PHYSICAL REVIEW D VOLUME 22, NUMBER 11 1 DECEMBER 1980

Spontaneous *CP* nonconservation and natural flavor conservation: A minimal model

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(Received 4 June 1980)

A minimal model based on the standard SU(2)×U(1) gauge theory, with spontaneous *CP* violation and natural flavor conservation, is presented in some detail. In particular it is shown that one on the minima of the classical Higgs potential leads to spontaneous *CP* breaking. The generalized Cabibbo-type matrix is real and there is no *CP* violation in the manner of Kobayashi-Maskawa. The *CP* nonconservation is entirely due to Higgs-boson exchange, and can be characterized by a single phase for an arbitrary number of quark generations.

I. INTRODUCTION

It has been recently pointed out[1] that the constraints of spontaneous *CP* breaking and natural flavor conservation[2] (NFC) lead to a class of theories where *CP* nonconservation[3] arises *exclusively* through Higgs boson exchange[4] (for an arbitrary number of fermion generations). A key problem in incorporating *CP* violation in a unified gauge theory is to understand the smallness of the violation.[5] The class of theories considered here provides an attractive scenario for understanding the strength of *CP* breaking: Since Higgs-boson exchange leads to a Fermi interaction of strength $G_F m_F^2/m_H^2$, the smallness of *CP* violation simply reflects the fact that Higgs bosons are much heavier[6] than light quarks.

In this paper, it will be shown how a minimal model with spontaneous *CP* breaking and NFC can be realized in the context of the standard SU(2) ×U(1) gauge theory. The present paper is organized as follows: In Sec. II we discuss the requirements which should be satisfied in order to have spontaneous *CP* breaking. In particular, it is demonstrated that if one imposes NFC, then at least three Higgs doublets are needed in order to violate *CP*. We then classify the various minima of the corresponding Higgs potential according to their *CP* transformation properties. It is shown that for an appropriate range of the free parameters of the Lagrangian the absolute minimum is not *CP* invariant. In Sec. III we exhibit explicitly the fermion interactions with the physical Higgs boson in the context of a minimal model. In particular, we compute the *CP*-violating component of the Fermi interaction originated through Higgs-boson exchange. Finally, in Sec. IV, we draw our conclusions.

II. A MINIMAL MODEL

Let us consider the standard $SU(2)_L\times U(1)$ gauge theory,[7] including an arbitrary number of quarks, with the left-handed components forming SU(2) doublets $\psi_{iL}=(\mathfrak{N}_i,\mathcal{P}_i)_L$, while the right-handed components are singlets. In this case, quarks acquire mass through their Yukawa couplings with Higgs doublets $\Phi_i=(\Phi_i^+,\Phi_i^0)$. It turns out that if one adheres to the principal of NFC in the Higgs sector, then a minimum of three Higgs doublets are necessary in order to have spontaneous *CP* violation.[8] This can be easily seen by considering a model with only two Higgs doublets. It is well known that in this case the constraints of NFC necessitate the introduction of some extra symmetry, the simplest choice being

$$R:\ \Phi_1\to-\Phi_1,\ \mathfrak{N}_{iR}\to-\mathfrak{N}_{iR}\,, \tag{1}$$

with all the other fields unchanged. The most general $SU(2)\times U(1)\times R$-invariant Yukawa interactions can then be written

$$\mathcal{L}=\sum_{i,j}(\bar\psi_{iL}\Gamma^1_{ij}\Phi_1\mathfrak{N}_{jR}+\bar\psi_{iL}\Gamma^2_{ij}\tilde\Phi_2\,\mathcal{P}_{jR}+\text{H.c.}, \tag{2}$$

where $\tilde\Phi_2=i\sigma_2\Phi_2^*$. The coupling constants Γ_{ij} are chosen to be real so that the Lagrangian is *CP* invariant. The most general Higgs potential consistent with (1) is given by

$$\begin{aligned}V(\Phi)&=m_1^2\Phi_1^\dagger\Phi_1+m_2^2\Phi_2^\dagger\Phi_2+a_{11}(\Phi_1^\dagger\Phi_1)^2+a_{22}(\Phi_2^\dagger\Phi_2)^2\\&+a_{12}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+b_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\&+[c_{12}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2)+\text{H.c.}]\,.\end{aligned} \tag{3}$$

