Spectrum of absorption strengths in diffraction scattering

K. Young
Department of Physics, The Chinese University of Hong Kong, Hong Kong
(Received 4 April 1980)

The attenuation of a hadron passing through nuclear matter is characterized by a spectrum of absorption strengths, corresponding to the eigenvalues of the absorption matrix. Analysis of nucleon-nucleus total cross sections shows that this spectrum (i) is broad, (ii) is skewed towards large values, and (iii) broadens with increasing energy in a manner consistent with the scaling of all eigenvalues.

I. INTRODUCTION

As a fast hadron passes through hadronic or nuclear matter, the amplitude for it to remain unscattered suffers an attenuation $Q=\exp[-\int U(\vec{r})dz]$, where U is the local absorption strength or optical potential. If $U(\vec{r})$ is proportional to the matter density $\rho(\vec{r})$,

$$U(\vec{r})=u\rho(\vec{r}), \tag{1}$$

then

$$Q=e^{-ut}, \tag{2}$$

where

$$t=\int \rho(\vec{r})dz \tag{3}$$

is the thickness of hadronic matter traversed. Such a picture of diffraction has provided valuable insights into the distribution of matter inside a hadron.[1]

The existence of diffraction dissociation implies the necessity of a multichannel approach[2]: u must now be replaced by a (Hermitian) matrix $\hat{u}$ in the space of states that can be diffractively produced from a given hadron. Such a generalization affects even diagonal processes. For example, a hadron $|i\rangle$ may pass straight through a piece of nuclear matter, with attenuation but no transition [Fig. 1(a)]; or it may make a transition to a different state $|j\rangle$, which propagates and later changes back to $|i\rangle$ [Fig. 1(b)]. The latter process, sometimes called "inelastic screening," increases the transmission and thereby lowers the cross section, especially in large nuclei. It accounts fairly well for the departure of measured hadron-nucleus and even photon-nucleus cross sections from calculations using single-channel Glauber theory.[3]

Even in diffraction dissociation, multistep processes may be quite important,[4] and the conventional Kolbig-Margolis formalism,[5] which retains only the lowest-order off-diagonal terms, is open to considerable doubt.[6]

An attractive method for discussing all these effects is to go to a basis in which $\hat{u}$ becomes diagonal.[7,8] Let the basis states $|n\rangle$ have eigenvalues λ_n:

$$\hat{u}\,|n\rangle=\lambda_n\,|n\rangle\,.$$

Then the differences among the various λ_n's will give rise to diffraction dissociation,[9] multistep processes, and inelastic screening. In particular, the attenuation of state $|i\rangle$ through a thickness t is now given by

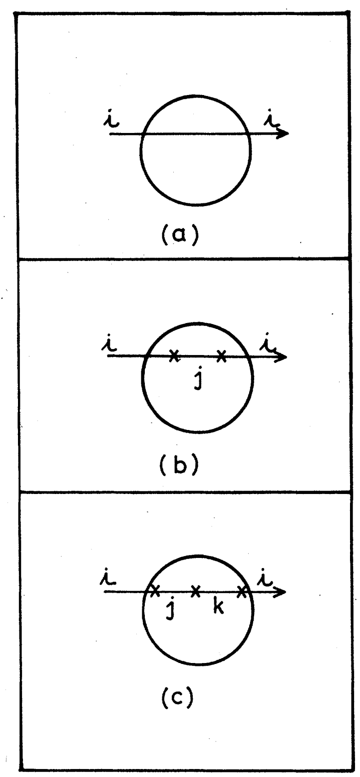

FIG. 1. Passage of a hadron through a nucleus (a) without transition, (b) by inelastic screening, (c) higher-order inelastic contribution.

$$Q = \langle i | e^{-\hat{u}t} | i \rangle$$
$$= \sum_n \langle i | n \rangle e^{-\lambda_n t} \langle n | i \rangle$$
$$= \int d\lambda\, P(\lambda) e^{-\lambda t}, \quad (4)$$

where

$$P(\lambda) = \sum_n |\langle i | n \rangle|^2 \delta(\lambda_n - \lambda)$$

is a normalized, non-negative weight function which we shall call the spectrum of absorption strengths. (Its dependence on i is suppressed.) A knowledge of $P(\lambda)$ is obviously crucial for the proper understanding of diffraction. Our object here is to show that total cross sections of hadron-nucleus scattering contains information on the moments of $P(\lambda)$ and to use the available data to determine the first few moments in the case of a nucleon passing through hadronic matter.

