Hadronic transitions for heavy quarkonia in the O-meson model

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Using the O-meson model of Freund and Nambu, we make predictions for the rates of $\psi''(4.032)\to\psi\pi\pi$ and $\psi'''(4.414)\to\psi\pi\pi$. We find that ψ'' and ψ''' should have significant branching ratios to $\psi\pi\pi$. With a broken-SU(5)-symmetry scheme, we extrapolate to the Υ's and find that the decays of Υ'' should be dominated by the two-pion transition $\Upsilon''\to\Upsilon\pi\pi$, contrary to other predictions. The scaling behavior of two-pion transitions is examined, assuming a logarithmic quark-antiquark potential. We also predict that transitions of the form $V'\to V\eta$ should be insignificant for the Υ's.

INTRODUCTION

The study of processes which violate the Okubo-Zweig-Iizuka (OZI) rule is of great interest, since an understanding of the mechanism by which the OZI rule is broken is vital to models of the interquark force.

With the discovery of ψ and ψ', whose decays are solely OZI-rule violating, a rich area was made available for the study of such processes. The simple O-meson model of Freund and Nambu has met with some success in explaining the variation in the degree to which the OZI rule can be violated.[1-7] In this approach, the OZI-rule-violating process is mediated by a quarkless state from dual dynamics, the O meson. For example, the decay $\psi\to\rho\pi$ is mediated by the O_V ($J^{PC}=1^{--}$), which is the "first daughter of the Pomeron" [Figs. 1(a) and 1(b)]. The O_V is an SU(4) singlet which mixes the two-quark mesons ψ ($c\bar{c}$) and $\omega[1/\sqrt{2})(u\bar{u}+d\bar{d})]$. Similarly, in the decay $\psi'\to\psi\pi\pi$, a broad scalar resonance O_s ($J^{PC}=0^{++}$) mixes ϵ_c ($c\bar{c}$) and $\epsilon[(1/\sqrt{2})(u\bar{u}+d\bar{d})]$. [See Figs. 1(c) and 1(d).]

Although its origins lie in dual dynamics, the O-meson model can be interpreted in quantum chromodynamics (QCD), where the O meson approximates the interacting gluons as a pole, and the SU(4)-singlet nature reflects flavor independence. Various QCD models predict stable states of gluons, "glueballs," suggesting that the pole approximation may be a good one.[8,9]

The discovery of the Υ system ($b\bar{b}$) has encouraged attempts to extrapolate from the known decays of ψ to those of Υ, where little is measured.[10-13] This has often involved scaling laws which are derived from a $q\bar{q}$ potential fitted to the ψ and Υ energy levels. It is therefore tempting to examine the predictions of the Freund-Nambu model, since it is independent of potentials. In this paper we predict rates for hadronic transitions between the radial excitations of Υ, and also for the high charmonium states $\psi''(4.032)$ and $\psi'''(4.414)$.

O_V MASS

Originally, Freund and Nambu[5] used the $J^{PC}=1^{--}$ state O_V, and an assumed mass range $\sqrt{2}<M_O<\sqrt{3}$ GeV to predict the ratio $\Gamma(\psi\to\rho\pi)/\Gamma(\phi\to\rho\pi)$. Assuming ideal mixing and an SU(4)-singlet O_V, the Lagrangian at the O_V-quark-meson vertices is

$$L=f_{O_V}O_V^\mu(\sqrt{2}\,\omega_\mu+\phi_\mu+\psi_\mu)\,, \tag{1}$$

which yields, with $M_\rho\simeq M_\omega$,

$$\frac{\Gamma(\psi\to\rho\pi)}{\Gamma(\phi\to\rho\pi)}=\left(\frac{M_\phi{}^2-M_O{}^2}{M_\psi{}^2-M_O{}^2}\right)^2\frac{M_\psi{}^2-M_\rho{}^2}{M_\phi{}^2-M_\rho{}^2}\;\frac{M_\phi{}^3}{M_\psi{}^3}\times\left[1-\frac{M_\pi{}^2}{(M_\phi-M_\rho)^2}\right]^{-3/2}. \tag{2}$$

Recent experimental statistics suggest $\Gamma(\phi\to\rho\pi)=603\pm60$ keV and $\Gamma(\psi\to\rho\pi)=0.75\pm0.25$ keV. Assuming the validity of (2), we calculate $1.11<M_O<1.16$ GeV (i.e., M_O must be close to M_ϕ), in agreement with Robson,[8] who has studied this problem in lattice gauge theory.

