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Constraints for $N=2$ superspace from extended supergravity in ordinary space

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The constraints on the supertorsion and Maxwell curvatures for $N=2$ supergravity are derived from $N=2$ extended supergravity in ordinary space with a closed gauge algebra. Nonlinear constraints are found.

I. INTRODUCTION

Supergravity can be formulated in ordinary spacetime or in superspace. In ordinary spacetime the theory obtains a closed gauge algebra if one adds auxiliary fields. For $N=1$ supergravity, the minimal set consists of a scalar S, a pseudoscalar P, and an axial vector A_m.[1] For $N=2$, one set is known[2-4] which is presumably minimal and consists of two spin-$\frac{1}{2}$ SO_2 doublets (λ^i, χ^i), two symmetric traceless SO_2 tensors (A_m^{ij}, P^{ij}), four antisymmetric SO_2 tensors $(V_m^{ij}, t_{mn}^{ij}, M^{ij}, N^{ij})$, and three SO_2 scalars (A_m, S, V_m). The gauge algebra of spacetime symmetries, SO_2 rotations, and supersymmetry closes, and the structure constants depend on physical as well as on auxiliary fields.

At this point we emphasize that the SO_2 group we consider acts only on the photon B_μ^{ij} but not on the indices i of ψ_μ^i, etc. In other words, this local SO_2 gauges the central charge of $N=2$ supersymmetry, but not the outside charge which rotates the generators Q^i and Q^j into each other. For the action this simply means that the photon only appears in the Maxwell curl, so that the action is locally SO_2 invariant, but not, for example, as $\partial_\mu \psi_\nu^i + e\epsilon^{ij} B_\mu^{jk} \psi_\nu^k - (\mu \longrightarrow \nu)$. As a result, local supersymmetry and SO_2 commute. The reason we make this choice is that in Ref. 3 only the results for this case have been given. The $N=2$ extended supergravity model was first found in Ref. 5 and an extension to a local SO_2 which acts on the indices for ψ_μ^i was given in Ref. 6 without auxiliary fields. This result agrees with a group-theoretical approach, in which one gauges $Sp(4) \times SO(2)$ (see Ref. 7). In this de Sitter algebra the photon charge appears of course on the left-hand and on the right-hand side. If one takes a Wigner-Inönü group contraction, one reverts to the SO_2 which is a central charge. Thus, we will consider a base manifold R^4 in which general coordinate transformations act, and a tangent manifold in which one has local Lorentz symmetry and local supersymmetry as well as SO_2 rotations of the photon alone. The latter still enable one to compute the parameter composition law for the SO_2 parameter. Also, in superspace we will restrict ourselves to this kind of SO_2 group.

In superspace one needs constraints on the supertorsions. For minimal $N=1$ supergravity, the constraints read (a, b, c are four-component spinor indices, and r, s, t bosonic indices)

$$T_{ab}^{r} = -\tfrac{1}{4}(C\gamma^r)_{ab}, \quad T_{rs}^{t} = T_{ab}^{c} = T_{ar}^{s} = 0. \tag{1}$$

They were found by a series of unrelated arguments,[8] but no systematic method seems to exist. However, one can establish a bridge between ordinary space and superspace by making a choice of gauge in superspace and requiring compatibility.[9-11] In this way it was found[12] that the set in Eq. (1) follows from the gauge algebra for $N=1$ theory with S, P, A_m. If one replaces in superspace at $\theta=0$ the ordinary spin connection $\omega_\mu^{ab}(e,\psi)$ by the improved spin connection, one finds[12] the constraints which also appear in the approach in which superspace consists of two small chirally conjugated superspaces.[13] Also for $N=1$ conformal supergravity, the constraints in Eq. (1) hold.[14] Moreover, the bridge between ordinary space and superspace yields here also the local scale and chiral transformations in superspace, and they agree with the ones previously proposed.[15]

For $N=2$ superspace, Wess has proposed a set of constraints on the supertorsion similar to Eq. (1).[16] In this article we construct these constraints by using the bridge between ordinary space and superspace and assuming the results of the $N=2$ auxiliary fields.[2,3] Our results are

$$\begin{aligned} &T_{ai,bj}^{r} = -\tfrac{1}{4}\delta_{ij}(C\gamma^r)_{ab}, \quad T_{rs}^{t} = T_{ai,s}^{t} = 0, \\ &T_{Ai,Bj}^{Ck} = -\tfrac{3}{8}\epsilon_{AB}\epsilon_{ij}(\epsilon^{kl}\lambda_l^C) - \tfrac{1}{8}\delta_{(A}^{C}\epsilon_{B)D}\delta_{(i}^{k}\lambda_{j)}^{D}, \\ &T_{Ai,Bj}^{\dot{C}k} = 0, \quad T_{\dot{A}i,Bk}^{Ck} = 0, \quad T_{\dot{A}i,Bj}^{\dot{C}k} = \tfrac{1}{2}\delta_{\dot{A}}^{\dot{C}} T_{\dot{D}i,Bj}^{\dot{D}k}, \end{aligned} \tag{2}$$

where the symbol λ_l^C in our method is the $N=2$ auxiliary field, and as we shall discuss, must be replaced in Eq. (2) by

$$\lambda_{Bl}=2T^{\dot{D}k}_{\dot{D}lBk}\,. \qquad (3)$$

The last two constraints in Eq. (2) are consequences of the following constraint which we will derive

$$T^{\dot{C}k}_{\dot{A}i,\,Bj}=\tfrac{1}{2}\delta^{\dot{C}}_{\dot{A}}(\delta_{ij}\delta^{kl}-\tfrac{1}{2}\delta^k_i\delta^l_j)\lambda_{Bl}\,. \qquad (4)$$

Just as in the $N=1$ nonminimal theory, one can introduce a spinor superfield T_{Bl} such that its $\theta=0$ component equals λ_{Bl}. Capital letters denote two-component spinors (see the Appendix). We used in Eq. (2) four-component notations where it led to a simplification. In addition, we find for the SO_2 curvatures in superspace the constraints

$$F^{ij}_{ai,\,s}=0,\quad F^{ij}_{ak,\,bl}=-\frac{1}{\sqrt{2}}C_{ab}\delta^{[i}_k\delta^{j]}_l\,.$$

