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Pionic corrections to the MIT bag model: The (3,3) resonance

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By incorporating chiral invariance in the MIT bag model, we are led to a theory in which the pion field is coupled to the confined quarks only at the bag surface. An equivalent quantized theory of nucleons and Δ's interacting with pions is then obtained. The pion-nucleon scattering amplitude in this model is found to give a good fit to experimental data on the (3,3) resonance, with a bag radius of about 0.72 fm.

I. INTRODUCTION

The problem of understanding pion-nucleon (πN) scattering in the energy region of the (3,3) resonance has had a long and fascinating history. Chew[1,2] showed that a field theory which involves pions and nucleons interacting via a Yukawa coupling could be used to explain the appearance of this resonance in πN scattering. The Chew theory consisted of summing (within the static model) the series of graphs of Fig. 1. Chew and Low[3] showed that a resonant scattering amplitude could also be obtained by solving a nonlinear integral equation (the Low equation) that was the forerunner of dispersion relations (e.g., Refs. 4 and 5). An expansion of the Low equation in powers of the coupling constant is the same as summing the series of Fig. 1, but it was also pointed out that there are an infinite number of solutions of the Low equation.[6] There has been much recent interest in pion-nucleus scattering as a probe of nuclear structure. The consequent need to understand πN scattering in a very precise fashion has led to a recent series of very sophisticated applications and modifications of the original Chew-Low theory.[7,8]

Shortly after the work of Chew and Low a vast number of πN resonances and other new particles were discovered. In order to find some order among all the particles Gell-Mann and Ne'eman[9] introduced the eightfold way. In this model the πN P_{33} resonance is essentially a stable particle (the Δ), which consists of three quarks. The corresponding πN t matrix can be calculated by defining Fig. 2(a) to be a K matrix. In this way the t matrix includes all the self-energy graphs of Fig. 2 with an on-energy-shell pion. There have been several recent calculations[10-12] of πN scattering using models of this kind.

The observed πN resonances can therefore be "explained" either in terms of pions and nucleons (Fig. 1), or in terms of Δ's that consist of quarks (Fig. 2). In the present work we unify these apparently contradictory views of the (3,3) resonance.

In our model, as in the work of Chodos and Thorn,[13] the Stony Brook group,[14-16] and Jaffe,[17] the baryon is regarded as consisting of three quarks confined in a bag that is surrounded by a cloud of pions (hence the name cloudy bag). We use the MIT bag model,[18-21] which has been very successful in describing hadronic structure.

In its simplest form the MIT bag model gives a degenerate nucleon and Δ, consisting of three massless up or down quarks moving freely in a spherical region of space of radius R, called a bag. The confinement of the quarks is guaranteed by demanding that no color-electric or -magnetic fields penetrate the surface of this region, that the quark wave functions are zero outside the bag, and that the pressure exerted by the quarks on the bag surface is balanced by an external pressure. The radius of the MIT bag is typically of the order of 1.2 fm, which yields an average nucleon and Δ mass of about 1.1 GeV. This degeneracy is removed by including the color-magnetic interaction between the quarks—essentially a spin-spin force. For a summary of the many achievements of this model we refer to several recent review articles.[17,19,22-24]

The MIT bag model raises a number of fascinating problems when looked at in the context of nuclear physics. In particular, there has been little effort to include the coupling of the pion to the nucleon in the MIT model, even though it is well established that the long-range part of the N-N force is given by one-pion exchange.[29] Even given some $NN\pi$ coupling, it is rather difficult to see how two nucleon bags in a nucleus, which would be touching, could easily interact through pion exchange. There is also the controversial question of the stability of nuclear matter against

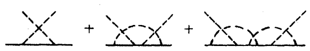

FIG. 1. The Chew series. Nucleons are represented by solid lines and pions by dashed ones.

percolation[25] if the nucleon bag has the MIT radius.

In an attempt to overcome these objections, Brown and Rho (BR) showed how the ideas of PCAC (partial conservation of axial-vector current) and the "Princeton bag"[26] could be used to derive an $NN\pi$ coupling. They obtained an equivalent Yukawa theory in which the parameters of the nucleon and the $NN\pi$ vertex could be related to the bag-model parameters. At large internucleon separation, this automatically yields the usual one-pion-exchange force. In an earlier report[27] we extended the BR model by observing that the equivalent Yukawa theory should include both nucleon and Δ bag states, and the appropriate interaction vertices.

In the present work we derive (Sec. II) the cloudy bag model in a much more rigorous way, by imposing chiral invariance on the MIT bag model. One advantage of the new derivation is that one obtains exact expressions for the $NN\pi$, $\Delta N\pi$, and $\Delta\Delta\pi$ vertex functions and coupling constants in a very straightforward manner.

In Sec. III formal expressions are obtained for the nucleon wave function and the πN scattering amplitude. The complete renormalization procedure is also discussed in some detail. An explicit expression for the πN scattering amplitude in the P_{33} channel, based on this formalism, is obtained in Sec. IV.

Numerical results are presented and discussed in Sec. V. There are two parameters in our model: R, the bag radius, and ω_Δ, the difference between the renormalized Δ and nucleon masses. The quantity ω_Δ is not necessarily the resonance energy (293 MeV) because the terms of Fig. 1 contribute to πN scattering. We find that the best fit to experimental data is obtained with $R = 0.72$ fm and $\omega_\Delta = 294$ MeV. With these parameters the effects of the pionic terms are relatively small: the Δ terms contain about 80% of the strength of the resonance. However, the pionic terms do contribute a non-negligible background. If they are completely neglected, but otherwise the same parameters are used, the position of the calculated resonance is shifted upward by 50 MeV.

Our results are summarized and plans for future work are discussed in Sec. VI.

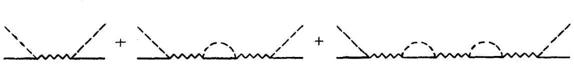

FIG. 2. The Δ model. The wiggly line is the bare Δ.

II. THEORETICAL FOUNDATION

As demonstrated by Chodos and Thorn,[13] it is possible to incorporate both the Dirac equation for massless quarks and the two-boundary conditions of the MIT bag model in a single Lagrangian density

$$\mathcal{L}(x) = \left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/}q_a(x) - B\right]\theta_v - \tfrac{1}{2}\sum_a \bar{q}_a(x)q_a(x)\Delta_s\,. \tag{2.1}$$

In this equation $q_a(x)$ is the usual Dirac field (color a), B a phenomenological energy density, θ_v a function which is one inside the confinement volume and zero outside [$\theta_v \equiv \theta(R-r)$ in the static case], and finally Δ_s is a surface δ function. By demanding that the action

$$S = \int d^4x\, \mathcal{L}(x) \tag{2.2}$$

be invariant under the variations of the fields and bag surface

$$q_a(x) \to q_a(x) + \delta q_a(x)\,, \tag{2.3a}$$

$$\bar{q}_a(x) \to \bar{q}_a(x) + \delta\bar{q}_a(x)\,, \tag{2.3b}$$

$$\theta_v \to \theta_v + \epsilon\Delta_s\,, \tag{2.3c}$$

$$\Delta_s \to \Delta_s - \epsilon n\cdot\partial\Delta_s\,, \tag{2.3d}$$

(where n^μ is an outward normal to the bag surface), we find

$$i\partial\!\!\!/ q_a(x) = 0\,, \quad x \in V \tag{2.4a}$$

$$i\gamma\cdot n q_a(x) = q_a(x)\,, \quad x \in S \tag{2.4b}$$

$$B = -\tfrac{1}{2}\, n\cdot\partial\left(\sum_a \bar{q}_a(x)q_a(x)\right) = P_D\,, \quad x \in S \tag{2.4c}$$

(where P_D is the Dirac pressure exerted on the bag surface).

The first boundary condition (2.4b) guarantees that there is no current flow through the bag surface, and the nonlinear relation (2.4c) expresses conservation of momentum at the bag boundary. Taking the static limit [$n = (0, \hat{r})$], we find that Eq. (2.4b) leads in the familiar way[19,22] to a set of quantized energy levels for the quarks, and (2.4c) provides a relation between B and R.

A. Chiral symmetry

Thus far we have been able to confine the quarks and guarantee energy and momentum conservation. Unfortunately, the necessary reflection of the quarks at the bag boundary violates chiral invariance, and the axial current associated with (2.1) is far from being conserved. Formally, this is equivalent to the observation that under the global chiral transformation

$$q_a(x) \to q_a(x) + \frac{i}{2}\epsilon\gamma_5 q_a(x) \tag{2.5}$$

the third term is not invariant, viz.,

$$\mathcal{L}(x) \to \mathcal{L}(x) - \tfrac{1}{2}\sum_a \bar{q}_a(x) i\epsilon\gamma_5 q_a(x)\Delta_s\,. \tag{2.6}$$

Indeed, as Jaffe[17] has observed, the linear boundary condition (2.4b) is not unique in guaranteeing vector current conservation. The most general condition which guarantees this is

$$i\gamma\cdot n q_a(x) = e^{i\alpha\gamma_5} q_a(x)\,,\quad x\in S \tag{2.7}$$

and our solution above corresponds to the choice $\alpha = 0$.

