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Neutrino masses, mixings, and oscillations in SU(2)×U(1) models of electroweak interactions

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We make a systematic investigation of all simple possibilities of having massive neutrinos in SU(2)×U(1) models of electroweak interactions, without the *ad hoc* imposition of lepton-number conservation. The minimal standard model is enlarged with triplet or singly or doubly charged singlet scalars as well as fermions in singlet and doublet representations. We find that in all cases the neutrino mass eigenstates are Majorana fields. This is so even though right-handed neutrino fields are added to the model. When mass terms of the Dirac type are also present (and if ν_R's also have small masses) neutrinos will oscillate into antineutrinos (which, we argue, are most likely "sterile"). General fermion mass terms of both Dirac and Majorana types are studied and the results are included in the Appendix.

I. INTRODUCTION

In the standard SU(2) × U(1) model of electromagnetic and weak interactions,[1] and other variations thereof, neutrinos are massless because they are described by two-component Weyl fields *and* because the simple Higgs structure of the theory leads to a global symmetry corresponding to lepton-number (L) conservation: There are no right-handed neutrinos that could combine with the left-handed neutrinos to form a Dirac mass term and lepton-number conservation forbids Majorana masses. This is also the case in the simplest grand unified model of strong and electroweak interactions: SU(5) with Higgs scalars (besides in $\underline{24}$) in $\underline{5}$ and/or $\underline{45}$ dimensional representations.[2] Here the restriction coming from lepton-number conservation is replaced by that of baryon minus lepton number ($B-L$).

On the other hand, studies of grand unified theories in general have led us to suspect that neutrino masses are not strictly zero. That they are massless in the two cases mentioned above is related to the restricted particle content being considered in such models. In the minimal SU(2) × U(1) model, given the known particle fields (left-handed doublets l_{aiL}, right-handed charged leptons l^-_{aR}, and a doublet scalar field ϕ_i), the lepton number is automatically conserved by the most general SU(2) × U(1) renormalizable interactions. When we consider a more complete unification it is inevitable that the number of fields will increase, with consequential breaking of all global symmetries. Thus the baryon number is generally not conserved in grand unified models and proton decays can take place; symmetries giving rise to L and $B-L$ conservation are similarly broken and neutrinos should be massive.[3-6]

Although recent interest in massive neutrinos is tied to the exploration of grand unifications,[6] we believe that it will still be very useful to have a systematic investigation of all reasonable possibilities of having massive neutrinos in SU(2) × U(1) models. All grand unified models must contain SU(2) × U(1) as a subtheory and it is much easier to study extensions of the minimal SU(2) × U(1) model directly.

From the above discussion it is clear that the procedure will be the addition of scalar and fermion fields to the minimal SU(2) × U(1) model. And we stipulate the rule that no *ad hoc* discrete or continuous global symmetries are to be imposed on the theory. The essential lessons we will learn from this catalog of models is that massive neutrinos invariably turn out to be Majorana particles—namely their mass eigenstates are always self-conjugate. This is so even though right-handed neutrino fields are introduced into the model.

Being massive particles, neutrinos of different flavors will mix, much in the same manner as different quark flavors mix through Cabibbo rotations. Thus a beam of neutrinos (produced through weak interactions, corresponding to some definite flavor) can oscillate in vacuum into neutrinos of a different flavor $\nu_{aL} \leftrightarrow \nu_{bL}$. Furthermore, neutrinos being Majorana particles, the presence of a mass term of the Dirac type causes mixings between the two two-component self-conjugate mass eigenstates associated with the four components of a fermion field. This will give rise to a new type of oscillation[7]: The left-handed neutrino can oscillate into a left-handed antineutrino $(\nu)_L \leftrightarrow (\nu^c)_L$. In most models $(\nu^c)_L$

or $(\nu_R)^c$, being totally neutral with respect to the $SU(2)\times U(1)$ gauge group, has only superweak Higgs couplings. We also study a class of models where such "sterile" neutrinos are avoided.

The plan for this paper is as follows. In Sec. II we present various $SU(2)\times U(1)$ models with enlarged particle content and massive neutrinos. In Sec. III we study the principal phenomenological consequences of massive Majorana neutrinos: lepton-number-nonconserving interactions (such as neutrinoless double-β decays) and neutrino oscillations. In Sec. IV we summarize all the models and their main features in a table. We also discuss briefly the cases where lepton-number conservation is imposed. Relation to grand unified models are also mentioned there. Finally, fermion mass terms of both Dirac and Majorana types are studied and the results are presented in the Appendix.