The symbols ( ) and [ ] denote symmetrization and antisymmetrization; for example, $A^{(i}B^{j)}=A^iB^j+A^jB^i$. For F^{ij}_{mn} we find the supercovariant photon curl, while T^{ck}_{rs} contains the supercovariant spin-$\frac{3}{2}$ curl and $T^{ck}_{ai,\,s}$ contains many auxiliary fields. Our results agree with Wess's results on many points, except that he proposes (in our notation)

$$\epsilon^{il}D^A{}_lT_{Ai,\,\dot{B}j}{}^{\dot{C}k}=0 \qquad (5)$$

which we do not find. (Note that Wess raises isospin indices with ϵ^{ij}, whereas we use δ^{ij}). Specifically Wess has proposed the constraints (in our notation)[17,18]

$$T^r_{ai,\,bj}+\tfrac{1}{4}\delta_{ij}(C\gamma^r)_{ab}=T^t_{rs}=T^t_{ai,\,s}=0\,, \qquad (6)$$

$$T_{Ai,Bj}{}^{\dot{C}k}=T_{Ai,Bj}{}^{\dot{C}j}=\oint_{ijk}\epsilon_{il}T_{Aj,\,\dot{B}}{}^{l,\dot{C}}{}_k=0\,, \qquad (7)$$

in addition to the constraint of Eq. (5). It can be seen that our results [first line of (2)] agree completely with Eq. (6) above. Similarly, the first two results in Eq. (7) agree. To see the equivalence of the remaining algebraic constraints, we note that the general solution of the "cyclicity" and traceless equations on $T_{Aj,\,\dot{B}l}{}^{\dot{C}k}$ is

$$T_{Aj,\,\dot{B}}{}^{l,\dot{C}}{}_k=(\delta^l_j\delta^m_k-\tfrac{1}{2}\delta^l_k\delta^m_j)f_{A\dot{B}}{}^{\dot{C}}{}_m \qquad (8)$$

for some spinorial quantity $f_{A\dot{B}}{}^{\dot{C}}{}_m$. Next there is a dimension-$\frac{1}{2}$ superspace Bianchi identity,

$$[[\mathfrak{D}_{Ai}\mathfrak{D}_{Bj}\},\bar{\mathfrak{D}}_{\dot{C}k}\}+[[\bar{\mathfrak{D}}_{\dot{C}k},\mathfrak{D}_{Bj}\},\mathfrak{D}_{Ai}\}+[[\bar{\mathfrak{D}}_{\dot{C}k},\mathfrak{D}_{Ai}\},\mathfrak{D}_{Bj}\}=0\,, \qquad (9)$$

which on using the constraints on $T^r_{ai,\,bj}$ and $T^t_{ai,\,s}$ yields

$$T_{Ai,\,Bj}{}^D{}_k(\sigma^r)_{\dot{C}D}+T_{\dot{C}k,\,Bj}{}^{\dot{D}}{}_i(\sigma^r)_{\dot{D}A}+T_{\dot{C}k,\,Ai}{}^{\dot{D}}{}_j(\sigma^r)_{\dot{D}B}=0 \qquad (10)$$

as the coefficient of D_r. This equation can be multiplied by $(\sigma_r)_{\dot{E}F}$. Utilizing the Fierz identity for two-component spinors (see Appendix) we obtain a result which contains two independent equations:

$$f_{A(\dot{B}\dot{C})m}=0\rightarrow f_{A\dot{B}}{}^{\dot{C}}{}_m=\tfrac{1}{2}\delta^{\dot{C}}_{\dot{B}}T_{Am}\,, \qquad (11)$$

$$T^{Ck}_{Ai,\,Bj}=-\tfrac{3}{8}\epsilon_{AB}\delta^k_{[i}T^C_{j]}-\tfrac{1}{8}\delta^C_{(A}\epsilon_{B)D}\delta^k_iT^D_{j)}\,. \qquad (12)$$

From Eq. (8) we thus obtain

$$T_{Ai,\,\dot{B}j}{}^{\dot{C}k}=\tfrac{1}{2}\delta^{\dot{C}}_{\dot{B}}(\delta_{ij}\delta^{kl}-\tfrac{1}{2}\delta^k_j\delta^l_i)T_{Al} \qquad (13)$$

and complete agreement has now been shown explicitly if one identifies $T_{Al}=\lambda_{Al}$.

The differential constraint, however, is in disagreement with our results. To show this, we use the fact that T_{Am}, the superfield defined in (11) and (8), is a good tensor. Hence, using the compatibility method followed in this paper, one finds

$$\delta(\epsilon)\lambda^{am}=[(\epsilon^{\beta j}\partial/\partial\vartheta^{\beta j})T^{am}(x,\vartheta)]_{\vartheta=0}\,. \qquad (14)$$

From the supersymmetry variation $\delta(\epsilon)$ of the auxiliary field λ^{Am} as given in Ref. 3, one finds that

$$D_{bj}T^a{}_i=[\eta_{ij}-\tfrac{1}{2}(\not{V}+i\gamma_5\not{A})_{ij}-\tfrac{1}{2}\not{V}\delta_{ij}-\tfrac{1}{2}(M+i\gamma_5N)_{ij}]^a{}_b-\lambda^a{}_i\bar{\lambda}_{bj}\,. \qquad (15)$$

The constraint in (5) becomes with (13)

$$D_{Am}\lambda^A{}_k+k\leftrightarrow m=0\,. \qquad (16)$$

Substituting (15) and using two-component notation [which means multiplying by $\frac{1}{2}(1+\gamma_5)^b{}_a=\delta^B_A$] we find

$$2S\delta^{ij}+2ip^{ij}-\lambda^{Ai}\lambda^{Bj}\epsilon_{AB}=0\,. \qquad (17)$$

This completes the proof that the formulation by Wess is *inequivalent* to that of de Wit and van Holten. We have not, however, shown that the formulation of Ref. 16 is inconsistent. This requires a detailed study of the superspace Bianchi identities. Perhaps it allows no action.

