=0.1\alpha_H$ and the broken curves are for $\alpha'_H=\alpha_H$ (see Sec. III). The damping occurs mostly near $z=1$ and is due solely to inverse decays. The results are independent of α^2/α_H.

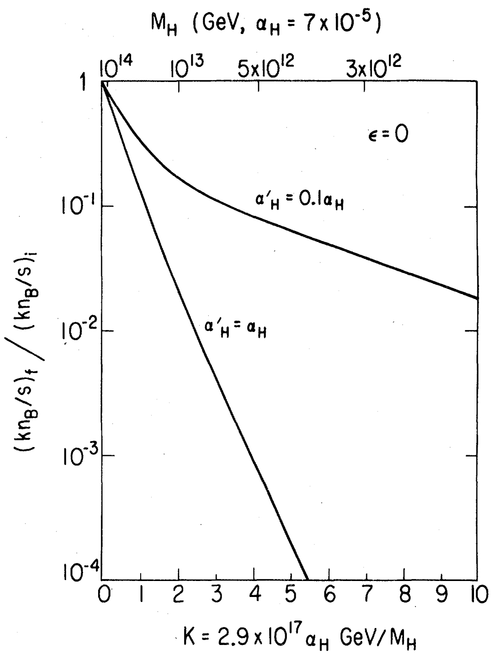

FIG. 2. The ratio of final to initial asymmetry (damping) with initial $B-L=0$ and $\epsilon=0$ is shown as a function of K for $\alpha'_H=0.1\alpha_H$ and $\alpha'_H=\alpha_H$. The change in slope near $K=1$ is due to the multiple modes of damping (see Sec. III). For large K, the damping (slowest mode) is $\sim\exp(-0.26K)$ for $\alpha'_H=0.1\alpha_H$ and $\sim\exp(-1.4K)$ for $\alpha'_H=\alpha_H$.

The evolution of the most general initial asymmetry with $B-L=Q=0$ can be written as

$$(kn_B/s)=(kn_B/s)_i\left\{C_3\exp\left[-\lambda_3K\int_0^z z'a(z')\gamma(z')dz'\right]+C_4\exp\left[-\lambda_4K\int_0^z z'a(z')\gamma(z')dz'\right]\right\}, \tag{3.2}$$

$$(kn_B/s)_f/(kn_B/s)_i=C_3\exp(-I\lambda_3K)+C_4\exp(-I\lambda_4K)\ ,$$

where C_3 and C_4 are the projections of the initial asymmetry in the 3 and 4 eigenmodes ($C_3+C_4=1$), and $I=\int_0^\infty a\gamma z\,dz\approx0.59$ has been evaluated numerically. The eigenvalues λ_3 and λ_4 depend upon α'_H/α_H. These coupling constants are given by $\alpha_H=\alpha_\lambda+\alpha_\xi$ and $\alpha'_H=\alpha_\lambda$. The two Higgs couplings α_λ and α_ξ are discussed in Sec. II (C), in the minimal SU(5) model with $m_t=15$ GeV, $\alpha_H=7\times10^{-5}$ and $\alpha'_H=7\times10^{-6}$ so that $\alpha'_H/\alpha_H\approx0.1$. In a more general theory α'_H/α_H can vary between 0 and 1.

For $\alpha'_H/\alpha_H=0.1$, $\lambda_3=2.68$, and $\lambda_4=0.45$, so that for small K the damping should be $\sim\exp(-4.6\times10^{17}\alpha_H/M_H)$ and for large $K\sim\exp(-7.7\times10^{16}\alpha_H/M_H)$. This agrees well with the slopes of the curve in Fig. 2. The big difference between the two eigenvalues λ_3 and λ_4 (a factor of 6) occurs because U, D, and L asymmetries are damped by interactions of $O(\alpha_H)$, while the interactions which damp ν asymmetries are of $O(\alpha'_H) \sim 0.1\, O(\alpha_H)$. The $\lambda_3 = 2.68$ eigenmode has only U, D, and L asymmetries, while the $\lambda_4 = 0.45$ eigenmode also has a ν asymmetry.

In the other extreme, $\alpha'_H/\alpha_H = 1.0$, the eigenvalues are $\lambda_3 = 3.68$ and $\lambda_4 = 2.45$, so that for small K the damping should be $\sim \exp(-6.3\times 10^{17}\alpha_H/M_H)$ and for large $K \sim \exp(-4.2\times 10^{17}\alpha_H/M_H)$. This agrees well with the slopes of the curve for $\alpha'_H/\alpha_H = 1$ in Fig. 2.

With α'_H/α_H between 0.1 and 1.0, the most general initial asymmetry (i.e., one which is not an eigenmode of the set of coupled equations) is reduced by $\sim \exp[-(5 \text{ to } 6)\times 10^{17}\alpha_H/M_H]$ for $K \lesssim 1$, and by $\sim \exp[-(0.8 \text{ to } 4)\times 10^{17}\alpha_H/M_H]$ for $K > 1$. In the minimal SU(5) model where $\alpha_H = 7\times 10^{-5}$ and $\alpha'_H/\alpha_H = 0.1$, when $M_H > 10^{13}$ GeV the damping is given by $\sim \exp(-3\times 10^{13}\ \text{GeV}/M_H)$, which is small (less than a factor of ~ 10). That is, Higgs bosons, unless they are very light, do not tend to damp preexisting asymmetries because of their weak coupling strength.