II. SU(2) × U(1) MODELS WITH MASSIVE NEUTRINOS

In the standard model of electroweak interactions, the lepton fields and Higgs scalars have the following $SU(2)\times U(1)$ transformation properties:

$$l_{aiL}=\begin{pmatrix}\nu_a\\ l_a^-\end{pmatrix}_L \sim (\underline{2},-\tfrac{1}{2}),\quad l_{aR}^-\sim(\underline{1},-1),\quad \phi_i=\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\sim(\underline{2},\tfrac{1}{2}), \tag{1}$$

where the second entries in the parentheses are the U(1) hypercharges; we adopt the convention of taking them to be $(Q-T_3)$. a is the flavor index that distinguishes the lepton families ($a=e,\mu,\tau,\ldots$), and i is the SU(2) index ($i=1,2$). Terms bilinear in fermion fields are given below (with $l^C=C\gamma^0 l^*$ and $\overline{l}^C=l^T C$, C being the Dirac charge-conjugation matrix):

$$\begin{aligned}\overline{l}_{iL}l_R^-&\sim(\underline{2},\tfrac{1}{2})\times(\underline{1},-1)=(\underline{2},-\tfrac{1}{2}),\\ \overline{l}_{iL}^C l_{jL}&\sim(\underline{2},-\tfrac{1}{2})\times(\underline{2},-\tfrac{1}{2})=(\underline{1},-1)+(\underline{3},-1),\\ \overline{l}_R^{-C}l_R^-&\sim(\underline{1},-1)\times(\underline{1},-1)=(\underline{1},-2).\end{aligned} \tag{2}$$

With $\phi_i\sim(\underline{2},\tfrac{1}{2})$, only the Yukawa couplings $\overline{l}_R l_{iL}\phi_i$ + H.c. are present in the standard model. We have a global symmetry corresponding to lepton-number conservation. Since ϕ_i does not carry a lepton number, this conservation law is not spoiled even after spontaneous symmetry breaking, with ϕ_i developing the vacuum expectation value (VEV),

$$\langle\phi_i\rangle_0=\frac{v_1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}. \tag{3}$$

In this section we shall enlarge the standard model by introducing additional scalar and/or fermion fields. Besides ϕ_i, other scalars that can join the above fermion bilinears to form $SU(2)\times U(1)$ gauge-invariant Yukawa couplings are

(1) triplet: $\vec{H}\sim(\underline{3},1)$,
(2) singly charged singlet: $h^+\sim(\underline{1},1)$,
(3) doubly charged singlet: $k^{++}\sim(\underline{1},2)$,

while lepton-number conservation can still be maintained in the Yukawa couplings by assigning appropriate letpon numbers to these scalar fields; this global symmetry will be broken by trilinear scalar couplings and/or broken spontaneously when $\vec{H}$ develop a VEV.

We shall also consider models with right-handed neutrino fields transforming as singlet and doublet under the weak SU(2).

A. Enlargement in the scalar sector only

1. Adding a triplet scalar field ($\vec{H}$)

Given the triplet scalar H with $Q-T_3=1$,

$$\vec{\tau}\cdot\vec{H}=\begin{pmatrix}H^+ & \sqrt{2}H^{++}\\ \sqrt{2}H^0 & -H^+\end{pmatrix}, \tag{4}$$

we have the additional Yukawa coupling

$$\mathcal{L}_Y'=f_{ab}\overline{l}_{aiL}^C l_{bjL}\vec{H}(\epsilon\vec{\tau})_{ij}+\text{H.c.}, \tag{5}$$

where $\epsilon=i\tau_2$. It should be noted that ϵ_{ij} and $(\epsilon\vec{\tau})_{ij}$ are, respectively, antisymmetric and symmetric tensors. It then follows from anticommutation of the fermion fields and the antisymmetric property of the charge conjugation matrix $C=i\gamma^2\gamma^0$ that the Yukawa coupling matrix f_{ab} is symmetric. Similarly we have the trilinear scalar coupling $\mu\phi_i\phi_j\vec{H}^*(\epsilon\vec{\tau})_{ij}$. When $\vec{H}$ develops a VEV,

$$\langle\vec{\tau}\cdot\vec{H}\rangle_0=\begin{pmatrix}0&0\\ v_2&0\end{pmatrix}, \tag{6}$$

a Majorana mass term for neutrinos $A_{ab}\overline{\nu}_{aL}^C\nu_{bL}$ results from Eq. (5) with

$$A_{ab}=v_2 f_{ab}=A_{ba}. \tag{7}$$

It should be noted that v_2 also contributes to W and Z masses. In fact we have

$$\rho\equiv(M_W/M_Z\cos\theta_W)^2=\frac{v_1^2+v_2^2}{v_1^2+2v_2^2}. \tag{8}$$

This implies that $\frac{1}{2}\leq\rho\leq1$. Phenomenologically we have the result[8] $\rho=0.981\pm0.037$. Allowing

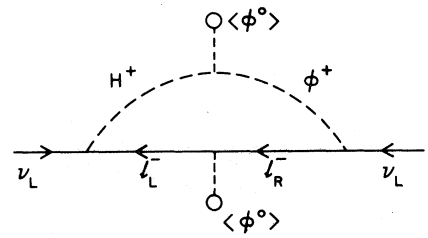

FIG. 1. Finite and calculable one-loop contribution to $\bar{\nu}_L(\nu_L)^c$ in the model with triplet scalar $\vec{H}$.