Fradkin and Vasiliev[2] have also given component results for SO_2 supergravity. Between their first and second works, they have made many field redefinitions. However, the net effect of these redefinitions is to bring their formulations into precise agreement with the form first presented by de Wit and van Holten. [The differences in notation used by these two sets of authors obscures this fact but, utilizing SU(2) Fierz identities, equivalence has been found.[19]]

We have not constructed the constraints on Lorentz supercurvatures, since they follow according to a general theorem from the Bianchi identities, once the supertorsion constraints are known.[20]

Many other authors[21] have studied constraints in superspace from points or view or by using methods which differ from ours. We intend to compare in the future their results to the results obtained by the method we follow, and to see whether proper choice of integration constants can simplify our or their results.

II. CONSTRAINTS IN FOUR-COMPONENT FORMALISM

We extend to $N=2$ supergravity a method used in Refs. 9–12 and 14 for finding superspace constraints. The SO_2 theory with auxiliary fields[2-4] is invariant under local supersymmetry s, Maxwell m, and Lorentz transformations l, all of which act in the tangent space; and under general coordinate transformations g acting in the base manifold. The closed gauge algebra[3] is given by

$$[\delta(\epsilon_1,\varphi_1,\lambda_1,\xi_1),\delta(\epsilon_2,\varphi_2,\lambda_2,\xi_2)] = \delta_s(\epsilon_{12})+\delta_m(\varphi_{12})+\delta_l(\lambda_{12})+\delta_g(\xi_{12}), \quad (18)$$

where

$$\begin{aligned}
\epsilon^i_{12} &= \xi^\nu_{[2}\partial_\nu\epsilon^i_{1]}+\tfrac{1}{2}\lambda^{rs}_{[2}\sigma_{rs}\epsilon^i_{1]}+\tfrac{1}{2}\bar\epsilon^j_1\gamma^\rho\epsilon^j_2(\psi^i_\rho+\gamma_\rho\lambda^i)-\tfrac{3}{8}(\bar\epsilon^{[i}_1\epsilon^{j]}_2+\bar\epsilon^{[i}_1\gamma_5\epsilon^{j]}_2\gamma_5)\lambda^j\\
&\quad -\tfrac{1}{8}(\bar\epsilon^{(i}_1\gamma^\rho\epsilon^{j)}_2\gamma_\rho+\bar\epsilon^{[i}_1\gamma^\rho\gamma_5\epsilon^{j]}_2\gamma_\rho\gamma_5)\lambda^j+\tfrac{1}{4}(\bar\epsilon^{(i}_1\sigma^{rs}\epsilon^{j)}_2\sigma_{rs})\lambda^j,\\
\varphi^{ij}_{12} &= \xi^\nu_{[2}\partial_\nu\varphi^{ij}_{1]}-\tfrac{1}{2}\bar\epsilon^\kappa_1\gamma^\rho\epsilon^\kappa_2 b^{ij}_\rho-\frac{1}{\sqrt2}\bar\epsilon^{[i}_1\epsilon^{j]}_2+\varphi^{ik}_{[2}\varphi^{kj}_{1]},\\
\lambda^{rs}_{12} &= \xi_{[2}\partial_\nu\lambda^{rs}_{1]}+\lambda^{rt}_{[2}\lambda^{ts}_{1]}-\tfrac{1}{2}\bar\epsilon^\kappa_1\gamma^\rho\epsilon^\kappa_2\omega^{rs}_\rho+\tfrac{1}{4}\bar\epsilon^i_1(T+\gamma_5\tilde T)^{rsij}\epsilon^j_2+\tfrac{1}{2}\bar\epsilon^i_1(\sigma^{rs}\eta^{ij}+\eta^{ij}\sigma^{rs})\epsilon^j_2,\\
\xi^\mu_{12} &= \xi^\nu_{[2}\partial_\nu\xi^\mu_{1]}+\tfrac{1}{4}\bar\epsilon^j_{[2}\gamma^\mu\epsilon^j_{1]}
\end{aligned} \quad (19)$$

and the supersymmetry variations of the physical fields are

$$\begin{aligned}
\delta e^r_\mu &= \tfrac{1}{2}\bar\epsilon^i\gamma^r\psi^i_\mu,\\
\delta b^{ij}_\mu &= -\frac{1}{\sqrt2}\bar\epsilon^{[i}\psi^{j]}_\mu,\\
\delta\psi^i_\mu &= D_\mu\epsilon^i+\frac{i}{2}A_\mu\gamma_5\epsilon^i+\tfrac{1}{2}(V^{ij}_\mu+\bar\psi^{[i}_\mu\lambda^{j]})\epsilon^j\\
&\quad -\frac{i}{2}(A^{ij}_\mu+i(\bar\psi^i_\mu\gamma_5\lambda^j)_s)\gamma_5\epsilon^j-\tfrac{1}{4}\sigma\cdot T^{ij}\gamma_\mu\epsilon^j\\
&\quad -\tfrac{1}{2}\gamma_\mu\eta^{ij}\epsilon^j-\tfrac{1}{2}\bar\epsilon^{[i}\lambda^{j]}\psi^j_\mu-\tfrac{1}{2}(\bar\epsilon^i\gamma_5\lambda^j)_s\gamma_5\psi^j_\mu.
\end{aligned} \quad (20)$$

In the case of $N=2$, the $\phi\phi$ term in (19) vanishes. The notations are as in Ref. 3, the gravitational coupling constant κ is taken equal to 1, and ϵ is normalized such that $\delta\psi^i_\mu=\partial_\mu\epsilon$ + more. Under Maxwell SO_2 rotations, $\delta b^{ij}_\mu=-\partial_\mu\phi^{ij}$ (thus our signs in ϕ^{ij}_{12} differ from Ref. 3).