A very natural way to make up for this lack of invariance is to introduce a compensating pointlike pseudoscalar field ϕ. Of course, this will eventually be identified as the pion, and we must therefore exclude the pion from those states described by $\mathcal{L}(x)$. Since our main interest is nuclear and intermediate-energy physics, we shall consider only two quark flavors, up and down. The new Lagrangian density is

$$\begin{aligned}\mathcal{L}_{\rm CBM}(x) = &\left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/} q_a(x) - B\right]\theta_v \\ &- \tfrac{1}{2}\sum_a \bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)\Delta_s \\ &+ \tfrac{1}{2}[\partial_\mu\vec{\phi}(x)][\partial^\mu\vec{\phi}(x)]\end{aligned} \tag{2.8}$$

(where the subscript CBM means "cloudy bag model"). Notice that for the moment the isovector pseudoscalar field $\vec{\phi}(x)$ is massless and that Eq. (2.8) reduces to (2.1) when $\vec{\phi}$ is zero. If one now performs a variation on the $\vec{\phi}$ field as well as the variations (2.3), one obtains the field equations

$$i\partial\!\!\!/ q_a(x) = 0\,,\quad x\in V\,, \tag{2.9a}$$

$$i\gamma\cdot n q_a(x) = e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)\,,\quad x\in S\,, \tag{2.9b}$$

$$B = -\tfrac{1}{2} n\cdot\partial\sum_a [\bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)]\,,\quad x\in S\,, \tag{2.9c}$$

$$\partial^2\vec{\phi}(x) = -\frac{i}{2f}\sum_a \bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f}\vec{\tau}\gamma_5 q_a(x)\Delta_s\,,\quad \forall x\,. \tag{2.9d}$$

Once again it is easy to show that the linear boundary condition (2.9b) implies current conservation at the surface, viz.,

$$\bar{q}_a(x) i\gamma\cdot n q_a(x) = n^\mu J_\mu^a(x) = 0\,,\quad x\in S\,. \tag{2.10}$$

The new equation (2.9d) shows explicitly that the $\vec{\phi}$ field is free except for a source term at the bag surface. Of course, the major reason for introducing $\vec{\phi}(x)$ is that the new Lagrangian density $\mathcal{L}_{\rm CBM}(x)$ is invariant under the global chiral transformation

$$q_a(x) \to q_a(x) + \frac{i}{2}\vec{\tau}\cdot\vec{\epsilon}\gamma_5 q_a(x)\,, \tag{2.11a}$$

$$\vec{\phi}(x) \to \vec{\phi}(x) - \vec{\epsilon} f\,. \tag{2.11b}$$

Associated with this invariance of the Lagrangian there is, of course, a conserved axial current. This can be shown in the standard way,[28] to have the explicit form

$$\vec{A}^\mu = \tfrac{1}{2}\sum_a \bar{q}_a\gamma^\mu\gamma_5\vec{\tau} q_a\theta_v + f\partial^\mu\vec{\phi}\,. \tag{2.12}$$

Of course, in the real world we want to identify $\vec{\phi}$ as the pion field. If we add a mass term $[-\frac{1}{2}m_\pi^2\vec{\phi}^2(x)]$ to the Lagrangian density (2.8), instead of the current (2.12) being exactly conserved ($\partial_\mu A^\mu = 0$), we find (since $\partial_\mu\partial^\mu\vec{\phi} = m_\pi^2\vec{\phi}$)

$$\partial_\mu A^\mu = f m_\pi^2\vec{\phi}\,. \tag{2.13}$$

This is exactly the form required for PCAC but derived at a somewhat deeper level than in the original work of Gell-Mann and Levy.[28]

Since we shall eventually deal with a Hamiltonian formulation of pion scattering, we now construct the Hamiltonian as

$$\hat{H} = \int d^3x\, T^{00}(x)\,, \tag{2.14}$$

where

$$T^{00}(x) = \sum_r \frac{\partial_0\mathcal{L}}{\partial(\partial_0\psi_r)}\partial^0\psi_r - \mathcal{L}g^{00}\,. \tag{2.15}$$

If we define $\vec{\pi}$ as the usual field conjugate to $\vec{\phi}$ ($\vec{\pi} = \partial^0\vec{\phi}$), Eq. (2.14) (with a pion-mass term) becomes

$$\begin{aligned}\hat{H} = \int d^3x\Bigg[&\left(\frac{i}{2}\sum_a q_a^\dagger\overleftrightarrow{\partial}_0 q_a + B\right)\theta_v \\ &+ \tfrac{1}{2}\sum_a \bar{q}_a e^{i\vec{\tau}\cdot\vec{\phi}\gamma_5/f} q_a\Delta_s \\ &+ \tfrac{1}{2}(\vec{\pi}\cdot\vec{\pi} + \vec{\nabla}\vec{\phi}\cdot\vec{\nabla}\vec{\phi} + m_\pi^2\vec{\phi}^2)\Bigg]\,.\end{aligned} \tag{2.16}$$

Up to this point our derivation has been exact. It may be possible to work directly with Eq. (2.16), and in the classical case we have made some progress which will be reported elsewhere. In the present work we intend to deal with a quantized pion field, and to make the calculations tractable we shall assume that the pion field is rather small. In that case, we can expand the exponential in Eq. (2.16) as

$$\tfrac{1}{2}\bar{q}_a e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a\Delta_s \simeq \tfrac{1}{2}\bar{q}_a q_a\Delta_s + \frac{i}{2f}\bar{q}_a\vec{\tau}\cdot\vec{\phi}(x)\gamma_5 q_a\Delta_s\,. \tag{2.17}$$

If we also neglect the second term in Eq. (2.17) in the linear boundary condition (2.9b), the quark fields will correspond to exactly the usual MIT model. Then we can write Eqs. (2.16) and (2.17) in the form

$$\hat{H}=\hat{H}_{\rm MIT}+H_\pi+\hat{H}_{\rm int}\,, \tag{2.18a}$$

where

$$\hat{H}_{\rm MIT}=\int d^3x\left(\frac{i}{2}\sum_a q_a^\dagger \overleftrightarrow{\partial}_0 q_a+B\right)\theta_v\,, \tag{2.18b}$$

$$H_\pi=\tfrac{1}{2}\int d^3x(\vec{\pi}\cdot\vec{\pi}+\vec{\nabla}\vec{\phi}\cdot\vec{\nabla}\vec{\phi}+m_\pi{}^2\vec{\phi}^2)\,, \tag{2.18c}$$

$$\hat{H}_{\rm int}=\frac{i}{2f}\int d^3x\sum_a \bar{q}_a\gamma_5\vec{\tau}\cdot\vec{\phi}q_a\Delta_s\,. \tag{2.18d}$$

Our procedure is to obtain the eigenvalues and other observables of $\hat{H}$. To do this we consider the sum ($\hat{H}_0$) of $\hat{H}_{\rm MIT}$ and H_π to be an unperturbed Hamiltonian, and work with matrix elements of $\hat{H}_{\rm int}$ in the representation of unperturbed direct product states. Let us examine the individual terms of Eq. (2.18a). The first, $\hat{H}_{\rm MIT}$, is simply the Hamiltonian describing the hadrons (excluding pions) of the original MIT bag model.[19,20] Consider the complete set of colorless baryonic bag states $|\alpha\rangle$. In this representation Eq. (2.18b) becomes

$$H_{\rm MIT}=\sum_\alpha m_\alpha|\alpha\rangle\langle\alpha|\,, \tag{2.19}$$

where m_α is the mass of the bare bag state.

Next we examine H_π which is simply the Hamiltonian for a quantized, free pion field. The eigenstates of H_π are described in terms of pion creation ($a_k^\dagger$) and destruction (a_k) operators. Then the free quantized field $\vec{\phi}$ is given by

$$\phi_j(\vec{x})=(2\pi)^{-3/2}\int\frac{d\vec{k}}{(2\omega_k)^{1/2}}(a_{j\vec{k}}e^{i\vec{k}\cdot\vec{x}}+a_{j\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{x}}) \tag{2.20}$$

and

$$H_\pi=\sum_j\int d\vec{k}\,\omega_{\vec{k}}a_{j\vec{k}}^\dagger a_{j\vec{k}}\,. \tag{2.21}$$

The interaction term in the Hamiltonian Eq. (2.18d) is particularly interesting in this representation, as nondiagonal matrix elements are not necessarily zero, viz,

$$H_{\rm int}=\sum_{\alpha,\beta}|\alpha\rangle\langle\beta|\left[\frac{i}{2f}\left\langle\alpha\right|\int d^3x\sum_a\bar{q}_a(x)\gamma_5\vec{\tau}q_a(x)\vec{\phi}(x)\Delta_s\left|\beta\right\rangle\right]. \tag{2.22}$$

To be more specific, not only will the matrix elements corresponding to $NN\pi$ and $\Delta\Delta\pi$ ($\alpha=\beta=N$ or Δ) vertices be defined by Eq. (2.22), but there will also be $\Delta N\pi$ and $N\Delta\pi$ vertices.

These interaction vertices can be calculated explicitly using the lowest-order bag-model wave functions. The latter are constructed in the usual way[22] in terms of the single-particle quark wave functions

$$q(\vec{r})=\frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0(\omega r/R)\\ ij_1(\omega r/R)\vec{\sigma}\cdot\hat{r}\end{pmatrix}v\,, \tag{2.23}$$

where v is a spin and isospin wave function, ω ($\equiv\omega_{1-1}$) $=2.04$, and R is the bag radius. The normalization constant N is given by

$$N^2=R^{-3}\{\omega^2+\omega/[2(\omega-1)]\}\,. \tag{2.24}$$

If we now substitute the usual expression for the quantized pion field from Eq. (2.20), the $NN\pi$ term in $H_{\rm int}$ becomes

$$H_{\rm int}^{NN\pi}=|N\rangle\langle N|(2\pi)^{-3/2}\sum_j\int\frac{d\vec{k}}{(2\omega_k)^{1/2}}(v_{j\vec{k}}^{NN}a_{\vec{k}}+v_{j\vec{k}}^{NN\dagger}a_{\vec{k}}^\dagger)\,, \tag{2.25}$$

where

$$v_{j\vec{k}}^{NN}=\frac{i}{2f}\frac{\omega}{(\omega-1)}\frac{j_1(kR)}{kR}\left\langle N\right|\sum_a\vec{\sigma}_a\cdot\vec{k}\tau_{aj}\left|N\right\rangle. \tag{2.26}$$

