Zeros in scattering amplitudes and the structure of non-Abelian gauge theories

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Some scattering amplitudes in non-Abelian gauge theories exhibit remarkable factorization properties. They can lead to the presence of zeros in the angular distribution which are in principle experimentally observable.

I. INTRODUCTION

Recently Mikaelian *et al.*[1] found a specific angle for which the cross section $q\bar{q}\to\gamma W$ vanishes, where W is the weak boson of the Weinberg-Salam model. It was pointed out that this characteristic phenomenon may be useful for checking the fractional charges of quarks. This zero actually appears in the amplitude.[2] It is a consequence of a factorization property of the $q\bar{q}$ annihilation amplitude; similar factorizations exist in the amplitudes for quark annihilation into gluons, annihilation of scalar fields, and gluon-gluon scattering.[2] We will discuss this problem in more detail. In Sec. II we formulate the problem of factorization of scattering amplitudes in gauge theories in general and discuss the conditions under which the factorizations hold. We shall then investigate the various conditions under which one can actually observe a zero direction connected with these factorization properties. This is done in Sec. III. Some specific processes where we can observe the zero direction are given in Sec. IV. Finally, in Sec. V we make a few concluding remarks.

II. FACTORIZATION

In non-Abelian theories the amplitudes of some processes can be written as products of two parts.[2] One part includes a certain combination of generators of the gauge group and kinematic invariants; the other part corresponds to the actual amplitudes of the Abelian fields. Zeros are associated with the first part.

This factorization is special to non-Abelian gauge fields. We will write down the factorization formulas for the annihilation of scalar fields, spinor fields, and for the scattering of gauge fields. We list for completeness the Feynman rules[3-5] in Table I. Our metric is that used by Bjorken and Drell.[6] The replacements to introduce gauge fields are

$$\partial^\mu\delta_{ij}\to D^\mu_{ij}\equiv\partial^\mu\delta_{ij}+igT^a_{ij}A^{a\mu}; \tag{1}$$

then the strengths of gauge fields are

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-gf_{abc}A^b_\mu A^c_\nu. \tag{2}$$

Here T^a and f_{abc} are, respectively, the generators and structure constants. We express the Feynman rule, following Cvitanovic,[5] as a product of two parts. One contains a purely gauge group factor; the other is purely dynamical. It is more convenient and explicit in this way to see the group-theoretical structure of S-matrix elements.

Let us calculate the matrix elements for the following processes:

$$\begin{aligned}&\phi_i\phi_j^*\to A^aA^b,\\&\psi_i\bar{\psi}_j\to A^aA^b,\\&A^cA^d\to A^aA^b.\end{aligned} \tag{3}$$

In the tree-diagram approximation the corresponding Feynman diagrams are shown in Fig. 1. The kinematic invariants are defined as

$$t=(p_1-q_1)^2,\quad u=(p_1-q_2)^2,\quad s=(p_1+p_2)^2. \tag{4}$$

We express the amplitude of channel p ($=t$, u, or s) in this way:

$$m_p=G_pD^p=G_p\frac{T^p}{C_p}. \tag{5}$$

Here G_p is the group factor, D^p is the dynamics part, and C_p is that part of the denominator due to propagators. T^p could be called the dynamic residue. The total amplitude is

$$m=\sum_p m^p =\sum_p\frac{G_pT^p}{C_p}. \tag{6}$$

We give the results for all processes in Table II. Note the kinematic relation

$$C_t+C_u+C_s=\tilde{t}+\tilde{u}+\tilde{s}=0 \tag{7}$$

and the group-theoretical relation

$$G_t-G_u=G_s. \tag{8}$$

We also have

$$T^{\tilde{t}}-T^{\tilde{u}}=T^{\tilde{s}}+\Delta, \tag{9}$$

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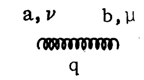
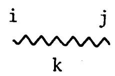
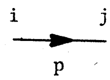
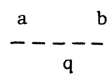
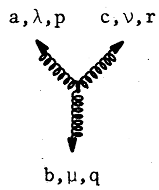
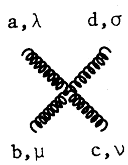
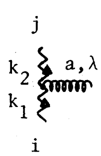
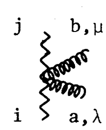
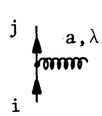
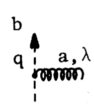
TABLE I. Feynman rules of non-Abelian gauge theories. (1) The vertices of four external lines are divided into several terms. This implies that the whole vertex is equal to the sum of products of the corresponding group factor and dynamics factor. (2) If we set $T=1, f=0$, then we get the usual Feynman rules of QED. (3) The differences between our Feynman rule and the ones generally used (Ref. 4) consist of a minus sign in the vertex of three gauge fields and the order of the subscripts of the generator in the spinor–gauge-field vertex is inverted.

