Branching ratios in baryon decay

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We estimate branching ratios for proton (and neutron) decay expected in the SU_5 grand unified theory of decay with a simple SU_6-invariant quark model for baryon structure and assuming several possible models for quark kinematics in the decay. Our branching ratios agree in the appropriate limit with two recent estimates. The proton (and neutron) decay into single pions (πe^+ plus $\pi\bar{\nu}$) more than 50% of the time in all models we consider.

I. INTRODUCTION

The most dramatic scenario raised in the context of spontaneously broken gauge theories is that of grand unification[1] with the attendant possibility of baryon decay. While the entire flurry may yet be short-lived—if no experimental evidence for decay will emerge—it seems useful to have reliable computations of the branching ratios expected for these decays. This is important both for the search and also—if the search is successful—for the interpretation of the observations. We discuss below calculations of branching ratios for proton and neutron decay on the basis of simple (SU_6-invariant) quark structure for baryons and simple models for decay (SU_5).

Several papers have already been written[2-5] on this topic. There is a wide range of disagreement in previous work—both in the results and also in the approximations used. Therefore, we make an effort to clarify the situation. The method we use is sufficiently general to cover several kinematic models, some of which have already been discussed: a static limit described in detail by Gavela *et al.*[5] and a relativistic limit which is close to the bag-model work of Donoghue.[4] Our branching ratios agree in the appropriate limit with these two calculations, which is somewhat reassuring. We also compute branching ratios at an intermediate point, called the recoil model, in which the initial quarks are at rest while the final antiquark recoils with $p/m = 0.75$, the appropriate kinematic value for a massless lepton. The branching ratios in this case are, not surprisingly, in between the static and the relativistic values.

We do not discuss in this note the proton lifetime because we feel that the uncertainties of extrapolation from the unification mass greatly overwhelm the effects we are worried about here. In addition we are not quite certain which of the three models discussed here is realistic. Thus, while we can see that the question of the rate is very important, we have nothing to offer here to improve existing estimates.

II. DETAILS OF COMPUTATION

Throughout this paper we compute matrix elements for the decay of a baryon at rest, with spin component $\pm\frac{1}{2}$ about the z axis, into a lepton which always moves along the positive z axis and a meson which always recoils along the negative z axis. The (total) rates of decay involve sums of squares of such matrix elements for both polarizations of the initial baryon. The matrix elements are computed by sandwiching the interaction responsible for the decay between appropriate initial and final states.

A. Interaction

We follow Weinberg[6] and Wilczek and Zee[7] to assume that the Lagrangian responsible for the decay is invariant under $SU_3 \times SU_2 \times U_1$ and therefore has the form

$$\begin{aligned}\mathcal{L} \equiv 2\sqrt{2}\,G[&\epsilon_{ijk}\bar{u}^c_{kL}\gamma_\mu u_{jL}(r\bar{e}^+_L\gamma_\mu d_{iL} - \bar{e}^+_R\gamma_\mu d_{iR} \\ &\qquad - \bar{\mu}^+_R\gamma_\mu s_{iR} - \bar{\mu}^+_L\gamma_\mu s_{iL}) \\ &+ \epsilon_{ijk}\bar{u}^c_{kL}\gamma_\mu d_{jL}(\bar{\nu}_e\gamma_\mu d_{iR} + \bar{\nu}_\mu\gamma_\mu s_{iR}) \\ &+ \mathrm{H.c.}]\,,\end{aligned} \tag{1}$$

where we have included terms corresponding to strange quarks following Gavela *et al.*[5]; otherwise the notation we follow is identical to that of Buras, Gaillard, Ellis, and Nanopoulos,[8] who chose $r = 2$ as appropriate to SU_5. In the Lagrangian (1) the fermion fields are expanded in terms of creation (and annihilation) operators $\hat{a}^\dagger$, $\hat{b}^\dagger$ appropriate to particles and antiparticles as, for example,

$$u_j \equiv \hat{a}(u_j)u + \hat{b}^\dagger(u_j)v\,, \tag{2}$$

where $\hat{a}(u_j)$ destroys an up quark of color j (and spinor u) and $\hat{b}^\dagger(u_j)$ creates an antiquark of flavor anti-up and color anti-j, with spinor v. The Dirac spinors u and v are defined by Bjorken and Drell[9] and in the charge-conjugate fields (u^c) the spinors u and v are interchanged:

$$u_k^c \equiv \hat{a}(u_k)v + \hat{b}^\dagger(u_k)u. \tag{3}$$

In using the Lagrangian (1) we assume all fermions to be free just before and just after the interaction and described by appropriate spinors u and v. The hadron wave function, to which we now turn, provides the amplitudes to find various flavors, colors, and spins of quarks in the hadron.