Using the explicit nucleon wave functions, one finds that the quark spin and isospin operators can be eliminated in favor of the nucleon operators. That is,

$$\left\langle N\right|\sum_a\vec{\sigma}_a\cdot\vec{k}\tau_{aj}\left|N\right\rangle=\tfrac{5}{3}v_N^\dagger\vec{\sigma}\cdot\vec{k}\tau_j v_N\,, \tag{2.27}$$

where v_N is the nucleon spin-isospin wave function. At last we have an NN vertex of the usual form

$$v_{j\vec{k}}^{NN}=i(4\pi)^{1/2}\left(\frac{f_{NN\pi}^{(0)}}{m_\pi}\right)u_N(k)v_N^\dagger\vec{\sigma}\cdot\vec{k}\tau_j v_N\,, \tag{2.28}$$

where the vertex function (normalized to unity at $k=0$) is

$$u_N(k)=j_0(kR)+j_2(kR)=3j_1(kR)/kR\,, \tag{2.29}$$

and the coupling constant is

$$(4\pi)^{1/2}\left(\frac{f_{NN\pi}^{(0)}}{m_\pi}\right)=\frac{5}{18}\left(\frac{\omega}{\omega-1}\right)\left(\frac{1}{f}\right). \tag{2.30}$$

It is interesting to notice that this value of $f_{NN\pi}^{(0)}$ is quite close to the observed $NN\pi$ coupling strength. Let us use the Goldberger-Treiman relation to replace f^{-1} on the right of (2.30). Then we obtain

$$\frac{f_{NN\pi}^{(0)}}{m_\pi}=\left(\frac{5}{9}\,\frac{\omega}{\omega-1}\,\frac{1}{g_A}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)$$
$$=\left(\frac{1.09}{g_A}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right), \tag{2.31}$$

and clearly the agreement is rather good. Note, however, that both $f_{NN\pi}^{(0)}$ and g_A will be affected by higher-order pion-quark interactions. For example, the πNN coupling constant will be renormalized (see Sec. III), and pion cloud contribution must be included in calculating g_A.

By an analogous procedure one can also establish the form of the $\Delta N\pi$ interaction term

$$H_{\rm int}^{\Delta N\pi}=\sum_j (2\pi)^{-3/2}\int d\vec{k}(v_{j\vec{k}}^{\Delta N}a_{j\vec{k}}+v_{j\vec{k}}^{N\Delta\dagger}a_{j\vec{k}}^\dagger)|\,\Delta\rangle\langle N|+\text{H.c}\,. \tag{2.32}$$

The coefficients in Eq. (2.32) are related to the transition spin and isospin operators ($\vec{S}$ and $\vec{T}$—see Ref. 10) by the equation

$$v_{j\vec{k}}^{\Delta N}=i(4\pi)^{1/2}\left(\frac{f_{\Delta N\pi}^{(0)}}{m_\pi}\right)u_\Delta(k)v_\Delta^\dagger\vec{S}\cdot\vec{k}T_j v_N\,. \tag{2.33}$$

This coupling constant can also be expressed in terms of the parameters of the bag model. However, since the coupling occurs only at the surface of the bag the details of the wave functions are irrelevant, and $[f_{\Delta N\pi}^{(0)}/f_{NN\pi}^{(0)}]$ takes the SU(6) value. For the same reason, if the nucleon and Δ bag radii are the same, the form factors $u_N(k)$ and $u_\Delta(k)$ will be identical:

$$u_N(k)=u_\Delta(k)=j_0(kR)+j_2(kR)\,. \tag{2.34}$$

These form factors provide a very natural high-momentum cutoff for the theory $[u(k)\sim k^{-2}$ as $k\to\infty]$.

The practical problem with Eqs. (2.19) and (2.22) is that in principle there are an infinite number of terms in the expansion. In the present work we shall be concerned with the energy region where, at most, one real pion is allowed. Highly excited bag states should be suppressed by large energy denominators. Therefore, we shall truncate the expansion after α equals N or Δ.

B. Summary of the cloudy bag model

Given the bag model of baryon structure, we have shown that from considerations of chiral invariance one is led to include pion coupling to the quarks at the bag surface. In our model the system is in fact described by the Hamiltonian

$$H=H_0+H_I\,, \tag{2.35}$$

$$H_0=\sum_\alpha m_b^{(\alpha)}\alpha^\dagger\alpha+\sum_k \omega_k a_k^\dagger a_k\,, \tag{2.36}$$

$$H_I=\sum_{\alpha,\beta,k}[(v_k^{\alpha\beta})\alpha^\dagger\beta a_k+\text{H.c.}]\,. \tag{2.37}$$

Here α ($\alpha^\dagger$) and β ($\beta^\dagger$) are annihilation (creation) operators for static nucleon (N) or Δ bag states of bare mass $m_b^{(\alpha)}\equiv m_b^{(N)}$ or $m_b^{(\Delta)}$. The boson operators a_k and $a_k^\dagger$ obey the usual commutation rules, and the sum over k is a formal way to represent a sum over pion isospin labels and an integral over pion momenta

$$\sum_k\equiv\sum_j\int\frac{d\vec{k}}{(2\pi)^3}\,. \tag{2.38}$$

Finally we can write interactions $v_k^{\alpha\beta}$ in terms of the microscopic form factors $u_N(k)$, $u_\Delta(k)$ of Eq. (2.34) as

$$v_k^{NN}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\,\frac{f_{NN\pi}^{(0)}}{m_\pi}u_N(k)\tau_k\vec{\sigma}\cdot\vec{k} \tag{2.39}$$

and

$$v_k^{\Delta N}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\,\frac{f_{\Delta N\pi}^{(0)}}{m_\pi}u_\Delta(k)T_k\vec{S}\cdot\vec{k}\,. \tag{2.40}$$

The ratio of $(f_{\Delta N\pi}^{(0)}/f_{NN\pi}^{(0)})^2$ can be obtained from an evaluation of the appropriate bag-model matrix elements. Because the pion interacts with quarks at the bag surface the ratio is the same as for the SU(6) model,[10] i.e.,

$$\left(\frac{f_{\Delta N\pi}^{(0)}}{f_{NN\pi}^{(0)}}\right)^2=\frac{72}{25}\,. \tag{2.41}$$

It is convenient to group the hadronic creation and annihilation operators with the interaction strengths v, so that

$$V_k^{NN}=v_k^{NN}N^\dagger N\,, \tag{2.42}$$

$$V_k^{\Delta N}=v_k^{\Delta N}\Delta^\dagger N\,, \tag{2.43}$$

and so on. Then the interaction Hamiltonian becomes

$$H_I=\sum_{\alpha,\beta\in(N,\Delta)}\sum_k(V_k^{\alpha\beta}a_k+\text{H.c.})\,. \tag{2.44}$$

This model is a *combination* of the Lee model[30] and the Chew-Low model. Note that whereas the free Hamiltonian H_0 has two stable particles, since the observed P_{33} resonance is unstable, H has only one discrete eigenvalue.

In concluding this section we wish to add one caution. We are in no way attempting to solve the bag model with pion coupling self-consistently as has been done by Chodos and Thorn[13] and by Vento *et al.*[16] (This because we neglect the influence of the pion field on the quark wave functions.) We simply assume that a self-consistent solution exists, and then examine its properties in a somewhat phenomenological way. It is nevertheless very interesting that the bag radius which we find, namely $R = 0.72$ fm, is within the range of solutions ($0.5 \leq R \leq 1.5$ fm) that the Stony Brook group has reported.

III. FORMAL DEVELOPMENTS

The effective Hamiltonian (2.35) is a combination of two textbook models, the Lee model and the Chew-Low model. In this section we extend standard treatments (see Refs. 31 and 32) of the Chew-Low model to include nucleon excitation. The key results are (i) an expression for the wave function of the physical (dressed) nucleon; (ii) an exact expression for the πN scattering amplitude, which is the basis for the developments of Sec. IV; and (iii) a proof that this scattering amplitude should obey the Low equation. (In Sec. IV we show that our solution does indeed obey the Low equation).[33] In view of point (iii), the CBM is an explicit (and we feel physically well motivated) example of the well known result that the Low equation does not have a unique solution.[6]

A. The physical nucleon

In developing perturbation expansions it is useful to use energy denominators involving physical nucleon masses. This is done [following Sec. XII(d) of Schweber[31]] by introducing a mass shift into H_0,

$$\delta m = \sum_{\alpha = N, \Delta} (m_\alpha - m_b^{(\alpha)})\alpha^\dagger \alpha \ , \tag{3.1}$$

where $m_b^{(\alpha)}$ is the bare (i.e., bag) mass, and m_α the mass of the physical particle. [The meaning of m_Δ is made clear in Sec. IV—see the discussion near Eq. (4.37).] Thus we find

$$H = \tilde{H}_0 + \tilde{H}_I \ , \tag{3.2}$$

$$\tilde{H}_0 = H_0 + \delta m \ , \tag{3.3}$$

$$\tilde{H}_I = H_I - \delta m \ , \tag{3.4}$$

and $\tilde{H}_0$ acting on the bag state ($|N\rangle$ or $|\Delta\rangle$) gives the physical mass

$$\tilde{H}_0 |\alpha\rangle = m_\alpha |\alpha\rangle \ . \tag{3.5}$$

Notice that the completeness relation for baryon number one is

$$1 = \sum_\alpha |\alpha\rangle\langle\alpha| + \sum_{\alpha, k} |\alpha, k\rangle\langle\alpha, k| + \sum_{\alpha, k, k'} |\alpha, k, k'\rangle\langle\alpha, k, k'| + \cdots \ , \tag{3.6}$$

or in a shorthand form

$$1 = \sum_n |n\rangle\langle n| \ , \tag{3.7}$$

where

$$\tilde{H}_0 |n\rangle = E_n |n\rangle \ . \tag{3.8}$$

For example, if $|n\rangle$ is $|\Delta, k\rangle$, the energy E_n is $(m_\Delta + \omega_k)$. (The kinetic energy of the Δ's and nucleons is neglected in our treatment.)

The eigenstates of H correspond to the physical nucleon $|\tilde{N}\rangle$, and the set of scattering states $|\tilde{N}, k\rangle$, $|\tilde{N}, k, k'\rangle$, and so on, corresponding to an incident pion of momentum k scattering from a real nucleon [total energy $(m_N + \omega_k)$], and two incident pions of momenta k and k' scattering from a real nucleon [total energy $(m_N + \omega_k + \omega_{k'})$]. Notice that whereas $H_0(\tilde{H}_0)$ has two discrete eigenstates, H has only one. The bare Δ becomes a resonance in the pion-nucleon system when $H_I(\tilde{H}_I)$ is turned on.