	Diagram	Rule	Group factor	Dynamics factor
Propagator of gauge field A		$\frac{-i}{q^2}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}(1-a)\right]\delta_{ab}$	δ_{ab}	$\frac{-i}{q^2}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}(1-a)\right]$
Propagator of scalar field ϕ		$\frac{i}{k^2-m^2}\delta_{ij}$	δ_{ij}	$\frac{i}{k^2-m^2}$
Propagator of spinor field ψ		$\frac{i(\not p+m)}{p^2-m^2}\delta_{ij}$	δ_{ij}	$\frac{i(\not p+m)}{p^2-m^2}$
Propagator of ghost field C		$\frac{-i}{q^2}\delta_{ab}$	δ_{ab}	$\frac{-i}{q^2}$
Vertex of three gauge fields AAA		$gf_{abc}C_{\lambda\mu\nu}(p,q,r)$ $C_{\lambda\mu\nu}(p,q,r)=(p-q)_\nu g_{\lambda\mu}+(q-r)_\lambda g_{\mu\nu}+(r-p)_\mu g_{\nu\lambda}$	$-if_{abc}$	$igC_{\lambda\mu\nu}(p,q,r)$ $=ig[(p-q)_\nu g_{\lambda\mu}+(q-r)_\lambda g_{\mu\nu}+(r-p)_\mu g_{\nu\lambda}]$
Vertex of four gauge fields $AAAA$		$-ig^2[f_{abe}f_{cde}(g_{\lambda\nu}g_{\mu\sigma}-g_{\lambda\sigma}g_{\mu\nu})$	$-f_{abe}f_{cde}$	$ig^2(g_{\lambda\nu}g_{\mu\sigma}-g_{\lambda\sigma}g_{\mu\nu})$
		$+f_{ace}f_{bde}(g_{\lambda\mu}g_{\nu\sigma}-g_{\lambda\sigma}g_{\mu\nu})$	$-f_{ace}f_{bde}$	$ig^2(g_{\lambda\mu}g_{\nu\sigma}-g_{\lambda\sigma}g_{\mu\nu})$
		$+f_{ade}f_{bec}(g_{\lambda\nu}g_{\mu\sigma}-g_{\lambda\mu}g_{\nu\sigma})]$	$-f_{ade}f_{cbe}$	$ig^2(g_{\lambda\nu}g_{\mu\sigma}-g_{\lambda\mu}g_{\nu\sigma})$
Vertex of scalar and gauge fields $\phi^*\phi A$		$-igT^a_{ji}(k_1+k_2)_\lambda$	T^a_{ji}	$-ig(k_1+k_2)_\lambda$
"Seagull" $\phi^*\phi AA$		$ig^2\{T^b,T^a\}_{ji}g_{\lambda\mu}$	$(T^bT^a)_{ji}$	$ig^2g_{\lambda\mu}$
		$=ig^2[(T^bT^a)_{ji}+(T^aT^b)_{ji}]g_{\lambda\mu}$	$(T^aT^b)_{ji}$	$ig^2g_{\lambda\mu}$
Vertex of spinor and gauge fields $\bar\psi\psi A$		$-igT^a_{ji}\gamma_\lambda$	T^a_{ji}	$-ig\gamma_\lambda$
Vertex of ghost and gauge fields $\bar cc A$		$-gf_{abc}q_\lambda$	$-if_{abc}$	$-igq_\lambda$