B. Baryon wave functions

We assume that only valence quarks are important in decay, and that the baryon wave functions are appropriate to the SU_6-symmetry limit. In particular, we assume that the orbital state of all three quarks is identical and is factored out for simplicity. In addition to spin and flavor the Lagrangian (1) acts also on the color indices of the quarks which have to be specified, and therefore we take for a proton of spin up,

$$|p_\uparrow\rangle \equiv \frac{1}{3\sqrt{2}}\,\epsilon_{ijk}(u_{i\uparrow}^\dagger d_{j\downarrow}^\dagger - u_{i\downarrow}^\dagger d_{j\uparrow}^\dagger)u_{k\uparrow}^\dagger\,|10\rangle\,, \tag{4}$$

where i,j,k are three-valued color indices, the arrows indicate spin components (not helicity), and each symbol denotes a (positive-energy) fermion operator in an abbreviated notation, i.e., $u_{i\uparrow}^\dagger \equiv a^\dagger(u_i)u_\uparrow$, where the spin component $\uparrow$ is reflected in the Dirac spinor of the state $u_\uparrow$. We are indebted to Donoghue[4] for this form, although we are using it in a slightly different context than his bag calculation. For example, we will consider a static case where all quark spinors are static.

C. Matrix elements

We compute matrix elements of the interaction (1) between an initial baryon [say $p_\uparrow$ as in (4)] and a final three-fermion state composed of an antilepton and a quark-antiquark pair in a color-singlet state. We give below the result for a particular example:

$$\langle \bar{e}_R;\bar{u}_\uparrow u_\downarrow\,|\,\mathcal{L}\,|\,p_\uparrow\rangle = \frac{2}{\sqrt{6}}\,(u_\uparrow\gamma u_\uparrow \bar{e}_R\gamma u_\uparrow + \bar{v}_\uparrow\gamma v_\uparrow \bar{e}_R\gamma u_\uparrow)\,, \tag{5}$$

where the proton state is given in (4), the Lagrangian $\mathcal{L}$ in (1) (a factor of $2\sqrt{2}\,G$ is omitted from the answer), and the state $\bar{u}_\uparrow u_\downarrow$ is more precisely the normalized color singlet: $(1/\sqrt{3})$ $\bar{u}_{p\uparrow}^\dagger u_{p\downarrow}^\dagger\,|0\rangle$, where p is a color index to be summed over. On the right-hand side of (5) we have used an abbreviation notation which omitted the space-time indices on the γ_μ matrices and the chirality projections, i.e.,

$$\bar{u}_\uparrow\gamma u_\uparrow \bar{e}_R\gamma u_\uparrow \equiv \tfrac{1}{4}[\bar{u}_\uparrow\gamma_\mu(1-\gamma_5)u_\uparrow \bar{e}_R\gamma_\mu(1+\gamma_5)u_\uparrow]. \tag{6}$$

The arrows represent quark spin and the u spinors refer to the particles in the order appearing in the Lagrangian (1); this also determines where the factors of $(1\pm\gamma_5)$ are located. For example, in the matrix element (5), which corresponds to the conversion of a proton with spin up into a $\bar{u}_\uparrow u_\downarrow$ pair, the basic process is $u_\uparrow d_\uparrow \to \bar{e}_R\bar{u}_\uparrow$; the quark $u_\downarrow$ survives from the initial state. Therefore, the relevant piece of the Lagrangian is $(\bar{u}^c\gamma u\bar{e}_R\gamma d)$ and the two terms on the right-hand side of (5) correspond, respectively, to the final anti-up quark $\bar{u}_\uparrow$ being created by the $\bar{u}^c$ field (with spinor $u_\uparrow$) and the initial up quark being destroyed by the second u field (with spinor $u_\uparrow$) while in the second term of (5) the final anti-up quark is created by the second u field (with spinor $v_\uparrow$) and the initial up quark is destroyed by the $\bar{u}_c$ field (with spinor $v_\uparrow$). In both terms the last spinor $u_\uparrow$ in the amplitude belongs to the initial d quark which is destroyed. The spinor belonging to the positron is explicitly marked. The choices of all quark and antiquark spinors will be discussed later.