The physical nucleon satisfies the equation

$$H |\tilde{N}\rangle = m_N |\tilde{N}\rangle \ . \tag{3.9}$$

To understand what are the components of $|\tilde{N}\rangle$ we rewrite $|\tilde{N}\rangle$ as

$$|\tilde{N}\rangle = Z^{1/2} |N\rangle + \Lambda |\chi\rangle \ , \tag{3.10}$$

where $\Lambda|\chi\rangle$ is to be determined—and the projection operator Λ is given by

$$\Lambda = 1 - |N\rangle\langle N| \ . \tag{3.11}$$

Thus $\Lambda|\chi\rangle$ includes the components other than the bare nucleon. To obtain $\Lambda|\chi\rangle$ use (3.2), (3.5), and (3.10) in (3.9) to find

$$\Lambda |\chi\rangle = (m_N - \tilde{H}_0)^{-1} \tilde{H}_I |\tilde{N}\rangle \ . \tag{3.12}$$

A useful integral equation for $|\tilde{N}\rangle$ may be obtained by using (3.12) in (3.10):

$$|\tilde{N}\rangle = Z^{1/2} |N\rangle + \Lambda (m_N - \tilde{H}_0)^{-1} \tilde{H}_I |\tilde{N}\rangle \ . \tag{3.13}$$

[In obtaining (3.13) the relationship $\Lambda^2 = \Lambda$ has been used.]

To appreciate (3.13) let us iterate (3.13) and keep terms of first order in $\tilde{H}_I$:

$$|\tilde{N}\rangle \simeq Z^{1/2} |N\rangle + Z^{1/2} \Lambda (m_N - \tilde{H}_0)^{-1} \tilde{H}_I |N\rangle \ . \tag{3.14}$$

However,

$$\tilde{H}_I |N\rangle = (H_I - \delta m)|N\rangle = \sum_k (v_k^{NN*} |Nk\rangle + v_k^{N\Delta*} |\Delta k\rangle) - (m_N - m_b^{(N)}) |N\rangle \ , \tag{3.15}$$

so that (3.14) may be written as

$$|\tilde{N}\rangle \simeq Z^{1/2} |N\rangle - Z^{1/2} \sum_k \left(\frac{v_k^{NN*} |N, k\rangle}{\omega_k} + \frac{v_k^{\Delta N*} |\Delta, k\rangle}{m_\Delta + \omega_k - m_N} \right). \tag{3.16}$$

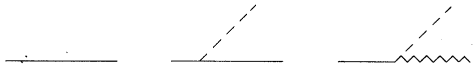

FIG. 3. The physical nucleon [from Eq. (3.16)].

Equation (3.16) is illustrated in Fig. 3. To the stated order, there is a probability Z,

$$Z^{-1}=1+\sum_k\left[\frac{v_k^{NN}v_k^{NN*}}{\omega_k{}^2}+\frac{v_k^{N\Delta}v_k^{\Delta N*}}{(m_\Delta+\omega_k-m_N)^2}\right], \quad (3.17)$$

that the physical nucleon is a bare three-quark state. In addition, there is some probability that the nucleon looks like either a nucleon or a Δ bag with a pion "in the air."

Finally we note that the mass shift can be obtained by considering the matrix element $\langle N|\tilde{H}_I|\tilde{N}\rangle$, which is zero because

$$\langle N|\tilde{H}_I|\tilde{N}\rangle=\langle N|H-\tilde{H}_0|\tilde{N}\rangle \quad (3.18a)$$

$$=0=\langle N|H_I-\delta m_N|\tilde{N}\rangle\,, \quad (3.18b)$$

where the last equation is obtained from (3.5). The use of the relation $\langle N|\tilde{N}\rangle=Z^{1/2}$ in (3.18a) then gives

$$\delta m_N=Z^{-1/2}\langle N|H_I|\tilde{N}\rangle\,. \quad (3.19)$$

To the lowest order in H_I, we find

$$\delta m_N^{(2)}=-\sum_k\left(\frac{v_k^{NN}v_k^{NN*}}{\omega_k}+\frac{v_k^{N\Delta}v_k^{\Delta N*}}{m_\Delta+\omega_k-m_N}\right), \quad (3.20)$$

which corresponds to the first two self-energy diagrams shown in Fig. 4(a).

B. Pion-nucleon scattering

Following Wick,[32] we suppose that the scattering wave function for a pion (k) incident on a nucleon leading to outgoing scattered waves is $|\tilde{N},k\rangle_+$. For this case the Schrödinger equation is

$$H|\tilde{N},k\rangle_+=(m_N+\omega_k)|\tilde{N},k\rangle_+\,. \quad (3.21)$$

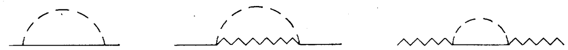

FIG. 4. Nucleon and Δ self-energy terms.

The boundary condition is imposed by writing

$$|\tilde{N},k\rangle_+=a_k^\dagger|\tilde{N}\rangle+|\chi\rangle_+\,, \quad (3.22)$$

where $|\chi\rangle_+$ has only outgoing waves in the asymptotic region. As usual this amounts to letting E become $(E+i\epsilon)$ and taking the limit $\epsilon\to 0+$.

By following analogous steps to those in Sec. IIIA, one can find an integral equation for $|\tilde{N}k\rangle_+$:

$$|\tilde{N},k\rangle_+=a_k^\dagger|\tilde{N}\rangle+(m_N+\omega_k+i\epsilon-H)\sum_{\alpha,\beta\in(N,\Delta)}V_k^{\alpha\beta}|\tilde{N}\rangle\,. \quad (3.23)$$

For ingoing boundary conditions we simply replace $+i\epsilon$ by $-i\epsilon$, so that the S matrix is

$$S(\tilde{N}'k';Nk)={}_-\langle\tilde{N}'k'|\tilde{N}k\rangle_+$$
$$=\delta_{kk'}\delta_{\tilde{N}\tilde{N}'}-2\pi i\delta(\omega_k-\omega_{k'})\,{}_-\Big\langle\tilde{N}'k'\Big|\sum_{\alpha\beta}V_k^{\alpha\beta}\Big|\tilde{N}\Big\rangle. \quad (3.24)$$

Therefore the *exact* expression for the πN t matrix in the CBM is

$$t(\tilde{N}'k',\tilde{N}k)={}_-\Big\langle\tilde{N}'k',\Big|\sum_{\alpha\beta}V_k^{\alpha\beta}\Big|\tilde{N}\Big\rangle. \quad (3.25)$$

The operator $\sum_{\alpha\beta}V_k^{\alpha\beta}$ is simply related to the pion current, i.e.,

$$[H,a_k^\dagger]-\omega_k a_k^\dagger=\sum_{\alpha\beta}V_k^{\alpha\beta}a_k\equiv J_k\,. \quad (3.26)$$

To obtain the Low equation for any other model simply replace J_k by the corresponding operator for the other model. Equation (3.25) is used to obtain the πN phase shifts in Secs. IV and V.

C. The Low equation

If we now use the integral equation for ${}_-\langle\tilde{N}'k'|$ in Eq. (3.25) we find

$$t(\tilde{N}'k',Nk)=\Big\langle\tilde{N}'\Big|\sum_{\alpha\beta}V_k^{\alpha\beta}a_{k'}\Big|\tilde{N}\Big\rangle+\Big\langle\tilde{N}'\Big|\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^\dagger(m_N+\omega_{k'}-H+i\epsilon)^{-1}\sum_{\mu\nu}V_k^{\mu\nu}\Big|\tilde{N}\Big\rangle, \quad (3.27)$$

where the relation $[J_k,a_{k'}]=0$ has been used. To simplify the quantity $a_{k'}|\tilde{N}\rangle$ consider $H\,a_{k'}|\tilde{N}\rangle$:

$$H\,a_{k'}|\tilde{N}\rangle=a_{k'}m_N|\tilde{N}\rangle+[H,a_{k'}]|\tilde{N}\rangle\,. \quad (3.28)$$

Using the definitions of H (2.35) and $V_k^{\mu\nu}$ [(2.42) and (2.43)] we find

$$a_{k'}|\tilde{N}\rangle=(m_N-\omega_{k'}-H)^{-1}\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^\dagger|\tilde{N}\rangle\,. \quad (3.29)$$

Using (3.29) in Eq. (3.27) gives

$$t(\tilde{N}'k',\tilde{N}k)=\left\langle\tilde{N}'\right|\sum_{\alpha\beta}V_{k}^{\alpha\beta}(m_N-\omega_{k'}-H)^{-1}\sum_{\mu\nu}(V_{k'}^{\mu\nu})^{\dagger}\left|\tilde{N}\right\rangle + \left\langle\tilde{N}'\right|\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^{\dagger}(m_N+\omega_{k'}-H+i\epsilon)^{-1}\sum_{\mu\nu}V_{k}^{\mu\nu}\left|\tilde{N}\right\rangle. \quad (3.30)$$

Equation (3.30) is the Low equation, as can be seen by inserting a complete set of eigenstates of H [c.f., Eq. (3.6)] with ingoing boundary conditions

$$1=\sum_{|\tilde{n}\rangle_-}|\tilde{n}\rangle_{-\,-}\langle\tilde{n}|\,. \quad (3.31)$$

Using Eq. (3.31) in (3.30) we find

$$t(\tilde{N}'k',\tilde{N}k)=\sum_{|\tilde{n}\rangle}\left[\frac{\langle\tilde{N}'|\sum_{\alpha\beta}V_{k}^{\alpha\beta}|\tilde{n}\rangle_{-\,-}\langle\tilde{n}|\sum_{\mu\nu}V_{k'}^{\mu\nu}|\tilde{N}\rangle}{m_N-\omega_{k'}-E_{\tilde{n}}}+\frac{\langle\tilde{N}'|\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^{\dagger}|\tilde{n}\rangle_{-\,-}\langle\tilde{n}|\sum_{\mu\nu}V_{k}^{\mu\nu}|\tilde{N}\rangle}{m_N+\omega_{k'}-E_{\tilde{n}}+i\epsilon}\right]. \quad (3.32)$$

However, from Eq. (3.25) (and its analogs for more incident mesons), this is simply

$$t(\tilde{N}'k',\tilde{N}k)=\sum_{|\tilde{n}\rangle}\left[\frac{t^{\dagger}(\tilde{N}'k,\tilde{n})t(\tilde{n},\tilde{N}k')}{m_N-\omega_{k'}-E_{\tilde{n}}}+\frac{t^{\dagger}(\tilde{N}'k',\tilde{n})t(\tilde{n},\tilde{N}k)}{m_N+\omega_{k'}-E_{\tilde{n}}+i\epsilon}\right], \quad (3.33)$$

which is the familiar form of the Low equation.