In superspace we *assume* three local symmetries[11]: general coordinate transformations $\delta_G(\Xi^\Pi)$ with $\Pi=(\mu,\alpha)$, local Lorentz rotations $\delta_L(\Lambda^{rs})$ and SO_2 Maxwell rotations $\delta_M(\Phi^{ij})$. The index convention is as in (12) and (14). The parameters $\Xi^\Pi(x,\theta)$, $\Lambda^{rs}(x,\theta)$, $\Phi^{ij}(x,\theta)$ depend on x^μ and $\theta^{\alpha i}$ with $\alpha=1,4$, $i=1,2$ since we consider $N=2$ superspace supergravity.[11] The superfields are the vielbein $E^A_\Pi(x,\theta)$, the Lorentz connection $\Omega^{mn}_\Pi(x,\theta)$, and the Maxwell connection $B^{ij}_\Pi(x,\theta)$. Their variations under the whole symmetry group are

$$\begin{aligned}
\delta(\Xi,\Lambda,\Phi)E^A_\Pi &= \Xi^\Lambda\partial_\Lambda E^A_\Pi+(\partial_\Pi\Xi^\Lambda)E^A_\Lambda\\
&\quad +\tfrac{1}{2}\Lambda^{rs}X_{rs}E^A_\Pi+\tfrac{1}{2}\Phi^{ij}Y_{ij}E^A_\Pi,\\
\delta(\Xi,\Lambda,\Phi)\Omega^{rs}_\Pi &= \Xi^\Lambda\partial_\Lambda\Omega^{rs}_\Pi+(\partial_\Pi\Xi^\Lambda)\Omega^{rs}_\Lambda\\
&\quad -\partial_\Pi\Lambda^{rs}-\Omega^{rt}_\Pi\Lambda^{ts}-\Omega^{st}_\Pi\Lambda^{rt},\\
\delta(\Xi,\Lambda,\Phi)B^{ij}_\Pi &= \Xi^\Lambda\partial_\Lambda B^{ij}_\Pi+(\partial_\Pi\Xi^\Lambda)B^{ij}_\Lambda\\
&\quad -\partial_\Pi\Phi^{ij}-B^{i\kappa}_\Pi\Phi^{\kappa j}-B^{j\kappa}_\Pi\Phi^{i\kappa},
\end{aligned} \quad (21)$$

where the generators of Lorentz and Maxwell transformations X_{rs} and Y_{ij} act on tensors as follows:

$$\begin{aligned}
&\tfrac{1}{2}\Lambda^{rs}X_{rs}E^m_\Pi=\Lambda^m{}_nE^n_\Pi,\quad \tfrac{1}{2}\Lambda^{rs}X_{rs}E^{ak}_\Pi=\tfrac{1}{2}(\Lambda\cdot\sigma)^{ab}E^{bk}_\Pi,\\
&\tfrac{1}{2}\Phi^{ij}Y_{ij}E^A_\Pi=0,\quad \tfrac{1}{2}\Phi^{ij}Y_{ij}B^{kl}_\Pi=\Phi^{kj}B^{jl}_\Pi+\Phi^{lj}B^{kj}_\Pi.
\end{aligned} \quad (22)$$

In $N=2$ supergravity, the $B\Phi$ terms in (21) and (22) vanish. From (21) one can extract the superspace gauge algebra[11]:

$$\begin{aligned}
&[\delta(\Xi_1,\Lambda_1,\Phi_1),\delta(\Xi_2,\Lambda_2,\Phi_2)]=\delta_G(\Xi_{12})+\delta_1(\Lambda_{12})+\delta_M(\Phi_{12}),\\
&\Xi^\Pi_{12}=\Xi^\Lambda_{[2}\partial_\Lambda\Xi^\Pi_{1]}+\delta_{[1}\Xi^\Pi_{2]},\\
&\Lambda^{rs}_{12}=\Lambda^{rt}_{[2}\Lambda^{ts}_{1]}+\Xi^\Pi_{[2}\partial_\Pi\Lambda^{rs}_{1]}+\delta_{[1}\Lambda^{rs}_{2]},\\
&\Phi^{ij}_{12}=\Phi^{ik}_{[2}\Phi^{kj}_{1]}+\Xi^\Pi_{[2}\partial_\Pi\Phi^{ij}_{1]}+\delta_{[1}\Phi^{ij}_{2]}.
\end{aligned} \quad (23)$$

Finally, we define covariant derivatives, supertorsions, and supercurvatures:

$$\begin{aligned}
&D_\Pi=\partial_\Pi+\tfrac{1}{2}\Omega^{rs}_\Pi X_{rs}+\tfrac{1}{2}B^{ij}_\Pi Y_{ij},\quad D_A\equiv E^\Lambda_A D_\Lambda,\\
&[D_A,D_B\}=-2T^C_{AB}D_C+\tfrac{1}{2}R^{mn}_{AB}X_{mn}+\tfrac{1}{2}F^{ij}_{AB}Y_{ij},
\end{aligned} \quad (24)$$

so that

$$T^{C}_{AB}=\tfrac{1}{2}(-)^{\Lambda(B+\Pi)}E^{\Lambda}_{A}E^{\Pi}_{B}[D_{\Lambda}E^{C}_{\Pi}-(-)^{\Lambda\Pi}D_{\Pi}E^{C}_{\Lambda}]\,,$$

$$R^{mn}_{AB}=(-)^{\Lambda(B+\Pi)}E^{\Lambda}_{A}E^{\Pi}_{B}[\partial_{\Lambda}\Omega^{mn}_{\Pi}-(-)^{\Lambda\Pi}\partial_{\Pi}\Omega^{mn}_{\Lambda}+\Omega^{ms}_{\Lambda}\Omega^{sn}_{\Pi}-(-)^{\Lambda\Pi}\Omega^{ms}_{\Pi}\Omega^{sn}_{\Lambda}]\,, \qquad (25)$$

$$F^{ij}_{AB}=-(-)^{\Lambda(B+\Pi)}E^{\Lambda}_{A}E^{\Pi}_{B}[\partial_{\Lambda}B^{ij}_{\Pi}-(-)^{\Lambda\Pi}\partial_{\Pi}B^{ij}_{\Lambda}+B^{ik}_{\Lambda}B^{kj}_{\Pi}-(-)^{\Lambda\Pi}B^{ik}_{\Pi}B^{kj}_{\Lambda}]\,.$$

We have chosen the same sign for the ΩX and BY terms in D_{Π}, so that our BY term has opposite sign to the one in Ref. 11.