QCD predictions of glueball masses range from 0.9 to 1.6 GeV.[8] This is then consistent with the identification of the O_V with a glueball.

TWO-PION TRANSITIONS

Hayashi *et al.*[2] and Pinsky *et al.*[3] have studied the decay $\psi'(3.684)\to\psi(3.095)\pi\pi$ in the Freund-Nambu model [Figs. 1(c) and 1(d)]. These authors use a broad resonance $O_S(J^{PC}=0^{++})$, assuming it to be in the same mass range as O_V, i.e., $\sqrt{2}<M_O<\sqrt{3}$ GeV. Ideal mixing implied

$$L=f_{O_S}O_S(\sqrt{2}\epsilon+\epsilon_s+\epsilon_c)\,, \tag{3}$$

and chiral symmetry at the $\epsilon\pi\pi$ vertex gives

$$L=g_{\epsilon\pi\pi}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\epsilon\,. \tag{4}$$

These two papers differ in their treatment of the $\psi'\psi\epsilon_c$ vertex. Pinsky *et al.* use

$$L=g_{\psi'\psi\epsilon_c}(\partial_\mu\psi'_\nu-\partial_\nu\psi'_\mu)(\partial^\mu\psi^\nu-\partial^\nu\psi^\mu)\epsilon_c\,, \tag{5}$$

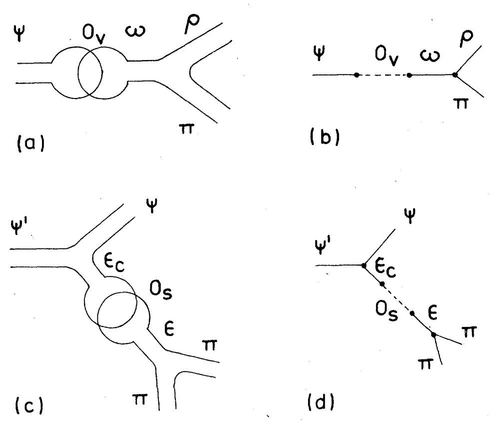

FIG. 1. ψ decays in the O-meson model.

while Hayashi *et al.* assume

$$L = M_{\psi'}\,\tilde{g}_{\psi'\psi\epsilon_c}\psi'_{\mu}\psi^{\mu}\epsilon_c\,. \qquad (6)$$

Equations (5) and (6) will hereafter be referred to as L_1 and L_2. The main advantage of L_2 appears to be its simplicity, whereas L_1 is in the spirit of vector dominance. We use both L_1 and L_2 in our analysis.

The main purpose of these papers was not to predict $\Gamma(\psi'\to\psi\pi\pi)$ but to determine the parameter f_{O_S} from the experimental value of $\Gamma(\psi'\to\psi\pi\pi)$, and thus predict the rates of other decays in the ψ region. Hence all free parameters except f_{O_S} were extracted from other data. In order to fix $g_{\psi'\psi\epsilon_c}$ or $\tilde{g}_{\psi'\psi\epsilon_c}$, comparison was made with the process $\rho'\to\rho\epsilon$ and a symmetry scheme was invoked. Pinsky *et al.* introduced perfect SU(4) symmetry by assuming $g_{\psi'\psi\epsilon_c}=\sqrt{2}\,g_{\rho'\rho\epsilon}$. Note that these g have dimension of (energy)$^{-1}$. Hayashi *et al.* used a broken-SU(4) symmetry by forming a dimensionless coupling $\tilde{g}$. This was clearly achieved by introducing the mass of one of the participant particles: $M_{\psi'}$ for the ψ' decay and $M_{\rho'}$ for ρ' decay. The rest of the Lagrangian is SU(4) symmetric: $\tilde{g}_{\psi'\psi\epsilon_c}=\sqrt{2}\,\tilde{g}_{\rho'\rho\epsilon}$. Both schemes would be identical if SU(4) were a perfect symmetry in nature, in which case $M_{\rho'}=M_{\psi'}$. The fact that SU(4) is badly broken in mass relations $[M_{\rho'}(1.6)\neq M_{\psi'}(3.7)]$ means that the two schemes are very different. We will be using our symmetry scheme to relate the ψ (3 to 4 GeV) and Υ (9 to 11 GeV) systems, so the choice between perfect or broken symmetry will be of great importance.

