Comments and Addenda

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Photon structure function as calculated using perturbative quantum chromodynamics

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(Received 28 January 1980)

The calculation of the moments of the photon structure function using perturbative quantum chromodynamics is briefly reviewed and extended with special emphasis being placed on the large magnitude of the next-to-leading-order corrections with respect to the leading-logarithm calculation. In addition, the moments are inverted in order to study the detailed x dependence of the structure function. It is found that even for large values of Q^2 the structure function is not positive definite when the next-to-leading-order corrections are included, indicating the unreliability of the calculation. This result is insensitive to the scheme used to define the coupling constant $\bar{g}^2$. A brief discussion of a vector-dominance estimate of possible background terms is also included.

I. INTRODUCTION

For some time it has been known that the point-like nature of the photon-quark interaction yields the unique result that the most significant part of the photon structure function can be calculated to leading-logarithm accuracy with the strong-interaction Q^2 scale Λ being the only required parameter.[1-4] It would seem, therefore, that a measurement of the photon structure function should provide a good test of quantum chromodynamics (QCD). However, this test is limited by the fact that Λ is not specified by a leading-logarithm calculation[5] and, therefore, the normalization of the structure function at finite values of Q^2 is not predicted by such calculations, although the shape of the structure function can be predicted. In order to determine Λ, and hence the overall normalization, it is necessary to include next-to-leading-order contributions. Such a calculation has been carried out in Ref. 6. The results of that calculation showed that the next-to-leading-order contributions to the even-n moments of the photon structure function F_2^γ were large and negative. There, too, an approximate inversion of the moments was given for the region $0.4 \leq x \leq 0.8$. This inversion was performed using only the even moments in the range $4 \leq n \leq 20$.

We have repeated the analysis of Ref. 6 and extended the results to include both odd-n values and $n=2$. Using the moments in the range $2 \leq n \leq 10$–20, the x dependence of F_2^γ has been obtained over the full x range for a variety of Q^2 values. The new main result presented here is that the large negative corrections to the leading-logarithm result yield a structure function which is not positive definite in the region $x \lesssim 0.2$.

In the next section the formalism for the next-to-leading-order calculation is reviewed and new results for the $n=2$ moment are presented. In Sec. III the inversion of the moments is discussed and the results for the x dependence of F_2^γ are presented. Section IV contains some brief remarks concerning a vector-dominance-based estimate of possible backgrounds to the above calculation. Our conclusions are given in Sec. V.

II. FORMALISM

In order to avoid a large number of formulas and definitions, we shall adopt directly the notation of Ref. 6 unless stated otherwise below. We have found that the full expression for the moments of $F_2^\gamma(x,Q^2)$ is simplest when a basis is chosen such that the one-loop anomalous-dimension matrix is diagonal. Thus, given

$$\underline{\gamma}^{(0,1)n} = \begin{pmatrix} \gamma_{GG}^{(0,1)n} & \gamma_{G\psi}^{(0,1)n} \\ \gamma_{\psi G}^{(0,1)n} & \gamma_{\psi\psi}^{(0,1)n} \end{pmatrix}$$

in the G,ψ basis, we transform to the $+,-$ basis using

$$\gamma_{ij}^{(0,1)n} = U^{-1}{}_{ia}\gamma_{ab}^{(0,1)n}U_{bj}\,, \quad i,j=+,-\,; \quad a,b=G,\psi$$

where $U_{G+} = U_{G-} = 1$, and

$$U_{\psi+} = \frac{d_+ - d_{GG}}{d_{G\psi}}\,, \quad U_{\psi-} = \frac{d_- - d_{GG}}{d_{G\psi}}\,,$$

with

$$d_\pm = \gamma_\pm^{(0)n}/2\beta_0\,,$$

where $\gamma_\pm^{(0)n}$ are the eigenvalues of $\underline{\gamma}^{(0)n}$. Similarly, we define (NS denotes nonsinglet)