$$\Delta=\begin{cases}-ig^2(q_{1\lambda}q_{1\mu}-q_{2\lambda}q_{2\mu})\epsilon_1^\lambda\epsilon_2^\mu \quad \text{(for scalar)},\\ -ig^2\bar{v}(p_2)[i\sigma_{\lambda\mu}(\not{p}_1-m)-i(\not{p}_2-m)\sigma_{\lambda\mu}+q_{2\mu}\gamma_\lambda-q_{1\lambda}\gamma_\mu]u(p_1)\epsilon_1^\lambda\epsilon_2^\mu \quad \text{(for spinor)},\\ -ig^2\{g_{\sigma\nu}[(p_2-p_1)_\lambda q_{2\mu}+q_{1\lambda}(p_1-p_2)_\mu]+g_{\sigma\lambda}[-p_{1\nu}(p_2+q_{1\mu})-(p_2+q_1)_\nu q_{2\mu}]\\ \quad +g_{\sigma\mu}[(2p_2-q_1)_\nu q_{1\lambda}+p_{1\nu}(2p_2+p_1)_\lambda]+g_{\nu\lambda}[p_{2\sigma}(p_1+q_1)_\mu+(p_1+q_1)_\nu q_{2\mu}]\\ \quad +g_{\nu\mu}[-p_{2\sigma}(p_1+p_2)_\lambda+(-2p_1+q_1)_\sigma q_{1\lambda}]\\ \quad +g_{\lambda\mu}[p_{2\sigma}-2q_1)_\nu+(-p_1+2q_1)_\sigma q_{1\nu}]\}\epsilon_1^\lambda\epsilon_2^\mu\epsilon_3^\nu\epsilon_4^\sigma \quad \text{(for gauge field)}.\end{cases} \tag{10}$$

Using the physical conditions of vector fields and equations of motion of spinor fields, we obtain the result

$$\Delta=0 \tag{11}$$

and then

$$T^{\tilde t}-T^{\tilde u}=T^{\tilde s}. \tag{12}$$

From (6), (7), (8), and (12) we obtain at last the factorization formula

$$m=\frac{G_tT^{\tilde t}}{C_t}+\frac{G_uT^{\tilde u}}{C_u}+\frac{G_sT^{\tilde s}}{C_s}$$

$$=Gm^A. \tag{13}$$

Here

$$G=\frac{C_uG_t+C_tG_u}{-C_s} \tag{14}$$

and

$$m^A=D^{\tilde t}+D^{\tilde u}=\frac{T^{\tilde t}}{C_t}+\frac{T^{\tilde u}}{C_u}. \tag{15}$$

In the scalar and spinor case m^A is just the Abelian amplitude (QED).

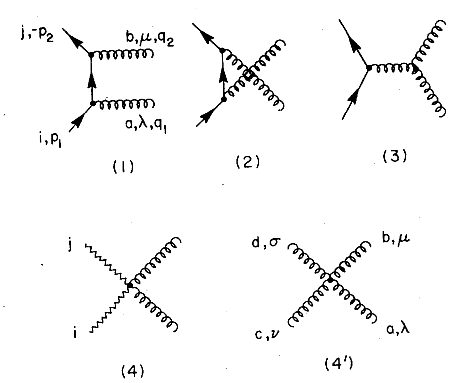


FIG. 1. The solid lines in the first three diagrams represent all kinds of fields. For the annihilation of spinor fields only the first three diagrams are present; for scalar fields or gauge fields we must add diagram (4) or (4′), respectively. From the Feynman rules we notice that diagrams (4) or (4′) can be separated into several parts and put into corresponding diagrams (1), (2), and (3). After such a procedure we only need consider three diagrams in any case.

In the conclusion of this section we emphasize several points:

(a) Obviously this factorization is characteristic for a non-Abelian gauge theory. In some other processes we also could have factorization (see Sec. IV).

(b) This factorization is a result of kinematics, gauge invariance, and dynamics [conditions (7), (8), (12)].

(c) At least one gauge field must be massless; otherwise, relation (7) breaks down and factorization no longer holds.

(d) This factorization also holds in some crossed processes such as spinor–gauge-field scattering. The only thing we need to do is to make certain substitution of invariants $\tilde t$, $\tilde u$, and $\tilde s$.

III. ZEROS IN THE ANGULAR DISTRIBUTION

First of all, we emphasize that zeros can only be associated with the group-theoretical factor. The Abelian amplitude, indeed, always gives a positive contribution to the cross section. We have

$$\overline{\sum_{\text{spin}}}|m^A|^2=\begin{cases}g^4 8\left[1-\dfrac{m^2\tilde s}{\tilde u\tilde t}+\left(\dfrac{m^2\tilde s}{\tilde u\tilde t}\right)^2\right] \quad \text{(for scalar)},\\ g^4 3\left[\dfrac{\tilde t^2+\tilde u^2}{\tilde u\tilde t}+\dfrac{4m^2\tilde s}{\tilde u\tilde t}\left(1-\dfrac{m^2\tilde s}{\tilde t\tilde u}\right)\right] \text{(for spinor)},\\ g^4 4\left[1+\tilde s^2\left(\dfrac{1}{\tilde u^2}+\dfrac{1}{\tilde t^2}\right)\right] \quad \text{(for gauge fields)}.\end{cases} \tag{16}$$

As usual, the notation $\overline{\sum}_{\text{spin}}$ stands for averaging over initial and summing over final spins. In the center-of-mass frame we have

$$\frac{m^2\tilde s}{\tilde t\tilde u}=\frac{1}{1+(|\vec p|^2/m^2)\sin^2\theta^*}\leqslant 1 \tag{17}$$

(θ^* is the angle between $\vec q_1$ an $\vec p_1$); therefore, (16) is always positive without any zero. Zeros must come from the group factor G.

