PHYSICAL REVIEW D VOLUME 22, NUMBER 11 1 DECEMBER 1980

Weak quark couplings induced by gluon corrections

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(Received 7 April 1980)

We compute the quark couplings in flavor-changing semileptonic transitions induced by lowest-order gluon corrections. We investigate the consequences of these radiative corrections for the quark axial-vector coupling, the deviations from Cabibbo universality for the axial-vector relative to the vector current, and the induced couplings (first-class pseudoscalar and anomalous magnetic moment, and second-class scalar and pseudotensor). The correction lowers the axial-vector coupling and increases the magnetic moment. We study the dependence of the couplings on the quark mass difference. Some of these results, true to all orders in α_s, generalize the theorem of Ademollo and Gatto. The effective current is pure $V-A$ to a very good approximation for transitions of heavy quarks ($m \gtrsim 5$ GeV).

I. INTRODUCTION

It is interesting to study the effects of the strong interactions on the quark current of gauge theories,

$$\bar{q}_2\gamma_\mu(1-\gamma_5)q_1 \, . \qquad (1)$$

In the case of semileptonic transitions the lowest-order gluon corrections in quantum chromodynamics (QCD) come from vertex, self-energy, and bremsstrahlung graphs (Fig. 1). Up to trivial color factors, everything will occur as in the radiative corrections to muon decay.[1] This has been pointed out by Suzuki,[2] and recently by other authors,[3] who have studied—within the quark-parton model—the lowest-order gluon corrections to the lepton spectrum and the total semileptonic width of heavy quarks. We are interested here in the "static" properties (form factors at $q^2=0$) induced by these radiative corrections. On general grounds we know that, starting from the current (1), strong interactions will produce an effective vertex of the form

$$\bar{q}_2(g_V\gamma_\mu - g_A\gamma_\mu\gamma_5 + g_S q_\mu - g_P q_\mu\gamma_5 - ig_M\sigma_{\mu\nu}q^\nu + ig_T\sigma_{\mu\nu}q^\nu\gamma_5)q_1 \, , \qquad (2)$$

where g_V, g_A, g_S, g_P, g_M, and g_T are, respectively, the vector, axial-vector, scalar, pseudoscalar, anomalous magnetic moment, and pseudotensor couplings.

Concerning the applicability of QCD to these low-momentum phenomena, we must make several remarks. On the one hand, being at low momentum transfer, the infrared-finite Feynman integrals will be dominated by regions of k^2 of the order of the (mass)^2 of the quarks involved in the vertex. Then, at least for light quarks, we cannot expect this lowest-order free-quark calculation to be very reliable. On the other hand, we have infrared (IR) singularities for which the running coupling constant $\alpha_s(k^2\to 0)$ is very large or becomes meaningless. However, at least *formally*—as we will proceed—these infrared divergences cancel out if we sum up incoherently the graphs of Fig. 1, $|(a)+(b)|^2+|(c)|^2$, as in QED.

In view of all these problems, we see at least three types of corrections to this calculation:

(i) *The effect of confinement* on these lowest-order graphs. At the end of the paper we will outline some remarks on this effect, which essentially amounts to cutting off the low-frequency gluons, $k_{\min}\sim 1/R$, R being the hadron size. We argue that this could be an alternative way of cur-

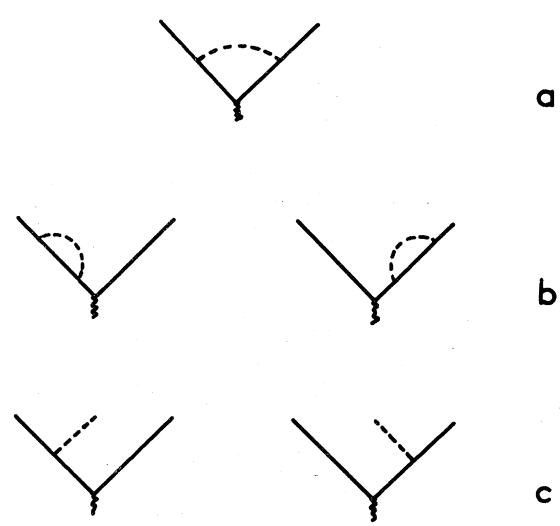

FIG. 1. (a) Vertex, (b) self-energy, (c) bremsstrahlung QCD corrections to the $V-A$ quark current.

ing the IR problem.