Using the SU(2) gauge invariance of the theory, one can assume, without loss of generality, that the minimum of the potential is at

$$\langle\Phi_1\rangle=\frac{1}{\sqrt2}\begin{pmatrix}0\\v_1e^{i\theta_1}\end{pmatrix},\quad\langle\Phi_2\rangle=\frac{1}{\sqrt2}\begin{pmatrix}w\\v_2\end{pmatrix}, \tag{4}$$

with v_i, w real. It can be easily shown that for an appropriate range of the parameters of the Higgs potential, the minimum occurs at $w=0$, as needed in order to have an unbroken U(1) for electromagnetism. Furthermore, it is clear from (3) that

$V(\Phi)$ depends on θ only through the last term. It follows then that

$$(\theta_1)_{\min} = n\pi \quad \text{for } c_{12} < 0, \tag{5}$$

$$(\theta_1)_{\min} = \left(\frac{2n+1}{2}\right)\pi \quad \text{for } c_{12} > 0 \,. \tag{6}$$

In the case of (5), there is clearly no spontaneous CP breaking. It turns out that solution (6) is also CP invariant. To show this, assume that $c_{12} > 0$ and let the tree-approximation minimum of $V(\Phi)$ be at

$$\langle 0|\phi_1^0|0\rangle = \frac{v_1}{\sqrt{2}}\, e^{i\pi/2}, \quad \langle 0|\phi_2^0|0\rangle = \frac{v_2}{\sqrt{2}} \,. \tag{7}$$

Under time reversal T, the Higgs fields transform as

$$T\phi_i^0 T^{-1} = e^{i\alpha_i}\,\phi_i^0 \,. \tag{8}$$

A T-invariant minimum (i.e., $T|0\rangle = |0\rangle$) implies

$$\langle 0|\phi_i^0|0\rangle = \langle 0|\, T\phi_i T^{-1}|0\rangle^* = e^{-i\alpha_i}\langle 0|\phi_i^0|0\rangle^* \,, \tag{9}$$

where we have used the fact that T is antiunitary. It follows then from (7) and (9) that ϕ_i^0 should transform as

$$T\phi_1^0 T^{-1} = -\phi_1^0 \,, \; T\phi_2^0 T^{-1} = \phi_2^0 \,. \tag{10}$$

Since the Higgs potential is invariant under (10), it follows that (7) corresponds to a CP-invariant minimum. We have thus shown that in a model with NFC and two Higgs doublets, CP is conserved by the tree-approximation minima. Furthermore, one can invoke the Georgi-Pais theorem[9] to conclude that CP will still be respected by higher orders in the perturbation expansion.

At this point, it is worth emphasizing the key role played by NFC in our analysis. Had we not insisted on NFC, $V(\Phi)$ would contain terms such as $(\Phi_i^\dagger\Phi_i)(\Phi_1^\dagger\Phi_2)$, which in general would lead to spontaneous CP breaking.

In view of the previous result, we consider next the case of three Higgs doublets, which turns out to be the minimal structure required to achieve spontaneous CP violation. The simplest way to conform[10] to NFC is by preventing the third Higgs doublet Φ_3 from coupling to quarks (this is easily achieved by introducing a reflection symmetry R' under which $\Phi_3 \to -\Phi_3$ while all other fields remain unchanged). The most general gauge-invariant Higgs potential consistent with $R\times R'$ can be written

$$\begin{aligned} V(\Phi) = \sum_{i=1}^{3} [m_i^2 \Phi_i^\dagger\Phi_i + a_{ii}(\Phi_i^\dagger\Phi_i)^2] &+ \sum_{i<j} \{ a_{ij}(\Phi_i^\dagger\Phi_i)(\Phi_j^\dagger\Phi_j) + b_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_j^\dagger\Phi_i) \\ &+ [c_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_i^\dagger\Phi_j) + \text{H.c.}]\} \,, \end{aligned} \tag{11}$$

where all the coupling constants are real, so that CP invariance holds at the Lagrangian level. We assume the minimum to be at

$$\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i e^{i\theta_i} \end{pmatrix} \tag{12}$$