We proceed now as in Refs. 12 and 14 by first choosing a gauge through the identifications at order θ^0:

$$\begin{aligned}&E^{r}_{\rho}(x,\vartheta=0)=e^{r}_{\rho}(x)\,,\quad \Xi^{\rho}(x,\vartheta=0)=\xi^{\rho}(x)\,,\\ &\Omega^{rs}_{\rho}(x,\vartheta=0)=\omega^{rs}_{\rho}(x)\,,\quad \Lambda^{rs}(x,\vartheta=0)=\lambda^{rs}(x)\,,\\ &E^{ai}_{\rho}(x,\vartheta=0)=\psi^{ai}_{\rho}(x)\,,\quad \Xi^{\alpha\iota}(x,\vartheta=0)=\epsilon^{\alpha\iota}(x)\,,\\ &B^{ij}_{\rho}(x,\vartheta=0)=b^{ij}_{\rho}(x)\,,\quad \Phi^{ij}(x,\vartheta=0)=\varphi^{ij}(x)\,.\end{aligned} \qquad (26)$$

Then the higher-order components follow by requiring consistency between the gauge algebras of SO_2 superspace supergravity and of SO_2 ordinary supergravity. As the purpose of this paper is to find constraints on gauge-invariant quantities, all the "integration constants" are set equal to zero. The results, to first order in θ, are given in Tables I and II.

Using (25) and Tables I and II, it is now straightforward to derive $T^{C}_{AB}(x,\theta=0)$. We get the following *tensor* relations (therefore true to all orders in θ):

$$T^{t}_{r,s}=T^{t}_{ai,s}=0\,,\quad T^{t}_{ai,bj}=-\tfrac{1}{4}\delta_{ij}(C\gamma^{t})_{ab}\,,$$

$$\begin{aligned}T^{ck}_{ai,bj}=&\tfrac{1}{4}\delta_{ij}(C\gamma^{\rho})_{ab}(\gamma_{\rho}\lambda^{k})^{c}\\ &-\tfrac{3}{16}[\delta^{k}_{[i}C_{ab}\lambda^{c}_{j]}+\delta^{k}_{[i}(C\gamma_5)_{ab}(\gamma_5\lambda_{j]})^{c}]\\ &-\tfrac{1}{16}[\delta^{k}_{(i}(C\gamma^{\rho})_{ab}(\gamma_{\rho}\lambda_{j)})^{c}+\delta^{k}_{[i}(C\gamma^{\rho}\gamma_5)_{ab}(\gamma_{\rho}\gamma_5\lambda_{j]})^{c}]\\ &+\tfrac{1}{8}\delta^{k}_{(i}(C\sigma^{\rho\sigma})_{ab}(\sigma_{\rho\sigma}\lambda_{j)})^{c}\,,\end{aligned} \qquad (27)$$

$$T^{ck}_{r,s}=\tfrac{1}{2}e^{\mu}_{r}e^{\nu}_{s}(\psi^{\mathrm{cov}}_{\mu\nu})^{ck}-\psi^{\alpha\iota}_{r}\psi^{\beta j}_{s}(T^{ck}_{\alpha\iota,\beta j})\,,$$

$$T^{ck}_{ai,s}=\psi^{\beta j}_{s}(T^{ck}_{ai,\beta j})+\tfrac{1}{2}H^{ck}_{s,ai}\,,$$

where $(\psi^{\mathrm{cov}}_{\mu\nu})^{ck}$ is the supercovariant spin-$\frac{3}{2}$ curl, i.e.,

$$(\psi^{\mathrm{cov}}_{\mu\nu})^{ck}=\tilde{D}_{[\mu}\psi^{ck}_{\nu]} \qquad (28)$$

and $\tilde{D}_{\mu}$ is defined by $\delta_s\psi^{k}_{\mu}=\tilde{D}_{\mu}\epsilon^{k}$ [see the supersymmetry variation of ψ^{ck}_{μ} in (20)]. $H^{ck}_{s,ai}$ is the tensor $e^{\nu}_{s}(D_{ai}E^{ck}_{\nu}-D_{\nu}E^{ck}_{ai})$, which can be written (order θ^0) as

$$e^{\nu}_{s}\partial_{ai}E^{ck}_{\nu}-\tfrac{1}{2}(\omega_{s}\cdot\sigma)^{cd}\delta^{d}_{a}\delta^{k}_{i} \qquad (29)$$

and from the explicit form of $E^{ck}_{\nu}(x,\theta^1)$ we see that the ω terms cancel, leaving

TABLE I. Superfields to order θ.[a]

$$E^{m}_{\mu}=e^{m}_{\mu}+\tfrac{1}{2}\bar{\vartheta}^{i}\gamma^{m}\psi^{i}_{\mu}$$

$$E^{m}_{\alpha\iota}=0-\tfrac{1}{4}(\bar{\vartheta}_{\iota}\gamma^{m})_{\alpha}$$

$$\begin{aligned}E^{ai}_{\mu}=&\psi^{ai}_{\mu}+\tfrac{1}{2}(\omega_{\mu}\cdot\sigma\vartheta^{i})^{a}+\tfrac{i}{2}A_{\mu}(\gamma_5\vartheta^{i})^{a}+\tfrac{1}{2}(V^{ij}_{\mu}+\bar{\psi}^{[i}\lambda^{j]})\vartheta^{aj}-\tfrac{i}{2}[A^{ij}_{\mu}+i(\bar{\psi}^{i}_{\mu}\gamma_5\lambda^{j})_s](\gamma_5\vartheta^{j})^{a}-\tfrac{1}{4}(\sigma\cdot T^{ij}\gamma_{\mu}\vartheta^{j})^{a}\\ &-\tfrac{1}{2}(\gamma_{\mu}\eta^{ij}\vartheta^{j})^{a}-\tfrac{1}{2}\bar{\vartheta}^{[i}\lambda^{j]}\psi^{aj}_{\mu}-\tfrac{1}{2}(\bar{\vartheta}^{i}\gamma_5\lambda^{j})_s(\gamma_5\psi^{j}_{\mu})^{a}\end{aligned}$$