Here is one other example:

$$\langle \bar{e}_R;\bar{d}_\downarrow d_\uparrow\,|\,\mathcal{L}\,|\,p_\uparrow\rangle = \frac{2}{\sqrt{6}}\,(\bar{v}_\uparrow\gamma u_\downarrow \bar{e}_R\gamma v_\downarrow + \bar{v}_\downarrow\gamma u_\uparrow \bar{e}_R\gamma v_\downarrow)\,, \tag{7}$$

where the basic process is $u_\uparrow u_\downarrow \to \bar{e}_R\bar{d}_\downarrow$. In both terms of (7) the last spinor $v_\downarrow$ belongs to the final down antiquark which is created, while the first two spinors belong to the initial up quarks destroyed. In the first term the first field destroys the spin-up quark while the second field destroys the spin-down quark; in the second term the roles are reversed. As can be noted from (2) and (3) the charge conjugate field u^c yields a v spinor when it annihilates a particle while the original field u yields a u spinor for the same purpose.

The results for all matrix elements for proton decay into right-handed leptons are collected in Table I. These matrix elements can be written entirely from the considerations discussed above, rather than by actual calculations. There are two further useful points. Firstly, there is an extra factor of 2 in the amplitude when there is an identical pair of quarks in the operative initial state (e.g., $u_\uparrow u_\uparrow$ in the proton or $d_\uparrow d_\uparrow$ in the neutron),

TABLE I. Matrix elements for proton decay into right-handed antileptons.

Initial state	Final state	Annihilating quarks	$(\frac{3}{2})^{1/2}$ amplitude
$p_\uparrow$	$\bar{e}_R\bar{u}_\uparrow u_\downarrow$	$u_\uparrow d_\uparrow$	$(\bar{u}_\uparrow\gamma u_\uparrow\bar{e}_R\gamma u_\uparrow+\bar{v}_\uparrow\gamma v_\uparrow\bar{e}_R\gamma u_\uparrow)$
$p_\uparrow$	$\bar{e}_R\bar{u}_\downarrow u_\uparrow$	$u_\uparrow d_\downarrow$	$(-2\bar{u}_\downarrow\gamma u_\uparrow\bar{e}_R\gamma u_\downarrow-2\bar{v}_\uparrow\gamma v_\downarrow\bar{e}_R\gamma_\downarrow$
		$u_\downarrow d_\uparrow$	$+\bar{u}_\downarrow\gamma u_\downarrow\bar{e}_R\gamma u_\uparrow+\bar{v}_\downarrow\gamma v_\downarrow\bar{e}_R\gamma u_\uparrow)$
$p_\uparrow$	$\bar{e}_R\bar{d}_\uparrow d_\downarrow$	$u_\uparrow u_\uparrow$	$-2\bar{v}_\uparrow\gamma u_\uparrow\bar{e}_R\gamma v_\uparrow$
$p_\uparrow$	$\bar{e}_R\bar{d}_\downarrow d_\uparrow$	$u_\uparrow u_\downarrow$	$(\bar{v}_\uparrow\gamma u_\downarrow\bar{e}_R\gamma v_\downarrow+\bar{v}_\downarrow\gamma u_\uparrow\bar{e}_R\gamma v_\downarrow)$
$p_\downarrow$	$\bar{e}_R\bar{u}_\downarrow u_\downarrow$	$u_\uparrow d_\downarrow$	$(-\bar{v}_\downarrow\gamma u_\uparrow\bar{e}_R\gamma u_\downarrow-\bar{v}_\uparrow\gamma v_\downarrow\bar{e}_R\gamma u_\downarrow$
		$u_\downarrow d_\uparrow$	$+2\bar{u}_\downarrow\gamma u_\uparrow\bar{e}_R\gamma u_\uparrow+2\bar{v}_\downarrow\gamma v_\downarrow\bar{e}_R\gamma u_\uparrow)$
$p_\downarrow$	$\bar{e}_R\bar{d}_\downarrow d_\downarrow$	$u_\uparrow u_\downarrow$	$-(\bar{v}_\uparrow\gamma u_\downarrow\bar{e}_R\gamma v_\downarrow+\bar{v}_\downarrow\gamma u_\uparrow\bar{e}_R\gamma v_\downarrow)$
$p_\uparrow$	$\bar{\nu}_R\bar{d}_\uparrow u_\downarrow$	$u_\uparrow d_\uparrow$	$-(\bar{v}_\uparrow\gamma u_\uparrow\bar{\nu}_R\gamma v_\uparrow+\bar{v}_\uparrow\gamma v_\uparrow\bar{\nu}_R\gamma u_\uparrow)$
$p_\uparrow$	$\bar{\nu}_R\bar{d}_\downarrow u_\uparrow$	$u_\uparrow d_\downarrow$	$(2\bar{v}_\uparrow\gamma u_\downarrow\bar{\nu}_R\gamma v_\downarrow+2\bar{v}_\uparrow\gamma v_\downarrow\bar{\nu}_R\gamma u_\downarrow$
		$u_\downarrow d_\uparrow$	$-\bar{v}_\downarrow\gamma u_\uparrow\bar{\nu}_R\gamma v_\downarrow-\bar{v}_\downarrow\gamma v_\downarrow\bar{\nu}_R\gamma u_\uparrow)$
$p_\downarrow$	$\bar{\nu}_R\bar{d}_\downarrow u_\downarrow$	$u_\uparrow d_\downarrow$	$(\bar{v}_\uparrow\gamma v_\downarrow\bar{\nu}_R\gamma u_\downarrow+\bar{v}_\uparrow\gamma u_\downarrow\bar{\nu}_R\gamma v_\downarrow$
		$u_\downarrow d_\uparrow$	$-2\bar{v}_\downarrow\gamma v_\downarrow\bar{\nu}_R\gamma u_\uparrow-2\bar{v}_\downarrow\gamma u_\uparrow\bar{\nu}_R\gamma v_\downarrow)$
$p_\uparrow$	$\bar{\mu}_R\bar{s}_\uparrow d_\downarrow$	$u_\uparrow u_\uparrow$	$2\bar{v}_\uparrow\gamma u_\uparrow\bar{\mu}_R\gamma v_\uparrow$
$p_\uparrow$	$\bar{\mu}_R\bar{s}_\downarrow d_\uparrow$	$u_\uparrow u_\downarrow$	$-(\bar{v}_\uparrow\gamma u_\downarrow\bar{\mu}_R\gamma v_\downarrow+\bar{v}_\downarrow\gamma u_\uparrow\bar{\mu}_R\gamma v_\downarrow)$
$p_\uparrow$	$\bar{\nu}_{\mu R}\bar{s}_\uparrow u_\downarrow$	$u_\uparrow d_\uparrow$	$-\bar{v}_\uparrow\gamma u_\uparrow\bar{\nu}_\mu\gamma v_\uparrow$
$p_\uparrow$	$\bar{\nu}_{\mu R}\bar{s}_\downarrow u_\uparrow$	$u_\uparrow d_\downarrow$	$(2\bar{v}_\uparrow\gamma u_\downarrow\bar{\nu}_\mu\gamma v_\downarrow$
		$u_\downarrow d_\uparrow$	$-\bar{v}_\downarrow\gamma u_\uparrow\bar{\nu}_\mu\gamma v_\downarrow)$
$p_\downarrow$	$\bar{\nu}_{\mu R}\bar{s}_\downarrow u_\downarrow$	$u_\uparrow d_\downarrow$	$(\bar{v}_\uparrow\gamma u_\downarrow\bar{\nu}_\mu\gamma v_\downarrow$
		$u_\downarrow d_\uparrow$	$-2\bar{v}_\downarrow\gamma u_\uparrow\bar{\nu}_\mu\gamma v_\downarrow)$