(ii) Higher-order effects contributing to the *vacuum polarization* on the gluon lines giving an effective $\alpha_s(m^2)$. To take into account this effect we will adopt an effective $\alpha_s(m^2)$, where m is the mass of the decaying quark.

(iii) Higher-order *vertex corrections* which, added to the lowest order, could be represented phenomenologically by a vector-meson-dominance model, which will also include, presumably, *nonperturbative* effects (confinement of the $q\bar{q}$ vector system). It is for the moment very difficult to have a simple and unambiguous answer for these corrections which could be the essential part of the whole strong interaction effect.

In spite of all these problems we hope that the lowest-order graphs will give us some useful qualitative indication on these effects, as, for instance, the sign of the renormalization for g_A, the order of magnitude of the deviation from Cabibbo universality, etc.

II. SELF-ENERGY AND VERTEX CORRECTIONS

We outline the calculations and give only the results. We adopt dimensional regularization and the prescription of Chanowitz *et al.*[4] for γ_5 since it preserves the Ward identities. The integrals appearing for g_S, g_P, g_M, and g_T are finite for both ends of the spectrum. As for g_V and g_A we have *ultraviolet* divergences which are absorbed by field and mass renormalization (Fig. 1) or which cancel out in the sum of the vertex and self-energy parts ($Z_1 = Z_2$ relation of QED).

Considering for the moment the graphs (a) and (b), g_V and g_A contain also IR singularities which can be expressed in terms of a small mass λ for the gluon:

$$\left.\begin{matrix} g_V \\ g_A \end{matrix}\right\} = 1 - \frac{4}{3}\frac{\alpha_s}{2\pi}\left[2 - \frac{2(m_1^2+m_2^2)+(m_1 \pm m_2)^2}{4(m_1^2-m_2^2)}\ln\left(\frac{m_1^2}{m_2^2}\right)\right] + \frac{4}{3}\frac{\alpha_s}{4\pi}\left[2 - \left(\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\right)\ln\left(\frac{m_1^2}{m_2^2}\right)\right]\ln\left(\frac{m_1 m_2}{\lambda^2}\right) . \tag{3}$$

The couplings which are IR finite are given by

$$\left.\begin{matrix} g_M \\ g_T \end{matrix}\right\} = \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1}{2}\frac{1}{(\pm m_1 + m_2)}\left[1 \pm \frac{m_1 m_2}{(m_1^2 - m_2^2)}\ln\left(\frac{m_1^2}{m_2^2}\right)\right], \tag{4}$$

$$\left.\begin{matrix} g_S \\ g_P \end{matrix}\right\} = \frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{\pm m_1^3 - m_2^3 + 2m_1^2 m_2 \mp 2m_1 m_2^2}{(m_1^2-m_2^2)^2} - \frac{m_1 m_2(3m_1^3 \mp 3m_2^3 \pm m_1^2 m_2 - m_1 m_2^2)}{2(m_1^2-m_2^2)^3}\times\ln\left(\frac{m_1^2}{m_2^2}\right)\right] . \tag{5}$$

m_1 and m_2 are the masses of the two fermions involved in the vertex. The couplings (4) and (5) were calculated by Halprin, Lee, and Sorba,[5] who were interested in the second-class currents g_S and g_T in nuclear β decay. We agree with their results. The factor $\frac{4}{3}$ comes from color:

$$\left\langle 3\left|\sum_\alpha \frac{\lambda^\alpha}{2}\frac{\lambda^\alpha}{2}\right|3\right\rangle = \frac{4}{3},$$

and the upper (lower) sign corresponds to vector (axial-vector) couplings. Note that we can obtain the axial-vector couplings [with their overall sign defined in (1)] by making $m_1 \to -m_1$ in the vector ones. This comes simply from the fact that the numerator of the Feynman integral can be written in the form

$$\gamma_\nu[(\not{p}_2 - \not{k}) + m_2]\gamma_\mu\gamma_5[(\not{p}_1 - \not{k}) + m_1]\gamma^\nu = \gamma_\nu[(\not{p}_2 - \not{k}) + m_2]\gamma_\mu[(\not{p}_1 - \not{k}) - m_1]\gamma^\nu\gamma_5 .$$