for one standard deviation, we must restrict

$$\frac{v_2}{v_1} < \frac{1}{4}. \tag{9}$$

One may be tempted to assume that scalar potential is of such structure that the triplet scalar actually does not develop a VEV, namely $v_2 = 0$ and the neutrinos gain masses through loop diagrams such as Fig. 1. However, it is easy to see that the requirement $v_2 = 0$ is not (in the technical sense) natural. Diagrams such as that in Fig. 2 (namely the neutrino lines in Fig. 1 are joined by H^0) are divergent. This necessitates a counter term and renders it impossible to preset $v_2 = 0$.[9]

2. *Adding a singly charged scalar field (h)*

This model has already been studied by Zee[10]; we include it here for completeness of presentation. The additional Yukawa coupling is

$$\mathcal{L}_Y' = f_{ab}\,\bar{l}^C_{aiL}\, l_{bjL}\, h\,\epsilon_{ij} + \text{H.c.} \tag{10}$$

Here $f_{ab} = -f_{ba}$; thus it connects leptons of different families.

If only this singlet scalar h is added in the minimal standard model with one doublet Higgs scalar, we still have lepton-number conservation. This is because h cannot develop VEV (so as not to break electromagnetic gauge invariance) and the trilinear scalar coupling $\epsilon_{ij}\phi_i\phi_j h^-$ vanishes because of the antisymmetric property of ϵ. To accomodate massive neutrinos, we must have at least two doublet scalars ϕ_{1i} and ϕ_{2i} forming a

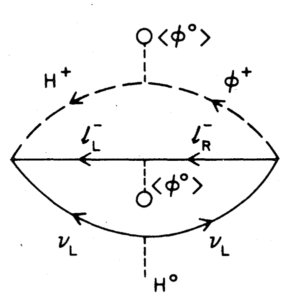

FIG. 2. A divergent contribution to $\langle H^0\rangle$.

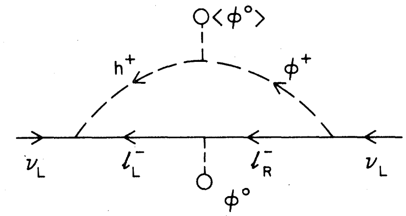

FIG. 3. Finite and calculable one-loop contribution to $\bar{\nu}_L(\nu_L)^c$ in the model with singly charged singlet scalar h.

lepton-number-nonconserving coupling,

$$\mu_{12}\,\phi_{1i}\,\phi_{2j}\,h^-\epsilon_{ij} + \text{H.c.} \tag{11}$$

Neutrinos can pick up masses through loop diagrams such as Fig. 3, which is finite and hence "calculable." We remark parenthetically that the diagram corresponding to Fig. 2 does not exist in this model, since h does not couple to the two neutrinos in Fig. 3.

The resulting neutrino mass matrix is symmetric (as it should be) and, if the coupling between the leptons and the Higgs doublets is assumed to conserve flavors, its diagonal elements vanish. The phenomenological consequences of such a structure has been discussed by Wolfenstein.[11]

3. *Adding a doubly charged scalar field (k^{++})*

We have the additional Yukawa coupling

$$\mathcal{L}_Y' = f_{ab}\,\bar{l}^{-C}_{aR}\, l^-_{bR}\, k^{++} + \text{H.c.} \tag{12}$$

with symmetric coupling matrix f_{ab}.

Again the model, as it stands, still conserves lepton number and has massless neutrinos—very much like the model in Sec. II A 2 with only one doublet scalar. To break this global symmetry we need to introduce a lepton number violating trilinear scalar coupling: $k\eta\eta$ with the new scalar η carrying $Q - T_3 = 1$. The model reduces to those in Secs. II A 1 and II A 2 if η transforms as a triplet or singlet, respectively. Here we will display the case of η being a $\underline{5}$ under SU(2) with its neutral member developing a VEV $\langle\eta^0\rangle_0 \neq 0$. The neutrinos then obtain masses through a highly convergent two-loop diagram in Figs. 4 and 5.

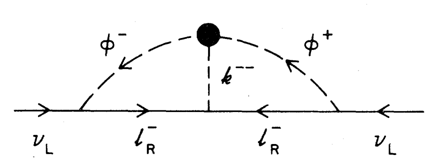

FIG. 4. Finite and calculable two-loop contribution to $\bar{\nu}_L(\nu_L)^c$ in the model with doubly charged singlet scalar k. The trilinear scalar vertex, represented by the black solid circle, is shown in Fig. 5.

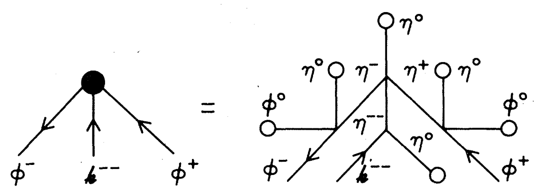

FIG. 5. Explicit representation for the effective trilinear scalar coupling $k\phi\phi$ of Figs. 4 and 8.