C. Possible complication

Above 100 GeV when the $SU(2)_L \times U(1)$ symmetry is restored and fermions are massless, the right- and left-helicity states of a particle are distinct and separate species. No interaction can transform one helicity state into the other. In this paper rather than consider seven species U_L, U_R, D_L, D_R, L_L, L_R, and ν_L, we have averaged over helicity states and considered only four species U, D, L, and ν.

There is one situation where we might run into difficulties with this procedure—finding the eigenmodes for damping asymmetries. This has been recently discussed by Treiman and Wilczek.[9] They point out that for gauge bosons there are seven damping modes, four of which have zero eigenvalues. These zero eigenvalues correspond to conservation laws; charge, weak isospin, $B-L$, and 5-ness. The first three are expected; the last, 5-ness, results because the interactions of the gauge bosons *do not* change the net number of particles in the $\underline{5}$ representation of SU(5)—ν_L, L_L, and $\overline{D}_L$. Because of this, gauge bosons *cannot* damp initial symmetries with net 5-ness, just as they cannot damp asymmetries with $B-L \neq 0$. (Note that one does not expect asymmetries with net charge or weak isospin since both these quantities are gauged.)

For the Higgs system we have derived the set of master equations for the seven quantities, U_{-L}, U_{-R}, etc., and then solved for the seven eigenvalues. We find that there are three zero eigenvalues corresponding to charge, weak isospin, and $B-L$. The interactions of the Higgs bosons *do not* have an additional hidden symmetry like 5-ness. By averaging over helicity states we lose two nonzero eigenvalues. However, the two we find are very representative of the four; for example, with $\alpha'_H = 0.1\,\alpha_H$ the four eigenvalues are 0.33, 0.40, 1.33, 3.67, and the two we obtain are 0.45, 2.68. The only asymmetries which Higgs bosons cannot erase have nonzero values of charge, weak isospin, or $B-L$.

IV. C AND CP VIOLATED ($\epsilon \neq 0$)

A. No initial asymmetry

The case where Higgs-boson decays violate both C and CP allows a baryon asymmetry to arise in an initially symmetrical universe. In most respects the baryon-violating Higgs particles are very similar to X- and Y-gauge bosons, with one potentially important quantitative difference. In the gauge system, the only important reactions are decays and inverse decays. Baryon-nonconserving fermion scatterings mediated by virtual X and Y bosons amount to only small corrections, and other second-order processes such as annihilations and Compton-type scatterings of gauge particles and fermions are even less important.[5,10] In the Higgs-boson system, however, annihilations cannot be neglected. This is because decays and inverse decays of Higgs particles proceed at a rate determined by $\alpha_H \simeq 10^{-4} - 10^{-6}$, while their annihilations are governed by $\alpha^2 = (\frac{1}{45})^2 = 5\times 10^{-4}$, since the Higgs particles we consider possess charge and color. The differences between the Higgs-boson and gauge systems caused by this are discussed below.

From (2.6a)–(2.6d), we see the source for generation of a baryon excess is proportional to the size of the CP violation ϵ and also $\Delta \equiv H_+ - 2a$, the amount by which the total number of H and $\overline{H}$ bosons differs from thermal equilibrium; $2a$ is the equilibrium value of H_+; for $z \ll 1$, $2a \simeq 2$, and for $z \gg 1$, $2a \simeq (\pi/2)^{1/2} z^{3/2} e^{-z}$. The source term Δ/H_+ is plotted in Fig. 3 as a function of $z = M_H/T$, while Fig. 4 shows the generated baryon excess kn_B/s also as a function of z, for three values of K. For small z, when annihilations are negligible, $2a \simeq 2(1-z^2/4)$ while $H_+ = 2 - O(z^5)$. Thus, $\Delta/H_+ \simeq z^2/4$. This is the source of the rise in Δ seen in Fig. 3 for $z \lesssim 1$.

For $z \gtrsim 1$, there are two cases depending on the ratio of reaction rates to expansion rate at $T = M_H$. If the mass is large or the coupling strength is small, more precisely, $K \ll 1$, then all rates are small and $H_+ = 2$ until $Kz^2 \simeq 1$. Meanwhile,

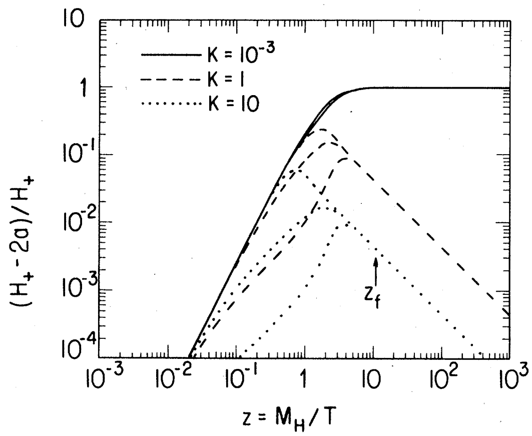

FIG. 3. The time evolution ($z \sim t^{1/2}$) of the source term for baryon generation, $\Delta = H_+ - 2a$, is shown normalized by H_+. The solid curves are for $K = 10^{-3}$, the broken curves for $K = 1$, and the dotted curves for $K = 10$; all values of K show the cases $\alpha^2/\alpha_H = 0$, 7, and 100 (the solid curves are indistinguishable). For $K \ll 1$, $H_+ \simeq 2$ until $Kz^2 \simeq 1$, while $2a$ falls exponentially to zero for $z \gtrsim 1$, so $\Delta/H_+ \to 1$. For $K \gtrsim 1$, decay rates are large enough to keep $H_+ \simeq 2a$ (Δ/H_+ is always small). Here, for $z \lesssim 1$, $\Delta/H_+ \simeq 1 - a = z^2/4$; for $z \gtrsim 1$, $\Delta/H_+ \simeq 1/9\gamma K z$ ($\gamma \simeq \frac{1}{4}$). Near $z = 1$ annihilations can reduce Δ, but they soon become ineffective relative to decays and the three curves merge. For K large enough, this happens before inverse decays freeze out (z_f for $K = 10$ is indicated). For further discussion see Sec. IV.

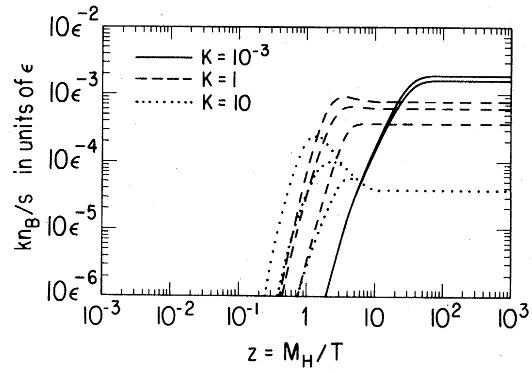

FIG. 4. The time evolution ($z \sim t^{1/2}$) of the baryon asymmetry kn_B/s generated when C and CP both are violated is show in units of ϵ for zero initial asymmetry. The solid curves are for $K = 10^{-3}$, the broken curves for $K = 1$, and the dotted curves for $K = 10$; for all values of K we show the cases $\alpha^2/\alpha_H = 0$, 7, and 100. For $K \ll 1$ the asymmetry is produced by late ($z \gtrsim K^{-1/2}$) free decays. For $K \gtrsim 1$, there is an initial period of nonequilibrium growth ($z \lesssim 1, kn_B/s \sim z^5$), followed by a quasiequilibrium state [$z \gtrsim 1, kn_B/s \sim (Kz)^{-1}$] which freezes out at z_f ($Kz_f^{7/2} e^{-z_f} = 1$). For $K \ll 1$, annihilations are always ineffective (corrections of order $K\alpha^2/\alpha_H \sim 10\%$ for $K = 10^{-3}$, $\alpha^2/\alpha_H = 100$), and for $K \gg 1$ annihilations become ineffective before quasiequilibrium freezes out. Only for $\alpha_H/\alpha^2 \lesssim K \lesssim 1$ is the final kn_B/s affected by annihilations.

the equilibrium value of H_+ drops as $2a \simeq (\pi/2)^{1/2} z^{3/2} e^{-z}$, and becomes effectively zero. Thus, for $z \gtrsim 1$, $\Delta \cong H_+$. This is the behavior of the solid curve(s) in Fig. 3, for $K = 10^{-3}$.

Numerically, in this case the behavior of H_+ is obtained from (2.6f), setting $2a = 0$ and ignoring annihilations,

$$H_+ = 2\exp(-9\gamma K z^2/2) = 2\exp(-t/\tau_H), \qquad (4.1)$$

where $\gamma \simeq \frac{1}{4}$ ($z \gtrsim 1$) and the decay rate $\Gamma_D \sim \gamma\alpha_H M_H$. We obtain $(kn_B/s) = 8 \times 10^{-3}(U_- + D_-)$ from (2.6a) and (2.6b); again setting $2a = 0$, we find

$$(kn_B/s) = 2 \times 10^{-3}\epsilon[1 - \exp(-9\gamma K z^2/2)]\,. \qquad (4.2)$$

The final, saturation value $(kn_B/s) = 2 \times 10^{-3}\epsilon$ is the largest than can be produced by this system. The effect of $H - \overline{H}$ annihilations here is to reduce H_+ slightly when $z \simeq 1$; this is the only time when annihilations can be effective since for large z $\gamma_A \sim z^{-3}$ (annihilation rate $\Gamma_A \sim \gamma_A \alpha_H M_H H_+$). The amount by which annihilations can reduce H_+ is of order $K\alpha^2/\alpha_H$, which is 10% for $K = 10^{-3}$, $\alpha^2/\alpha_H = 100$. This behavior is seen in the solid curves in Fig. 4, which show kn_B/s vs. z for $K = 10^{-3}$ and $\alpha^2/\alpha_H = 0$, 7, and 100 (the first two are indistinguishable).

For $K \gtrsim 1$ decays happen rapidly enough to keep H_+ always near its equilibrium value $2a$. The broken ($K = 1$) and dotted ($K = 10$) curves in Fig. 3 show that even for $K = 1$, Δ/H_+ is never more than about 25%. Thus we expect the baryon excess produced here to be smaller. Again for $z \lesssim 1$, $\Delta \simeq z^2/4$. Ignoring annihilations (2.6f) can be written as $\Delta' = -9\gamma K z \Delta - 2a'$ for $z \gtrsim 1$ where $\gamma \simeq \frac{1}{4}$. Recall, for large z, a falls off as e^{-z}, so the asymptotic solution to this is $\Delta = 2a/9\gamma K z + O(a/z^2)$, but when they are effective, annihilations can reduce the value of Δ significantly. The curves in Fig. 3 show this for $\alpha^2/\alpha_H = 0$, 7, and 100.

Again, given Δ from (2.6a) and (2.6b) we can find kn_B/s. Near $z = 1$, both production and destruction rates in (2.6a) and (2.6b) are large, and U_- and D_- will adjust so that the positive and negative terms nearly cancel (U_-' and $D_-' \simeq 0$). We obtain

$$(kn_B/s)=3\times10^{-3}\epsilon\,\Delta/a \tag{4.3}$$

until inverse decays are no longer effective. This occurs at z_f such that $K\,z_f^{7/2}\exp(-z_f)=1$, and kn_B/s "freezes out" at its value at $z=z_f$ ($z_f=10.5$ for $K=10$).

When annihilations are effective, kn_B/s can be much smaller for $z\lesssim z_f$ because Δ, the source term, is smaller; H_+ is nearer to equilibrium. However, if K is large enough, a curious thing happens. The effectiveness of annihilation relative to decay (Γ_A/Γ_D) behaves as $\alpha^2/\alpha_H\, z^{-3/2}\exp(-z)$, and when this becomes less than 1 they are not important. [Annihilation requires two Higgs bosons while decay requires only one—as Higgs bosons become scarce annihilations become rare compared to decays by a factor of their number density $\sim\exp(-z)$]. In Fig. 3, we see that for K large enough (such that $z_f^5>\alpha^2/\alpha_H$) this occurs before inverse decays freeze out, and kn_B/s attains the quasiequilibrium value it would have had if annihilations were not considered at all.

Thus we have seen that the differences caused by large annihilation rates compared to decay rates are only manifested for intermediate values of K; for small K there are corrections of order $K\alpha^2/\alpha_H$ and for large K no difference is seen at all.

The final value of kn_B/s generated is shown as a function of K in Fig. 5 for $\alpha^2/\alpha_H=0$, 7, 100, and 500. For $K<\alpha_H/\alpha^2$ and 1 the saturation value $2\times10^{-3}\epsilon$ is produced, while for $K\gg1$ the production, about $8\times10^{-3}\epsilon/3Kz_f$, is reasonably well fit by $(kn_B/s)=2\times10^{-3}\epsilon\,(M_H/M_c)^{1.2}$, with $M_c=1\times10^{18}\,\alpha_H$ GeV for $10\lesssim K\lesssim100$. Interpolating, we approximate

$$(kn_B/s)\cong2\times10^{-3}\epsilon(M_H/M_c)^{1.2}/[1+(M_H/M_c)^{1.2}]\,. \tag{4.4}$$

Annihilations reduce the final baryon excess for $K\simeq1$, but for the best value in SU(5) $\alpha^2/\alpha_H=7$, these corrections are minimal, and even for $\alpha^2/\alpha_H=500$ the reduction is never as much as a factor 10.

For a Higgs-boson mass in the range 10^{14}–10^{15} GeV, $\alpha_H=7\times10^{-5}$ and $\alpha^2/\alpha_H=7$, the values of K (0.02–0.2) are in the saturation regime, and the observed $(kn_B/s)\simeq10^{-9.8}$ could be produced by Higgs bosons for $\epsilon\simeq10^{-7}$.

B. With an initial asymmetry

When a baryon asymmetry exists before $T=M_H$, perhaps one generated by gauge bosons, the Higgs bosons can both add to it by CP violations in their own system and damp the initial asymmetry by their decays and inverse decays.