We have chosen broken SU(4), and its SU(5) extension for Υ, as the natural symmetry at the $V'V\epsilon_V$ vertex for the following reason: Assuming that the underlying quark-gluon dynamics contains dimensionless couplings, a coupling constant in a Lagrangian such as L_1 or L_2 which is not dimensionless must contain the results of various energy and momentum transfers amongst the quarks and gluons involved. For the $V'V\epsilon_V$ vertex, these energies will tend to be characteristic of the particle masses. Hence separating out the mass of the decaying particle, as in broken symmetry, is a reasonable approximation. If the quark mass is the only flavor-dependent property, then the rest of the Lagrangian should remain SU(N) symmetric, where N is the number of flavors.

Using the broken symmetry, we adapted the formulas of Pinsky *et al.* to the use of both L_1 and L_2. The new results for f_{O_S} were 0.6 GeV^2 for L_1 and 1.1 GeV^2 for L_2, compared with the two papers' original predictions of 0.4 and 0.7 GeV^2, respectively. Our purpose is to predict the general decay $V'\to V\pi^+\pi^-$ from

$$\Gamma(\psi'\to\psi\pi^+\pi^-)=75\pm 25\ \text{keV}. \qquad (7)$$

This means that f_{O_S} or $g_{\epsilon\pi\pi}$ are not needed, nor the absolute value of $\tilde{g}_{\psi'\psi\epsilon_c}$, etc. If $\tilde{g}_{V'V\epsilon_V}=\tilde{g}_{\psi'\psi\epsilon_c}$, we obtain

$$\Gamma(V'\to V\pi^+\pi^-)=\frac{3.07\pm 30\%}{M_{V'}{}^5}\int_{2m_\pi}^{M_{V'}-M_V}\rho\,dx \quad \text{for } L_1, \qquad (8)$$

$$=\frac{1.09\pm 30\%}{M_V{}^2 M_{V'}{}^3}\int_{2m_\pi}^{M_{V'}-M_V}\rho^*dx \quad \text{for } L_2, \qquad (9)$$

where

$$\rho(x)=x\left(1-\frac{4m_\pi{}^2}{x^2}\right)^{1/2}\left(\frac{x^2}{2}-m_\pi{}^2\right)[x^4-2x^2(M_V{}^2+M_{V'}{}^2)+(M_{V'}{}^2-M_V{}^2)^2]^{1/2}[(M_{V'}{}^2+M_V{}^2-x^2)^2+2M_{V'}{}^2M_V{}^2]$$
$$\times[(x^2-M_\epsilon{}^2)^2+M_\epsilon{}^2\Gamma_\epsilon{}^2]^{-1}[(x^2-M_{\epsilon_c}{}^2)^2+M_{\epsilon_c}{}^2\Gamma_{\epsilon_c}{}^2]^{-1}[(x^2-M_O{}^2)^2+M_O{}^2\Gamma_O{}^2]\ ,$$

and $\rho^*(x)$ is the same as $\rho(x)$, except that $[(M_{V'}{}^2+M_V{}^2-x^2)^2+8M_{V'}{}^2M_V{}^2]$ replaces $[(M_{V'}{}^2+M_V{}^2-x^2)^2+2M_{V'}{}^2M_V{}^2]$. With an isospin-zero $\pi\pi$ system,

$$\Gamma(V'\to V\pi\pi)=\Gamma(V'\to V\pi^+\pi^-)+\Gamma(V'\to V\pi^0\pi^0)$$
$$=\tfrac{3}{2}\Gamma(V'\to V\pi^+\pi^-)\,. \qquad (10)$$

We use

$$M_\epsilon=0.7\ \text{GeV},\quad \Gamma_\epsilon=0.6\ \text{GeV},$$
$$M_O=1.11\ \text{GeV},\quad \Gamma_O=1\ \text{GeV}, \qquad (11)$$
$$M_{\epsilon_c}=3.5\ \text{GeV},\quad \Gamma_{\epsilon_c}=0.001\ \text{GeV}.$$

Because the O_S and ϵ are broad, the particular choice of their masses does not affect the calculation greatly.