B. Enlargement in the fermion sector only

The only simple scheme that can produce a neutrino mass term without necessitating a simultaneous enlargement of scalar fields is the addition of a neutral singlet fermion field ν_{aR} for each family a. Then we have in the Lagrangian,

$$\mathcal{L}' = f_{ab}\bar{l}_{aiL}\nu_{bR}\phi_i^* + B_{ab}\bar{\nu}_{aR}^c\nu_{bR} + \text{H.c.} \tag{13}$$

$$\xrightarrow[\langle\phi^0\rangle_0\neq 0]{} D_{ab}\bar{\nu}_{aL}\nu_{bR} + B_{ab}\bar{\nu}_{aR}^c\nu_{bR} + \text{H.c.}, \tag{14}$$

where $D_{ab} = (1/\sqrt{2})v_1 f_{ab}$. A Majorana bare mass term B is present because ν_{aR} is totally neutral with respect to the $SU(2)\times U(1)$ group and we do not impose lepton-number conservation. Thus in this model we are naturally led to consider neutrino mass terms of Dirac and Majorana types. A study of such a general mass term is given in the Appendix. It is shown that for each family there will be two massive Majorana neutrinos. The mass term is

$$\mathcal{L}_M = \sum_{a=1}^{N}(m_{a1}\bar{\eta}_{a1}\eta_{a1} + m_{a2}\bar{\eta}_{a2}\eta_{a2}), \tag{15}$$

with

$$\eta_{a1,2} = \eta_{a1,2}^c$$

and

$$m_{a1,2} = \tfrac{1}{2}[b_a \pm (b_a{}^2 + d_a{}^2)^{1/2}], \tag{16}$$

where b_a and d_a correspond to eigenvalues of B and D matrices, respectively. The original weak-interaction eigenstates $\nu_{aL,R}$ are then superpositions of the corresponding Majorana fields $\eta_{a1,2}$.

C. Enlargement with scalars and fermions

1. Adding singlet fermions and triplet scalars

The models studied in Secs. II B and II A 1 can easily be combined. The neutrino mass term is then of the most general type:

$$\mathcal{L}_M = D\bar{\nu}_L\nu_R + A\bar{\nu}_L^c\nu_L + B\bar{\nu}_R^c\nu_R + \text{H.c.} \tag{17}$$

The Dirac mass term D arises from the VEV of the doublet scalar ϕ_i, the Majorana mass term A arises from the VEV of the triplet scalar $\vec{H}$, and B is the bare Majorana mass term. The particle spectrum is the same as the model in Sec. II B.

2. Adding right-handed doublet fermions

In order for ν_R to participate in weak interactions, it must be a member of a nontrivial $SU(2)\times U(1)$ representation: A doublet is clearly the simplest possibility. However, this doublet l_{iR} cannot have the same U(1) charge as l_{iL} (namely $Q - T_3 = -\frac{1}{2}$) since such a "vectorlike model" will not be compatible with known experimental results on neutral-current processes.[12] Thus we consider the following case:

$$l_{aiL} \sim (\underline{2}, -\tfrac{1}{2});\quad l_{aR}^- \sim (\underline{1}, -1),$$
$$r_{aiR} \sim (\underline{2}, +\tfrac{1}{2});\quad r_{aL}^+ \sim (\underline{1}, +1). \tag{18}$$

With only a doublet scalar ϕ_i, the only allowed Yukawa couplings are

$$\mathcal{L}_Y = f_{ab}^{(1)}\bar{l}_{aiL}l_{bR}^-\phi_i + f_{ab}^{(2)}\bar{r}_{aiR}r_{bL}^+\phi_i^* + \text{H.c.} \tag{19}$$

Clearly this model still has massless neutrinos: Neither Dirac nor Majorana masses can spring from the above interactions. In fact, there is separate conservation of two types of lepton numbers L_l and L_r corresponding to the l-type and r-type leptons. In order to have massive neutrinos we must enlarge the scalar sectors as well, much in the same manner as in Sec. II A. We can again consider three basic cases corresponding to triplet and singly or doubly charged singlets. Here we shall collectively refer to them as S field. In each case we have Yukawa couplings as $\bar{l}_L r_R S$, $\bar{l}_L^c l_L S^*$ and $r_R^c r_R S$. We note that such couplings conserve $(L_l - L_r)$ with $L_l - L_r = 2$ for the S field.

(i) S is a triplet and $\langle S^0\rangle_0 \neq 0$. Thus $L_l - L_r$ conservation is also spontaneously broken and all neutrino mass terms [Eq. (17)] are proportional to this VEV.

(ii) S is a singly charged singlet. Again as in Sec. II A 2, we need to have more than one set of doublet scalars. Their trilinear coupling with S then breaks $L_l - L_r$ conservation. Neutrino masses (both Dirac and Majorana types) obtain their nonzero values through one loop diagrams similar to Fig. 3.