In order to make (3.33) tractable some standard approximations are made. First, only the nucleon and one-meson–nucleon states are included in the sums over $\tilde{n}$. Thus inelasticities in the πN amplitude are ignored. This should be a reasonable approximation for the (3.3) resonance region as the phase shift is real up to pion laboratory energies of about 500 MeV. We also keep only the nucleon-pole contribution in the first term on the right-hand side of Eq. (3.33). Since a solution of (3.33) that includes complete crossing symmetry has *never* been found, this seems to be reasonable for an initial study of our model. With these two approximations we find

$$t(\tilde{N}'k',\tilde{N}k)=\sum_{\tilde{N}''}\frac{\langle\tilde{N}'|\sum_{\mu\nu}V_{k'}^{\mu\nu}|\tilde{N}''\rangle\langle\tilde{N}''|\sum_{\mu\nu}V_{k}^{\mu\nu}|\tilde{N}\rangle}{-\omega_{k'}}+\sum_{\tilde{N}''p}\frac{t^{*}(\tilde{N}''p,\tilde{N}'k')t(\tilde{N}''p,\tilde{N}k)}{\omega_{k'}-\omega_p+i\epsilon}. \quad (3.34)$$

The zero meson term arising from the second term of (3.33) has been ignored because it gives no contribution to scattering in the (3,3) channel.

IV. THE P_{33} RESONANCE

With the theoretical basis described fully in Secs. II and III, it is relatively straightforward to derive equations for πN scattering in the (3,3) channel. Our proof relies heavily on the renormalization techniques of Dyson[34] as applied by Chew[2] to the static model of the πN system. We briefly review Chew's arguments in Sec. IV A, before proceeding to the analogous treatment of the CBM in Sec. IV B. We show that with a small number of very reasonable assumptions a simple formula for the P_{33} scattering amplitude can be obtained.

A. The Chew model

This model is defined by our Eqs. (2.35)–(2.44), provided all mention of the Δ is omitted. That is,

$$H_{\text{Chew}}=\sum_k\omega_k a_k^{\dagger}a_k+m_0N^{\dagger}N+\sum_k(V_ka_k+V_k^{\dagger}a_k^{\dagger}),\quad (4.1)$$

where V_k ($\equiv V_k^{NN}$) includes a phenomenological (sharp) cutoff to eliminate ultraviolet divergences. Following Dyson, Chew grouped together all self-energy graphs (see e.g., Fig. 5) as $\Sigma(E)$. The full nucleon propagator $S(E)$ is therefore

$$S(E)=[E-m_0-\Sigma(E)]^{-1}.\quad (4.2)$$

At this stage it is customary to assume that the theory makes sense. That is, if $\Sigma(E)$ was evaluated exactly, to all orders, that $S(E)$ would have a pole at the nucleon mass. In practice, one can only evaluate the lowest-order terms, so that it is helpful to impose this pole on the approximate solution. Thus one expands $\Sigma(E)$ about the real nucleon mass m as

$$S(E)=\{E-[m_0+\Sigma(m)]+(E-m)\Sigma'(m)+\Sigma^R(E)\}^{-1},\quad (4.3)$$

where $\Sigma^R(E)$ vanishes at least as fast as $(E-m)^2$ at $E=m$. Clearly we must now identify

$$m=m_0+\Sigma(m).\quad (4.4)$$

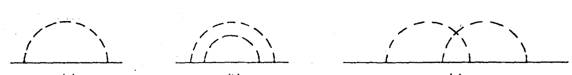

FIG. 5. Some contributions to $\Sigma(E)$ in Chew's model (Refs. 1 and 2).

If $\Sigma(E)$ was very slowly varying, Eq. (4.3) would be simply $S(E)=(E-m)^{-1}$. Indeed the usual assumption, introduced by Chew, is that higher-order graphs such as Figs. 5(b) and 5(c) vary slowly with energy, and therefore $\Sigma'(E)$ and $\Sigma^R(E)$ get their major contribution from Fig. 5(a). That is,

$$\Sigma(E)=\Sigma(m)+[\Sigma_{N\pi}(E)-\Sigma_{N\pi}(m)]\,, \tag{4.5}$$

where $\Sigma_{N\pi}$ denotes the self-energy contribution of Fig. 5(a).

At this stage one has a choice. Since $\Sigma_{N\pi}(E)$ and its derivatives are all finite, one can work with the propagator of Eqs. (4.3), (4.4), and (4.5), viz.,

$$S(E)=\{(E-m)[1+\Sigma'_{N\pi}(m)]-\Sigma^R_{N\pi}(E)\}^{-1}\,. \tag{4.6}$$

However, it is more conventional to define a renormalized propagator

$$S'(E)=Z_2{}^{-1}S(E)\,, \tag{4.7}$$

and for consistency a renormalized coupling constant

$$f'=Z_2f\,. \tag{4.8}$$

As mentioned in Sec. III, Z_2 is the probability that the dressed nucleon looks like a bare nucleon and is therefore less than 1. Thus, as observed by Chew there are two reasons for performing the mass renormalization: (i) It leads to a simpler propagator

$$S'(E)=[E-m-Z_2{}^{-1}\Sigma^R_{N\pi}(E)]^{-1}\,, \tag{4.9}$$

because as Chew demonstrated numerically this is very well approximated by

$$S'(E)\simeq(E-m)^{-1}\,, \tag{4.10}$$

for low energy pion scattering. (ii) Since $Z_2<1$, renormalizing the coupling constant reduces its magnitude, so that an expansion in powers of the coupling constant is more convergent.

In this theory there is no pion coupling to a nucleon-antinucleon pair, and therefore no renormalization of the pion propagator. Thus the only renormalization remaining is the inclusion of processes as in Fig. 6. As shown by Chew, this leads to a redefinition of the coupling constant

$$f_r=Z_2Z_1{}^{-1}f\,. \tag{4.11}$$

Once again $Z_1<1$, but the lowest-order contribution of Fig. 6(a) has only $\frac{1}{9}$ of the effect in increasing $Z_1{}^{-1}$ that Fig. 5(a) has in lowering Z_2—that is Z_2 is significantly less than Z_1 (indeed Z_1 is very nearly one in Chew's model).

(a) (b)

FIG. 6. Contributions to vertex renormalization.

With this renormalization, one has to calculate fewer diagrams in studying πN scattering. Since the renormalized $NN\pi$ coupling constant was relatively small, Chew argued that an expansion in powers of $f_r{}^2$ would make sense. The one additional observation which he made was that the pole in diagrams with only one pion in an intermediate state would effectively lower it by a power $f_r{}^2$. Thus each term in the infinite series of graphs in Fig. 1 is formally of order $f_r{}^2$, whereas those in Fig. 7 are of order $f_r{}^4$ or higher, and are dropped.

It is well known that the series of Fig. 1, with $f_r{}^2\sim 0.08$ and suitable choice of vertex function

$$v_{\text{Chew}}(k)\simeq\theta(m-k)\,, \tag{4.12}$$

leads to the Chew-Low effective range formula, and in particular to a resonance in the P_{33} channel (see, for example, Ref. 36).

B. The cloudy bag model

This involves a very straightforward extension of the theory of Sec. IV A to the more general Hamiltonian (2.35)–(2.44), which was dictated by our considerations of PCAC and the bag model in Sec. II. The key results which we need are the formal expressions (3.14) and (3.25) of Sec. III for the physical nucleon and the πN scattering amplitude.

The nucleon

If for clarity we retain only the two lowest-order nucleon self-energy graphs of Fig. 4 explicitly, and call the rest $\Sigma^{(N)}_{\text{HO}}(E)$ (HO = higher order), the nucleon propagator will be

$$S_N(E)=[E-m_b^{(N)}-\Sigma^{(N)}_{N\pi}(E)-\Sigma^{(N)}_{\Delta\pi}(E)-\Sigma^{(N)}_{\text{HO}}(E)]^{-1}\,. \tag{4.13}$$

The large number of virtual pions in $\Sigma^{(N)}_{\text{HO}}(E)$ means that it will be effectively constant in the energy region of interest. Thus these terms will shift the mass down, but [cf., the discussion near Eq. (4.6)] have a negligible affect on the coupling constant. With this assumption the renormalization can be carried out as before, with

$$S'_N(E)\simeq(E-m_N)^{-1}\,, \tag{4.14}$$

and

$$f'_m=Z_2f\,, \tag{4.15a}$$

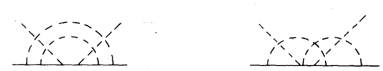

FIG. 7. Typical higher-order irreducible diagrams contributing to πN scattering.

$$Z_2^{(N)} = [1 + \Sigma_{N\pi}'^{(N)}(m) + \Sigma_{\Delta\pi}'^{(N)}(m)]^{-1} . \tag{4.15b}$$

Once again Z_2 is the probability that the physical nucleon looks like the three-quark bag. (Technically it also includes the possibility that the nucleon looks like a bag with *more than one* virtual pion, but by assumption this is small.)