$$\begin{aligned}E^{ai}_{\alpha\iota}=&\delta^{a}_{\alpha}\delta^{i}_{\iota}+\tfrac{1}{4}(\bar{\vartheta}_{\iota}\gamma^{\rho})_{\alpha}(\gamma_{\rho}\lambda^{i})^{a}-\tfrac{3}{16}[\bar{\vartheta}^{[i}_{\alpha}\delta^{k]}_{\iota}\lambda^{ak}+(\bar{\vartheta}^{[i}\gamma_5)_{\alpha}\delta^{k]}_{\iota}(\gamma_5\lambda^{k})^{a}]\\ &-\tfrac{1}{16}[(\bar{\vartheta}^{(i}\gamma^{\rho})_{\alpha}\delta^{k)}_{\iota}(\gamma_{\rho}\lambda^{k})^{a}+(\bar{\vartheta}^{[i}\gamma^{\rho}\gamma_5)_{\alpha}\delta^{k]}_{\iota}(\gamma_{\rho}\gamma_5\lambda^{k})^{a}]+\tfrac{1}{8}(\bar{\vartheta}^{(i}\sigma^{\rho\sigma})_{\alpha}\delta^{k)}_{\iota}(\sigma_{\rho\sigma}\lambda^{k})^{a}\end{aligned}$$

$$B^{ij}_{\mu}=b^{ij}_{\mu}-\frac{1}{\sqrt{2}}(\bar{\vartheta}^{[i}\psi^{j]}_{\mu})$$

$$B^{ij}_{\alpha\iota}=0+\frac{1}{2\sqrt{2}}(\bar{\vartheta}^{[i}_{\alpha}\delta^{j]}_{\iota})$$

$$\begin{aligned}\Omega_{\mu,rs}=&\omega_{\mu,rs}+\tfrac{1}{4}\bar{\vartheta}^{i}[-\gamma_r(\psi^{\mathrm{cov}}_{\mu s})^{i}+\gamma_s(\psi^{\mathrm{cov}}_{\mu r})^{i}-\gamma_{\mu}(\psi^{\mathrm{cov}}_{rs})^{i}]+\tfrac{1}{2}\bar{\vartheta}^{i}(\sigma_{rs}\tilde{\eta}^{ij}+\tilde{\eta}^{ij}\sigma_{rs})\psi^{j}_{\mu}+\tfrac{1}{4}\bar{\vartheta}^{i}(-t^{ij}_{rs}+\tilde{t}^{ij}_{rs}\gamma_5)\psi^{j}_{\mu}\\ &+\tfrac{1}{8}\bar{\vartheta}^{i}(T^{ij}_{[re}\gamma_e\gamma_{s]}+\tilde{T}^{ij}_{[re}\gamma_5\gamma_e\gamma_{s]})\psi^{j}_{\mu}\end{aligned}$$

$$\Omega_{\alpha\iota,rs}=0-\tfrac{1}{8}[\bar{\vartheta}^{i}(T+\gamma_5\tilde{T})_{rs,}{}^{i}{}_{\iota}]_{\alpha}-\tfrac{1}{4}[\bar{\vartheta}^{i}(\sigma_{rs}\eta^{i}_{\iota}+\eta^{i}_{\iota}\sigma_{rs})]_{\alpha}$$

[a] $\tilde{\eta}^{ij}$ contains only the S, P, A fields ($\tilde{\eta}^{ij}\equiv S\delta^{ij}+i\gamma_5 p^{ij}-i\gamma_5 A\!\!\!/\,\delta^{ij}$).

TABLE II. Superparameters to order θ.[a]

$\Xi^\mu=$	$\xi^\mu-\frac{1}{4}\bar\epsilon^j\gamma^\mu\vartheta^j$
$\Xi^i=$	$\epsilon^i-\frac{1}{2}\lambda^{rs}\vartheta^i+\frac{1}{4}\bar\epsilon^j\gamma^\rho\vartheta^j(\psi_\rho^i+\gamma_\rho\lambda^i)$ $-\frac{3}{16}(\bar\epsilon^{[i}\vartheta^{j]}+\bar\epsilon^{[i}\gamma_5\vartheta^{j]}\gamma_5)\lambda^j-\frac{1}{16}(\bar\epsilon^{(i}\gamma^\rho\vartheta^{j)}\gamma_\rho+\bar\epsilon^{[i}\gamma^\rho\gamma_5\vartheta^{j]}\gamma_\rho\gamma_5)\lambda^j+\frac{1}{8}(\bar\epsilon^{(i}\sigma^{\rho\sigma}\vartheta^{j)})\sigma_{\rho\sigma}\lambda^j$
$\Phi^{ij}=$	$\varphi^{ij}+\frac{1}{4}(\bar\vartheta^k\gamma^\rho\epsilon^k)b_\rho^{ij}+\frac{1}{2\sqrt2}\bar\vartheta^{[i}\epsilon^{j]}$
$\Lambda^{rs}=$	$\lambda^{rs}-\frac{1}{4}(\bar\epsilon^i\gamma^\rho\vartheta^i)\omega_\rho^{rs}+\frac{1}{8}\bar\epsilon^i(T+\gamma_5\tilde T)^{rs,ij}\vartheta^j+\frac{1}{4}\bar\epsilon^i(\sigma^{rs}\eta^{ij}+\eta^{ij}\sigma^{rs})\vartheta^j$

[a] A term $-\Phi^{ij}\vartheta^j$ in Ξ^i present in Ref. 1 is absent here, because our SO_2 gauges the central charge.