$$d_{\rm NS} = \gamma_{\rm NS}^{(0)n}/2\beta_0\,,$$

$$h_{\pm,\,\rm NS} = K_{\pm,\,\rm NS}^{(0)n}/2\beta_0\,,$$

$$H_{\pm,\,\rm NS} = K_{\pm,\,\rm NS}^{(1)n}/2\beta_0 - \beta_1 K_{\pm,\,\rm NS}^{(0)n}/2\beta_0{}^2\,,$$

$$G_{\pm,\,\rm NS} = \gamma_{\pm\pm,\rm NS}^{(1)n}/2\beta_0 - \beta_1\gamma_{\pm,\,\rm NS}^{(0)n}/2\beta_0{}^2\,,$$

$$G_{+-} = \gamma_{+-}^{(1)n}/(\gamma_-^{(0)n} - \gamma_+^{(0)n} + 2\beta_0)\,,$$

with G_{-+} obtained by interchanging + and − in the last line. A superscript n should be understood on each of the quantities U, d, h, H, K, and G. Here we have used

$$K_i^{(0,1)n} = U^{-1}{}_{ia}K_a^{(0,1)n}\,, \quad i=+,-\,; \quad a=G,\psi$$

where $K_\psi^{(0,1)n}$, $K_G^{(0,1)n}$ are defined as in Ref. 6 except that we omit all electric charge factors $\langle e^2\rangle^2$, $\langle e^4\rangle$ from the $K_a^{(0,1)n}$ and include them explicitly in our final equation. Finally, if we define

$$B_i = B_a^n U_{ai}\,, \quad i=+,-\,; \quad a=G,\psi$$

with the coefficient functions B_G^n, B_ψ^n as given in Ref. 6, then we find that the moments of the pointlike part of the photon structure function may be expressed as

$$M_n^\gamma(Q^2) = \int_0^1 dx\,x^{n-2}F_2^\gamma(x,Q^2) = \frac{e^2}{\beta_0\bar g^2}\left[a_n + \frac{\beta_0\bar g^2}{16\pi^2}b_n + \delta_{n,2}a_2'\frac{\beta_0\bar g^2}{16\pi^2}\ln\bar g^2\right], \tag{2.1}$$

where (in order to avoid a $\ln\ln Q^2$ term) we obtain $\bar g^2$ by solving numerically

$$\frac{16\pi^2}{\beta_0\bar g^2} - \frac{\beta_1}{\beta_0{}^2}\ln\left(\frac{16\pi^2}{\beta_0\bar g^2} + \frac{\beta_1}{\beta_0{}^2}\right) = \ln\frac{Q^2}{\Lambda^2}\,. \tag{2.2}$$

Note that our definition of the structure function F_2^γ lacks a factor e^2 as compared to Ref. 6. Thus our definition is more in analogy with hadron scattering. In Eq. (2.1) we have, for $n\geq 2$,

$$a_n = \beta_0\left(\frac{U_{\psi+}h_+}{1+d_+} + \frac{U_{\psi-}h_-}{1+d_-}\right)\langle e^2\rangle^2 + \beta_0\frac{h_{\rm NS}}{1+d_{\rm NS}}(\langle e^4\rangle - \langle e^2\rangle^2)\,, \tag{2.3}$$

while, for $n>2$,

$$b_n = \left[\frac{B_+h_+}{1+d_+} + \left(\frac{H_+}{d_+} - \frac{h_+G_+}{d_+(1+d_+)} - \frac{h_-G_{+-}}{d_+(1+d_-)}\right)U_{\psi+}\right]\langle e^2\rangle^2 + \left[\frac{B_-h_-}{1+d_-} + \left(\frac{H_-}{d_-} - \frac{h_-G_-}{d_-(1+d_-)} - \frac{h_+G_{-+}}{d_-(1+d_+)}\right)U_{\psi-}\right]\langle e^2\rangle^2$$
$$+ \left[\frac{B_{\rm NS}h_{\rm NS}}{1+d_{\rm NS}} + \frac{H_{\rm NS}}{d_{\rm NS}} - \frac{h_{\rm NS}G_{\rm NS}}{d_{\rm NS}(1+d_{\rm NS})}\right](\langle e^4\rangle - \langle e^2\rangle^2) + 6B_G^n\langle e^4\rangle\,. \tag{2.4}$$