TWO-PION TRANSITIONS OF THE HIGHER STATES OF ψ

We assume the decay proceeds via the ϵ and O_S for the higher states, and that $\tilde{g}_{\psi'\psi\epsilon_c} = \tilde{g}_{\psi'\psi\epsilon_c}$, or in general that $\tilde{g}_{V'V\epsilon_V}$ is independent of which S state the quarks occupy. Hayashi *et al.* and Pinsky *et al.* both assume that the magnitude of $\tilde{g}_{\psi\psi\epsilon_c}$, $\tilde{g}_{\psi'\psi\epsilon_c}$, and $\tilde{g}_{\psi'\psi\epsilon_0}$ (or g) are the same, Hayashi *et al.* finding that this is roughly consistent with data on ψ'. The general assumption above is therefore likely to work. Using (8)–(11), we estimate

$$\Gamma(\psi''(4.032) \to \psi(3.095)\pi\pi) = 6.7 \pm 2 \text{ MeV for } L_2$$
$$= 5.6 \pm 2 \text{ MeV for } L_1, \quad (12)$$

compared with $\Gamma(\psi' \to \psi\pi\pi) \simeq 0.11$ MeV. The increased mass difference between the two ψ states has caused a 60-fold increase in the width. This is a consequence of the final-state interaction at the $\epsilon\pi\pi$ vertex.[14] The derivative coupling introduces a term in the integrand of $(x^2/2 - m_\pi^2)^2$ (see definition of ρ), where x is the invariant mass of the $\pi\pi$ system. The participation of the ϵ with its chiral-symmetric coupling has been used to explain the observed invariant-mass distribution of the $\pi\pi$ system in $\psi' \to \psi\pi\pi$.[3,4] Figure 2 illustrates that this high-power behavior is the major contributor to the vast increase.

We also estimate

$$\Gamma(\psi'''(4.414) \to \psi\pi\pi) = 43 \pm 13 \text{ MeV for } L_2$$
$$= 10 \pm 3 \text{ MeV for } L_1. \quad (13)$$

ψ'' and ψ''' are above the OZI threshold. Two experimental values of the total width of ψ''' are 33 ± 10 MeV (Ref. 15) and 66 ± 15 MeV.[16] Thus the OZI-rule violating decay $\psi''' \to \psi\pi\pi$ should be of some importance in the decays of ψ''', and not just the OZI-rule-allowed $\psi''' \to D\bar{D}$.

The Υ's

The broken-SU(5) relation is $\tilde{g}_{\Upsilon'\Upsilon\epsilon_b} = \tilde{g}_{\psi'\psi\epsilon_c}$, and using

$$M_\Upsilon = 9.46 \text{ GeV}, \quad M_{\Upsilon'} = 10.02 \text{ GeV}, \quad (14)$$
$$M_{\Upsilon''} = 10.40 \text{ GeV}, \quad M_{\epsilon_b} = 9.7 \text{ GeV},^{17} \quad \Gamma\epsilon_b \simeq 0,$$

we obtain

$$\Gamma(\Upsilon' \to \Upsilon\pi\pi) = 1.5 \pm 0.5 \text{ keV for } L_1$$
$$= 1.3 \pm 0.5 \text{ keV for } L_2. \quad (15)$$

Assuming $\tilde{g}_{\Upsilon''\Upsilon\epsilon_b} = \tilde{g}_{\Upsilon'\Upsilon\epsilon_b}$ (see earlier),

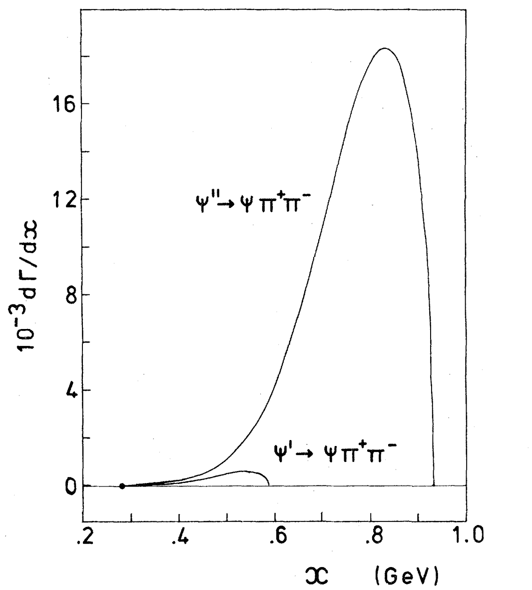

FIG. 2. Predicted invariant-mass distribution of the two-pion system for ψ' (3.684) $\to \psi\pi^+\pi^-$ and ψ'' (4.032) $\to \psi\pi^+\pi^-$, using L_2.

$$\Gamma(\Upsilon'' \to \Upsilon\pi\pi) = 150 \pm 50 \text{ keV for } L_1,$$
$$= 120 \pm 50 \text{ keV for } L_2. \quad (16)$$