$$H^{ck}_{r,ai}(x,\vartheta=0)=\frac{i}{2}A_r(\gamma_5)^c{}_a\delta^k_i+\tfrac12(\gamma_r^{kj}+\bar\psi_r^{[k}\lambda^{j]})\delta^c_a\delta^j_i$$
$$-\frac{i}{2}(A_r^{kj}+i(\bar\psi_r^k\gamma_5\lambda^j)_s)(\gamma_5)^c{}_a\delta^j_i-\tfrac14(\sigma\cdot T^{kj}\gamma_r)^c{}_a\delta^j_i$$
$$-\tfrac12(\gamma_r\eta^{kj})^c{}_a\delta^j_i-\tfrac12\delta_i^{[k}\lambda_a^{j]}\psi_r^{cj}-\tfrac12(\delta^k_i(\gamma_5\lambda^j)_a)_s(\gamma_5\psi_r)^{cj}\,. \quad (30)$$

We turn now to the B-field strength F^{ij}_{AB}, and we derive at order θ^0:

$$F^{ij}_{rs}=e^\mu_r e^\nu_s(\partial_{[\mu}b^{ij}_{\nu]})+\frac{1}{\sqrt2}\bar\psi_r^{[i}\psi_s^{j]}\,,$$
$$F^{ij}_{al,s}=0\,,\quad F^{ij}_{ak,be}=-\frac{1}{\sqrt2}C_{ab}\delta_k^{[i}\delta_e^{j]}\,. \quad (31)$$

Notice that the right-hand side of the first relation is the supercovariant curl of b [cf. the supersymmetry variation of b in (20)].

III. CONSTRAINTS IN TWO-COMPONENT FORMALISM

We can now rewrite the constraints using the explicit representation of γ matrices given in the Appendix, and we immediately get for their dotted and undotted components

$$T^r_{mn}=T^r_{Ai,m}=T^r_{Ai,Bj}=0\,,$$
$$T^r_{\dot Ai,Bj}=-\tfrac14(i\sigma^r)_{\dot AB}\delta_{ij}\,,$$
$$T^{Ck}_{Ai,Bj}=-\tfrac38\epsilon_{AB}\delta^k_{[i}\lambda^C_{j]}-\tfrac18\delta^C_{(A}\epsilon_{B)D}\delta^k_{(i}\lambda^D_{j)}\,,$$
$$T^{Ck}_{\dot Ai,Bj}=\tfrac12\delta^C_B(\delta_{ij}\delta^{k\ell}-\tfrac12\delta^k_i\delta^\ell_j)\lambda_{\dot A\ell}\,,$$
$$T^{\dot Ck}_{\dot Ai,Bj}=\tfrac12\delta^{\dot C}_{\dot A}(\delta_{ij}\delta^{k\ell}-\tfrac12\delta^k_i\delta^\ell_j)\lambda_{B\ell}\,,$$
$$T^{Cj}_{\dot Ai,Bj}=0\,,\quad T^{Ck}_{\dot Ai,Bj}-\tfrac12\delta^C_BT^{Dk}_{\dot Ai,Dj}=0\,, \quad (32)$$
$$T^{\dot Ck}_{Ai,Bj}=0\,,$$
$$F^{ij}_{Ak,\dot B\ell}=F^{ij}_{\dot Ak,B\ell}=F^{ij}_{Ak,s}=F^{ij}_{\dot Ak,s}=0\,,$$
$$F^{ij}_{Ak,B\ell}=-\frac{1}{\sqrt2}\epsilon_{AB}\delta_k^{[i}\delta_\ell^{j]}\,,$$
$$F^{ij}_{\dot Ak,\dot B\ell}=-\frac{1}{\sqrt2}\epsilon_{\dot A\dot B}\delta_k^{[i}\delta_\ell^{j]}\,,$$

$F^{ij}_{m,n}=$ supercovariant photon curl.

Note that $\lambda_{B\ell}$ can be replaced by $2T^{\dot Dk}_{\dot D\ell,Bk}$, as we see from the expression of $T^{\dot Ck}_{\dot Ai,Bj}$. Hence, $\lambda_{B\ell}$ is the $\theta=0$ component of a superfield $T_{B\ell}$.

Note added. Breitenlohner and Sohnius in Ref. 4 have also derived a set of constraints on supertorsions and supercurvatures in superspace with (gauged) SU_2 internal symmetry. The torsion constraints read

$$T_{ai,bj}{}^r=\tfrac14\epsilon_{ij}(C\gamma_5\gamma^r)_{ab}\,,T_{ai,m}{}^r=0\,,$$

$T_{ai,bj}{}^{ck}$ should not contain spin or isospin $\frac32$:

$$(\vec\tau\sigma_{mn}C^{-1}\gamma_5)^{ai,bj}T_{bj,ai}{}^{ck}=0\,,\quad T_{mn}{}^r=0\,,$$
$$F_{ai,bj}=\epsilon_{ij}(C\gamma_5)_{ab}\exp(C)\,.$$

They do not impose a differential constraint and state that their formulation is equivalent to that of Refs. 2 and 3, the difference being a redefinition of fields and connections.

Recently, J. Wess replaced his differential constraint in Eq. (5) by

$$D^C_iT_{Ci,\dot Bj}{}^{\dot Bk}=D^{\dot C}_iT_{\dot Ci,Bj}{}^{Bk}\,.$$

With Eqs. (13) and (15) one may verify that this constraint is not in agreement with our results. However, we can propose an alternative to Eq. (5) which is satisfied. We find

$$\epsilon^{i\ell}[(D^A{}_\ell-2T^A{}_{m,\dot B\ell,}{}^{\dot Bm})T_{Ai,\dot Cj,}{}^{\dot C}{}_k]+\epsilon^{i\ell}[(\bar D^{\dot A}{}_l-2T^{\dot A}{}_{m,Bl,}{}^{Bm})T_{\dot Ai,Ck,}{}^C{}_j]=0$$

To lowest order in θ this equation merely implies the reality of S and P_{ij}.

Stelle and West (Ref. 21) applied an algebraic method of determining constraints to $N=2$ superspace supergravity. All constraints were deduced, with the exception of the differential constraint.

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APPENDIX: TWO-COMPONENT FORMALISM

We give here our conventions for two-component spinors, and the explicit representation of γ matrices used to translate from four-component to two-component formalism:

$$\gamma_k = \begin{pmatrix} & -i\sigma_k \\ i\sigma_k & \end{pmatrix}, \quad \sigma_k \text{ are Pauli matrices}$$

$$\gamma_4 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix},$$

$$X^A \equiv \left(\frac{1+\gamma_5}{2}\right) X, \quad X_{\dot{A}} \equiv \left(\frac{1-\gamma_5}{2}\right) X,$$

$$X^{\dot{A}} \equiv (X^A)^*, \quad X_A \equiv (X_{\dot{A}})^* .$$

Lorentz transformations on spinors

$$X_a' = (e^{(1/2)\omega\cdot\sigma})_a{}^b X_b ,$$

$$X'^A = \{\exp[(\vec{\omega} + i\vec{v}) i\vec{\sigma}]\}^A{}_B X^B ,$$

$$X_{\dot{A}}' = \{\exp[(\vec{\omega} - i\vec{v}) i\vec{\sigma}]\}_{\dot{A}}{}^{\dot{B}} X_{\dot{B}} ,$$

$$\omega_k = \tfrac{1}{4}\epsilon_{ijk}\omega^{ij}, \quad v^k = \frac{i\omega^{k4}}{2} .$$

Invariants:

$$\epsilon^{AB}, \quad \epsilon_{AB} \quad (\epsilon_{12} = 1) ,$$

$$\epsilon^{\dot{A}\dot{B}}, \quad \epsilon_{\dot{A}\dot{B}} \quad (\epsilon_{\dot{1}\dot{2}} = -1) ,$$

$$\delta^A_B, \quad \delta^{\dot{A}}_{\dot{B}} .$$

Contraction rule:

$$X_A \equiv X^B \epsilon_{BA}, \quad X^A \equiv \epsilon^{AB} X_B$$

and similar for dotted indices. From the definitions

$$(\sigma_\mu)^{A\dot{B}} \equiv (\vec{\sigma}, i1)^{A\dot{B}} ,$$

$$(\sigma_\mu)_{\dot{A}B} \equiv (\vec{\sigma}, -i1)_{\dot{A}B} ,$$

it follows that $(\sigma_\mu)_{A\dot{B}} = (\sigma_\mu)_{\dot{B}A}$. Conversely, given $(\sigma_\mu)^{A\dot{B}}$ and defining $(\sigma_\mu)_{\dot{A}B} = (\sigma_\mu)_{B\dot{A}}$, where $(\sigma_\mu)_{B\dot{A}} = (\sigma_\mu)^{C\dot{D}}\epsilon_{CB}\epsilon_{\dot{D}\dot{A}}$ one finds the quoted result for $(\sigma_\mu)_{\dot{A}B}$. Since the $(\sigma_\mu)_{\dot{A}B}$ are Hermitian (except for $\mu=4$), $(\sigma_\mu^*)_{A\dot{B}} = (\sigma_\mu)_{\dot{A}B}(-)^{\mu 4}$ as the indices suggest.

γ matrices in two-component notation:

$$\gamma_\mu = \begin{pmatrix} & -i(\sigma_\mu)^{A\dot{B}} \\ i(\sigma_\mu)_{\dot{A}B} & \end{pmatrix}, \quad C = \gamma_4\gamma_2 = \begin{pmatrix} \epsilon_{AB} & \\ & \epsilon^{\dot{A}\dot{B}} \end{pmatrix},$$

$$\sigma_{\mu\nu} = \tfrac{1}{4}[\gamma_\mu, \gamma_\nu] = \begin{pmatrix} \tfrac{1}{4}[\sigma_\mu, \sigma_\nu]^A{}_B & \\ & \tfrac{1}{4}[\sigma_\mu, \sigma_\nu]_{\dot{A}}{}^{\dot{B}} \end{pmatrix}.$$

Note the useful properties (due to $\epsilon_{\dot{1}\dot{2}} = -1$)

$$(\sigma_\mu)_{\dot{A}B}(\sigma^\nu)^{B\dot{A}} = 2\delta^\nu_\mu ,$$

$$(\sigma_\mu)_{\dot{A}B}(\sigma^\mu)_{\dot{C}D} = 2\epsilon_{\dot{A}\dot{C}}\epsilon_{BD} ,$$

$$(\sigma_\mu)_{\dot{A}B}(\sigma^\mu)^{C\dot{D}} = 2\delta^{\dot{D}}_{\dot{A}}\delta^C_B ,$$

$$(\sigma^\nu)^{A\dot{B}}(\sigma_\mu)_{\dot{B}C} = \delta^\nu_\mu\delta^A_C + 2(\sigma^\nu{}_\mu)^A{}_C .$$

Our conventions are further that $C\gamma_\mu C^{-1} = -\gamma_\mu^T$ and $\bar{\theta}_\alpha = \theta^\beta C_{\beta\alpha}$ for a Majorana spinor. [Since in a general representation $C^{-1} = -C$ does not always hold, one should not define $\theta^\alpha = C^{\alpha\beta}\bar{\theta}_\beta^T$. This is already obvious from the indices of $(\gamma_\mu)^\alpha{}_\beta$ because then C has indices $C_{\alpha\beta}$ and not $\underline{C}^{\alpha\beta}$.] Requiring that $C\sigma_{\mu\nu}C^{-1} = -\sigma_{\mu\nu}^T$ (i.e., that θ transforms as $\theta^+\gamma_4$) leads to the general solution $C = A(1+\gamma_5)\sigma_2 + B(1-\gamma_5)\sigma_2$ since X^A and $X_{\dot{A}}$ transform independently, but invariance of the Dirac equation equates A to B.

The four-component contraction $\bar{\chi}\lambda = \chi^\beta C_{\beta\alpha}\lambda^\alpha$ becomes in two-component notation $-\chi^A\lambda_A - \chi^{\dot{A}}\lambda_{\dot{A}} = X_A\lambda^A + \chi_{\dot{A}}\lambda^{\dot{A}}$ since $C = (\epsilon_{AB}, \epsilon^{\dot{A}\dot{B}})$ and one always contracts as $\lambda^A\epsilon_{AB}$ and $\epsilon^{\dot{A}\dot{B}}\chi_{\dot{B}}$.

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