(iii) S is a doubly charged singlet. We need other scalars (a $\underline{5}$, for example) for it to form a $(L_l - L_r)$-nonconserving interaction. Neutrinos pick up Dirac and Majorana masses through two-loop diagrams similar to Fig. 4.

III. PHENOMENOLOGICAL CONSEQUENCES

A. Lepton-number-nonconserving interactions

As we can see from the above catalog of SU(2) $\times$ U(1) models of electroweak interactions with massive neutrinos, lepton-number conservation is always broken—even in theories that have "right-handed neutrinos." For the neutrino mass terms, this means that a term of the Majorana type is present and the neutrino mass eigenstates are the self-conjugate Majorana fields. The implications of such mass terms for neutrino propagation in vacuum will be discussed in the next subsection. Here we comment on other lepton-number-nonconserving interactions such as neutrinoless double-β decay, $nn \to ppe^-e^-$,[13] and $\mu^- \to e^+$ conversion in nuclei.[14]

If the Majorana mass term is present at the tree-level perturbation, then it itself is the dominant contribution to such reactions. However, if the neutrino mass results from higher-order loop effects, then the lepton-number-nonconserving trilinear scalar couplings are primarily responsible for such interactions (usually at the tree levels). We illustrate these remarks with Figs. 6–8.

B. Neutrino mixings and oscillations

The neutrino mass term has the following general form:

$$\mathcal{L}_M = A\,\overline{\nu}_L^c\,\nu_L + B\,\overline{\nu}_R^c\,\nu_R + D\overline{\nu}_L\,\nu_R + \text{H.c.}\,, \tag{20}$$

where A, B, and D are $N\times N$ matrices in a theory with N flavors of neutrinos. The Majorana terms A and B, in particular, are symmetric matrices. The above expression may be written in the basis of self-conjugate fields,

$$\chi = \nu_L + \nu_L^c;\quad \omega = \nu_R + \nu_R^c \tag{21}$$

(our notations and conventions are stated in the Appendix),

$$\begin{aligned}\mathcal{L}_M &= A\overline{\chi}\chi + B\overline{\omega}\omega + \tfrac{1}{2}D\overline{\chi}\omega + \tfrac{1}{2}D^T\overline{\omega}\chi\\ &= (\overline{\chi}, \overline{\omega})\begin{pmatrix} A & \tfrac{1}{2}D\\ \tfrac{1}{2}D^T & B\end{pmatrix}\begin{pmatrix}\chi\\ \omega\end{pmatrix}\\ &\equiv \overline{n}Mn\,.\end{aligned} \tag{22}$$

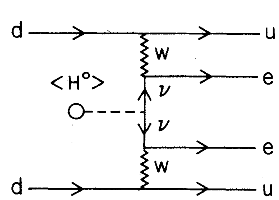

FIG. 6. Neutrinoless double-β decay effective quark amplitude $dd \to uuee$ via a neutrino Majorana mass term.

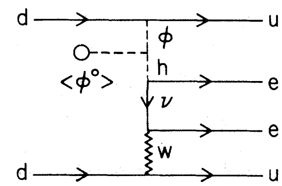

FIG. 7. Neutrinoless double-β decay effective quark amplitude $dd \to uuee$ via trilinear scalar coupling $h\phi\phi$.

The usual weak-interaction eigenstates are simply the chiral-projected components of the n vectors: $\nu_{eL} = \frac{1}{2}(1-\gamma_5)n_1$, $\nu_{\mu L} = \frac{1}{2}(1-\gamma_5)n_2$, $\nu_{eR} = \frac{1}{2}(1+\gamma_5)n_{N+1}$ [or $(\nu_{eR})^c = \frac{1}{2}(1-\gamma_5)n_{N+1}$], etc. Thus for our purposes we can simply take n_a ($a = 1, \ldots, 2N$) to be our weak eigenstates. The neutrino mass M is a $(2N\times 2N)$ symmetric matrix. It can be diagonalized by a unitary transformation (U_{ai}) yielding $2N$ mass eigenvalues $m_i (i = 1, \ldots, 2N)$. We shall simply label the corresponding mass eigenstates by $|m_i\rangle$. The neutrino states produced in weak interactions, $|n_a\rangle$ (say with definite momentum), are then actually superpositions of states with different masses (hence different energies).

$$|n_a\rangle = \sum_i U_{ai}|m_i\rangle\,. \tag{23}$$

A neutrino beam of definite momentum ($p \equiv |\vec{p}|$) is actually composed of states with different propagation properties: $|m_i(t)\rangle = e^{iE_it}|m_i(0)\rangle$; their quantum-mechanical interference produces neutrino oscillations. Thus in an initially pure n_a beam there is a finite probability after time t of finding n_b,

$$\begin{aligned}P_t(n_a \to n_b) &= |\langle n_b|n_a(t)\rangle|^2\\ &= \sum_{i,j} e^{-i(E_i-E_j)t}(U_{ai}U_{bi}^* U_{aj}^* U_{bj})\,.\end{aligned} \tag{24}$$

In particular, when $p \gg m_i$, we have $E_i - E_j \simeq (m_i^2 - m_j^2)\,(2p)^{-1}$.