Let us analyze in general where we can observe zeros in the differential cross section. In scattering processes such as $A\psi\to A\psi$ the group factor is given by

TABLE II. Matrix elements of annihilation.

	Scalar field	Spinor field	Gauge field
C_t	$\tilde{t}=t-m^2$	$\tilde{t}=t-m^2$	$\tilde{t}=t$
C_u	$\tilde{u}=u-m^2$	$\tilde{u}=u-m^2$	$\tilde{u}=u$
C_s	$\tilde{s}=s$	$\tilde{s}=s$	$\tilde{s}=s$
G_t	$(T^bT^a)_{ji}$	$(T^bT^a)_{ji}$	$(F^bF^a)_{dc}=-f_{bde}f_{aec}$
G_u	$(T^aT^b)_{ji}$	$(T^aT^b)_{ji}$	$(F^aF^b)_{dc}=-f_{ade}f_{bec}$
G_s	$-if_{abc}T^c_{ji}$	$-if_{abc}T^c_{ji}$	$-if_{abe}F^e_{dc}=f_{abe}f_{edc}$
$T^{\tilde{t}}$	$-ig^2[(2p_1-q_1)_\lambda(q_2-2p_2)_\mu$ $-\tilde{t}g_{\lambda\mu}]\epsilon_1^\lambda\epsilon_2^\mu$	$-ig^2\overline{v}(p_2)\gamma_\mu[\not{p}_1-\not{q}_1+m]$ $\times\gamma_\lambda u(p_1)\epsilon_1^\lambda\epsilon_2^\mu$	$ig^2[C_{\sigma\tau\mu}(-p_2,p_2-q_2,q_2)$ $\times C^\tau{}_{\nu\lambda}(p_1-q_1,-p_1,q_1)$ $+\tilde{t}(g_{\mu\nu}g_{\lambda\sigma}-g_{\lambda\mu}g_{\nu\sigma})]$ $\times\epsilon_1^\lambda\epsilon_2^\mu\epsilon_3^\nu\epsilon_4^\sigma$
$T^{\tilde{u}}$	$-ig^2[(2p_1-q_2)_\mu(q_1-2p_2)_\lambda$ $-\tilde{u}g_{\lambda\mu}]\epsilon_1^\lambda\epsilon_2^\mu$	$-ig^2\overline{v}(p_2)\gamma_\lambda[\not{p}_1-\not{q}_2+m]$ $\times\gamma_\mu u(p_1)\epsilon_1^\lambda\epsilon_2^\mu$	$ig^2[C_{\sigma\tau\lambda}(-p_2,p_2-q_1,q_1)$ $\times C^\tau{}_{\nu\mu}(p_1-q_2,-p_1,q_2)$ $+\tilde{u}(g_{\lambda\nu}g_{\mu\sigma}-g_{\lambda\mu}g_{\nu\sigma})]$ $\times\epsilon_1^\lambda\epsilon_2^\mu\epsilon_3^\nu\epsilon_4^\sigma$
$T^{\tilde{s}}$	$-ig^2[-(p_1-p_2)_\lambda(2q_1+q_2)_\mu$ $+(p_1-p_2)_\mu(q_1+2q_2)_\lambda$ $+(\tilde{u}-\tilde{t})g_{\lambda\mu}]\epsilon_1^\lambda\epsilon_2^\mu$	$-ig^2\overline{v}(p_2)\gamma^\tau$ $\times C_{\tau\lambda\mu}(-(q_1+q_2),q_1,q_2)$ $\times u(p_1)\epsilon_1^\lambda\epsilon_2^\mu$	$ig^2[C_{\sigma\nu\tau}(-p_2,-p_1,p_1+p_2)$ $\times C^\tau{}_{\lambda\mu}(-(q_1+q_2),q_1,q_2)$ $+\tilde{S}(g_{\lambda\sigma}g_{\mu\nu}-g_{\lambda\nu}g_{\mu\sigma})]$ $\times\epsilon_1^\lambda\epsilon_2^\mu\epsilon_3^\nu\epsilon_4^\sigma$