as required by charge conservation. Since only relative phases are physically meaningful, we set $\theta_2 = 0$. From (11) and (12) one obtains the following stationarity conditions:[8]

$$\begin{aligned} &m_1^2 + a_{11}v_1^2 + \tfrac{1}{2}a_{12}v_2^2 + \tfrac{1}{2}a_{13}v_3^2 + \tfrac{1}{2}b_{12}v_2^2 + \tfrac{1}{2}b_{13}v_3^2 + c_{12}v_2^2\cos 2\theta_1 + c_{13}v_3^2\cos^2(\theta_1-\theta_3) = 0, \\ &m_2^2 + a_{22}v_2^2 + \tfrac{1}{2}a_{12}v_1^2 + \tfrac{1}{2}a_{23}v_3^2 + \tfrac{1}{2}b_{12}v_1^2 + \tfrac{1}{2}b_{23}v_3^2 + c_{12}v_1^2\cos(2\theta_1) + c_{23}v_3^2\cos(2\theta_3) = 0 \,, \\ &m_3^2 + a_{33}v_3^2 + \tfrac{1}{2}a_{13}v_1^2 + \tfrac{1}{2}a_{23}v_2^2 + \tfrac{1}{2}b_{13}v_1^2 + \tfrac{1}{2}b_{23}v_2^2 + c_{13}v_1^2\cos 2(\theta_1-\theta_3) + c_{23}v_2^2\cos 2\theta_3 = 0 \,, \\ &c_{12}v_2^2\sin 2\theta_1 + c_{13}v_3^2\sin 2(\theta_1-\theta_3) = 0, \\ &c_{13}v_1^2\sin 2(\theta_1-\theta_3) - c_{23}v_2^2\sin 2\theta_3 = 0 \,, \end{aligned} \tag{13}$$

We now classify the various solutions of (13), according to their CP transformation properties.

(a) CP-conserving solutions. These correspond to

$$\theta_1 = \tfrac{1}{2}n\pi, \quad \theta_3 = \tfrac{1}{2}m\pi \,, \tag{14}$$

with n, m integers. It can be easily verified, following arguments entirely analogous to those leading to Eqs. (9) and (10), that these solutions are CP conserving.

(b) CP-violating solution. Fortunately, there is another solution of (13), namely,

$$\begin{aligned} \cos 2\theta_1 &= \frac{1}{2}\left(\frac{D_{13}D_{23}}{D_{12}^2} - \frac{D_{23}}{D_{13}} - \frac{D_{13}}{D_{23}}\right) , \\ \cos 2\theta_3 &= \frac{1}{2}\left(\frac{D_{13}D_{12}}{D_{23}^2} - \frac{D_{12}}{D_{13}} - \frac{D_{13}}{D_{12}}\right) , \end{aligned} \tag{15}$$

where $D_{ij} = c_{ij}v_i^2 v_j^2$. Since in general $2\theta_i \neq n\pi$, this solution leads to spontaneous CP breaking. In order that this solution exists, the following inequalities have to be satisfied:

$$
\begin{aligned}
&|D_{13}D_{23}| < |D_{12}D_{23}| + |D_{13}D_{12}| , \\
&|D_{12}D_{23}| < |D_{13}D_{23}| + |D_{13}D_{12}| , \\
&|D_{13}D_{12}| < |D_{13}D_{23}| + |D_{12}D_{23}| .
\end{aligned}
\tag{16}
$$

These relations have a simple geometrical interpretation: The three quantities $|D_{12}D_{23}|$, $|D_{12}D_{13}|$, and $|D_{13}D_{23}|$ should form a closed triangle. In the Appendix we show that if $V(\Phi)$ is bounded below and if we assume all D_{ij} to be positive, then the CP-violating solution is the absolute minimum of $V(\Phi)$.

III. INTERACTION OF HIGGS BOSONS AND FERMIONS

The quark Yukawa interactions are still given by Eq. (2), since the third Higgs doublet does not couple to quarks. From (2) and (12) one obtains, after spontaneous symmetry breaking, the following mass terms:

$$
\sum_{i,j} \frac{1}{\sqrt{2}} [\bar{\mathfrak{N}}_{iL}(v_1\Gamma^1_{ij})e^{i\theta_1}\,\mathfrak{N}_{jR} + \bar{\mathcal{P}}_{iL} v_2 \Gamma^2_{ij})\mathcal{P}_{jR}] .
\tag{17}
$$

As noticed before,[1] by making a phase redefinition

$$
n'_{jR} = e^{i\theta_1}\mathfrak{N}_{jR}
\tag{18}
$$

one obtains real mass matrices, which are diagonalized by the usual biorthogonal transformations:

$$
\begin{aligned}
&\mathfrak{N}_{iL} = (O^d_L)_{ij}\, d_{jL} ,\quad \mathfrak{N}'_{iR} = (O^d_R)_{ij}\, d_{jR} , \\
&\mathcal{P}_{iL} = (O^u_L)_{ij}\, u_{jL} ,\quad \mathcal{P}_{iR} = (O^u_R)_{ij} u_{jR} .
\end{aligned}
\tag{19}
$$

The weak charged current is then given by

$$
J_{\mu L} = \bar{u}_{iL}\gamma_\mu (O_C)_{ij}\, d_{jL} ,
\tag{20}
$$

with the generalized Cabibbo matrix given by $O_C = (O^u_L)^T O^d_L$. The reality of O_C guarantees that the vector gauge interactions conserve CP, for an arbitrary number of quark generations. It has been previously shown[1] that this result is much more general and applies to *any model* based on SU(2)×U(1), provided one conforms to NFC and spontaneous symmetry breaking. We now exhibit the Yukawa interactions with the charged Higgs boson. Using (2) and (19), one obtains

$$
\mathcal{L}(\phi^\pm) = \sqrt{2}\left(\bar{u}_L O_C M_d\, d_R \frac{\phi_1'^+}{v_1} - \bar{u}_R M_u O_C\, d_L \frac{\phi_2'^+}{v_2} + \text{H.c.}\right) ,
\tag{21}
$$

where $\phi_i'^+ = e^{-i\theta_i}\phi_i^+$ and M_u, M_d are diagonal matrices:

$$
M_d = \begin{pmatrix} m_d & & & \\ & m_s & & \\ & & m_b & \\ & & & \ddots \end{pmatrix} ,\quad M_u = \begin{pmatrix} m_u & & & \\ & m_c & & \\ & & m_t & \\ & & & \ddots \end{pmatrix} .
\tag{22}
$$

There are three charged[11] Higgs states. One component $G^\pm$ remains massless and is just the Goldstone boson which is absorbed as the longitudinal component of $W^\pm_\mu$. The other two components $H^\pm_i$ acquire mass and remain as physical charged scalars. In order to find their Yukawa interactions, we analyze next the Higgs-boson mass matrix. From (11), (12), and (13) one obtains the following mass matrix in the basis of Φ'_j:

$$
M_H = \begin{pmatrix}
\frac{1}{v_1^2}(X_{12}+X_{12}) & -\frac{1}{v_1 v_2}(X_{12}+iY) & -\frac{1}{v_1 v_3}(X_{13}-iY) \\
-\frac{1}{v_1 v_2}(X_{12}-iY) & \frac{1}{v_2^2}(X_{12}+X_{23}) & -\frac{1}{v_2 v_3}(X_{23}+iY) \\
-\frac{1}{v_1 v_3}(X_{13}+iY) & -\frac{1}{v_2 v_3}(X_{23}-iY) & \frac{1}{v_3^2}(X_{13}+X_{23})
\end{pmatrix} ,
\tag{23}
$$

where

$$
X_{ij} = [\tfrac{1}{2}b_{ij} + c_{ij}\cos 2(\theta_i - \theta_j)v_i^2 v_j^2]
\tag{24}
$$

and

$$
\begin{aligned}
Y &= c_{12}v_1^2 v_2^2 \sin 2\theta_1 \\
&= -c_{13}v_1^2 v_3^2 \sin 2(\theta_1 - \theta_3) \\
&= -c_{23}v_2^2 v_3^2 \sin 2\theta_3 .
\end{aligned}
\tag{25}
$$