The Δ

As we have defined it, the physical Δ is a resonance in the πN P_{33} scattering amplitude. Thus we are led to consider the series of diagrams generated by the perturbative expansion of the exact scattering amplitude (3.25). This is done by using the formally exact expression for the wave function $|\tilde{N}'k'\rangle_-$,

$$|\tilde{N}'k'\rangle_- = a^\dagger_{k'} |N'\rangle Z^{1/2} + \frac{1}{m_N + \omega_k' - \tilde{H}_0} \tilde{H}_I |\tilde{N}'k'\rangle_- . \tag{4.16}$$

As Eq. (3.25) represents the solution to the Low equation, the correct solution of the linear equation (4.16) along with a solution of Eq. (3.10) for the physical nucleon must yield a solution of the Low equation. As we discuss below, our t matrix is indeed a solution of the Low equation.

Some terms of order coupling constant to the fourth (or lower) are shown in Fig. 8. Note that the nucleon mass renormalization is assumed done, in the manner described above.

By the criterion suggested by Chew for treating low energy pion-nucleon scattering, all of the graphs in Fig. 8 [except Fig. 8(g)] are formally of order coupling constant squared. That is, all those with four vertices, except Fig. 8(g), have one pion which can be on shell in an intermediate state. Terms like Fig. 8(g) can easily be retained as an essentially energy-independent shift in the bare-Δ mass. If apart from such higher-order self-energy graphs we adopt Chew's one-meson approximation, the πN t matrix is easily seen to be the solution of effectively a two-potential problem

$$t(E) = (v_{\rm CL} + v_\Delta) + (v_{\rm CL} + v_\Delta) G_0(E) t(E) . \tag{4.17}$$

Here $v_{\rm CL}$ (CL ≡ Chew Low) is the Chew driving term of Fig. 9(a), and v_Δ involves formation and decay of a Δ bag [Fig. 9(b)],

$$v_\Delta = g_{\pi N\Delta}(\vec{k}') S_\Delta^{(0)}(E) g_{\Delta N\pi}(\vec{k}) , \tag{4.18}$$

with

$$S_\Delta^{(0)}(E) = [E - m_b^{(\Delta)} - \Sigma_{\rm HO}^\Delta(E)]^{-1} . \tag{4.19}$$

As the higher-order Δ self-energy terms [$\Sigma_{\rm HO}^\Delta(E)$—see e.g., Figs. 8(g) and 8(h)] contain many virtual pions, they should be essentially independent of energy in the low-energy region. Thus we can define

$$m_0^{(\Delta)} = m_b^{(\Delta)} + \Sigma_{\rm HO}^\Delta , \tag{4.20}$$

and hence

$$S_\Delta^{(0)}(E) = (E - m_0^{(\Delta)})^{-1} . \tag{4.21}$$

Although we could solve Eq. (4.17) as it stands, the problem is greatly simplified by using the approximation[35,36] for the nucleon propagator in $v_{\rm CL}$ [Fig. 9(a)]:

$$\begin{aligned}(E - m_N - \omega_k - \omega_{k'})^{-1} &= (\omega - \omega_k - \omega_{k'})^{-1} \\ &= -\frac{\omega}{\omega_k \omega_{k'}} + \frac{(\omega - \omega_k)(\omega - \omega_{k'})}{\omega_k \omega_{k'} (\omega - \omega_k - \omega_{k'})} , \qquad (4.22) \\ &\simeq -\frac{\omega}{\omega_k \omega_{k'}} . \qquad (4.22')\end{aligned}$$

Note that the correction term in (4.22) vanishes when either the incident or outgoing pion is on shell. For fully one-shell kinematics our crossed Born term is proportional to $1/\omega_{k'}$, and gives the pole term of the Low equation (3.34). In the usual Chew model, Eq. (4.22′) leads to the standard Chew-Low effective range formula.[35] With this approximation $v_{\rm CL}$ is also separable, and t is the solution of the Schrödinger equation for a rank-2 separable potential, which can be written analytically.[37]

In fact, with the usual Chew-Low normalization conventions,[31]

$$v_\Delta(\vec{k}', \vec{k}; \omega) = 4\pi P_{33} v_\Delta(k', k; \omega) , \tag{4.23}$$

with P_{33} the usual projection operator,[31] and

$$v_\Delta(k', k; \omega) = \frac{k' u_\Delta(k') u_\Delta(k) k}{(2\omega_{k'} 2\omega_k)^{1/2}} \frac{f_{\Delta N\pi}{}^2}{3 m_\pi{}^2} S_\Delta^{(0)}(\omega) . \tag{4.24}$$

The potential $v_{\rm CL}$ with approximation (4.22) is

$$v_{\rm CL}(\vec{k}', \vec{k}; \omega) = 4\pi P_{33} v_{\rm CL}(k', k; \omega) , \tag{4.25}$$

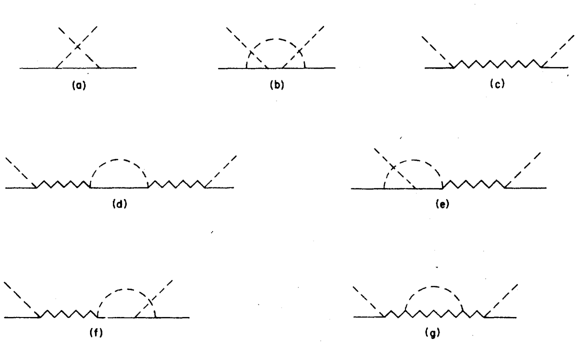

FIG. 8. Terms of Eq. (3.25) after renormalization.

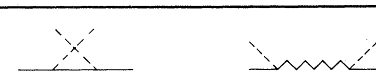

FIG. 9. Born terms of Eq. (4.17).

and

$$v_{\rm CL}(k',k;\omega)=-\frac{4}{3}\frac{f_{NN\pi}{}^2}{m_\pi{}^2}\frac{k'u_N(k')u_N(k)k}{(2\omega_{k'}2\omega_k)^{1/2}}\frac{\omega}{\omega_k\omega_{k'}}\,. \tag{4.26}$$

The nucleon and Δ form factors are related to the Fourier transform of the quark wave functions in the bag by Eq. (2.34). From Eqs. (4.24), (4.26), and (4.21) it is easily seen that $(v_{\rm CL}+v_\Delta)$ is a rank-2, energy-dependent, separable potential

$$v_{\rm CL}(k',k;\omega)+v_\Delta(k',k;\omega)=\omega g(k')g(k)+(\omega-\omega_\Delta^{(0)})^{-1}h(k')h(k)\,, \tag{4.27}$$

$$\omega_\Delta^{(0)}=m_\Delta^{(0)}-m_N\,. \tag{4.28}$$

The solution to the Lippmann-Schwinger equation (4.17) for a rank-2 separable potential can easily be obtained [cf., Eq. (9) of Mongan[37]] as

$$t(k',k;\omega)=N(k',k;\omega)/D(\omega)\,, \tag{4.29}$$

with

$$N(k',k;\omega)=g(k')g(k)D_2(\omega)+h(k')h(k)D_1(\omega)+\omega[g(k')h(k)+h(k')g(k)]D_3(\omega)\,, \tag{4.30a}$$

$$D(\omega)=D_1(\omega)D_2(\omega)-\omega D_3{}^2(\omega)\,. \tag{4.30b}$$

Here D_1 is very closely related to the Chew-Low propagator,

$$D_1(\omega)=1-\frac{2\omega}{\pi}\int_0^\infty\frac{dq\,q^2g^2(q)}{\omega^+-\omega_q}\,, \tag{4.31}$$

and D_2 is the propagator for the dressed Δ:

$$D_2(\omega)=\omega-\omega_\Delta^{(0)}-\frac{2}{\pi}\int_0^\infty\frac{dq\,q^2h^2(q)}{\omega^+-\omega_q} \tag{4.32}$$

$$=S_\Delta^{-1}(\omega)=\omega-\omega_\Delta^{(0)}-\frac{f_{\Delta N\pi}{}^2}{3m_\pi{}^2\pi}\int_0^\infty\frac{dq\,q^4}{\omega_q}\frac{u_\Delta{}^2(q)}{\omega^+-\omega_q}\,. \tag{4.33}$$

Finally, D_3 involves the interference between Chew-Low and Δ terms

$$D_3(\omega)=\frac{2}{\pi}\int_0^\infty\frac{dq\,q^2g(q)h(q)}{\omega^+-\omega_q}\,. \tag{4.34}$$

As for the Chew model of Sec. IV A, all the quantities in the cloudy bag model are finite and no renormalization is absolutely necessary. However, just as for that case, there are advantages to carrying out the Δ mass renormalization here. In particular, we readily identify the term $S_\Delta(\omega)$ in Eq. (4.33) as the Δ propagator. Formally,

$$S_\Delta(E)=[E-m_b^{(\Delta)}-\Sigma_{\rm HO}^{(\Delta)}-\Sigma_{N\pi}^{(\Delta)}(E)]^{-1}\,, \tag{4.35}$$

where $\sum_{N\pi}^{(\Delta)}$ is given by the self-energy diagram of Fig. 4(c) involving a nucleon and a pion in the intermediate state. The renormalization consists of replacing $S_\Delta(\omega)$ by $S'_\Delta(\omega)$:

$$S'_\Delta(\omega)=Z_2^{(\Delta)-1}S_\Delta(\omega)\,, \tag{4.36}$$

where

$$S'_\Delta(E)=[\omega-\omega_\Delta-\tilde{\Sigma}_{N\pi}^{(\Delta)}(E)]^{-1}\,, \tag{4.37}$$

and

$$Z_2^\Delta=[1+\Sigma_{N\pi}^{(\Delta)\prime}(\omega_\Delta)]^{-1}\,. \tag{4.38}$$