$$G=\frac{\tilde{u}G_s+\tilde{s}G_u}{-\tilde{t}}\,. \tag{18}$$

To get a zero, we must have $G=0$. But we know that $\tilde{u}$ and $\tilde{s}$ have opposite signs in the physical region. Therefore, G_s and G_u are required to have the same sign. G_s, G_t, and G_u are some kind of charges (electric charge, isospin, color, etc). For integral or half-integral charges we often have $|G_s|=|G_u|$. The requirement for a zero is

$$G=G_s\frac{\tilde{u}+\tilde{s}}{-\tilde{t}}=G_s\,,$$

and therefore the zero cannot appear. It is of course possible to find a zero if G_s and G_u have the same sign but are not equal.

If one of the scalar or spinor particles is chargeless, then G_u vanishes, with

$$G=G_s\frac{\tilde{u}}{-\tilde{t}}$$

and a factor $\tilde{u}^2G_s^{\,2}/\tilde{t}^{\,2}$ appears in the cross section. From (16) we know there is only a single pole in the Abelian cross section if the fields are massless. So we get a zero at $\tilde{u}=0$; i.e., $\theta^*=\pi$. The differential cross section vanishes in the backward direction.

In annihilation or its inverse process the group factor is

$$G=\frac{\tilde{u}G_t+\tilde{t}G_u}{-\tilde{s}}\,. \tag{19}$$

Because $\tilde{u}$ and $\tilde{t}$ have the same signs, we will get a zero if G_t and G_u have opposite signs. We expect a zero at $\tilde{u}=\tilde{t}$; i.e., $\theta^*=\pi/2$ when particles with opposite charges annihilate. In addition, it is possible, as in the scattering case, to have a zero in the forward or backward direction when one of the massless scalar or spinor particles has no charge.

Next, we investigate the effect of gauge-group indices on the zero direction. We shall call these indices gauge polarizations. For simplicity we only consider annihilation. We denote the process illustrated in Fig. 1 as $(i\bar{j}\to ab)$ and the corresponding amplitude as $m(i\bar{j}\to ab)$. The group factor in the cross section can be written as

$$F=\sum|G|^2$$

$$=\frac{A}{2\tilde{s}^2}\left[\left(\frac{\tilde{u}}{\tilde{t}}^{\;2}+2B\,\frac{\tilde{u}}{\tilde{t}}\right)+C\right]\,. \tag{20}$$

Then the condition for a zero to exist is

TABLE III. Reactions with zeros. Processes are symmetric under $j \leftrightarrow i, b \leftrightarrow a$. v^* is the incident velocity of the particle in the center-of-mass frame. $v^*=1$, if we ignore the mass.

Gauge group	j	i	b	a	θ_0^*
SU(2)	1	1	1	2	$\frac{1}{2}\pi$
	2	2	1	2	$\frac{1}{2}\pi$
	1	2	1	3	$\frac{1}{2}\pi$
	1	2	2	3	$\frac{1}{2}\pi$
SU(3)	1	1	1	2	$\frac{1}{2}\pi$
	2	2	1	2	$\frac{1}{2}\pi$
	1	2	1	3	$\frac{1}{2}\pi$
	1	2	2	3	$\frac{1}{2}\pi$
	1	1	4	5	$\frac{1}{2}\pi$
	3	3	4	5	$\frac{1}{2}\pi$
	2	3	6	7	$\frac{1}{2}\pi$
	1	3	4	8	$\cos\theta_0^* = -\frac{1}{3}\frac{1}{v^*} \to -\frac{1}{3}$
	1	3	5	8	$\cos\theta_0^* = -\frac{1}{3}\frac{1}{v^*} \to -\frac{1}{3}$
	2	3	6	8	$\cos\theta_0^* = -\frac{1}{3}\frac{1}{v^*} \to -\frac{1}{3}$
	2	3	7	8	$\cos\theta_0^* = -\frac{1}{3}\frac{1}{v^*} \to -\frac{1}{3}$

Gauge group	j	i	b	a	θ_0^*
SU(3)	1	2	4	6	π
	1	2	4	7	π
	1	2	5	6	π
	1	2	5	7	π
	1		1	6	π
	1		1	7	π
	1		2	6	π
	1		2	7	π
	1		3	4	π
	1		3	5	π
	2	3	1	4	π
	2	3	1	5	π
	2	3	2	4	π
	2	3	2	5	π
	2	3	3	6	π
	2	3	3	7	π

$$C>0, \quad B \leq -\sqrt{C}, \qquad \frac{\tilde{u}}{\tilde{t}} = -B \pm (B^2 - C)^{1/2}. \tag{21}$$

We have one zero when $B=-\sqrt{C}$, but there are two zeros if $B<-\sqrt{C}$. When $C=1$, these two zero directions θ_1^* and θ_2^* are mutually complementary; i.e., $\theta_2^* = \pi - \theta_1^*$. An alternative condition for a zero is

$$C<0, \qquad \frac{\tilde{u}}{\tilde{t}} = (B^2 - C)^{1/2} - B. \tag{22}$$

We only have one zero.

Let us discuss various alternatives separately:

(a) All gauge polarizations are observed. Then

$$B = (T^aT^b)_{ji}/(T^bT^a)_{ji}, \qquad C = B^2. \tag{23}$$

We have one zero direction provided that $(T^aT^b)_{ji}$ and $(T^bT^a)_{ji}$ have opposite signs. In Table III we list all reactions that possess zeros; we also list the corresponding angle θ_0^* for SU(2) and SU(3).

(b) If we do not measure the gauge polarization of one particle in the initial or final states, the situation is the same because of the symmetry of the theory. So we directly consider the case where the gauge polarizations of two particles are not measured.

(b1) We do not measure polarizations of one initial state (i) and one final state (a). Then we have

$$B = \sum_a D^{ab}_{A,jj} \Big/ \sum_a D^{ab}_{F,jj} \,,$$
$$C = \sum_a D^{ba}_{F,jj} \Big/ \sum_a D^{ab}_{F,jj} \,. \tag{24}$$

Here the matrices are

$$D^{ab}_F = 2(T^a T^b T^b T^a) \,,$$
$$D^{ab}_A = 2(T^a T^b T^a T^b) \,. \tag{25}$$

For SU(2),

$$D^{ab}_{F,jj} = \tfrac{1}{8} = D^{ba}_{F,jj} \,,$$
$$D^{ab}_{A,jj} = \tfrac{1}{8}(2\delta^{ab} - 1) \,, \tag{26}$$

$$B = 1 , \quad C = 1 \,. \tag{27}$$

So there is no zero in this case. For SU(3) there is a zero direction (Table IV).

(b2) If we do not measure all gauge polarizations of initial states, then we have

$$B = \frac{\mathrm{tr} D^{ab}_A}{\mathrm{tr} D^{ab}_F} \,,$$
$$C = 1 \,. \tag{28}$$

The condition that guarantees the zero direction is

$$B \leqslant -1 \,. \tag{29}$$

For SU(2),

$$B = 2\delta_{ab} - 1 \,. \tag{30}$$

So when $a \neq b$, we get a zero at

$$\theta^*_0 = \frac{\pi}{2} \,. \tag{31}$$

This means all zero directions of case (a) remain.

For SU(3), we have

$$B \geqslant \tfrac{7}{8} \,. \tag{32}$$

There is no zero direction here.

(b3) If all gauge polarizations of gauge fields are not observed, then

$$B = \frac{1}{2} \frac{\sum_{a,b,k,l} (T^b_{jk} T^b_{il} T^a_{ki} T^a_{lj} + T^b_{ki} T^b_{lj} T^a_{jk} T^a_{il})}{\sum_{a,b,k,l} (T^b_{jk} T^b_{lj} T^a_{ki} T^a_{il})} \,,$$
$$C = \frac{\sum_{a,b,k,l} (T^a_{jk} T^a_{lj} T^b_{ki} T^b_{il})}{\sum_{a,b,k,l} (T^b_{jk} T^b_{lj} T^a_{ki} T^a_{il})} \,. \tag{33}$$

Using projection formulas, we can calculate (33). For SU(n) we get

$$B = \frac{(n^2 - n)\delta_{ij}}{n(n-2) + \delta_{ij}} \,,$$
$$C = 1 \,. \tag{34}$$

The condition $B \leqslant -1$ cannot be satisfied. For SO(n)

$$B = \frac{(n-1)\delta_{ij}}{(n-2) + \delta_{ij}} \,,$$
$$C = 1 \,. \tag{35}$$

The situation is the same as SU(n).

(c) It is easy to deduce that we cannot find any zero direction if we do not measure any gauge polarization. Actually, in this case we have

$$B = 1 - \frac{1}{2} \frac{C_A}{C_F} \,,$$
$$C = 1 \,. \tag{36}$$

Here C_F and C_A are the eigenvalues of the quadratic Casimir operators for the fundamental and adjoint representations, respectively. The condition $B \leqslant -1$ cannot hold for ordinary simple or semisimple Lie groups [SU(n), SO(n), Sp($2n$), G_2, E_6, E_7, E_8, F_4] because

$$C_A \leqslant \tfrac{8}{3} C_F \,. \tag{37}$$

To summarize this paragraph, we emphasize that the best way to observe the zero direction is to leave gauge indices exposed; if all gauge polarizations are hidden, there is no way to find the zero direction. This is similar to polarized-beam experiments. We know that the scattering process $\bar{\nu}_e e \to \bar{\nu}_e e$ has a zero direction if we measure the helicity of the electron. But if we do not measure the polarization of any electron state, the zero direction disappears (in the Weinberg-Salam model).

IV. PROCESSES WITH A ZERO IN THE DIFFERENTIAL CROSS SECTION

Mikaelian *et al.* have provided two examples of processes. One is $\bar{p} + p \to W^{\pm} + \gamma + X$; the other is $\bar{\nu}_e e \to W^- \gamma$. In the subprocess $q_i \bar{q}_i \to W^- \gamma$ the vertex $WW\gamma$ is equivalent to a three-gauge-field vertex when we choose an anomalous magnetic moment of the W boson to be $\kappa = 1$, as is the case in the Weinberg-Salam model. The photon is

TABLE IV. Reactions with zeros of SU(3) gauge symmetry. The gauge polarization of one initial and one final state are averaged.

j	b	θ^*_0
1	6	π
	7	π
2	4	π
	5	π
3	1	π
	2	π
	3	π

massless; therefore, condition (7) is satisfied ($\tilde{s}=s-m_W{}^2$). The corresponding group factors are

$$G_t=g_{ji}Q_j,\quad G_t=g_{ji}Q_i,\quad G_s=Q_W g_{ji} \tag{38}$$

and therefore, condition (8) also holds because of charge conservation. We have factorization and the group factor in the amplitude is

$$G=\frac{\tilde{u}G_t+\tilde{t}G_u}{-\tilde{s}}=\frac{g_{ji}}{-\tilde{s}}(\tilde{u}Q_i+\tilde{t}Q_j)\,. \tag{39}$$

There is a zero when

$$\frac{\tilde{u}}{\tilde{t}}=-\frac{Q_j}{Q_i} \tag{40}$$

or

$$\cos\theta_0^*=\frac{Q_i+Q_j}{Q_W}\,. \tag{41}$$

For $u\bar{d}\to W^-\gamma$,

$$\cos\theta_0^*=-\tfrac{1}{3}\,. \tag{42}$$

For the case $\bar{\nu}_e e\to W^-\gamma$, the situation is the same—the zero direction is at

$$\cos\theta_0^*=1\,. \tag{43}$$

It should be pointed out that there is factorization in this model if and only if one uses a three-gauge-field vertex, as Ref. 1 has shown no zero unless $\kappa=1$.

In this model the photon couples to the electric charge of quarks in a pointlike manner, so the scattering channel $\gamma p\to W^+n$ does not have a zero direction.[7] But there is a zero in the backward direction if we think of a photon interacting with the nucleon as a whole point particle (Fig. 2). The recent observations of prompt photons are supporting the pointlike interaction of photon-quark.[8] If so, observing no zero in $\gamma p\to W^+n$ will present further evidence.

Until now we have not found the W boson. It would therefore be desirable to find some experimentally accessible processes to check the theory.

The best candidate of a non-Abelian gauge field among existing particles is the ρ meson. There are models[9,10] that describe the ρ meson as a SU(2) gauge field which couples to pseudoscalar mesons and nucleons. In such models some processes will develop zeros in the differential cross section. These zero directions only appear at $\theta_0^*=0$, $\pi/2$, or π, because the isospin is integral or half-integral.

(1) The processes

$$\begin{aligned}&\bar{p}n\to\rho^-\rho^0\,,\\ &K^-K^0\to\rho^-\rho^0\,,\end{aligned} \tag{44}$$

(and their charge-conjugated processes) develop a zero at

$$\theta_0^*=\frac{\pi}{2}\,. \tag{45}$$

The amplitudes of these processes can be factorized when we ignore the mass of the neutral ρ meson. The group factor is

$$\begin{aligned}G&=\frac{\tilde{u}(T^3T^+)_{12}+\tilde{t}(T^+T^3)_{12}}{-\tilde{s}}\\ &=\frac{1}{2}\frac{\tilde{u}-\tilde{t}}{-\tilde{s}}\,.\end{aligned} \tag{46}$$

Here T^a ($a=1,2,3$) are isospin matrices, $T^\pm\equiv T^1\pm iT^2$. The zero direction is fixed at $\theta_0^*=\pi/2$.

(2) Consider the processes

$$\begin{aligned}&\bar{p}n\to\pi^-\rho^0,\rho^-\pi^0\,,\\ &\theta_0^*=\frac{\pi}{2}\,.\end{aligned} \tag{47}$$

In most models the π meson interacts with the nucleon by a pseudoscalar coupling. The process $N_i\bar{N}_j\to\pi^a\rho^b$ also factorizes if we ignore the mass of the ρ meson. The Feynman diagrams are illustrated in Fig. 3, and the amplitude is

$$m(N_i\bar{N}_j\to\pi^a\rho^b)=\frac{\tilde{u}G_t+\tilde{t}G_u}{-\tilde{s}}m^A\,. \tag{48}$$

Here

$$m^A=-ig_\rho g_\pi\bar{v}(p_2)\left[\frac{1}{\tilde{t}}\gamma_\mu(\not{p}_1-\not{q}_1+m_N)\gamma_5+\frac{1}{\tilde{u}}\gamma_5(\not{p}_1-\not{q}_2+m_N)\gamma_\mu\right]u(p_1)\epsilon_2^\mu \tag{49}$$

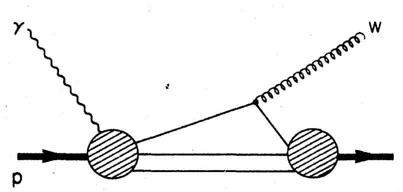


FIG. 2. Feynman diagram for the reaction $\gamma p\to W^+n$.

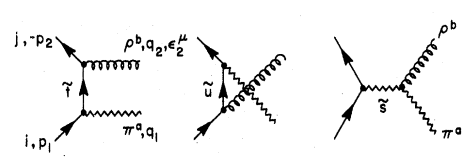


FIG. 3. Feynman diagrams for $N_i\bar{N}_j\to\pi^a\rho^b$.

and g_π and g_ρ are, respectively, the pseudoscalar and vector coupling constants. For $\bar{p}n \to \pi^-\rho^0$,

$$G_t = (T^3T^+)_{12} = \tfrac{1}{2}\,,$$
$$G_u = (T^+T^3)_{12} = -\tfrac{1}{2}\,.$$

So $\theta_0^* = \pi/2$ is a zero direction.

(3) Consider the processes

$$\bar{p}n \to \gamma\rho^-\,,\quad K^-K^0 \to \gamma\rho^-\,,\quad \theta_0^* = \pi\,. \tag{50}$$

These processes are similar to Mikaelian's process involving quarks and a W. If we consider that the ρ meson has an anomalous moment $k=1$, or introduces the electromagnetic interaction as in the Weinberg-Salam model, the above processes can be factorized. Because of the integral charges the zero direction necessarily corresponds to forward or backward scattering.

(4) Scattering processes

$$\gamma e \to W^-\nu_e\,,\quad \gamma p \to \rho^+ n\,,\quad \gamma K^+ \to \rho^+ K^0\,, \tag{51}$$

have zero at

$$\theta_0^* = \pi\,.$$

These processes are the crossed reactions of the previous set. The argument is therefore the same. Indeed for $\gamma p \to \rho^+ n$ we have

$$\overline{\overline{\sum}}\,|m|^2 = \frac{g_\rho^{\,2}e^2}{2}\,\frac{u^2}{t^2}\left(\frac{u^2+s^2+2m_\rho^{\,2}t}{-su}\right). \tag{52}$$

So $u=0$ or $\theta_0^* = \pi$ is a zero direction. Of course, we have to ignore all masses of p, K, e (high-energy approximation).

(5) Consider the process

$$\bar{p}n \to \pi^-\pi^0\,. \tag{53}$$

The Feynman diagrams which describe the process

$$N_i\bar{N}_j \to \pi^a\pi^b \tag{54}$$

are illustrated in Fig. 4. Different from other processes, this process has no gauge field outgoing, but it also can be factorized if we assume

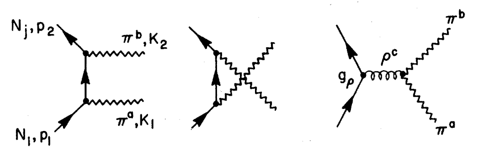


FIG. 4. