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t-quark mass predicted from a sum rule for lepton and quark masses

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It is pointed out that the sum rule for lepton and quark masses previously derived in our unified model of the Nambu–Jona-Lasinio type predicts the t-quark mass to be $m_t \cong \sqrt{8/3}\,m_W \cong 148$ GeV (where m_W is the weak-boson mass) if there exist only three generations of leptons and quarks. Also, if this is the case, the Higgs-scalar mass is predicted to be $m_H \cong \sqrt{32/3}\,m_W \cong 296$ GeV.

There seem to exist at least three generations of the Glashow-Weinberg-Salam multiplets of leptons and quarks

$$\begin{pmatrix}\nu_i\\ l_i\end{pmatrix} \equiv \begin{pmatrix}\nu_e\\ e\end{pmatrix}, \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}, \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix},$$

$$\begin{pmatrix}u_i\\ d_i\end{pmatrix} \equiv \begin{pmatrix}u\\ d\end{pmatrix}, \begin{pmatrix}c\\ s\end{pmatrix}, \begin{pmatrix}t\\ b\end{pmatrix},$$

although the existence of the t quark has not yet been found. No evidence for the $t\bar{t}$ threshold has been observed in e^+e^- experiments at PETRA for energies below 31 GeV.[1] What is the mass of the t quark? The purpose of this short Comment is to point out that the sum rule for lepton and quark masses previously derived in our unified model of the Nambu–Jona-Lasinio type[2] predicts the t-quark mass to be

$$m_t \cong \sqrt{8/3}\, m_W \cong 148 \text{ GeV} \tag{1}$$

(where m_W is the weak-boson mass) if there exist only three generations of leptons and quarks.

In our unified model of the Nambu–Jona-Lasinio model for all elementary-particle forces, the gauge bosons as well as the Higgs scalars appear as collective excitations of lepton-antilepton or quark-antiquark pairs. As a result, we have derived a simple sum rule for lepton and quark masses:

$$(\langle m^2 \rangle)^{1/2} = m_W/\sqrt{3}\ , \tag{2}$$

where $\langle\ \rangle$ denotes the arithmetic average over all leptons and quarks, i.e., for N generations

$$\langle m^2 \rangle = \frac{1}{8N} \sum_{i=1}^{N} (m_{\nu_i}{}^2 + m_{l_i}{}^2 + 3m_{u_i}{}^2 + 3m_{d_i}{}^2)\ . \tag{3}$$

The sum rule becomes useful for predicting the mass of the last member of leptons and quarks to be found, since the mass of W is predicted to be $m_W \cong 80$ GeV for $\sin^2\theta_W \cong 0.22$ in the Weinberg-Salam model.

If there exist only three generations of leptons and quarks, i.e., $N = 3$, the t quark is the last member to be found. In this case, since the masses of all leptons and quarks except for the t quark can be neglected compared to m_W, the sum rule (2) simply becomes the prediction presented in (1). If this is the case, the possible ($t\bar{t}$ bound state) and the $t\bar{t}$ threshold are to be found at much higher energies (~300 GeV) than usually expected, far beyond the Z-boson peak expected at around 90 GeV. It would be very unfortunate if even the next generation of e^+e^- colliding-beam machines such as LEP could not reach that high-energy region.

In the same model, the mass of the physical Higgs scalar H is related to those of leptons and quarks as

$$m_H = 2\left(\sum m^4 \Big/ \sum m^2\right)^{1/2}\ . \tag{4}$$

If there are only three generations of leptons and quarks, the t-quark mass dominates the sums in the right-hand side of this relation to give

$$m_H \cong 2m_t \cong \sqrt{32/3}\, m_W \cong 296 \text{ GeV}\ . \tag{5}$$

This indicates that the Higgs scalar may be located very close to the $t\bar{t}$ threshold and that it may behave as if it were a scalar $t\bar{t}$ bound state.

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[1] See, for example, G. Wolf, Report No. DESY 80/13, 1980 (unpublished).

[2] H. Terazawa, Y. Chikashige, and K. Akama, Phys. Rev. D 15, 480 (1977).