The Higgs-boson mass matrix is diagonalized by performing a unitary transformation

$$
\begin{pmatrix} G^+ \\ H_1^+ \\ H_2^+ \end{pmatrix} = U_H \begin{pmatrix} \phi_1' \\ \phi_2' \\ \phi_3' \end{pmatrix} .
\tag{26}
$$

From (23), one obtains by inspection

$$G^{\pm}=\frac{1}{v}\,(v_1\phi_1'+v_2\,\phi_2'+v_3\phi_3')\ , \qquad (27)$$

where $v^2=\sum v_i^2$. By appropriate choice of the H_i^+ phases, the matrix U_H assumes the conventional Kobayashi-Maskawa[12] (KM) form, with the first row and first column real. Two of the KM angles[12] θ_i are determined by (26):

$$\sin\theta_1=\frac{(v_2{}^2+v_3{}^2)^{1/2}}{v}\ ,$$
$$\sin\theta_3=\frac{v_3}{(v_2{}^2+v_3{}^2)^{1/2}}\ . \qquad (28)$$

Similarly, $\sin\theta_2$ and the CP-violating phase δ can be determined in terms of the parameters of M_H. Using (21) and (26), one obtains the quark interactions with the physical Higgs boson H_i. Note that since the generalized Cabibbo matrix O_C is real, the only CP-violating phase comes from U_H. Exchange of charged Higgs bosons leads to an effective Fermi interaction, whose CP-violating component is given by

$$\mathcal{H}^{CP}=-\,\mathrm{Im}\left(\frac{m^2}{(m^4-\Delta^4)}\,A_+ +\frac{2}{(m^4-\Delta^4)}\,A_-\right)\frac{2}{v_1v_2}\ H\ , \qquad (29)$$

with

$$H=(\bar{d}_R M_D\, O_C\, U_L)(\bar{u}_R M_u O_C\, d_L) \qquad (30)$$

and

$$A_{\pm}=u^*_{21}u_{22}\pm u^*_{31}u_{32}\ , \qquad (31)$$

$$m^2=\tfrac{1}{2}(m_{H_1}{}^2+m_{H_2}{}^2)\ , \qquad (32)$$

$$\Delta^2=\tfrac{1}{2}(m_{H_2}{}^2-m_{H_1}{}^2)\ , \qquad (33)$$

where m_{H_i} are the charged-Higgs boson masses and u_{ij} denote the elements of u_H. Using the fact that u_H is unitary, it follows that $A_+=-v_1v_2/v_2$, and A_+ is therefore real. The CP-violating part of the amplitude comes only from the A_- term, and vanishes in the limit of degenerate Higgs-boson masses.[13] Computing the imaginary part of A_- from (23), (26), and (31), one finally obtains for the CP-violating component of the Fermi interaction

$$\mathcal{H}^{CP}=2\sqrt{2}\ G_F\,\frac{vv_3}{v_1v_2}\,\left(\frac{\Delta^2}{m^4-\Delta^4}\right)\cos\xi H\ , \qquad (34)$$

where

$$\sin^2\xi=\frac{\Sigma^2}{\Sigma^2+(4v^2/v_1{}^2v_2{}^2v_3{}^2)Y^2}\ ,$$

and Σ^2 is a quadratic form of the X_{ij}. Note that in expression (34), $|\cos\xi|$ can vary from zero to one, independently of the v_i. It is clear that $\mathcal{H}^{CP}$ can alternatively by expressed in terms of the KM-type mixing angles θ_i and the CP-violating phase δ. If one assumes that only one of the Higgs bosons is relatively light (i.e., $m_{H_1}{}^2 \ll m_{H_2}{}^2$), expression (34) becomes

$$\mathcal{H}^{CP}\cong\frac{\sqrt{2}\ G_F}{m_{H_1}{}^2}\left(\frac{vv_3}{v_1v_2}\right)(\cos\xi)H\ . \qquad (35)$$

IV. CONCLUSION

We have shown how a minimal model with spontaneous CP breaking and NFC can be realized in the context of a SU(2)×U(1) gauge theory. In particular, we have verified that there is a solution for the minimum of the Higgs potential which leads to spontaneous CP violation. The generalized Cabibbo matrix is real, and there is no CP violation in the manner of Kobayashi-Maskawa.[12] The breaking of CP invariance is solely generated by Higgs-boson exchange and can be characterized by a *unique phase* for an *aribtrary* number of fermion generations.

ACKNOWLEDGMENTS

I wish to thank Professor Lincoln Wolfenstein for his valuable advice and for a careful reading of the manuscript. I have also benefited from discussions with Professor Ling-Fong Li. This work was supported in part by the U. S. Department of Energy.

APPENDIX

We show here that there is a range of the free parameters of the Higgs potential for which the CP-violating solution (15) is indeed an absolute minimum of the potential. We do this by proving the following theorem:

Theorem. If the function $V(\Phi)$ is bounded below and if all the c_{ij} are positive, then the CP-violating solution given by Eq. (15) is the absolute minimum of $V(\Phi)$.

Proof. From (11) and (12), it follows that $V(v_i,\theta_i)$ can be written

$$V=U(v_i)+W(v_i,\theta_i)\ , \qquad (A1)$$

where U depends only on v_i and W is given by

$$W=\tfrac{1}{2}[c_{12}v_1^2v_2{}^2\cos 2\theta_1+c_{13}v_1{}^2v_3{}^2\cos 2(\theta_1-\theta_3)$$
$$+c_{23}v_2{}^2v_3{}^2\cos 2\theta_3]\ . \qquad (A2)$$

In the case of the CP-breaking solution, one obtains, using Eq. (15),

$$W=-\frac{1}{4}\left(\frac{c_{13}c_{12}}{c_{23}}\,v_1{}^4+\frac{c_{12}c_{23}}{c_{13}}\,v_2{}^4+\frac{c_{13}c_{23}}{c_{12}}\,v_3{}^4\right)\ . \qquad (A3)$$

If the function $V(\Phi)$ is bounded below, one of its stationary points has to be an absolute minimum.

It can be easily shown that if $c_{ij}>0$, none of the solutions corresponding to (13) can be an absolute minimum. Consider, for example, the stationary point

$$V^{(0)}=V(v_i^{(0)},\ \theta_1=\theta_3=0)$$
$$=U(v_i^0)+\tfrac{1}{2}c_{12}v_1^{(0)2}v_2^{(0)2}$$
$$+\tfrac{1}{2}c_{13}v_1^{(0)2}v_3^{(0)2}+\tfrac{1}{2}c_{23}v_2^{(0)2}v_3^{(0)2}\ . \qquad \text{(A4)}$$

It is clear that for $c_{ij}>0$, $V^{(0)}$ cannot be the absolute minimum, since V assumes a lower value at any point with $v=v_i^{(0)}$, $\theta_i\neq 0$. Consider another stationary point, corresponding to (13):

$$V^{(1)}=V(v_i^{(1)},\ \theta_1=0,\ \theta_3=\tfrac{1}{2}\pi)$$
$$=U(v_i^{(1)})+\tfrac{1}{2}c_{12}v_1^{(1)2}v_2^{(1)2}$$
$$-\tfrac{1}{2}c_{13}v_1^{(1)2}v_3^{(1)2}-\tfrac{1}{2}c_{23}v_2^{(1)2}v_3^{(1)2}\ . \qquad \text{(A5)}$$

It turns out that V assumes a lower value when $v_i=v_i^{(1)}$ and θ_i assume the values of Eq. (15). This is seen by observing that

$$(\tfrac{1}{2}c_{12}v_1^2v_2^2-\tfrac{1}{2}c_{13}v_1^2v_3^2-\tfrac{1}{2}c_{23}v_2^2v_3^2)+\frac{1}{4}\left(\frac{c_{13}c_{12}}{c_{23}}v_1^4+\frac{c_{12}c_{23}}{c_{13}}v_2^4+\frac{c_{13}c_{23}}{c_{12}}v_3^4\right)$$
$$=\frac{1}{4c_{12}c_{13}c_{23}}\left[c_{12}(c_{13}v_1^2+c_{23}v_2^2)-c_{23}c_{13}v_3^2\right]^2\geq 0\ . \qquad \text{(A6)}$$

The same argument can be applied, *mutatis mutandis*, to the other CP-conserving solutions given by (13). Therefore, we conclude that Eq. (15) corresponds to the absolute minimum.

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