Experiment and phenomenological predictions have suggested that $\Gamma(\Upsilon' \to \Upsilon\pi\pi)$ should indeed be small compared with $\Gamma(\psi' \to \psi\pi\pi)$.[12,18] It is clear that the value obtained is dependent on the broken-SU(5) symmetry. That $\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$ is large compared with $\Gamma(\Upsilon' \to \Upsilon\pi\pi)$ is again a consequence of the final-state interaction at the $\epsilon\pi\pi$ vertex. Recently a QCD calculation was done which treats such hadronic transitions as $V' \to V + 2$ gluons.[12] This analysis predicts $\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$ to be small. The qualitative disagreement with our prediction is not surprising since their assumption that "the 2 gluons turn into hadrons with unit probability" excludes final-state effects which, in the O-meson model, dominate the process.

TWO-PION TRANSITIONS AND SCALING

In order to extend calculations to possible heavier quarks, an estimate of how energy levels scale with quark mass is needed. The logarithmic $q\bar{q}$ potential has had some success in the ψ and Υ systems. For this potential, the excited states have the same level spacings for all quark masses M_q, which is in accord with $M_{\Upsilon'} - M_\Upsilon \simeq M_{\psi'} - M_\psi$. Quigg *et al.*[19] have computed how the OZI threshold should scale with quark mass, so we used their calculations to obtain expected energy levels, for logarithmic potential, between $M_q = 1.5$ and 15 GeV. Further, assuming $M_{\epsilon_Q} = M_{1^3S_1}$ (this is good for

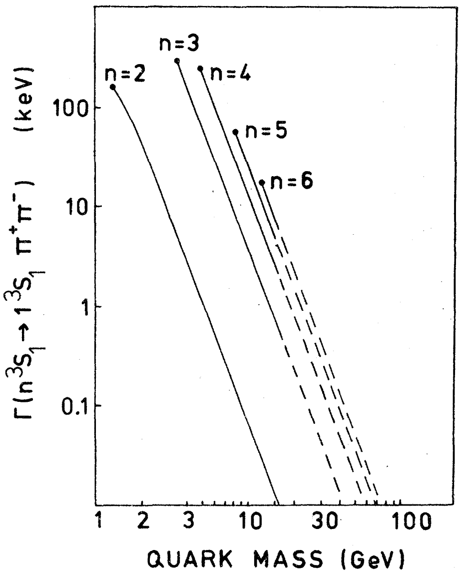

FIG. 3. Scaling behavior of $V' \to V\pi^+\pi^-$ for V' a radial excitation below OZI threshold. The OZI threshold is indicated by the solid dots. ψ is at $M_q = 1.5$ GeV.

large M_q but leads to slight discrepancies around ψ), and using L_2, we obtain in Fig. 3 a plot of $\Gamma(N^3S_1 \to 1^3S_1\pi\pi)$ for N a state below the OZI threshold. The width quickly settles down to the asymptotic M^{-4} behavior which is expected for both L_1 and L_2, using the broken symmetry. M^{-4} scaling is in disagreement with an asymptotic M^{-2} scaling proposed by Gottfried[10] for logarithmic potential, using a different approach. However, he notes that the resulting prediction of $\Gamma(\Upsilon' \to \Upsilon\pi\pi) = 14.5$ keV (assuming asymptotic behavior) is probably too large.

$V' \to V\eta$

Using the Freund-Nambu model, a $J^{PC} = 0^{-+}$ glueball, and broken SU(5), we find that the two-body decay $V' \to V\eta$ is insignificant for the Υ's, and any higher quarkonia. Thus two-pion decay should dominate the decays of Υ''.

CONCLUSION

We have found that the O-meson model of Freund and Nambu predicts dramatic effects in transitions of the form $V' \to V\pi\pi$, arising from the presence of the ϵ and its chiral-symmetric interaction. The usual expectation that higher momentum transfer means a smaller rate is not realized, and in particular the states $\psi''(4.032)$ and $\psi'''(4.414)$ should decay to $\psi\pi\pi$ with substantial branching ratios, suggesting a large-scale violation of the OZI rule. Our assumption of a broken-SU(N) symmetry leads to qualitative agreement with previous predictions for $\Gamma(\Upsilon' \to \Upsilon\pi\pi)$, but contrary to other estimates we predict that $\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$ should be large and dominate the decays of Υ'. We await data on decays of the Υ system and the higher states of ψ with great interest.

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