II. FORMALISM

Analysis of the data directly using the exact attenuation factor (4) is, of course, complicated. But suppose, for a given thickness t, we try to replace (4) by the single-channel formula (2) with an *effective* absorption coefficient u_e:

$$\int d\lambda\, P(\lambda) e^{-\lambda t} = e^{-u_e t} \; . \quad (5)$$

Let $\langle \; \rangle_P$ denote an average with weight P, e.g.,

$$\langle \lambda^n \rangle_P = \int d\lambda\, \lambda^n P(\lambda) ,$$

and let

$$\Delta\lambda = \lambda - \langle \lambda \rangle_P .$$

Then by expanding both sides of (5) about $\langle \lambda \rangle_P$, we find that u_e is given (for each t) by

$$u_e = \langle \lambda \rangle_P - \tfrac{1}{2}\langle \Delta\lambda^2 \rangle_P t + \tfrac{1}{6}\langle \Delta\lambda^3 \rangle_P t^2 + \cdots . \quad (6)$$

So by fitting data to the single-channel formula (2) and studying the thickness dependence of the resultant absorption coefficient u_e, the moments of $P(\lambda)$ can be determined. Note that the second term in (6) tends to decrease the effective absorption coefficient, in agreement with the increased transmission due to inelastic screening alluded to earlier.

A purely technical complication is that nature provides the experimenter with nuclei whose thickness t is not a constant but a function of the impact parameter b. However, as far as the total cross section is concerned, it is sufficient if (5) holds when integrated over the impact parameter. Then it is easy to show that (6) remains valid provided each nucleus is assigned a suitable average value of t^n:

$$u_e = \langle \lambda \rangle_P - \tfrac{1}{2}\langle \Delta\lambda^2 \rangle_P \langle t \rangle + \tfrac{1}{6}\langle \Delta\lambda^3 \rangle_P \langle t^2 \rangle + \cdots , \quad (7)$$

where

$$\langle t^n \rangle = \int d^2b\, t(b)^{n+1} e^{-\langle\lambda\rangle_P t(b)} \Big/ \int d^2b\, t(b) e^{-\langle\lambda\rangle_P t(b)} . \quad (8)$$

Equation (7) is the basic formula we shall employ. We take neutron-nucleus scattering from 34–273 GeV/c incident momentum as an example.[10] For each nucleus and at each incident momentum, u_e is calculated from the total cross section σ by

$$\sigma = 2 \int d^2b [1 - e^{-u_e t(b)}] , \quad (9)$$

where $t(b)$ is given by (3) and $\rho(r)$ is assumed to be a Woods-Saxon density

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/s}} , \quad R = R_0 A^{1/3} .$$

The values of the parameters are taken to be[11]

$$R_0 = 1.12 \text{ fm}, \quad s = 0.545 \text{ fm} ,$$

and ρ_0 is determined by normalization

$$\int d^3r\, \rho(r) = \int d^2b\, t(b) = A .$$

Since $u_e \to \langle \lambda \rangle_P$ as $\langle t^n \rangle \to 0$, by making an extrapolation to $A \to 0$, $\langle \lambda \rangle_P$ is estimated to be 2.1 fm^2 and this is used in (8) to calculate $\langle t^n \rangle$ for each nucleus. These are listed in Table I. (For this purpose an accurate value of $\langle \lambda \rangle_P$ is not required.) Roughly speaking, $\langle t^n \rangle$ is proportional to R^n for small R, but never exceeds order $\langle \lambda \rangle_P^{-n}$ even for large nuclei, since the exponential in (8) prevents large thicknesses from being sampled.

The result of such an analysis may be summar-

TABLE I. $\langle t \rangle$ and $\langle t^2 \rangle$ for various nuclei used in Eq. (7).

	$\langle t \rangle$ (fm^{-2})	$\langle t^2 \rangle$ (fm^{-4})
H	0.05	0.004
D	0.09	0.01
Be	0.24	0.08
C	0.28	0.11
Al	0.40	0.24
Fe	0.52	0.41
Cu	0.54	0.45
Cd	0.63	0.62
W	0.69	0.77
Pb	0.70	0.81
U	0.71	0.85

ized by a plot of u_e versus $\langle t\rangle$ at each incident momentum. Typical results are shown in Fig. 2.