whether both these quarks are destroyed or only one of them is. Secondly, the phases of all terms are controlled by the relative phase of the two terms in Eq. (4). This means, for proton decay, that all processes involving an initial $d_\downarrow$ have amplitudes of opposite sign to those involving an initial $d_\uparrow$. Finally, the normalization factor $2/\sqrt{6}$ was obtained by direct computation.

D. Meson wave functions

The physical quark-antiquark states are linear combinations of the states listed in Table I, with definite values for spin or isospin, e.g.,

$$|\omega^0\rangle=\tfrac{1}{2}(\bar{u}_\uparrow u_\downarrow+\bar{u}_\downarrow u_\uparrow+\bar{d}_\uparrow d_\downarrow+\bar{d}_\downarrow d_\uparrow)\,, \tag{8}$$

$$|\rho^0\rangle=\tfrac{1}{2}(\bar{u}_\uparrow u_\downarrow+\bar{u}_\downarrow u_\uparrow-\bar{d}_\uparrow d_\downarrow-\bar{d}_\downarrow d_\uparrow)\,, \tag{9}$$

$$|\pi^0\rangle=\tfrac{1}{2}(\bar{u}_\uparrow u_\downarrow-\bar{u}_\downarrow u_\uparrow-\bar{d}_\uparrow d_\downarrow+\bar{d}_\downarrow d_\uparrow)\,, \tag{10}$$

$$|\eta^0\rangle=\frac{1}{2\sqrt{3}}(\bar{u}_\uparrow u_\downarrow-\bar{u}_\downarrow u_\uparrow+\bar{d}_\uparrow d_\downarrow-\bar{d}_\downarrow d_\uparrow+2\bar{s}_\uparrow s_\downarrow-2\bar{s}_\downarrow s_\uparrow)\,, \tag{11}$$

where we have chosen for η^0 the member of the octet rather than Isgur's[10] η^0.

We are now in a position to compute amplitudes for physical processes, provided we know what to take for the spinors u and v listed in Table I. This is what we discuss next.

III. PHYSICAL APPROXIMATIONS

It is important to realize that the decay amplitudes of Table I still have freedom for further physical assumptions for the annihilation process. This freedom amounts to making choices for the u and v spinors of the quarks involved, corresponding to the kinematics of the four-fermion process underlying the baryon decay. We shall assume in *all cases* that the *antileptons* are fully relativistic and have spinors

$$e_R\equiv\nu_R\equiv\frac{1}{\sqrt{2}}\begin{bmatrix}\chi_\uparrow\\ \chi_\uparrow\end{bmatrix},\quad e_L\equiv\frac{1}{\sqrt{2}}\begin{bmatrix}\chi_\downarrow\\ -\chi_\downarrow\end{bmatrix}, \tag{12}$$

and in fact consider *only right-handed* antilepton emission; the rates for left-handed positron emission can be obtained with the identities of Weinberg and Wilczek and Zee, which depend only on the value of the parameter r in the Lagrangian (1).

There are three types of matrix element $\bar{u}\gamma u\bar{l}_R\gamma u$, $\bar{v}\gamma v\bar{l}_R\gamma u$, and $\bar{v}\gamma u\bar{l}_R\gamma v$ and *a priori* there are three spin configurations for each if we conserve helicity, namely, one $S_z=1$ configuration and two $S_z=0$ configurations (corresponding to where the down arrow is put for the initial quark). Of these nine possible matrix elements we find three vanishing in all cases because the two currents have identical lightlike form. This is the case, for example, with

$$u_\downarrow\gamma_\mu(1-\gamma_5)u_\downarrow l_R\gamma_\mu(1+\gamma_5)u_\uparrow\propto(1,0,0,1)\cdot(1,0,0,1)$$

The remaining six matrix elements have to be computed.

In the *static* approximation both the initial quarks and the final antiquark are at rest, with spinors

$$u_\uparrow\equiv\begin{bmatrix}\chi_\uparrow\\ 0\end{bmatrix},\quad u_\downarrow\equiv\begin{bmatrix}\chi_\downarrow\\ 0\end{bmatrix},\quad v_\uparrow\equiv\begin{bmatrix}0\\ -\chi_\downarrow\end{bmatrix},\quad v_\downarrow\equiv\begin{bmatrix}0\\ \chi_\uparrow\end{bmatrix}. \tag{13}$$

The matrix elements are listed in Table II. The static model can only be valid if the antiquark emmitted in the final state is almost as massive as the sum or the masses of the two initial quarks.

Another extreme model corresponds to *relativistic recoil* for the antiquark. The spinors of the antiquarks relevant to this case are

TABLE II. Nonvanishing four-fermion matrix elements in three models (noncovariant normalization).

Amplitude	Static model	Recoil model	Relativistic model
$\bar{u}_\uparrow\gamma u_\uparrow\bar{l}_R\gamma u_\uparrow$	$1/\sqrt{2}$	$2/\sqrt{5}$	2
$\bar{u}_\downarrow\gamma u_\uparrow\bar{l}_R\gamma u_\downarrow$	$1/\sqrt{2}$	$1/\sqrt{5}$	0
$\bar{v}_\downarrow\gamma v_\downarrow\bar{l}_R\gamma u_\uparrow$	$1/\sqrt{2}$	$2/\sqrt{5}$	2
$\bar{v}_\uparrow\gamma v_\downarrow\bar{l}_R\gamma u_\downarrow$	$-1/\sqrt{2}$	$-2/\sqrt{5}$	-2
$\bar{v}_\uparrow\gamma u_\uparrow\bar{l}_R\gamma v_\uparrow$	$-1/\sqrt{2}$	$-2/\sqrt{5}$	-2
$\bar{v}_\downarrow\gamma u_\uparrow\bar{l}_R\gamma v_\downarrow$	$-1/\sqrt{2}$	$-1/\sqrt{5}$	0

$$u_L(-) \equiv u_\uparrow = \frac{1}{\sqrt{2}}\begin{bmatrix} \chi_\uparrow \\ \chi_\uparrow \end{bmatrix},$$

$$v_L(-) \equiv v_\downarrow = \frac{1}{\sqrt{2}}\begin{bmatrix} \chi_\uparrow \\ -\chi_\uparrow \end{bmatrix},$$

and (14)

$$v_R(-) \equiv v_\uparrow = \frac{-1}{\sqrt{2}}\begin{bmatrix} \chi_\downarrow \\ \chi_\downarrow \end{bmatrix},$$

where we indicate by a minus argument that the antiquark recoils into the minus z direction by definition. Of the six matrix elements shown in Table II, two vanish in this limit because the recoiling antiquark is constrained to have spin projection $+\frac{1}{2}$ if its spinor is u_L or v_R. Therefore, the matrix elements in which the antiquark has $s_z = -\frac{1}{2}$ must vanish; this is a case of helicity suppression. The two initial quarks in this model are still at rest, and their wave functions are given in Eq. (13). This model applies if the recoiling antiquark has much smaller mass than the initial quarks *or* if all quarks and antiquarks are massless as is the case for nonstrange hadrons in the *bag model*. The matrix elements we find are listed in Table II.