Although m_Δ in Eqs. (4.37) and (4.38) will not necessarily be the exact position of the observed P_{33} resonance, because of the interference with the Chew-Low-type graphs, we expect it to be rather close. The one minor difficulty with Eqs. (4.37) and (4.38) is that m_Δ is above the $N\pi$ threshold, so that in fact we must carry out the subtraction procedure on the principal-value part of the self-energy integral only:

$$Z_2^\Delta(\omega)=\left[1+\frac{{\rm P}\Sigma_{N\pi}^{(\Delta)}(\omega)-{\rm P}\Sigma_{N\pi}^{\Delta}(\omega_\Delta)}{\omega-\omega_\Delta}\right]^{-1}, \tag{4.39a}$$

$$Z_2^\Delta=\lim_{\omega\to\omega_\Delta}Z_2^\Delta(\omega)\,, \tag{4.39b}$$

where P means that only the Cauchy principal value

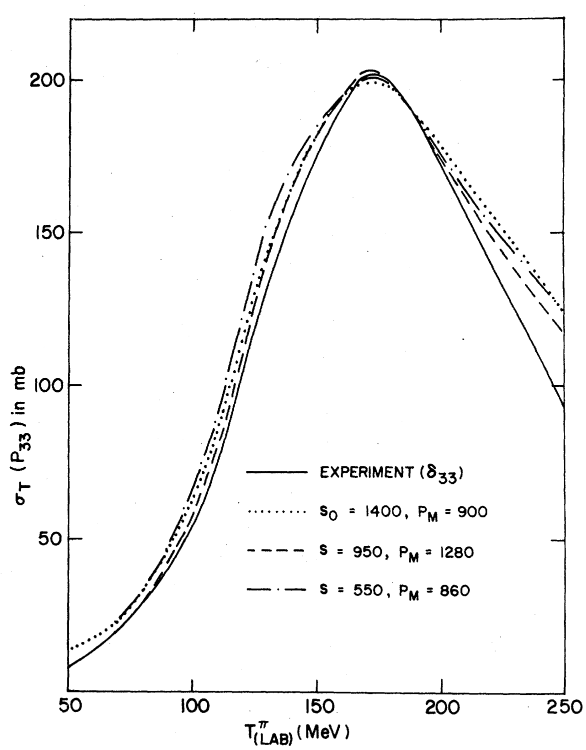

FIG. 10. Multiplicity of solutions of Ref. 27. The vertex function was a simple cutoff at p_M, and s (s_0) was the renormalized (unrenormalized) Δ mass, with respect to the mass of the nucleon.

of the integral is included. We also have

$$\tilde{\Sigma}_{N\pi}^{(\Delta)}(\omega)=\frac{Z_2^{\Delta}(\omega)-Z_2^{\Delta}(\omega_\Delta)}{\omega-\omega_\Delta}-i\frac{f_{\Delta N\pi}{}^2}{3m_\pi{}^2}u^2(p)p^3, \quad (4.40)$$

where

$$p=(\omega^2-m_\pi{}^2)^{1/2}.$$

Finally we observe that this renormalization of the Δ mass also leads to a renormalization of the $\Delta N\pi$ coupling constant.

C. Summary

In terms of the renormalized Δ propagator $S'_\Delta(\omega)$ of Eq. (4.37), and the renormalized coupling constants[39] $(f_{NN\pi}, f_{\Delta N\pi})$, the pion-nucleon P_{33} scattering amplitude for the CBM may be written analytically [using the standard approximation (4.22′)] as

$$t_{\rm CBM}(k',k;\omega)=\frac{g(k')g(k)[S'_\Delta(\omega)]^{-1}+h(k')h(k)D_1(\omega)+[g(k')h(k)+h(k')g(k)]D_3(\omega)}{D_1(\omega)[S'_\Delta(\omega)]^{-1}-\omega D_3{}^2(\omega)}\ . \quad (4.41)$$

All the quantities in Eq. (4.41) were defined in Sec. IV B, but for convenience we recall that $g(k)$ represents the $NN\pi$ vertex, $h(k)$ the $\Delta N\pi$ vertex, D_1 is effectively the Chew-Low propagator, and D_3 represents the interference between the Chew-Low and Δ type of graphs.

The parameters in Eq. (4.41) are ω_Δ, $f_{NN\pi}$, $f_{\Delta N\pi}$, and implicitly the bag radius R. While we cannot fix the renormalized Δ mass at the position of the experimental resonance because of the interference from $v_{\rm CL}$, we nevertheless expect ω_Δ $(\equiv m_\Delta - m_N)$ to be in the region of 290 MeV. The overall magnitude of $f_{NN\pi}$ and $f_{\Delta N\pi}$ is to be determined, but we do not expect the ratio $(f_{\Delta N\pi}/f_{NN\pi})$ to be altered much from the bag-model values. Finally the bag radius appearing in $u(k)$ must be considered an unknown, although everyone has his own prejudices.

Now that we have our solution we can show that it is a solution of the Low equation (3.34). The amplitude (4.41) satisfies the criteria of Castillejo *et al.*[6] for an amplitude to be a solution of the Low equation. This solution is different from the Chew-Low solution, but it has long been known that there are many such solutions. Indeed the fact that different choices for the discrete spectrum of states of the unperturbed Hamiltonian lead to different solutions of the Low equation was pointed out by Dyson[38] in 1957.

V. NUMERICAL RESULTS

As we explained in Sec. IV, the parameters of our theory are the mass of the dressed Δ bag $(\omega_\Delta = m_\Delta - m_N)$, which we expect to be near 290 MeV, the strength of the renormalized $\Delta N\pi$ coupling constant $f_{\Delta N\pi}$, and the bag radius R. The latter, through the form factor (2.34), serves to cut off the contribution of the high-energy virtual pions.

In our first calculations,[27] we followed the suggestion of Brown and Rho[14] by using simply a sharp cut off, $\theta(1/R-k)$, at the $\Delta N\pi$ and $NN\pi$ vertices. This gave a multiplicity of solutions, each of which fit the P_{33} scattering data equally well. For example, Fig. 10 shows the fits to the experimental P_{33} total cross section for two possible combinations of (ω_Δ, R), namely (950 MeV, 0.15 fm) and (550 MeV, 0.23 fm). In general, as ω_Δ decreased, the bag radius for the best fit increased. In the limit of very large Δ mass, the solution was essentially the Chew-Low result, and the percentage of Δ in the observed P_{33} resonance [as measured by the relative strength of the *gg* and *hh* terms in Eq. (4.41) at the pole] decreased to zero.

From many points of view this multiplicity of solutions was unsatisfactory. We needed some constraint other than πN scattering to choose between the solutions. Fortunately, this problem disappears when the theoretically derived form factor (2.34) is used. Indeed, in that case it is very hard to find a solutions. With $f_{\Delta N\pi}$ anywhere near the usually accepted range [and $(f_{\Delta N\pi}/f_{NN\pi})^2 = \frac{72}{25}$] we were able to find only *one* acceptable solution. This fit is shown in Fig. 11. It is an extremely good fit, corresponding to (ω_Δ, R) equal to (294 MeV, $R=0.72$ fm). The coupling constant $f_{\Delta N\pi}$ is 0.42, and the delta carries about 80% of the strength at the P_{33} resonance. We stress that this minimum in χ^2 space corresponding to this fit was quite sharp, and to the best of our knowledge it is unique.

The bag radius for the CBM fit is intermediate between the Brown-Rho suggestion of ~0.3 fm and the MIT value of about 1 fm. It is more in line with the suggestions of many of the early papers dealing with quark confinement. The mass $\omega_\Delta = 294$ MeV corresponds to a dressed Δ mass very close to the observed P_{33} resonance position, but slightly lighter ($m_\Delta \sim 1232$ MeV, compared with $m_R = 1236$ MeV from experiment).

In our model the bag radius is extremely well determined. (A shift of only one-tenth of a fermi would destroy the fit.) However, there are many features of a complete theory of πN scattering which we have omitted in this initial work. In par-

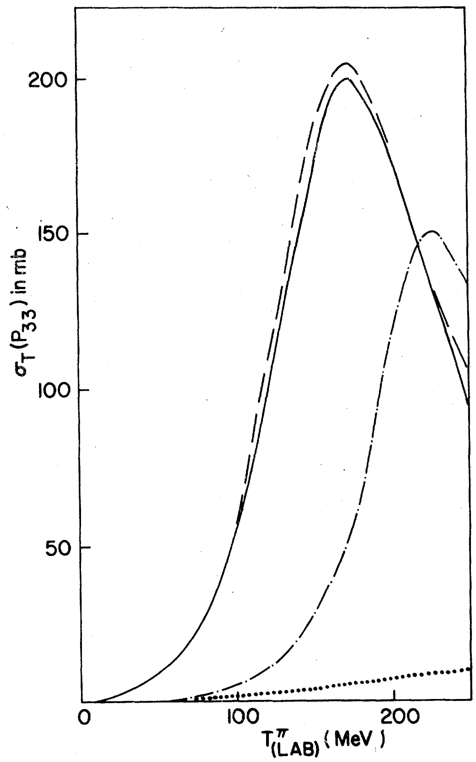

FIG. 11. Best fit in the cloudy bag model (dashed curve) to the experimental P_{33} total cross section (solid). The dash-dotted line shows the effect of arbitrarily setting $f_{NN\pi}(f_{\Delta N\pi})$ to zero, with all other parameters unchanged.

ticular, the inclusion of crossing and inelasticities would probably increase the size of the source somewhat. From our experience with the Chew-Low model, this could increase R by as much as 20%. Thus if forced to quote some estimate of the possible systematic error in the determination of R in the CBM, we would guess 0.72 ± 0.14 fm.

We also note that the $NN\pi$ coupling constant for the CBM solution is about 10% lower than the experimental value $f_{NN\pi}{}^2 = 0.06$. [Recall that $f_{NN\pi}$ is given in terms of $f_{\Delta N\pi}$, by demanding that $(f_{\Delta N\pi}/f_{NN\pi})^2$ be $\frac{72}{25}$.] Since we do not claim better than perhaps 20% accuracy in the determination of the bag radius, this level of agreement is acceptable for the moment. Future work may involve the explicit calculation of vertex corrections like Fig. 12.

The essential feature of the CBM is that one must keep both couplings $f_{\Delta N\pi}$ and $f_{NN\pi}$ nonzero. Nevertheless, it is interesting to turn one of these off to obtain either an elementary Δ model or an equivalent Chew-Low model. In both cases only one good solution could be found. For the "elementary Δ model" ω_Δ was 265 MeV, and R was 0.16 fm. In the effective Chew-Low case, R was 0.22 fm. These two cases are shown in Fig. 13.

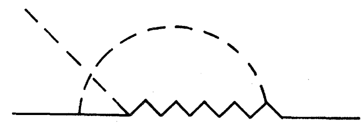

FIG. 12. A possible higher-order vertex correction to the $NN\pi$ coupling constant obtained in this work.

As we have emphasized our model keeps both pionic and Δ terms. One may investigate the relative importance of the two kinds of effects by setting $f_{NN\pi}$ or $f_{\Delta N\pi}$ equal to zero. This is shown in Fig. 11. If $f_{NN\pi} = 0$, the position of the calculated resonance peak moves up by about 50 MeV. Thus pionic terms are important. If $f_{\Delta N\pi} = 0$ the calculated resonance goes away; hence, Δ terms are much more important than pionic terms.

VI. CONCLUSION

By incorporating chiral invariance in the MIT bag model, we obtain a theory in which the pion field is coupled to the confined quarks only at the bag surface. This leads us naturally to a theory of bare (bag-state) nucleons and Δ's interacting with a quantized pion field. Renormalization of

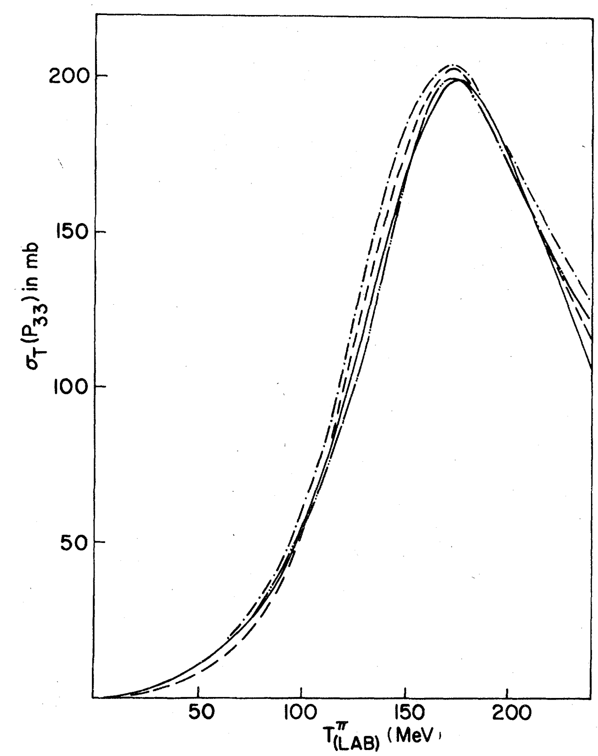

FIG. 13. Best-fit calculations using the CBM form factor but retaining only the delta graphs (dash-dot-dot curve, $R = 0.16$ fm), or only Chew-type graphs (dot-dash, $R = 0.22$ fm)—the solid line is the experimental result, and the dashed curve is the full CBM curve of Fig. 11.

this theory is necessary and is carried out. Explicit equations were derived for the physical nucleon and the πN scattering amplitude. This scattering amplitude satisfies the Low equation.

In the present model the Δ resonance is given by the coherent contribution of elementary Δ and Chew-Low-type graphs. Although the Δ is not an exact eigenstate of the Hamiltonian, by examining the residue of the (3,3) t matrix at the pole it is found that 80% of the strength is carried by the elementary Δ contributions, and only 20% by Chew-Low. This is a very satisfying result, because it unifies the two apparently contradictory theories of πN scattering, namely the quark model and the Chew-Low model, which have existed side by side in the literature for many years. In a later paper we intend to examine the consequences of this new model of the off-shell behavior of the πN interaction for pion-nucleus scattering—particularly the Lorentz-Lorenz effect.

The bag radius which we obtain (0.72 fm) is interesting for a number of reasons. It lies below the MIT value ($R \gtrsim 1$ fm) but considerably above the value of ~0.3 fm oringinally suggested by Brown and Rho.[14] (However, recent self-consistent calculations by the Stony Brook group[16] have suggested that any value of R from 0.5–1.5 fm could be acceptable.)

One of the most fascinating observations concerns the charge distribution of the neutron. In our model (to lowest order) the physical nucleon is 61% of the time a nucleon bag, 25% an $N\pi$ state, and 14% a $\Delta\pi$ state. In the absence of quark-quark interactions the neutron bag has no charge distribution. In higher order their spin-spin interaction would tend to give a negative charge radius, but with R=0.72 fm this effect is far too small. On the other hand, the N-bag-plus-pion state has a probability $\frac{2}{3}$ of being a proton bag with a π^- cloud at the surface (hence the cloudy bag model). Since this cloud is very much localized at the surface, we see that there is a very natural explanation of the positive charge core of the neutron and its negative tail. Most important for the moment, we see that the *bag radius* will be very naturally associated with the *zero* of the *neutron charge distribution.* Experimentally this occurs at about 0.8 fm, which is surprisingly close to our bag radius. Detailed calculations of the nucleon (and Δ) charge form factors and magnetic moments will be reported in a forthcoming paper, but there is reason to believe that this relatively small pion admixture will help to cure a number of quantitative failures in the pure quark bag models.

Another interesting feature of this bag radius is that it is no longer so difficult to accommodate classical nuclear physics with the bag model of nucleon structure. Our nucleon bags will only occupy about 35% of the typical nuclear volume, and there will certainly be long-range pion exchange forces between them. In addition, the lower limit on the critical density for percolation, as discussed by Baym,[25] is $\rho_c = 0.34/\frac{1}{3}4\pi R^3 = 0.22$ nucleons/fm^3, which is some 30% above normal nuclear densities.

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[1]G. F. Chew, Phys. Rev. 94, 1748 (1954).
[2]G. F. Chew, Phys. Rev. 94, 1755 (1954).
[3]G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956).
[4]G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1337 (1957).
[5]J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963).
[6]L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).
[7]A. W. Thomas and R. H. Landau, Phys. Rep. 58, 121 (1980).
[8]G. A. Miller, Phys. Rev. C 14, 2230 (1976); D. J. Ernst and M. B. Johnson, *ibid.* C 17, 247 (1978); D. J. Ernst and M. B. Johnson, Phys. Rev. C 22, 651 (1980); F. Myhrer and A. W. Thomas, Nucl. Phys. A326, 497 (1979); M. K. Banerjee, in *Meson-Nuclear Physics—1979*, proceedings of the 2nd International Topical Conference, Houston, edited by E. V. Hungerford III (AIP, New York, 1979), p. 30.
[9]M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964).
[10]G. E. Brown and W. Weise, Phys. Rep. 22C, 280 (1975).
[11]E. J. Moniz, in *Meson-Nuclear Physics—1976*, proceedings of the International Topical Conference, Pittsburgh, edited by P. D. Barnes, R. A. Eisenstein, and L. S. Kisslinger (AIP, New York, 1976), p. 105.
[12]L. C. Liu and C. M. Shakin, Phys. Rev. C 18, 604 (1978).
[13]A. Chodos and C. B. Thorn, Phys. Rev. D 12, 2733 (1975).
[14]G. E. Brown and M. Rho, Phys. Lett. 82B, 177 (1979).

[15]G. E. Brown, M. Rho, and V. Vento, Phys. Lett. 84B, 383 (1979).
[16]V. Vento, J. G. Jun, E. M. Nyman, M. Rho, and G. E. Brown, Stony Brook report, 1980 (unpublished).
[17]R. L. Jaffe, lectures at the 1979 Erice Summer School "Ettore Majorana," 1979 (unpublished).
[18]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).
[19]K. Johnson, Acta Phys. Pol. B6, 865 (1975).
[20]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974).
[21]T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).
[22]A. J. G. Hey, in *Topics in Quantum Field Theory and Gauge Theories*, proceedings of the 8th International Seminar on Theoretical Physics, Salamanca, Spain, 1977 (Springer, Berlin/Heidelberg, 1978), p. 155.
[23]P. Hasenfratz and J. Kuti, Phys. Rep. 40, 76 (1978).
[24]F. E. Close, *An Introduction to Quarks and Partons* (Academic, New York, 1979), Chap. 18.
[25]G. Baym, Physica (Utrecht) 96A, 131 (1979).
[26]C. G. Callen, R. F. Dashen, and D. J. Gross, Phys. Rev. D 19, 826 (1979).
[27]G. A. Miller, A. W. Thomas, and S. Théberge, Phys. Lett. 91B, 192 (1980).
[28]M. Gell-Mann and M. Levy, Nuovo Cimento 16, 53 (1960).
[29]F. Gross, in *Proceedings of the Workshop on Baryonium and Other Unusual Hadron States, Orsay, 1979*, edited by B. Nicolescu, J. M. Richard, and R. Vinh Mau (Division de Physique Theorique, Institut de Physique Nucleaire, Orsay, 1979), p. XXV-1.
[30]T. D. Lee, Phys. Rev. 95, 1329 (1954).
[31]S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1964).
[32]G. C. Wick, Rev. Mod. Phys. 27, 339 (1955).
[33]We thank M. K. Banerjee for pointing out that our scattering amplitude should obey the Low equation.
[34]F. J. Dyson, Phys. Rev. 75, 486, 1736 (1949).
[35]G. A. Miller, in *Meson-Nuclear Physics—1979*, proceedings of the 2nd International Topical Conference, Houston, edited by E. V. Hungerford III (AIP, New York, 1979), p. 561.
[36]G. A. Miller and E. M. Henley (unpublished).
[37]T. R. Mongan, Phys. Rev. 175, 1260 (1968).
[38]F. J. Dyson, Phys. Rev. 104, 157 (1957).
[39]We use the same symbol here as in Eqs. (4.24) and (4.26), but the $f_{\Delta N\pi}$ here includes an extra factor of $Z_2^{(\Delta)1/2}$.