Here we shall briefly comment upon the types of neutrino oscillations that can occur in the SU(2) $\times$ U(1) models as listed in Sec. II. In Section II A no ν_R's are added to the minimal model and we have only term A in Eq. (20). Thus there are

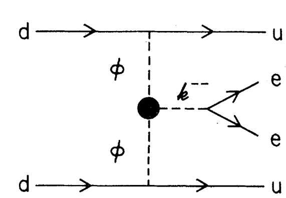

FIG. 8. Neutrinoless double-β decay effective quark amplitude $dd \to uuee$ via trilinear scalar coupling $k\phi\phi$ of Fig. 5.

only N flavor states and only flavor oscillations of the type $\nu_{eL} \leftrightarrow \nu_{\mu L}$ etc. In all other models we have ν_R's and the Dirac mass term $D \neq 0$. Thus there will be flavor as well as particle-antiparticle oscillations such as $(\nu_e)_L \leftrightarrow (\nu_{eR})^c$ [$= (\nu_e^c)_L$] or $(\nu_{\mu R})^c$, if all neutrino masses are assumed to be relatively small. Among these models we can further differentiate two subclasses. The first class (Secs. II B, II C 1) have SU(2) × U(1) neutral ν_R's. These neutral leptons have only Higgs couplings which are superweak, being of the order $m_\nu G_F^{1/2}$. Thus effectively they are noninteracting ("sterile") neutrinos. The second class of models (Sec. II C 2) have ν_R's that are in nontrivial representations of SU(2) × U(1). Thus they can interact with full weak-interaction strength to produce $r^\pm$ leptons via the charged current or produce their own neutral-current reactions.

IV. DISCUSSIONS AND SUMMARY

The principal features of the SU(2) × U(1) models with massive neutrinos discussed in this paper are summarized in Table I.

Models with massive Dirac neutrinos. It is interesting to note that in all these models the neutrino mass eigenstates are Majorana fields. Lepton-number-nonconserving reactions such as neutrinoless double-β decay and its flavor-changing analog of $\mu^- \to e^+$ conversion in a nucleus can in principle take place. This is the case even though right-handed neutrino fields are added to the model. In such models the Majorana mass terms A and B in Eq. (20) could only be banished by an *ad hoc* global symmetry corresponding to lepton-number conservation. The models in Sec. II B, II C can still have neutrino masses (of the Dirac type). The above mentioned $\beta\beta$ decays and $\nu_L \leftrightarrow (\nu_R)^c$ [$= (\nu^c)_L$] oscillations are then forbidden. However such a procedure seems to us to be unwarranted. It cannot be justified even if we regard lepton-number conservation as a good symmetry that should be valid in a low-energy effective theory. This would imply that the fields we have added to the minimal SU(2) × U(1) model (to bring about the neutrino mass and the $\Delta L \neq 0$ couplings) actually correspond to some superheavy particles and that they would therefore decouple from low-energy processes[15] (and should be left out in the effective Lagrangian).

An *ad hoc* imposition of lepton-number conservation can give a rather misleading approximation to the true physical theory.

GUT generalization. Generalization of our listing of SU(2) × U(1) models to grand unified theories (GUT's) of strong and electroweak interactions is fairly straightforward. The simplest (and the most economical) GUT is the SU(5) model with fermions in the reducible $\underline{5}^* + \underline{10}$ representation. Thus fermion bilinears are

$$\begin{aligned} &\underline{5}^* \times \underline{5}^* = \underline{10}^* + \underline{15}\,, \\ &\underline{5}^* \times \underline{10} = \underline{5} + \underline{45}\,, \\ &\underline{10} \times \underline{10} = \underline{5}^* + \underline{45}^* + \underline{50}\,, \end{aligned} \tag{25}$$

while it is well known that the familiar doublet Higgs can come from either $\underline{5}$ or $\underline{45}$. The scalars

TABLE I. Summary of SU(2) × U(1) models with massive neutrinos. The asterisks in the columns headed by neutrino mass terms indicate the presence of such terms in the theory.