An intermediate model, probably more relevant than either of the above limiting cases is a *recoil* model, where two static quarks of identical mass annihilate into a *recoiling* antiquark of the same mass as the initial quarks and a relativistic antilepton. From kinematics, the recoiling antiquark has $(p/m) = 0.75$ and therefore $[p/(E+m)] = 3^{-1}$. The relevant antiquark spinors are

$$u_\uparrow(-) = \frac{1}{\sqrt{10}}\begin{bmatrix} 3\chi_\uparrow \\ -\chi_\uparrow \end{bmatrix}, \quad u_\downarrow(-) = \frac{1}{\sqrt{10}}\begin{bmatrix} 3\chi_\downarrow \\ \chi_\downarrow \end{bmatrix},$$

and (15)

$$v_\uparrow(-) = \frac{-1}{\sqrt{10}}\begin{bmatrix} \chi_\downarrow \\ 3\chi_\downarrow \end{bmatrix}, \quad v_\downarrow(-) = \frac{1}{\sqrt{10}}\begin{bmatrix} -\chi_\uparrow \\ 3\chi_\uparrow \end{bmatrix}.$$

Using the appropriate spinor for the recoiling antiquark and static spinors for the initial quarks we obtain the amplitudes listed under the recoil-model column of Table II. It is interesting to note that in the recoil model the helicity suppression (of rows 2 and 6) is exactly *halfway* between the static model (where row 1 equals row 2 and row 5 equals row 6) and the relativistic model with full suppression, where rows 2 and 6 vanish. In the recoil model the matrix elements of rows 2 and 6 are exactly half the matrix elements of rows 1 and 5, respectively.

IV. RESULTS AND DISCUSSION

Given the four-fermion matrix elements of Table II we can substitute these values into the forms of Table I to obtain numerical estimates for amplitudes into specified quark-antiquark final states; these are listed in Table III for the three different models we discuss. Then we take linear combinations of these amplitudes appropriate to the physical mesons ρ, ω, π, η according to the meson wave function of Sec. II*D*. Some of the results for right-handed antilepton production are listed in Table IV. The sums of squares of matrix elements (into the same final state) from the two-proton polarizations are proportional to the decay rates and are given in Table V.

We can check directly with the spin-averaged rates of Table I of Gavela *et al.* that their results coincide with the static-model column of Table V. Similarly, the ω^0/ρ^0 ratio of Donoghue (for the bag model) is very close to the ratio (27:11) we find in the relativistic column of Table V; note also the very small rate of η production in the bag model as against the zero value in the relativistic column of Table V. The discrepancy with the bag model in the case of the pion stems from an additional form factor used by Donoghue to suppress large momentum transfers. Note

TABLE III. Numerical amplitudes for proton decay to specified quark-antiquark states (arbitrary units).

Process	Static model	Recoil model	Relativistic model
$p_\uparrow \to \bar{e}_R \bar{u}_\uparrow u_\downarrow$	$1/\sqrt{3}$	$4/\sqrt{30}$	$4/\sqrt{6}$
$p_\uparrow \to \bar{e}_R \bar{u}_\downarrow u_\uparrow$	$1/\sqrt{3}$	$8/\sqrt{30}$	$12/\sqrt{6}$
$p_\uparrow \to \bar{e}_R \bar{d}_\uparrow d_\downarrow$	$2/\sqrt{3}$	$8/\sqrt{30}$	$8/\sqrt{6}$
$p_\uparrow \to \bar{e}_R \bar{d}_\downarrow d_\uparrow$	$-1/\sqrt{3}$	$-2/\sqrt{30}$	0
$p_\downarrow \to \bar{e}_R \bar{u}_\downarrow u_\downarrow$	$2/\sqrt{3}$	$10/\sqrt{30}$	$12/\sqrt{6}$
$p_\downarrow \to \bar{e}_R \bar{d}_\downarrow d_\downarrow$	$1/\sqrt{3}$	$2/\sqrt{30}$	0
$p_\uparrow \to \bar{\mu}_R \bar{s}_\uparrow d_\downarrow$	$-2/\sqrt{3}$	$-8/\sqrt{30}$	$-8/\sqrt{6}$
$p_\uparrow \to \bar{\mu}_R \bar{s}_\downarrow d_\uparrow$	$1/\sqrt{3}$	$1/\sqrt{30}$	0
$p_\uparrow \to \bar{\nu}_R \bar{s}_\uparrow u_\downarrow$	$1/\sqrt{3}$	$4/\sqrt{30}$	$4/\sqrt{6}$
$p_\uparrow \to \bar{\nu}_R \bar{s}_\downarrow u_\uparrow$	$1/\sqrt{3}$	$2/\sqrt{30}$	0
$p_\downarrow \to \bar{\nu}_R \bar{s}_\downarrow u_\downarrow$	$2/\sqrt{3}$	$4/\sqrt{30}$	0

TABLE IV. Some amplitudes for proton decay to physical mesons, in three models (arbitrary units).

Process	Static model	Recoil model	Relativistic model
$p_\uparrow \to \bar{e}_R \omega$	$3/2\sqrt{3}$	$9/\sqrt{30}$	$12/\sqrt{6}$
$p_\uparrow \to \bar{e}_R \rho^0$	$1/2\sqrt{3}$	$3/\sqrt{30}$	$4/\sqrt{6}$
$p_\uparrow \to \bar{e}_R \pi^0$	$-3/2\sqrt{3}$	$-7/\sqrt{30}$	$-8/\sqrt{6}$
$p_\uparrow \to \bar{e}_R \eta_8$	$1/2$	$1/\sqrt{10}$	0
$p_\downarrow \to \bar{e}_R \omega^0_{-1}$	$3/\sqrt{6}$	$6/\sqrt{15}$	$\sqrt{12}$
$p_\downarrow \to \bar{e}_R \rho^0_{-1}$	$1/\sqrt{6}$	$4\sqrt{2}/\sqrt{30}$	$\sqrt{12}$
$p_\uparrow \to \bar{\mu}_R K^0$	$-\sqrt{3/2}$	$-9/\sqrt{60}$	$-4/\sqrt{3}$
$p_\uparrow \to \bar{\nu}_R K^+$	0	$1/\sqrt{15}$	$2/\sqrt{3}$
$p_\downarrow \to \bar{\nu}_R K^{*+}$	$2/\sqrt{3}$	$4/\sqrt{30}$	0
$p_\uparrow \to \bar{\nu}_R K^{*+}$	$2/\sqrt{6}$	$3/\sqrt{15}$	$2/\sqrt{3}$

TABLE V. Some proton decay rates in three kinematic models (with SU_6 for hadrons) (arbitrary units).

Process	Static model	Recoil model	Relativistic model
$p \to \bar{e}_R \omega$	$\frac{9}{4}$	$\frac{153}{30}$	36
$p \to \bar{e}_R \rho^0$	$\frac{1}{4}$	$\frac{41}{30}$	$\frac{44}{3}$
$p \to \bar{e}_R \pi^0$	$\frac{3}{4}$	$\frac{49}{30}$	$\frac{32}{3}$
$p \to \bar{e}_R \eta_8$	$\frac{1}{4}$	$\frac{3}{30}$	0
$p \to \bar{\mu}_R K^0$	$\frac{3}{2}$	$\frac{81}{60}$	$\frac{16}{3}$
$p \to \bar{\nu}_R K^+$	0	$\frac{2}{30}$	$\frac{4}{3}$
$p \to \bar{\nu}_R K^{*+}$	2	$\frac{34}{30}$	$\frac{4}{3}$

that all three sets of rates of Table V refer to an SU_6-invariant world in which all mesons have the same mass; the differences are due entirely to the treatment of antiquark recoil in the basic four-fermion process. To get somewhat closer to the real world we allow for SU_6 breaking in the meson masses by introducing the phase-space correction $(1-x^2)(1-x^4)$, where x is the ratio of the meson mass to the proton mass; this is equivalent to the prescription of Gavela *et al.* We also have to include left-handed fermion production. This we do for definiteness by specializing to SU_5 [$r=2$ in the Lagrangian (1)]. Using the formulas of Wilczek and Zee and Weinberg, which amount to multiplying the right-handed positron rates of Table V by a factor of 5 [equals $(1+r^2)$], our final branching ratios for proton decay are in Table VI. We have ignored the K^{*+} final state which has very small phase space resulting in a branching ratio of less than 1% (0.7%) in the static case, which is most favorable.

TABLE VI. Branching ratios for proton decay in SU_5 in three kinematic models.

Channel	Static model	Recoil model	Relativistic model
$e^+\omega$	21.4%	24.9%	25.9%
$e^+\rho^0$	2.4%	6.6%	10.5%
$e^+\pi^0$	35.7%	39.8%	38.4%
$e^+\eta_8$	6.9%	1.5%	0
$\bar{\nu}\rho^+$	1.0%	2.6%	4.2%
$\bar{\nu}\pi^+$	14.3%	15.9%	15.4%
μ^+K^0	18.3%	8.4%	4.9%
$\nu_\mu K^+$	0	0.2%	0.6%
Total	100.1%	99.9%	99.9%

TABLE VII. Neutron-decay branching ratios in SU_5 in three models.

Channel	Static model	Recoil model	Relativistic model
$\bar{\nu}\omega$	4.7%	4.5%	4.6%
$\bar{\nu}\rho^0$	0.6%	1.2%	1.8%
$\bar{\nu}\pi^0$	7.8%	7.3%	6.8%
$\bar{\nu}\eta_8$	1.5%	0	0
$e^+\rho-$	5.5%	11.8%	18.5%
$e^+\pi-$	78.7%	72.2%	67.8%
$\bar{\nu}_\mu K^0$	1.1%	3.0%	0.6%
Total	99.9%	100.0%	100.1%

In all three models we note that the decay into $e^+\pi^0$ (which is experimentally important) varies rather little and is sizable, between 36–40%; in addition the decay into the charged pion is an extra 15% and also quite model independent. This is rather fortunate from an experimental viewpoint. Similarly, the $\omega^0 e^+$ mode is fairly model independent and large. The model-dependent channels are $e^+\eta$, $e^+\rho^0$, and μ^+K^0, where we do not know which model is closer to real life. Charged-kaon production appears quite insignificant in all cases. Given the proton-decay branching ratios one can compute the values appropriate for neutron decay by isospin arguments already published.[6,7] The only exception is the $\nu_\mu K^0$ channel which we computed explicitly. Our results are given in Table VII for the three kinematic models discussed. It is again noteworthy that the decays involving single pions ($e^+\pi^-$ and $\bar{\nu}\pi^0$) form about 75% of all neutron decays in each of the three kinematic models discussed. This again is a favorable experimental feature.

V. OTHER COMMENTS

In our computations above we obtained the decay rates by squaring the probability amplitude for the elementary processes $qq \to \bar{l}\bar{q}$ weighted with the amplitude of finding the specific qq pair in the proton. Since we were interested only in branching ratios we dropped a common factor (from *all* amplitudes) involving the amplitude of finding the two quarks qq at the same point in the proton. This involves dropping a factor of $|\psi(0)|^2 \equiv \langle\psi_p|\delta^3(\vec{r}_{12})|\psi_p\rangle$ from the rate of decay. We discuss this approximation in more detail here.

This approximation assumes that the factor $|\psi(0)|^2$ is in fact identical for all possible flavor and spin orientations of the annihilating quark pair qq. This is the case in the SU_6 limit when the spin-flavor wave function factorizes from the orbital wave function of a proton which is a pure 56-plet. As discussed elsewhere[11] this approximation is not good since SU_6 is violated by color hyperfine interactions, which give rise for example to a nonvanishing charge radius for the neutron.[12] We therefore expect distortions at short distances which favor spin singlet over spin triplet and change the branching ratios computed in the SU_6 limit. A more elaborate computation should include SU_6 violations.

In addition, even in the SU_6-invariant world, the prescription of multiplying by $|\psi(0)|^2$ is not quite correct in that it ignores the fact that the "spectator" quark has to move in the right direction and with the correct momentum to join the antiquark into a physical meson. In other words, there is additional momentum dependence in the amplitudes, over and above the correct choice of Dirac spinors, which we emphasized. A more correct calculation takes into account the third quark through a complete three-fermion to three-fermion amplitude. This can be carried through for the harmonic-oscillator wave function for the initial baryon and final meson. The answer we find is

$$|\mathrm{Amp}|^2 = \mathrm{const} \times \exp[-3k^2/(8\alpha^2+6\beta^2)],$$

where α^2 and β^2 are harmonic-oscillator parameters for the baryon and meson, respectively, while $\vec{k}$ is the three-momentum of the meson in the proton rest frame. The exponential does not vary too strongly from unity even at the largest momentum transfer $\vec{k}$ corresponding to the channel πl, as correctly noted already by Gavela *et al.*,[5] who choose $\alpha^2 \approx 0.166\ \mathrm{GeV}^2$ when the diminution in the production rate for pions (relative to vector mesons, in the approximation $\alpha^2=\beta^2$) is about 15%; even for $\alpha^2 \simeq 0.1\ \mathrm{GeV}^2$ the reduction in the rate is 25%. Therefore, it is a reasonable approximation to neglect the variation of this factor with momentum, as we have done. We apologize for having thought otherwise at an initial stage of our investigations and communicated the erroneous conclusion to J. F. Donoghue.

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