The second-class couplings g_S and g_T are *odd* under the exchange $m_1 \leftrightarrow m_2$, a sufficient condition for their vanishing when $m_1 = m_2$. On the other hand, the correction to the first-class form factors is *even* under $m_1 \leftrightarrow m_2$. In the case of the *vector* coupling g_V we know moreover that the renormalization is at least of order $(m_1 - m_2)^2$ (Ademollo-Gatto theorem[6]). A finite expansion of g_V in (3) in $(m_1 - m_2)^2$ shows indeed this behavior.

When $m_1 = m_2 = m$, g_V and g_A are free from infrared singularities and we get

$$\begin{aligned} g_V &= 1 , \\ g_A &= 1 - \frac{4}{3}\frac{\alpha_s}{2\pi} , \\ g_M &= \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1}{2m} , \\ g_T &= 0 , \\ g_S &= 0 , \\ g_P &= -\frac{4}{3}\frac{\alpha_s}{2\pi}\frac{7}{6m} , \end{aligned} \tag{6}$$

so that the second-class form factors disappear, and g_V is not renormalized as expected.

III. DISCUSSION OF THE INFRARED CANCELLATION

For unequal masses, g_V and g_A present IR singularities. At present, and *due to confinement*, there is not in QCD an unambiguous way of curing this problem. *Formally*, it is clear at this order that everything will occur as in QED: The IR singularities will cancel out if, to the preceding contributions, we sum up incoherently the bremsstrahlung rate $|(a)+(b)|^2+|(c)|^2$. At $q^2=0$ the rate $|(a)+(b)|^2$ is given by

$$P=\sum_{\sigma}(1+a_\sigma)P_\sigma^{(0)}\ , \tag{7}$$

where $\sigma = VV,\ AA,\ VA,\ AV$, $P_\sigma^{(0)}$ is the probability without radiative corrections, and $a_\sigma=2(g_V-1)$, $2(g_A-1)$, and $(g_V-1)+(g_A-1)$ for $\sigma=VV,\ AA$, and VA or AV, respectively, and g_V, g_A are given by (3). The bremsstrahlung rate is given by

$$P'=\sum_{\sigma} b_\sigma P_\sigma^{(0)}\ , \tag{8}$$

where (in the rest frame of the decaying quark)

$$b_\sigma=\frac{4}{3}\frac{\alpha_s}{2\pi}\left\{8+4\left(\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\right)\left[F\left(\frac{m_2^2-m_1^2}{m_1^2}\right)-\frac{1}{4}\ln\left(\frac{m_1^2}{m_2^2}\right)\right]\right.$$
$$\left.-\left[2-\left(\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\right)\ln\left(\frac{m_1^2}{m_2^2}\right)\right]\left[\ln 4+\ln\left(\frac{k_0^{\max}}{\lambda}\right)^2\right]\right\}, \tag{9}$$
$$\left(F(\xi)=\int_0^{\xi}\frac{\ln(1+x)}{x}\,dx\right).$$

We can see that the λ^2 singularity in b_σ cancels out with the ones in a_σ. Moreover, each term in (9) vanishes separately in the equal-mass limit. Note that the formula (9) is approximate, it is only valid for an upper limit of the real gluon energy, $k_0^{\max}\ll m_2$. One may wonder, however, if this is the right way—on physical grounds—of treating the IR singularity in QCD. On the one hand, as in QED, when one measures for instance the *nucleon* axial-vector coupling—the absolute magnitude by a rate and its sign by a polarization—one is certainly taking into account both the virtual gluon corrections to the fundamental vertex and the bremsstrahlung. This is so because gluons will take part in the true nucleon wave function, which will have a component in the three quarks plus one gluon sector. Then, the sum of both contributions to these *measurable* quantities is free from IR singularities, and everything seems consistent. But here $k_0^{\max}$ cannot play the same role as in QED. It cannot be the experimental uncertainty on the energy of the particles in some process since the *quark momentum is not observed,* and *gluons are confined.* Then, in principle, $k_0^{\max}$ could take, not a small value, but the maximum value consistent with the kinematics *at $q^2=0$ for the current*, $k_0^{\max}=(m_1-m_2)/2$. In this situation, the approximate result (9) is not valid since one should integrate over the whole phase space. Moreover, and due to the spatial distribution of quarks within the hadron, not all gluon momenta will be equally probable, and one should take into account the wave function of three quarks in a color-octet state plus one gluon. We are presently studying this possibility in detail.