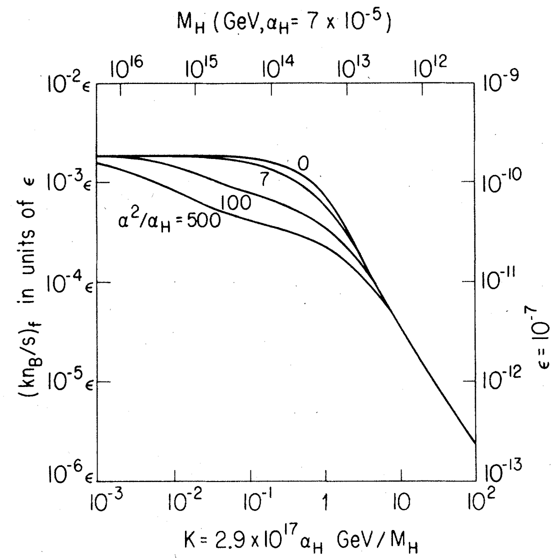

FIG. 5. The final baryon asymmetry kn_B/s in units of ϵ is shown as a function of K when the initial asymmetry is zero for $\alpha^2/\alpha_H=0$, 7, 100, and 500. For $K\ll1$, delayed free decays produce $kn_B/s=2\times10^{-3}\epsilon$. For $K\gtrsim1$, a period of quasiequilibrium freezes out with $(kn_B/s)\simeq3\times10^{-3}\epsilon/Kz_f$ ($Kz_f^{7/2}e^{-z_f}=1$). For $\alpha_H/\alpha^2\lesssim K\gtrsim1$, H-$\bar{H}$ annihilations can keep the Higgs bosons closer to equilibrium and reduce the baryon asymmetry generated, but even for $\alpha^2/\alpha_H=500$ this reduction is never as much as by a factor of 10, and for $K\alpha^2/\alpha_H\lesssim1$ and $K\gg1$ the final results are essentially independent of annihilations.

Figure 6(a) shows the time dependence of kn_B/s for $K=0.2$ and initial asymmetries of $2\times10^{-2}\epsilon$, $2\times10^{-3}\epsilon$, $2\times10^{-4}\epsilon$, and zero [recall, $2\times10^{-3}\epsilon$ is the maximum (saturation) asymmetry that can be produced by decays of H bosons]; Fig. 6(b) shows the same initial conditions with $K=10$. For $K=0.2$ the damping is small and the final asymmetry is just given by the initial asymmetry plus $2\times10^{-3}\epsilon$, the asymmetry produced by the decays of the H bosons. However, for $K=10$ there is substantial damping of the initial asymmetry and the final asymmetry is nearly independent of the size of the initial asymmetry. In general, since the set of equations is linear in the − quantities we expect the final asymmetry to be given by

$$(kn_B/s)_f=(kn_B/s)_i\exp(-0.3K\text{ to }-2.0K)+2\times10^{-3}\epsilon/[1+(3K)^{1.2}], \tag{4.5}$$

where the exponent of the damping factor was discussed in Sec. III. This is verified in Fig. 7 which shows $(kn_B/s)_f$ as a function of K for $(kn_B/s)_i=2\times10^{-2}\epsilon$ and $(kn_B/s)_i=0$.

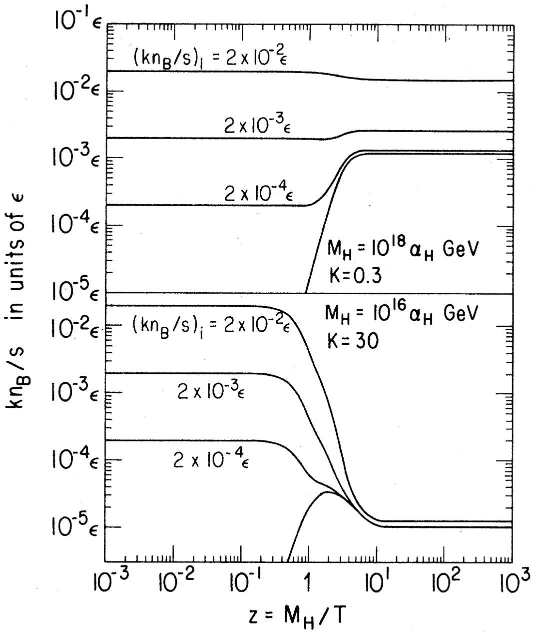

FIG. 6.(a) The time evolution ($z \sim t^{1/2}$) of the baryon asymmetry for $K=0.3$ (for $\alpha_H=7\times10^{-5}$, $M_H=7\times10^{13}$ GeV) is shown in units of ϵ for initial asymmetries of $2\times10^{-2}\epsilon$, $2\times10^{-3}\epsilon$, $2\times10^{-4}\epsilon$, and zero. Little damping occurs, and the final asymmetry is essentially the initial value plus $2\times10^{-3}\epsilon$ generated by delayed free decays. (b) The time evolution of kn_B/s is shown in units of ϵ for $K=30$ (for $\alpha_H=7\times10^{-5}$, $M_H=7\times10^{11}$ GeV) for the same initial conditions as above. The final asymmetry is essentially independent of the initial value, and equal to the value generated for zero initial asymmetry.

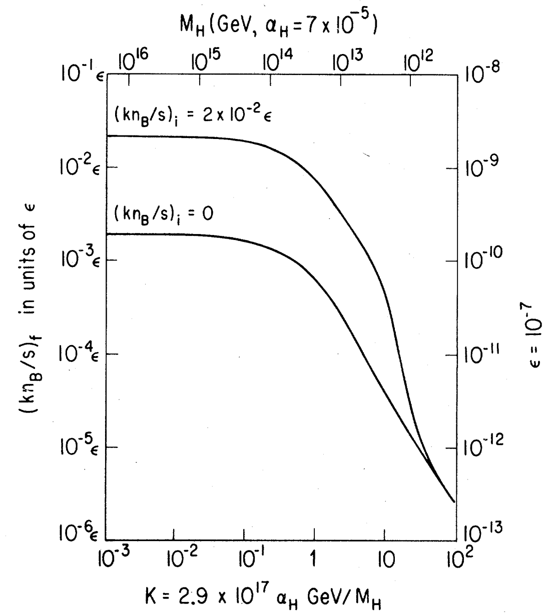

FIG. 7. The final baryon asymmetry kn_B/s in units of ϵ is shown as a function of K for $\alpha^2/\alpha_H=7$, with initial asymmetries of $2\times10^{-2}\epsilon$ (ten times the maximum asymmetry that can be generated by decays) and zero. For $K \ll 1$, the initial asymmetry is damped only slightly, and the excess produced by H decay adds to the initial asymmetry but is insignificant. For $K \gg 1$, the initial asymmetry is damped sufficiently so that the final value is independent of the initial conditions.

V. CONCLUSIONS

As one would expect, the evolution of baryon asymmetries due to the actions of Higgs bosons and gauge bosons are qualitatively very similar. In the Higgs-boson system, when C and CP are not both violated, no baryon excess is produced and any initial asymmetry is reduced. For an arbitrary initial asymmetry, the ratio of final to initial baryon excess is $\exp(-0.3K$ to $-2.0K)$. This is less than the amount of damping we would expect by just using the results for gauge bosons (in FOT) and substituting $\alpha\to\alpha_H$ and $M\to M_H$, which predicts damping by $\sim\exp(-5.5K)$. This is easily understood; there are two species of gauge bosons (X and Y), and each species has three helicity states. These effects increase the rate of inverse decays which are responsible for the damping by a factor of about 6.

When $\epsilon \neq 0$, the saturation asymmetry for the Higgs boson (achieved for $K \ll 1$) is $(kn_B/s) = 2\times10^{-3}\epsilon$, while for gauge bosons it was $8\times10^{-3}\epsilon$. The difference again is accounted for by the statistical factor of three spin states for the vector gauge particles, combined with the difference that here we have used Maxwell-Boltzmann statistics (when quantum statistics are used there are relatively $\frac{4}{3}$ as many bosons as there are fermions).

For large K, the situation is more complicated as the quasiequilibrium state depends on both decay and inverse decay reaction rates, and both systems give the result $(kn_B/s)\simeq 3\times10^{-3}\epsilon/Kz_f$. For intermediate values of K ($K\sim1$) in the Higgs-boson case annihilations slightly reduce the final baryon excess by allowing the number of H bosons to remain closer to the equilibrium value. For plausible values of α^2/α_H, the relative effectiveness of annihilations compared to decays, this reduction is less than a factor of 10. For all values of K and $\alpha^2/\alpha_H \lesssim 10$, the production is well represented by $kn_B/s \simeq 2\times10^{-3}\epsilon/[1+(3K)^{1.2}]$. With $\alpha_H = 7\times10^{-5}$ and $M_H > 3\times10^{-13}$ GeV the observed $kn_B/s \simeq 10^{-9.8}$ can be produced by Higgs bosons alone if $\epsilon \simeq 10^{-7}$.

Because the Higgs bosons are expected to be lighter than the gauge bosons of the theory, it has often been said that the Higgs bosons alone will determine the final kn_B/s of the universe since their effect on kn_B/s will occur when $T \simeq M_H$, after the actions of all the other superheavy gauge bosons have occurred. However, for $\alpha_H \lesssim 10^{-4}$ and $M_H \gtrsim 5\times10^{13}$ GeV we have $K \lesssim 0.6$ (for $\alpha_H = 7\times10^{-5}$ and $M_H = 10^{14}$ GeV $K = 0.02$), which is in the saturation production range and leads to *very little* damping of preexisting asymmetries, less than a factor of 4. Thus we expect the baryon excess produced by Higgs bosons to simply add on to that produced by gauge bosons. Only if the Higgs boson is *very* light ($M_H < 10^{13}$ GeV) or if α_H is *big* ($>10^{-4}$) will K be large enough so that preexisting asymmetries will be erased by the Higgs bosons. In this situation its actions alone will determine the final value of kn_B/s.

ACKNOWLEDGMENTS

We are pleased to acknowledge the useful discussions we have had with M. Crawford, T. Curtright, H. Georgi, S. Love, M. Machacek, and D. Schramm. This work was supported in part by M.S.T.'s Fermi Fellowship, K.A.O.'s Hertz Fellowship, J.N.F.'s McCormick Fellowship, NSF Grants Nos. AST 78-20402 and AST 78-20392.

APPENDIX

The superheavy boson H which mediates B- and L-nonconserving interactions in the most general theory which breaks down to $SU(3)_c \times SU(2)_L \times U(1)$ is an isosinglet, color triplet with charge $\pm\frac{1}{3}$. In the minimal SU(5) theory its couplings to B- and L-violating currents are given by the interaction Lagrangian[11]

$$\mathcal{L} = \sum_\alpha [(\lambda_\alpha/\sqrt{2})H_k(\epsilon_{ijk}\bar{d}_{i\alpha}u^c_{j\alpha L} + e^+_\alpha u_{k\alpha L} + \bar{\nu}^c_\alpha d_{k\alpha L}) + (\xi_\alpha/\sqrt{2})H^c_k(-\epsilon_{ijk}\bar{u}^c_{j\alpha}d_{i\alpha L} + \bar{u}_{k\alpha}e^+_{\alpha L})] + \text{H.c.}, \quad \text{(A1)}$$

where α is the generation index ($\alpha = 1,2,3$); i,j,k, are color indices, and superscript c denotes charge conjugation. The coupling strengths λ_α and ξ_α are given by

$$\lambda_\alpha = M_{D\alpha}g/2M_W, \quad \xi_\alpha = M_{U\alpha}g/2M_W, \quad \text{(A2)}$$

where $M_{U\alpha}$ and $M_{D\alpha}$ are the current-algebra masses of the up and down quarks of the specified generation, M_W is the mass of the W boson $\simeq 80$ GeV, and $g^2 = 4\pi\alpha$ (α = gauge coupling constant). The u and d masses are ~5 MeV and ~8 MeV; the c and s masses are ~1 GeV and ~150 MeV; the b mass is ~5 GeV, while the t mass is $\gtrsim 15$ GeV. In any other theory which breaks down to $SU(3)\times SU(2)\times U(1)$ the interaction Lagrangian will have the same structure, however, the coupling strengths λ and ξ might be very different.

1. Decays and inverse decays

From (A1) it follows that the decay channels of the H are $\bar{U}_\alpha + \bar{D}_\alpha$, $U_\alpha + L_\alpha$, and $D_\alpha + \nu_\alpha$. The matrix elements for these processes averaged over initial and final spins, colors, and generations are needed. Since the coupling constants λ and ξ are very mass dependent, and the first- and second-generation masses are so light, only third-generation processes will be included (although averaging is done over all three generations).

(*i*) $H \to \bar{U}_\alpha + \bar{D}_\alpha$. The matrix element for this process is given by

$$|\mathfrak{M}|^2 = (\pi\alpha_H/27)M_H^2, \quad W = (\pi\alpha_H/216)M_H^2(2\pi)^4\delta^4\left(\sum p\right), \quad \text{(A3)}$$

where

$$\alpha_H \equiv (\alpha/12)(m_t^2 + m_b^2)/M_W^2.$$

(*ii*) $H \to U_\alpha + L_\alpha$. The matrix element for this process is given by

$$|\mathfrak{M}|^2 = (\pi\alpha_H/18)M_H^2, \quad W = (\pi\alpha_H/144)M_H^2(2\pi)^4\delta^4\left(\sum p\right). \quad \text{(A4)}$$

(*iii*) $H \to D_\alpha + \nu_\alpha$. The matrix element for this process is given by

$$|\mathfrak{M}|^2 = (\pi\alpha'_H/9)M_H^2, \quad W = (\pi\alpha'_H/72)M_H^2(2\pi)^4\delta^4\left(\sum p\right), \quad \text{(A5)}$$

where

$$\alpha'_H \equiv (\alpha/12)m_b^2/M_W^2.$$

2. Annihilations

Because H bosons are charged and colored they can annihilate via photons and gluons (above ~100 GeV, the U(1) gauge particle is not the same as the U(1) gauge particle below 100 GeV—the photon). More exotic annihilation channels involving the B- and L-violating couplings of the H are much less important since they are second order in α_H rather than α. The annihilation channels of the H are $H + \bar{H} \to U + \bar{U}$, $D + \bar{D}$, $L + \bar{L}$, $\nu + \bar{\nu}$, $G + G$, $\gamma + \gamma$, $\gamma + G$, where G is a gluon. Note that since the H boson is an isosinglet it does not couple to the W bosons. In any theory which breaks down to $SU(3)\times SU(2)\times U(1)$ the necessary couplings are contained in the strong SU(3) and weak U(1) parts of the Lagrangian,[12]

$$\mathcal{L}=gG_\mu \sum_q \bar{q}\gamma^\mu \tfrac{1}{2}\lambda q + (\tfrac{3}{5})^{1/2} g B_\mu \sum_f \bar{f}\gamma^\mu (Q_f - I_{3f}) f\,, \qquad \text{(A6)}$$

where q is a quark, f is a fermion, Q_f is its charge, and I_{3f} its third component of weak isospin.

(i) and (ii) $H+\bar{H}\to U+\bar{U}$ *or* $D+\bar{D}$. For each of these processes there are two Feynman diagrams—one with a virtual photon and one with a virtual gluon. Also note that the effective charge ($\equiv Q_f - I_{3f}$) is different for the right- and left-helicity states. When the appropriate averaging has been done the matrix elements for the $U+\bar{U}$ and $D+\bar{D}$ channels are, respectively.

$$|\mathfrak{M}|^2 = 0.137\pi^2\alpha^2[-2(p_1\cdot q)^2 - p_1\cdot p_2 q^2]/s^2\,, \quad |\mathfrak{M}|^2 = 0.133\pi^2\alpha^2[-2(p_1\cdot q)^2 - p_1\cdot p_2 q^2]/s^2\,, \qquad \text{(A7)}$$

where p, p', p_1, p_2 are the four-momenta of the H, $\bar{H}$, U or D, $\bar{U}$ or $\bar{D}$; $q=p-p'$ and $s=(p+p')^2$.

(iii) and (iv) $H+\bar{H}\to L+\bar{L}$ *or* $\nu+\bar{\nu}$. For each of these processes there is one Feynman diagram which has a virtual photon. Once again, the effective charge depends upon the helicity state. When the appropriate averaging has been done the matrix elements for the $L+\bar{L}$ and $\nu+\bar{\nu}$ channels are, respectively,

$$|\mathfrak{M}|^2 = 0.044\pi^2\alpha^2[-2(p_1\cdot q)^2 - p_1\cdot p_2 q^2]/s^2\,, \quad |\mathfrak{M}|^2 = 0.0356\pi^2\alpha^2[-2(p_1\cdot q)^2 - p_1\cdot p_2 q^2]/s^2\,, \qquad \text{(A8)}$$

where p, p', p_1, p_2 are the four-momenta of the H, $\bar{H}$, L or ν, $\bar{L}$ or $\bar{\nu}$; $q=p-p'$ and $s=(p+p')^2$.

(v) $H+\bar{H}\to G+G$. For this process there are four Feynman diagrams including a seagull diagram and a diagram with a three-gluon vertex. When the appropriate averaging has been done, the matrix element is

$$\begin{aligned}|\mathfrak{M}|^2 = (\pi^2\alpha^2/27) &\times\{11.5 + 4M_H^4[(p\cdot k_1)^{-2} + (p\cdot k_2)^{-2}] \\ &+ p\cdot p'[(p\cdot k_1)^{-1} + (p\cdot k_2)^{-1}] + 18(p\cdot k_1)^2/(k_1\cdot k_2)^2 \\ &- 18(p\cdot k_1)/(k_1\cdot k_2) \\ &- 9M_H^2(k_1\cdot k_2)^{-1}(p\cdot k_1/p\cdot k_2 + p\cdot k_2/p\cdot k_1) \\ &- (p\cdot p')^2(p\cdot k_1)^{-1}(p\cdot k_2)^{-1}\}\,, \end{aligned} \qquad \text{(A9)}$$

where p, p', k_1, k_2 are the four-momenta of the H, $\bar{H}$, and the two gluons, respectively.

(vi) and (vii) $H+\bar{H}\to\gamma+\gamma$ *or* $\gamma+G$. For each of these processes there are three Feynman diagrams, including a seagull diagram. When the appropriate averaging has been done, the matrix elements for the $\gamma+\gamma$ and $\gamma+G$ channels are, respectively,

$$\begin{aligned}|\mathfrak{M}|^2 &= 0.0948\pi^2\alpha^2\{1 + (M_H^4/4)[(p\cdot k_1)^{-2} + (p\cdot k_2)^{-2}] - \tfrac{1}{2}M_H^2 p\cdot p'(p\cdot k_1)^{-1}(p\cdot k_2)^{-1}\}\,, \\ |\mathfrak{M}|^2 &= 0.237\pi^2\alpha^2\{1 + (M_H^4/4)[(p\cdot k_1)^{-2} + (p\cdot k_2)^{-2}] - \tfrac{1}{2}M_H^2 p\cdot p'(p\cdot k_1)^{-1}(p\cdot k_2)^{-1}\}\,,\end{aligned} \qquad \text{(A10)}$$

where p, p', k_1, k_2 are the four-momenta of the H, $\bar{H}$, γ or G, γ.

The elementary transition rates W for these processes are given by

$$W = \frac{1}{16}\frac{1}{m!}|\mathfrak{M}|^2(2\pi)^4\delta^4\left(\sum p\right)\,. \qquad \text{(A11)}$$

For processes (v) and (vi) the factor of $1/2!$ must be included because of the identical particles—$\gamma+\gamma$ and $G+G$.

[1] T. J. Goldman and D. A. Ross, Phys. Lett. 84B, 208 (1979).
[2] S. Weinberg, Phys. Rev. Lett. 42, 850 (1979).
[3] D. Toussaint, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 19, 1036 (1979).
[4] D. V. Nanopoulos and S. Weinberg, Phys. Rev. D 20, 2484 (1979).
[5] J. N. Fry, K. A. Olive, and M. S. Turner, preceding paper, Phys. Rev. D 22, 2953 (1980).
[6] J. Ellis and G. Steigman, Phys. Lett. 89B, 186 (1980).
[7] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1964).
[8] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1964).
[9] S. B. Treiman and F. Wilczek, Phys. Lett. 95B, 222 (1980).
[10] E. W. Kolb and S. Wolfram, Phys. Lett. 91B, 217 (1980); Nucl. Phys. B172, 224 (1980).
[11] M. Machacek, Nucl. Phys. B159, 37 (1979).
[12] A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978).