For $n=2$ we have

$$a_2' = \left(\frac{h_+G_{-+}}{1+d_+} - H_- + h_-G_-\right)U_{\psi-}\langle e^2\rangle^2\,, \tag{2.5}$$

and the second $\langle e^2\rangle^2$ term in Eq. (2.4) is replaced by

$$\left[B_-h_- + \left(h_-G_- + \frac{h_+G_{-+}}{(1-d_+)^2}\right)U_{\psi-}\right]\langle e^2\rangle^2\,. \tag{2.6}$$

The special treatment required for b_2 results from the vanishing of $\gamma_-^{(0)n}$ for $n=2$ as a result of energy conservation. Our equations for a_n, b_n agree numerically with the values given in Ref. 6. Our expression (2.6) is new. Furthermore, we have used the analytic values for the two-loop anomalous dimensions $\gamma^{(1)n}$ given in Ref. 7 to compute b_n for odd values of n.[8]

Since the anomalous dimension $\gamma_-^{(0)n}$ vanishes for $n=2$, there is actually an additional contribution to $M_{n=2}^\gamma(Q^2)$ at the level of b_2, namely, that contribution due to the matrix elements of the quark operator with photon states.[1,6] This contribution reflects the hadronic components of the photon. We shall discuss an estimate for the effect of this contribution in Sec. IV.

We have found that the moments $M_n^\gamma(Q^2)$ are more easily inverted if we separate them into parts proportional to $\langle e^4\rangle$ and $\langle e^2\rangle^2$. This separation corresponds roughly to the usual valence-sea decomposition of hadronic structure functions. This is most clearly demonstrated by considering the calculation of the structure functions according to diagrammatic techniques.[2,3] We note that in the leading-logarithm approximation, just those graphs of the form shown in Fig. 1(a) contribute terms proportional to $\langle e^4\rangle$. These diagrams we call valence diagrams since the struck quark originated directly from the target photon. Graphs of the form of Fig. 1(b) we call sea diagrams since the