				Neutrino mass terms			
Models (section no.)	Fields added to minimal model	Mechanism for L nonconservation	Order of contributions	$\bar\nu_L(\nu_L)^c$	$\bar\nu_R(\nu_R)^c$	$\bar\nu_L\nu_R$	Features of ν oscillations [$(\nu_R)^c = (\nu^c)_L$]
Adding scalars only							
II A 1	$\vec{\mathrm{H}} = (H^{++}, H^+, H^0)$	$\langle H^0\rangle \neq 0$	tree	*			
II A 2	h^+, $\phi_{\alpha i}$	$h\phi\phi$ coupling	one loop	*			$\nu_{aL} \leftrightarrow \nu_{bL}$
II A 3	k^{++}, η_{ij}	$k\eta\eta$ coupling	two loop	*			
Adding fermions only							$\nu_{aL} \leftrightarrow \nu_{bL}$
II B	ν_R	$\Delta L = 2$ mass	tree		*	*	$\nu_L \leftrightarrow (\nu_R)^c$ (sterile)
Adding fermions and scalars		$\langle H^0\rangle \neq 0$					$\nu_{aL} \leftrightarrow \nu_{bL}$
II C 1	ν_R, $\vec{\mathrm{H}}$	$\Delta L = 2$ mass	tree	*	*	*	$\nu_L \leftrightarrow (\nu_R)^c$ (sterile)
II C 2	$(r^+, \nu)_R$, r_L^+ and $\vec{\mathrm{H}}$	$\langle H^0\rangle \neq 0$	tree	*	*	*	
	h^+, $\phi_{\alpha i}$	$h\phi\phi$ coupling	one loop	*	*	*	$\nu_{aL} \leftrightarrow \nu_{bL}$
	k^{++}, η_{ij}	$k\eta\eta$ coupling	two loop	*	*	*	$\nu_L \leftrightarrow (\nu_R)^c$ (full strength)

discussed in this paper (Secs. II A and II C 2), triplet, or singly or doubly charged singlets are contained in $\underline{15}$, $\underline{10}$, and $\underline{50}$ representations of SU(5), respectively.[16] It is also well known that ν_R is an SU(5) singlet and it can combine with $\underline{5}^* + \underline{10}$ fermions to form the $\underline{16}$ spinorial representation of an SO(10) GUT. Since

$$\underline{16} \times \underline{16} = \underline{10} + \underline{120} + \underline{126}$$

and only $\underline{126}$ contains an SU(5) singlet (as well as $\underline{15}$), our model in Sec. II C 1 corresponds to the $\underline{126}$ scalar developing a VEV and giving rise to the Majorana mass terms A and B for neutrino. One possible GUT generalization for the class of models in Sec. II C 2) would be an SU(5) theory with each fermion family in the reducible $\underline{5} + \underline{5}^* + \underline{10} + \underline{10}^*$ representation (and a correspondingly complicated Higgs structure). Besides the familiar

$$\left[d^c, \begin{pmatrix} \nu \\ e \end{pmatrix}\right]_L + \left[u_c, \begin{pmatrix} u \\ d \end{pmatrix}, e^+\right]_L$$

in $\underline{5}^* + \underline{10}$, we must also have

$$\left[x^c, \begin{pmatrix} r^+ \\ \nu \end{pmatrix}\right]_R + \left[y^c, \begin{pmatrix} x \\ y \end{pmatrix}, r^-\right]_R$$

in $\underline{5} + \underline{10}^*$ representation. x, y are a set of new quarks with $(x, y)_R$ transforming as an SU(2) doublet.

Sterile right-handed neutrinos. We have studied the models in Sec. II C 2 as the simplest examples of the case where ν_R's have full-strength weak interactions. The discussion in the previous paragraph indicates that a richer structure for the quark sector is required in these models.[17] This complication makes such theories perhaps less attractive. Furthermore, if one accepts the validity of the cosmological constraint that there can be at most three to four flavors of two-component neutrino fields with full-strength weak interactions,[18] we can rule out such models with ν_R's corresponding to light mass particles. We can thus conclude that if $\nu_L \to (\nu_R)^c$ $[=(\nu^c)_L]$ oscillations can take place, laboratory neutrino beams will oscillate into sterile neutrinos.

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APPENDIX: FERMION MASS TERMS OF DIRAC AND MAJORANA TYPE

Our conventions with respect to charge-conjugation (C) and helicity-projection operations are

$$\psi^c = C\gamma^0\psi^* = i\gamma^2\psi^*, \tag{A1}$$

$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\psi; \quad \psi_R = \tfrac{1}{2}(1+\gamma_5)\psi. \tag{A2}$$

We adopt the notation[19]

$$\psi_L^c \equiv (\psi_L)^c = \tfrac{1}{2}(1+\gamma_5)\psi^C = (\psi^c)_R. \tag{A3}$$

The fermion mass terms connect left- and right-handed fields. A Dirac-type mass connects the L and R components of the same field,

$$\mathcal{L}_D = D(\bar\psi_L\psi_R + \bar\psi_R\psi_L) = D\bar\psi\psi. \tag{A4}$$

Namely, the mass eigenstate is $\psi = \psi_L + \psi_R$. A Majorana-type mass term connects the L and R components of conjugate fields. In the notation of Eq. (A3), we can have

$$\mathcal{L}_{MA} = A(\bar\psi_L^c\psi_L + \bar\psi_L\psi_L^c) = A\bar\chi\chi, \tag{A5}$$

$$\mathcal{L}_{MB} = B(\bar\psi_R^c\psi_R + \bar\psi_R\psi_R^c) = B\bar\omega\omega. \tag{A6}$$

The mass eigenstates are then self-conjugate fields: $\chi = \psi_L + \psi_L^c$ and $\omega = \psi_R + \psi_R^c$. Conversely,

$$\begin{aligned} \psi_L &= \tfrac{1}{2}(1-\gamma_5)\chi; \quad \psi_L^c = \tfrac{1}{2}(1+\gamma_5)\chi, \\ \psi_R &= \tfrac{1}{2}(1+\gamma_5)\omega; \quad \psi_R^c = \tfrac{1}{2}(1-\gamma_5)\omega. \end{aligned} \tag{A7}$$

When the γ_5 matrix is applied to the ψ, χ, and ω fields, we have [see Eqs. (A2) and (A7)]

$$\begin{pmatrix} \psi \\ \chi \\ \omega \end{pmatrix} \to \begin{pmatrix} \psi' \\ \chi' \\ \omega' \end{pmatrix} = \begin{pmatrix} -\psi_L + \psi_R \\ -\psi_L + \psi_L^c \\ +\psi_R - \psi_R^c \end{pmatrix}. \tag{A8}$$

They are interpreted as the correct mass eigenstates for the minus values of fermion masses.

When both Dirac and Majorana mass terms are simultaneously present we have

$$\begin{aligned} \mathcal{L}_{DM} &= D\bar\psi_L\psi_R + A\bar\psi_L^c\psi_L + B\bar\psi_R^c\psi_R + \text{H.c.} \\ &= \tfrac{1}{2}D(\bar\psi\omega + \bar\omega\chi) + A\bar\chi\chi + B\bar\omega\omega \\ &= (\bar\chi, \bar\omega)\begin{pmatrix} A & \tfrac{1}{2}D \\ \tfrac{1}{2}D & B \end{pmatrix}\begin{pmatrix} \chi \\ \omega \end{pmatrix}, \end{aligned} \tag{A9}$$

which can be diagonalized to yield two mass eigenvalues

$$M_{1,2} = \tfrac{1}{2}\{(A+B) \pm [(A-B)^2 + D^2]^{1/2}\} \tag{A10}$$

corresponding to Majorana mass eigenstates

$$\eta_1 = (\cos\theta)\chi - (\sin\theta)\omega ,$$
$$\eta_2 = (\sin\theta)\chi + (\cos\theta)\omega \tag{A11}$$

with

$$\tan 2\theta = D/(A-B) . \tag{A12}$$

We can easily invert Eqs. (A10) and (A12) and obtain

$$D = (M_1 - M_2)\sin 2\theta ,$$
$$A = M_1 \cos^2\theta + M_2 \sin^2\theta , \tag{A13}$$
$$B = M_1 \sin^2\theta + M_2 \cos^2\theta .$$

Thus the most general mass term Eq. (A8) for a four-component fermion field actually describes two Majorana particles with distinctive masses.

It is interesting to see how the usual four-component Dirac field formalism can be recovered in the limit of $A = B = 0$. When $\theta = \pi/4$, we have mass eigenstates $(1/\sqrt{2})(\chi \pm \omega)$ corresponding to eigenvalues $\pm\frac{1}{2}D$. To flip the sign of the negative mass, we need to apply a chiral transformation as in Eq. (A8). Thus the fields

$$\frac{1}{\sqrt{2}}(\chi + \omega) = \frac{1}{\sqrt{2}}(\psi_L + \psi_L^c + \psi_R + \psi_R^c) \equiv \xi_1$$

and

$$\frac{1}{\sqrt{2}}(\chi' - \omega') = \frac{1}{\sqrt{2}}(-\psi_L + \psi_L^c - \psi_R + \psi_R^c) \equiv \xi_2 \tag{A14}$$

have the same mass eigenvalue $\frac{1}{2}D$. Because of this degeneracy we are free to use any new combinations of fields so long they represent rotations in the ξ_1-ξ_2 plane. Namely,

$$\mathcal{L}_{DM}(A = B = 0) = \frac{1}{2}D(\bar{\xi}_1\xi_1 + \bar{\xi}_2\xi_2)$$
$$= \frac{1}{2}D(\bar{\xi}_1'\xi_1' + \bar{\xi}_2'\xi_2') . \tag{A15}$$

For the particular linear combinations (i.e., another 45° rotation)

$$\xi_1' = \frac{1}{\sqrt{2}}(\xi_1 - \xi_2) = \psi_L + \psi_R ,$$
$$\xi_2' = \frac{1}{\sqrt{2}}(\xi_1 + \xi_2) = \psi_L^c + \psi_R^c , \tag{A16}$$

it is obvious that Eq. (A15) reduces to $D\bar{\psi}\psi$ with $\psi = \psi_L + \psi_R$.

Thus a Dirac fermion really corresponds to a limit in the more general case of two Majorana particles with distinctive masses. But all known elementary fermions, except neutrinos, are forced into this limit because of charge conservation and they are prevented to have Majorana mass terms A or B.

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