On the other hand, and due also to confinement, the expression (3) will be modified. Confinement of gluons will mean that gluons of wavelengths $>R$ do not contribute appreciably, R being the hadron radius. The low frequencies in the Feynman integrals will be cut off by a $k^{\min}\sim 1/R$ (this is found indeed in the QED Lamb-shift calculation). The binding may thus make the IR problem less severe, although the framework is theoretically less well defined than in the preceding formal point of view.

All this means is that unless we have a way of handling this IR problem, we cannot say anything about the gluon corrections to Cabibbo universality, i.e., deviations of the relation

$$G_{\text{eff}}^2(d\to u)+G_{\text{eff}}^2(s\to u)=G^2\ , \tag{10}$$

i.e., deviations of $g_V(d\to u)/g_V(s\to u)$ from one due to the strong interactions. We can, however, consider quantities which *at least formally at this lowest order are free from* IR *singularities*, as the ratio of axial-vector to vector couplings for the quarks

$$\frac{g_A}{g_V}\cong 1-\frac{4}{3}\frac{\alpha_s}{2\pi}\left(\frac{m_1m_2}{m_1^2-m_2^2}\right)\ln\left(\frac{m_1^2}{m_2^2}\right)\ . \tag{11}$$

There is an interesting quantity which is also free from IR divergences: the ratio of axial-vector to vector Cabibbo angles. Defining

$$\tan\theta_C^{V,A}=\tan\theta_C\left(\frac{g_{V,A}(s\to u)}{g_{V,A}(d\to u)}\right)\ , \tag{12}$$

we get

$$\frac{\tan\theta_C^A}{\tan\theta_C^V}\cong 1-\frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{m_sm_u}{(m_s^2-m_u^2}\ln\left(\frac{m_s^2}{m_u^2}\right)-(s\to d)\right]\ . \tag{13}$$

This ratio is second order in the SU(3) breaking $\Delta m = m_s - m$,

$$\frac{\tan\theta_C^A}{\tan\theta_C^V} \cong 1 + \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1}{6}\left(\frac{\Delta m}{m}\right)^2 . \tag{14}$$

As we will see in the next section this result [the expansion beginning in $(\Delta m)^2$] is true to all orders in perturbative theory.

IV. DEPENDENCE OF THE COUPLINGS ON THE QUARK MASS DIFFERENCE

Let us make an expansion in $(m_1 - m_2)$ of the results (3), (4), (5), and (9) up to second order. Calling $\Delta m = m_1 - m_2$ and $m = \frac{1}{2}(m_1 + m_2)$ we get

$$g_V = 1 + \frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{5}{12}\left(\frac{\Delta m}{m}\right)^2 - \frac{2}{3}\left(\frac{\Delta m}{m}\right)^2 \ln\left(\frac{m^2}{\lambda^2}\right)\right] + \cdots , \tag{15}$$

$$g_A = 1 - \frac{4}{3}\frac{\alpha_s}{2\pi} + \frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{7}{12}\left(\frac{\Delta m}{m}\right)^2 - \frac{2}{3}\left(\frac{\Delta m}{m}\right)^2 \ln\left(\frac{m^2}{\lambda^2}\right)\right] + \cdots$$

and for the dimensionless quantities

$$\begin{aligned} g_M(m_1+m_2) &= \frac{4}{3}\frac{\alpha_s}{2\pi}\left[1 - \frac{1}{12}\left(\frac{\Delta m}{m}\right)^2\right] + \cdots , \\ g_T(m_1-m_2) &= -\frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1}{12}\left(\frac{\Delta m}{m}\right)^2 + \cdots , \\ g_S(m_1-m_2) &= \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1}{12}\left(\frac{\Delta m}{m}\right)^2 + \cdots , \\ g_P(m_1+m_2) &= -\frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{7}{3} + \frac{23}{240}\left(\frac{\Delta m}{m}\right)^2\right] + \cdots . \end{aligned} \tag{16}$$

For the bremsstrahlung rate we get

$$b_\sigma = \frac{4}{3}\frac{\alpha_s}{2\pi}\left[-\frac{32}{9}\left(\frac{\Delta m}{m}\right)^2 + \frac{2}{3}\left(\frac{\Delta m}{m}\right)^2 \ln\left(\frac{2k_0^{\max}}{\lambda}\right)^2\right] + \cdots . \tag{17}$$

Let us call

$$\begin{aligned} G_i &= g_i \quad (i = V, A) , \\ G_i &= g_i(m_1 + m_2) \quad (i = M, P) , \\ G_i &= g_i(m_1 - m_2) \quad (i = T, S) . \end{aligned} \tag{18}$$

We see that all G_i are *even* up to second order in perturbation in α_s due to T invariance. We get (see for example Ref. 7) from T invariance

$$g_i(m_1, m_2) = \epsilon_i g_i^*(m_2, m_1) , \tag{19}$$

where

$$\epsilon_i = +1 \text{ for } i = V, A, M, P, \quad \epsilon_i = -1 \text{ for } i = T, S.$$

To all orders in α_s the Feynman graphs give real results since there are no physical cuts. It then follows from (18) and (19) that

$$G_i(m_1, m_2) = G_i(m_2, m_1) . \tag{20}$$

This implies that G_i as a function $\tilde{G}_i$ of $\frac{1}{2}(m_1 + m_2) \equiv m$ and $m_1 - m_2 \equiv \Delta m$ satisfies

$$\tilde{G}_i(m, \Delta m) = \tilde{G}_i(m, -\Delta m) . \tag{21}$$

As a particular case, we get the Ademollo-Gatto theorem

$$g_V = 1 + O((\Delta m)^2) . \tag{22}$$

But we also get an analogous result for g_A:

$$g_A = (g_A)_0 + O((\Delta m)^2), \tag{23}$$

where

$$(g_A)_0 = 1 - \frac{4}{3}\frac{\alpha_s}{2\pi}$$

is the equal mass result. This means that *although the nonconservation of the axial-vector current implies $(g_A)_0 \neq 1$, it does not imply a stronger* SU(3) *breaking for g_A than for g_V.* This means, in particular, a breaking of the ratio of axial-vector to vector Cabibbo angles of *second order* in Δm:

$$\frac{\tan\theta_C^A}{\tan\theta_C^V} = 1 + O((\Delta m)^2) , \tag{24}$$

a generalization to all orders of relation (14).

Strictly speaking, these results are valid only as long as we keep λ^2 finite. Let us now discuss how IR singularities can modify this result. For the bremsstrahlung contribution we cannot say anything beyond first order in α_s and $(\Delta m)^2$ because of the factor $\ln(k_0^{\max})^2$. Up to this order, if we take $k_0^{\max} \propto (m_1 - m_2)$ for kinematical reasons, we get

$$b_\sigma = O((\Delta m)^2 \ln(\Delta m)) . \tag{25}$$

This behavior confirms that IR singularities disappear when $\Delta m \to 0$. However, the behavior (25) is disturbing because it seems to violate the Ademollo-Gatto theorem for g_V by logarithmic factors. This leads us to assume[8] that to all orders the IR singularities do *exponentiate*. Then

$$g_V = \{\exp[O((\Delta m)^2 \ln(\Delta m))]\}[1 + O((\Delta m)^2) + \cdots]$$

and if the exponential factor is the same for g_A,

$$\frac{g_A}{g_V} = \left(\frac{g_A}{g_V}\right)_0 + O((\Delta m)^2) ,$$

and the universality of Cabibbo angles is broken only by *second-order* SU(3) breaking [relation (24)].

V. QUANTITATIVE RESULTS

Let us now discuss quantitatively relations (4), (5), (6), (11), and (13). In order to do that we need an estimation of the quark masses. The results are very sensitive to the choice of these parameters, mainly for light quarks. Notice the interesting point that the true development parameter in these expressions is not $\frac{4}{3}(\alpha_s/\pi)$ but

$$\frac{4}{3}\frac{\alpha_s}{\pi}\frac{m_1 m_2}{(m_1^2-m_2^2)}\ln\left(\frac{m_1^2}{m_2^2}\right). \tag{26}$$

We think that our calculations are not reliable if this number is not small. Expression (26) is a decreasing function of the ratio (m_1/m_2), so that the development parameter remains small even for $m_1 \gg m_2$ if α_s/π is small enough. For α_s we will take the running coupling constant at the heaviest quark mass in the process

$$\alpha_s(m_1^2)=\frac{1}{[(33-2F)/12\pi]\ln(m_1^2/\Lambda^2)},$$

with $\Lambda = 0.5$ GeV. This gives the following variation of α_s (for $F=6$):

$\alpha_s(m_c^2)=0.7$ for $m_c=1.5$ GeV,

$\alpha_s(m_b^2)=0.4$ for $m_b=5$ GeV,

$\alpha_s(m_t^2)=0.2$ for $m_t=30$ GeV

for light quarks, strange ($m_s=0.54$ GeV) and non-strange ($m=0.3$ GeV), relation (26) cannot be extrapolated. To illustrate our results we will however adopt two values of α_s, $\alpha_s=1$, and $\alpha_s=2.2$. This last figure is the value needed in the bag model to adjust the hyperfine structure of low-lying hadrons.

We plot in Table I g_A/g_V, g_M, g_T, g_P, and g_S for the transitions $d\to u$, $s\to u$, $c\to s$, $c\to d$, $b\to c$, $b\to u$, $t\to b$, $t\to s$, and $t\to d$. We take $m_t=30$ GeV just as an example, without theoretical motivation. For light quarks we take an effective mass $m=0.3$ GeV instead of the current masses $m_d\cong 7$ MeV, $m_u\cong 4$ MeV.

It makes sense to adopt a constituent quark mass since we are not at small distances: Due to confinement, the low-frequency end is suppressed at $1/R\sim 300$ MeV. This is equivalent to adopting constituent masses instead of current masses. This point has been studied by Halprin *et al.*[5] We will, moreover, neglect in our calculation the isospin-violating effects and we will take $m_u=m_d$. From Table I we see that the effective coupling (2) is very close to pure $V-A$ for heavy quarks for quark masses $m>m_b$. This is due to the combined effect of the decreasing of α_s and of the function (26). For charm we have effects of the order of 10% in the anomalous magnetic moment and axial-vector coupling. However, the signs of the correction to g_A/g_V and of the anomalous magnetic moment are interesting.

As is well known, the static value of the nucleon axial-vector coupling and the proton total magnetic moment are lowered by relativistic corrections due to the quark Fermi motion[9,11]

$$\frac{G_A}{G_V}=\tfrac{5}{3}(1-\delta),\quad \mu_p^{\text{tot}}=\frac{1}{2m}(1-\delta'), \tag{27}$$

TABLE I. Value of the induced quark couplings for different transitions. We adopt as an illustration the masses: $m_d=m_u=0.3$ GeV, $m_s=0.54$ GeV, $m_c=1.5$ GeV, $m_b=5$ GeV, and $m_t=30$ GeV. For transitions involving heavy quarks ($m>m_c$) we adopt the value of α_s given by the running coupling constant. For light quarks we adopt, as examples, $\alpha_s=1$ and $\alpha_s=2.2$, this last value being the one adopted in the bag model.

Transition	α_s	g_A/g_V	$g_M(m_1+m_2)$	$g_T(m_1+m_2)$	$g_P(m_1+m_2)$	$g_S(m_1+m_2)$
$d\to u$	1	0.79	0.21	0	0.49	0
	2.2	0.53	0.46	0	1.09	0
$s\to u$	1	0.80	0.20	0.02	0.48	0.02
	2.2	0.56	0.45	0.04	1.05	0.05
$c\to d$	0.7	0.90	0.12	0.03	0.28	0.05
$c\to s$	0.7	0.88	0.13	0.02	0.32	0.03
$b\to c$	0.4	0.93	0.08	0.01	0.18	0.02
$b\to u$	0.4	0.97	0.06	0.03	0.13	0.05
$t\to b$	0.2	0.97	0.03	0.01	0.07	0.02
$t\to s$	0.2	0.99	0.02	0.02	0.05	0.03
$t\to d$	0.2	0.99	0.02	0.02	0.05	0.04

where δ, δ' are positive and of order $v^2/c^2 \sim 1/R^2m^2$. It is hard to fit the experimental values of μ_p^{tot} and G_A/G_V with the relations (27). In the bag model there is a similar problem. One gets[10]

$$\left(\frac{G_A}{G_V}\right)_{bag} = 1.09\ , \quad \left(\mu_p^{tot}\right)_{bag} = \frac{1.9}{2M_p} \tag{28}$$

to be compared, respectively, with 1.25 and 2.79. These expressions assume $g_A/g_V = 1$ and $g_M = 0$ for the quarks. We should modify them by

$$\frac{G_A}{G_V} = \left(\frac{G_A}{G_V}\right)_0 \left(\frac{g_A}{g_V}\right), \quad \mu_p^{tot} = (\mu_p^{tot})_0 + g_M\ , \tag{29}$$

where the subscript 0 means the uncorrected value ($\alpha_s = 0$). We see that since $(g_A/g_V) < 1$ and $g_M > 0$, the computed strong-interaction effects would make the situation worse, lowering G_A/G_V too much.[11] In connection with this problem it is worth mentioning that in the Nambu–Jona-Lasinio model[12] of spontaneous breaking of chiral symmetry (and presumably also in the σ model[13]), the pion radiative corrections give $g_A/g_V > 1$ for the fundamental fermion. This could be a possible way out, since we are at large distances and these effects could be comparable to gluon corrections. Another possible way out could be to take seriously into account confinement: Maybe the equal-mass result,

$$\frac{g_A}{g_V} = 1 - \frac{4}{3}\frac{\alpha_s}{2\pi}\ ,$$

could be changed by cutting off the low-frequency gluons. A possible calculational scheme could be the QCD Feynman rules modified by confinement formulated by T. D. Lee.[14] We think that unless these effects could change the situation, the bag model is in serious difficulty. Using $\alpha_s = 2.2$ we get $G_A/G_V = 0.51$ and $\mu_p^{tot} = 3.3/2M_p$ (taking $m = M_p/3$ for the correction). The magnetic moment goes in the right direction but the axial coupling is terribly small.

Let us finally consider the deviations from Cabibbo universality for vector Cabibbo angle relative to axial-vector Cabibbo angle [relation (13)]. For quarks heavier than the charm, we have seen that due to the smallness of (25), the effect is negligible. Defining the ratio (13) in the charm sector, we get

$$\left.\frac{\tan\theta_C^A}{\tan\theta_C^V}\right|_u = 1.012 \text{ or } 1.026, \quad \left.\frac{\tan\theta_C^A}{\tan\theta_C^V}\right|_c = 1.026\ ,$$

if $\alpha_s = 1$ or $\alpha_s = 2.2$, respectively, and $\alpha_s(m_c^2) = 0.7$. The subscript means u- or c-quark transitions.

VI. CONCLUSION

In conclusion, we have seen that for heavy-quark transitions $m \gtrsim m_b$ strong interactions do not renormalize appreciably the $V - A$ structure of the $SU(2)_L \times U(1)$ model. For charm we get $\sim 10\%$ effects. For light quarks the effect can be large but it is uncontrollable. The renormalization seems to lower the nucleon axial-vector coupling too much.[15] The sign of the anomalous magnetic moment goes on the contrary in the right direction.[16] Concerning Cabibbo universality, we find $\theta_C^V = \theta_C^A$ up to corrections that are second order in the SU(3) breaking, the study of deviations to the relation $G_{eff}^2(d \to u) + G_{eff}^2(s \to u) = G^2$ due to the strong interactions require a better understanding of the confinement of gluons produced by the bremsstrahlung.

ACKNOWLEDGMENTS

One of us (L. O.) is indebted to Professor M. Suzuki for many useful and friendly discussions, and to Professor G. Chew and Professor J. D. Jackson for hospitality at the Lawrence Berkeley Laboratory, where part of this work was performed. We are also grateful to J. P. Leroy and J. Micheli for interesting remarks on the exponentiation of infrared divergences.

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