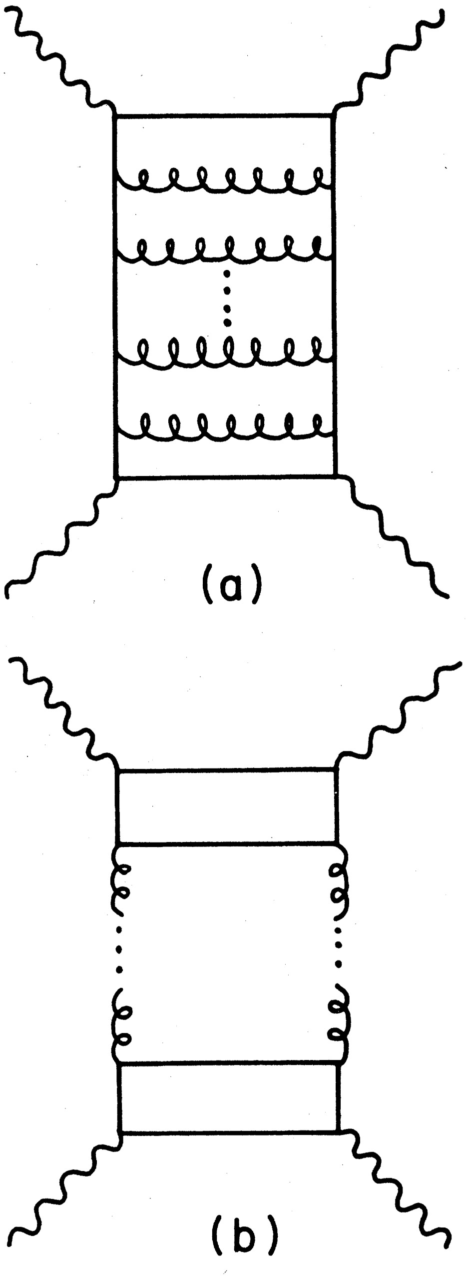

FIG. 1. Typical leading-logarithm contributions to (a) the valence or $\langle e^4\rangle$ and (b) the sea or $\langle e^2\rangle^2$ components of the photon structure function.

struck quark did not originate directly from the photon. Such diagrams clearly contribute terms proportional to $\langle e^2\rangle^2$. The utility of this decomposition rests upon the fact that the shapes in x of the valence and sea parts are distinctly different. Furthermore, we have found that in the leading-logarithm approximation the sea contribution is small and very sharply peaked near $x=0$, while the valence contribution is large and vanishes at $x=0$, in analogy with results from hadronic targets. When we include the terms b_n in Eq. (2.1) we are going beyond the ladder approximation, but the valence-sea decomposition remains equally useful.

TABLE I. Numbers necessary to compute $M_n^\gamma(Q^2)$ using Eqs. (2.1)–(2.6). In addition, $a_2'=0.3911$. We have used four flavors and the $\overline{\rm MS}$ scheme of Ref. 9.

N	$a_n^V\langle e^4\rangle$	$a_n^s\langle e^2\rangle^2$	$b_n^v\langle e^4\rangle$	$b_n^s\langle e^2\rangle^2$
2	1.177	0.6764×10^{-1}	−2.064	0.9327
3	0.7052	0.5649×10^{-2}	−1.240	−0.1735
4	0.5026	0.1473×10^{-2}	−0.9523	-0.7541×10^{-1}
5	0.3895	0.5713×10^{-3}	−0.7930	-0.4443×10^{-1}
6	0.3171	0.2738×10^{-3}	−0.6858	-0.3002×10^{-1}
7	0.2667	0.1497×10^{-3}	−0.6063	-0.2198×10^{-1}
8	0.2296	0.8967×10^{-4}	−0.5441	-0.1697×10^{-1}
9	0.2012	0.5741×10^{-4}	−0.4936	-0.1361 ± 10^{-1}
10	0.1787	0.3867×10^{-4}	−0.4517	-0.1123×10^{-1}

We write the valence and sea parts of a_n and b_n as

$$a_n=a_n^v\langle e^4\rangle+a_n^s\langle e^2\rangle^2\,,\qquad b_n=b_n^v\langle e^4\rangle+b_n^s\langle e^2\rangle^2\,. \tag{2.7}$$

Of course, a_2' contributes only to the sea. We give in Table I the values of $a_n^v\langle e^4\rangle$, $a_n^s\langle e^2\rangle^2$, $b_n^v\langle e^4\rangle$, $b_n^s\langle e^2\rangle^2$, and a_2' for $2\le n\le10$ computed assuming four flavors. We have followed Ref. 6 in using the modified minimal-subtraction ($\overline{\rm MS}$) scheme[9] for calculating B_ψ^n and B_G^n. The value of $\Lambda_{\overline{\rm MS}}$ appropriate to this scheme is known from deep-inelastic hadron scattering to be approximately $\Lambda_{\overline{\rm MS}}=0.5$ GeV.[9,10] We discuss in Sec. III the effect of changing our choice of scheme.

Finally, we recall that Witten's original result,[1] which is equivalent to the leading-logarithm approximation,[2,3] is obtained by keeping only the a_n term in Eq. (2.1) while setting $\beta_1=0$ in Eq. (2.2). Also, the parton-model (PM) result is simply (for f flavors)

$$M_n^\gamma(Q^2)|_{\rm PM}=\frac{e^2}{\beta_0\bar g_{\rm PM}{}^2}\,\frac{12(n^2+n+2)}{n(n+1)(n+2)}f\langle e^4\rangle\,, \tag{2.8}$$

where the magnitude of $\bar g_{\rm PM}{}^2$ is uncertain. For definiteness we shall use the same Λ for all of our calculations and we also take $\bar g_{\rm PM}{}^2=16\pi^2/\beta_0\ln(Q^2/\Lambda^2)$.

III. x DEPENDENCE

The method chosen for inverting the moments was to first parametrize a function of x and then to fit the moments of this function to the theoretical moments of F_2^γ. Both even and odd moments were fitted over the interval $2\le n\le n_{\rm max}$ where $n_{\rm max}$ was chosen to be sufficiently large that the fitting results were stable under small variations of n_{max}. The results shown here are stable for $10 \leq n_{max} \leq 20$. As discussed in the previous section, the shapes for the valence and sea functions are quite different. Therefore, it is advantageous to fit the two functions separately and then add the results to obtain F_2^γ. For both the valence and sea terms in the leading-logarithm case, and for the valence term in the next-to-leading-order case, the fitting function was chosen to be

$$F(x) = x^\alpha (1-x)^\beta \sum_{n=0}^{4} C_n x^n .$$

This function is perfectly adequate except for x very near unity, where the moments $M_n^\gamma(Q^2)$ of Eq. (2.1) become negative for sufficiently large n. This particular breakdown of perturbation theory is not an issue discussed in this paper. The sign change in the moments of the sea term in the next-to-leading-order calculation results in a negative function over most of the x range with a sharp positive spike at small x. The fitting function chosen for this case was

$$F(x) = [x^\alpha (1-x)^\beta (\ln x/a)/\ln a] \sum_{n=0}^{4} C_n x^n .$$

In Fig. 2 the results for the leading-logarithm calculation are shown together with the separate valence and sea contributions. Also shown for comparison is the result of the parton model [see Eq. (2.8)]. These results are in agreement with one's intuition in that the QCD corrections have softened the x dependence of F_2^γ and built up a spike at small x. This spike comes entirely from what we have called the sea term, as one would expect since it results from quarks which have been pair-produced from the gluons which, in turn, were radiated from the valence quarks. The structure is entirely analogous to that observed for the nucleon structure function.

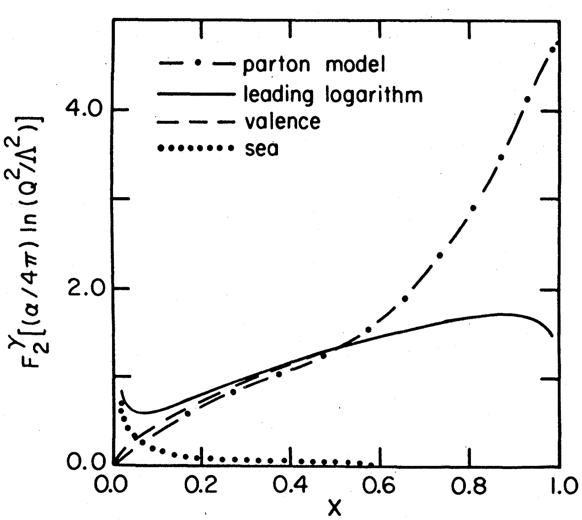

FIG. 2. The valence and sea components of the leading-logarithm contribution to F_2^γ. Also shown for comparison is the parton-model result for F_2^γ, which is purely valence in our language.

In Fig. 3 the results are shown for the next-to-leading-order calculation at $Q^2 = 3$ (GeV/c)^2. Again, the separate sea and valence contributions are shown together with the leading-logarithm result from Fig. 2. As in the leading-logarithm calculation, the valence piece is positive (except for x very near to unity) and provides the bulk of the structure function except for small x values. The sea term, however, is negative for all but very small x values and it is large enough in size to cause the structure function itself to be negative for $0.02 \lesssim x \lesssim 0.2$. This situation is clearly unphysical and signals a breakdown in the perturbation-theory expansion. This point is further reinforced by comparing the results with the leading-logarithm curve. The next-to-leading-order corrections are large, particularly for large and small x. Clearly, the perturbation series is converging slowly, if at all, in these regions of x for this value of Q^2.

The rate of convergence of the expansion depends somewhat on the scheme used to define the coupling $\bar{g}^2$.[9,11-13] We have so far used the $\overline{\text{MS}}$ scheme. We can change schemes by changing b_n and Λ according to

$$b_n \to b_n' = b_n + \delta a_n ,$$

$$\Lambda \to \Lambda' = \Lambda e^{\delta/2} .$$

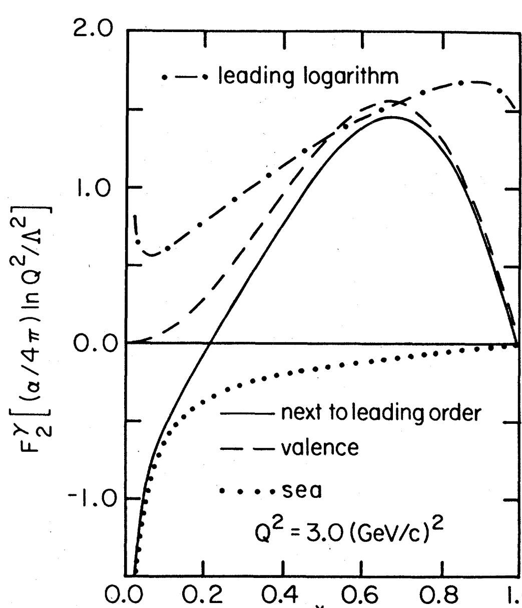

FIG. 3. The valence and sea components of F_2^γ including the first nonleading corrections at $Q^2 = 3$ (GeV/c)^2.

For example, one obtains the minimal-subtraction (MS) scheme[9] by choosing $\delta=\gamma_E-\ln 4\pi\simeq -1.95$, whereas the choice $\delta=1.55$ yields the momentum-space (MOM) scheme.[11-13] One can render all the b_n's positive for $n\leqslant 10$ by choosing $\delta=2.6$. This latter scheme corresponds to a large $\Lambda\gtrsim 1$ GeV[(14)] and would imply very large perturbative corrections to deep-inelastic hadron scattering, yet even this $\delta=2.6$ scheme still yields a negative F_2^γ for $x<0.2$ and $Q^2\simeq 3$ $(\text{GeV}/c)^2$. In fact, F_2^γ is very stable under a change in scheme for $\bar{g}^2$ over the entire range of x.

An additional ambiguity in the $n=2$ moment of F_2^γ is apparent in Eq. (2.1). The a_2' term contains $\ln\bar{g}^2$, and clearly a rescaling of $\bar{g}^2$ will generate modifications to b_2^s. This ambiguity results from the mixing between the hadronic and photon $n=2$ operators. We have determined that this ambiguity does not appreciably affect our results, since the value of b_2^s affects only the height of a very narrow positive spike in F_2^γ very near to $x=0$ (this spike is not shown in Figs. 3–5). The unphysical behavior of the sea contribution for $x<0.2$ is caused entirely by $n\geqslant 3$ moments.

In order to illustrate the Q^2 dependence of the next-to-leading-order results, Fig. 4 shows the results at $Q^2=3$ and 20 $(\text{GeV}/c)^2$ together with the leading-logarithm curve. Even at the larger value of Q^2 the corrections to the leading-logarithm result are still large.

The observable $F_2^\gamma(x,Q^2)$ will contain contributions other than the pointlike contribution we have so far discussed. We will estimate the hadronic contribution in the next section. Our main conclusion, however, is that the perturbative corrections to the pointlike part of F_2^γ are much too large to be reliable except, perhaps, in a small region around $x=0.6$. Therefore, measurements of F_2^γ at high-energy e^+e^- machines will *not* be particularly critical quantitative tests of QCD.

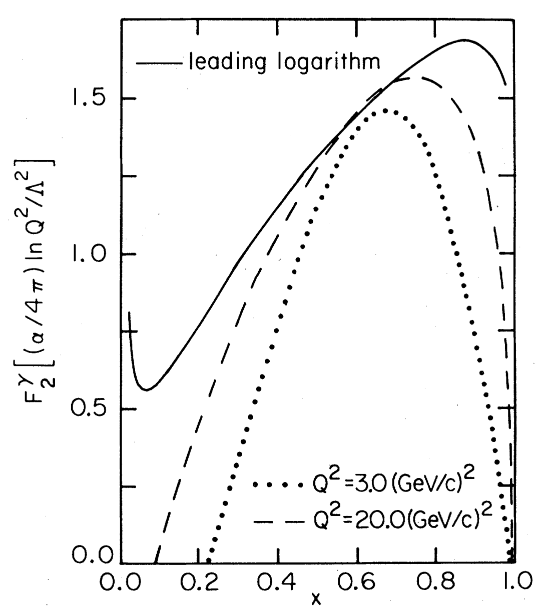

FIG. 4. The Q^2 dependence of F_2^γ including nonleading corrections.

IV. VECTOR-DOMINANCE ESTIMATE

It is well known[1,6] that the photon matrix elements of the quark and gluon operators contribute a term to b_n for $n=2$, but for $n>2$ the contributions are down by a factor $(\ln Q^2/\Lambda^2)^{-\gamma}$. In order to complete the discussion presented in Secs. II and III it is necessary to give an estimate of the contribution of these terms. For this purpose, an argument based on vector dominance will suffice. The estimate presented here is similar in spirit to that given in Ref. 3.

According to the vector-dominance model the photon structure function can be written as

$$F_2^\gamma(x,Q^2)=\sum_V \frac{4\pi\alpha}{f_V{}^2}\sum_i e_i{}^2 x G_{q_i/V}(x,Q^2)\,, \qquad (4.1)$$

where e_i is the fractional quark charge, $4\pi\alpha/f_V{}^2$ is the coupling of the photon to the vector meson V, and $G_{q_i/V}(x,Q^2)$ is the distribution function for the quark q_i in the vector meson V. For real photons this sum is dominated by $V=\rho^0$. The ρ^0 distribution functions can be estimated by employing the

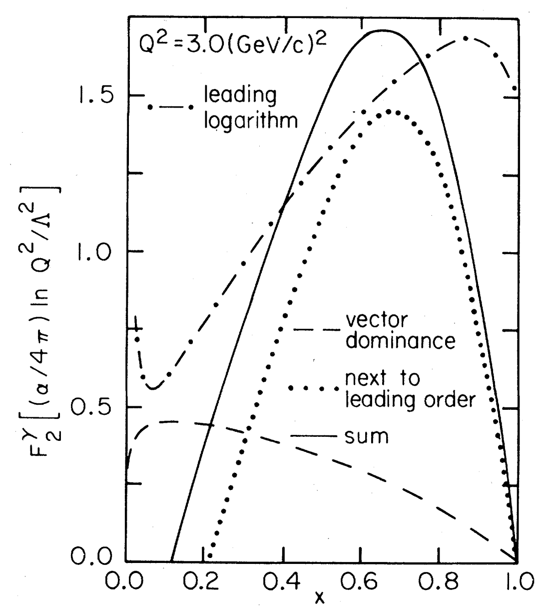

FIG. 5. A comparison of a vector-dominance estimate for F_2^γ, with the QCD contribution including nonleading corrections. Also shown for comparison is the leading-logarithm result.

dimensional-counting rules and also by requiring that

$$\int dx\,[G_{u/\rho^+}(x,Q^2)-G_{\bar{u}/\rho^+}(x,Q^2)]=1\,.$$

In addition, the sea has been normalized so as to carry 10% of the momentum of the ρ. The results are (the x and Q^2 dependence has been dropped for notational convenience)

$$G_{u/\rho^0}=G_{\bar{u}/\rho^0}=G_{d/\rho^0}=G_{\bar{d}/\rho^0}$$
$$=\frac{3}{8}\frac{1}{\sqrt{x}}(1-x)+\frac{1}{10x}(1-x)^5$$

and

$$G_{s/\rho^0}=G_{\bar{s}/\rho^0}=\frac{1}{10x}(1-x)^5\,.$$

A possible charm-quark contribution has been neglected. Inserting these expressions in Eq. (4.1) yields

$$\sum_i e_i^{\,2}xG_{q_i/V}=0.417\sqrt{x}(1-x)+0.133(1-x)^5.$$

For the vector-dominance coupling the value $f_\rho^{\,2}/4\pi=2.2$ was chosen. These estimates for $G_{q_i/\rho}$ are assumed to be valid at some input value of $Q^2=Q_0^{\,2}$. For different values of Q^2 the distribution functions can be calculated, assuming ordinary QCD scaling violations.

In order to compare with the results of the preceding section we assume that the above estimates are valid for $Q^2\simeq 3$ (GeV/c)^2. The resulting contribution to F_2^γ is shown in Fig. 5 together with the next-to-leading-order result and the total of the two. For reference, the leading-logarithm result is also included. It is clear that the vector-dominance contribution provides a negligible background in the large-x region.

Formally, of course, we are only required to include the $n=2$ moment of the vector-dominance contribution in Eq. (2.1). In this circumstance we find that the predicted $F_2^\gamma(x,Q^2)$ is still negative for $x\lesssim 0.2$. In any event, however, the pointlike contribution to Eq. (2.1) should always be positive definite on its own.

V. CONCLUSIONS

In this analysis we have reviewed and extended the calculation of the next-to-leading-order corrections to the photon structure function. The main conclusion is that the large negative corrections to F_2^γ yield a function which is not positive definite. This indicates that, at least for $x<0.3$, the calculation is not reliable. Furthermore, for $x>0.8$ the corrections are so large as to render the final result meaningless. In such circumstances the perturbative agreement for $0.4\leqslant x\leqslant 0.7$ might be fortuitous.

ACKNOWLEDGMENTS

We wish to thank W. A. Bardeen and A. J. Buras for very helpful and critical comments. This work was supported in part by the U. S. Department of Energy.

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[8]Expressing the two-loop anomalous dimensions as (see Ref. 7)

$$\gamma_{ab}^{(1)n}=\gamma_{ab,\alpha}^{(1)n}+(-1)^n\gamma_{ab,\beta}^{(1)n},$$

we have also computed $\gamma_{ab}^{(1)n}$ for odd n as

$$\gamma_{ab}^{(1)n}=\gamma_{ab,\alpha}^{(1)n}+\gamma_{ab,\beta}^{(1)n},$$

which is the appropriate analytic continuation of $\gamma_{ab}^{(1)n}$ for s-u-crossing-even structure functions such as F_2^γ [see, e.g., D. A. Ross and C. T. Sachrajda, Nucl. Phys. B149, 497 (1979)]. The effect of this subtlety on our numerical results is practically negligible.

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[13]When changing schemes we actually use a conservative estimate of the change in Λ. Thus, assuming $\Lambda_{\overline{MS}}=0.5$ GeV, we have used $\Lambda_{MOM}=0.85$ GeV (see Ref. 11) and $\Lambda(\delta=2.6)=1.10$ GeV. Our results, however, are completely insensitive to these estimates.