The error in u_e (Δu_e) arises through (at least) three sources. (i) The experimental error in σ: Typically a 1% error in σ will cause a Δu_e of 0.02 fm^2 for H, 0.03 fm^2 for Fe, and 0.07 fm^2 for U. (ii) We assume R_0 may be in error by 0.02 fm.[11] This leads to Δu_e of 0.001 fm^2 for H, 0.04 fm^2 for Fe, and 0.15 fm^2 for U. (iii) We also assume an error of 0.02 fm in the nuclear skin thickness s, resulting in the following Δu_e: 0.007 fm^2 for H, 0.04 fm^2 for Fe, 0.06 fm^2 for U. The errors shown in Fig. 2 are obtained by directly adding (ii) and (iii), and combining with (i) quadratically. For large nuclei, the large thickness $[\langle\lambda\rangle_P t(b) \gg 1]$ at all $b \lesssim R$ implies $\sigma \simeq 2\pi R^2$, i.e., σ is sensitive to R (and in fact also s) but not to u_e, and this is the reason for the very large Δu_e in these cases.

The use of a Woods-Saxon density for small nuclei (e.g., H and D) may be questioned. Fortunately, however, from (9) it can be seen that for small nuclei,

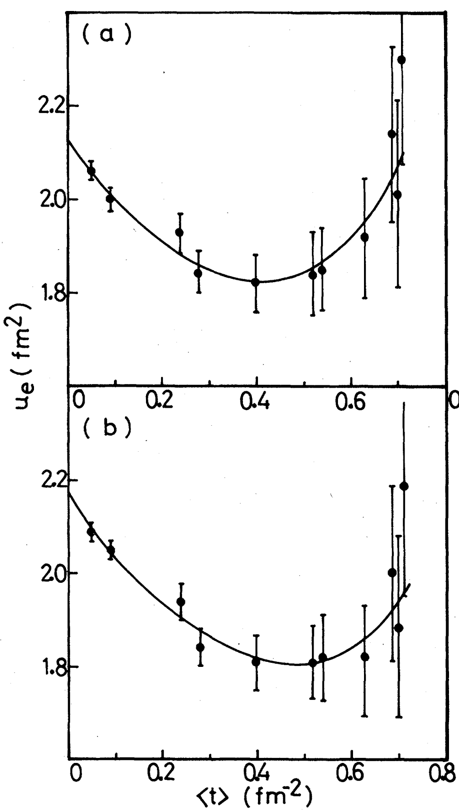

FIG. 2. Effective absorption coefficient u_e versus average nuclear thickness $\langle t\rangle$ at incident momentum (a) 80 GeV/c, (b) 240 GeV/c. The nuclei are H, D, Be, C, Al, Fe, Cu, Cd, W, Pb, U.

$$\sigma = 2\int d^2b\, u_e t(b) + O(t^2)$$
$$= 2u_e A + O(t^2)$$

so that the determination of u_e is insensitive to the shape chosen. This can also been seen from the very weak dependence of u_e on R_0 and s quoted in the case of H.

Besides the multichannel generalization, our formalism differs from the usual Glauber theory in one way. Since the thickness t is nonzero even for a hydrogen target, multiple scattering is possible (though numerically not very important as far as total cross sections are concerned). Thus the "elementary" scattering process is not nucleon-nucleon scattering but parton-parton scattering. In this way, the formalism used here embodies the compositeness and finite spatial extension of hadrons.[1]

III. RESULT AND DISCUSSION

Figure 2 also shows the curve obtained by fitting u_e to (7). From such a fit, $\langle\lambda\rangle_P$, $\langle\Delta\lambda^2\rangle_P$, $\langle\Delta\lambda^3\rangle_P$ can be determined for each incident momentum k. Note that (7) is not a quadratic since $\langle t^2\rangle \neq \langle t\rangle^2$.

First moment

The values of $\langle\lambda\rangle_P$ thus obtained are shown in Fig. 3. (Errors are about the size of each dot.) Clearly, $\langle\lambda\rangle_P$ shows an increase by about 5% over the interval 34–273 GeV/c. It should be noticed (e.g., see Fig. 2) that $\langle\lambda\rangle_P$ is larger than the effective absorption coefficient for hydrogen by a few percent, indicating the effect of multiple scattering in this case, and providing an estimate

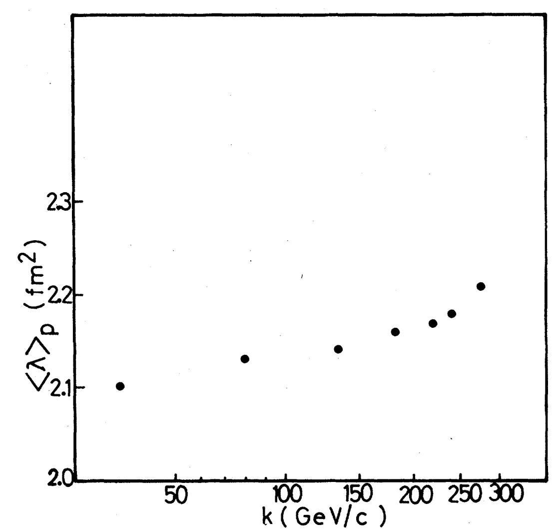

FIG. 3. Energy dependence of $\langle\lambda\rangle_P$. Horizontal scale is linear in $(\ln k)^2$.

of the error that would be involved if nucleon-nucleon scattering were used as the "elementary" amplitude in Glauber theory.

Second moment

We find $\langle\Delta\lambda^2\rangle_P \simeq 2.9\ \mathrm{fm}^4$, corresponding to $\alpha \equiv \Delta\lambda_{\mathrm{rms}}/\langle\lambda\rangle_P \simeq 0.8$. Thus the spectrum of absorption strengths is quite broad, and in particular extends to very small values (existence of "transparent states").

This has crucial implications on the matter density of a nucleon in the context of the Chou-Yang model.[1] This has been discussed in some detail in Ref. 8 by using a specific model for $P(\lambda)$. Here we simply wish to note some general features. Expand the attenuation factor (4) in powers of t:

$$Q = 1 - \langle\lambda\rangle_P t + \tfrac{1}{2}\langle\lambda^2\rangle_P t^2 - \cdots .$$

The scattering amplitude $T(q)$ is then

$$T = 1 - Q = \langle\lambda\rangle_P \tilde{t} - \tfrac{1}{2}\langle\lambda^2\rangle_P \tilde{t} \otimes \tilde{t} + \cdots ,$$

where a tilde denotes a Fourier transform and $\otimes$ is a convolution. As is well known, the location of the first dip in the differential cross section is determined largely by the cancellation of the first two terms. However, our results on the second moment imply that the usual single-channel formalism errs in the second term by a factor $\langle\lambda^2\rangle_P/\langle\lambda\rangle_P^2 = 1 + \alpha^2 \simeq 1.65$. Fairly large effects on the matter density deduced from $T(q)$ should therefore come as no surprise.[8]

Energy dependence

Figure 4 shows that the width of the spectrum, $\Delta\lambda_{\mathrm{rms}}$, increases with incident momentum k, but $\alpha = \Delta\lambda_{\mathrm{rms}}/\langle\lambda\rangle_P$ is apparently constant. In other words, the spectrum broadens, but in a manner consistent with the scaling of all eigenvalues λ_n:

$$\lambda_n(k) = \langle\lambda(k)\rangle_P f_n$$

or, in terms of the distribution P,

$$P(\lambda; k) = \frac{1}{\langle\lambda(k)\rangle_P} F\left(\frac{\lambda}{\langle\lambda(k)\rangle_P}\right) .$$

Third moment

The upward bend of the curves in Fig. 2 indicates a positive value of $\langle\Delta\lambda^3\rangle_P$, i.e., the spectrum is skewed towards large value. Unfortunately, the determination of $\langle\Delta\lambda^3\rangle_P$ is sensitive to the values of u_e for the larger nuclei and these cannot be determined with great accuracy, for reasons discussed earlier. Moreover, the neglect of $\langle\Delta\lambda^4\rangle_P, \ldots$ introduces systematic uncertainties. Thus the fitted result for $\langle\Delta\lambda^3\rangle_P$ can only be regarded as an

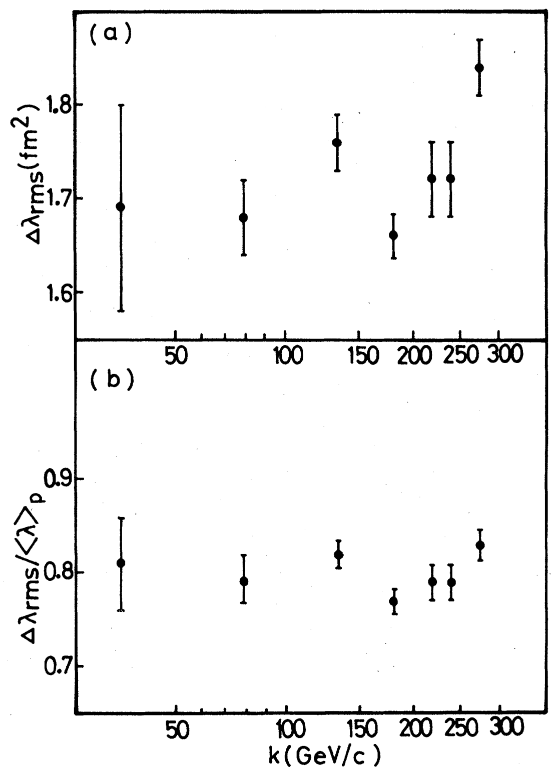

FIG. 4. Energy dependence of (a) $\Delta\lambda_{\mathrm{rms}}$ and (b) $\alpha = \Delta\lambda_{\mathrm{rms}}/\langle\lambda\rangle_P$. Horizontal scale is linear in $(\ln k)^2$.

estimate. The result is most conveniently expressed in terms of $\beta \equiv \langle\Delta\lambda^3\rangle_P^{1/3}/\langle\lambda\rangle_P$ and we find $\beta \simeq 0.8$–0.9 and consistent with being constant with respect to energy.

The usual inelastic screening [Fig. 1(b)] is essentially an $O(\Delta\lambda^2)$ correction to the single-channel approximation. The fact that β is by no means small shows that higher-order contributions [Fig. 1(c)] are not always negligible.

The estimate of the moments of $P(\lambda)$ is, of course, dependent upon the nuclear parameters R_0 and s, for which there are two possible types of errors. First, these errors could be uncorrelated from nucleus to nucleus (e.g., fluctuations about an $A^{1/3}$ law). The error bars displayed (about 5% for both α and β) have already taken this into account under assumption that R_0 and s may each be in error by 0.02 fm. Secondly, there could by a systematic error in R_0 or s. To estimate the effect of this, we have performed the analysis by changing R_0 by 0.02 fm for every nucleus, which results in changes of 0.2% in $\langle\lambda\rangle_P$, 1% in α, and 5% in β. Likewise, changing s by 0.02 fm for every nucleus results in changes of 0.4% in $\langle\lambda\rangle_P$, 3% in α, and 1% in β. None of these are at a level which would bring our qualitative conclusions into doubt.

IV. CONCLUSION

We have obtained, in a model-independent way, the first three moments of the spectrum of absorption strengths. The spectrum is broad and skewed, and moreover broadens with energy in a manner consistent with "scaling." A more accurate characterization of the spectrum (e.g., higher moments) would be possible if nuclear densities were known with greater precision.

It would be interesting to apply these model-independent results to calculate the matter density in hadrons.

[1] T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1968).
[2] T. T. Chou and C. N. Yang, Phys. Rev. 175, 1832 (1968).
[3] V. A. Karmanov and L. A. Kondratyuk, Zh. Eksp. Teor. Fiz. Pis'ma Red. 18, 451 (1973) [JETP Lett. 18, 266 (1973)]; A. Gsponer *et al.*, Phys. Rev. Lett. 42, 9 (1979); D. O. Caldwell, in *Proceedings of the 1978 International Meeting on Frontiers of Physics, Singapore*, edited by K. K. Phua *et al.* (Singapore National Academy of Science, Singapore, 1979).
[4] G. Faldt and P. Osland, Nucl. Phys. B87, 445 (1975).
[5] K. S. Kolbig and B. Margolis, Nucl. Phys. B6, 85 (1968).
[6] H. I. Miettinen and J. Pumplin, Phys. Rev. Lett. 42, 204 (1979).
[7] L. Van Hove, Nucl. Phys. B46, 75 (1972); H. Fujisaki *et al.*, Prog. Theor. Phys. 61, 682 (1979).
[8] H. I. Miettinen and G. H. Thomas, Nucl. Phys. B166, 365 (1980).
[9] M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960).
[10] M. J. Longo *et al.*, Phys. Rev. Lett. 33, 725 (1974); L. W. Jones *et al.*, *ibid.* 33, 1440 (1974).
[11] H. Alvensleben *et al.*, Phys. Rev. Lett. 24, 792 (1970).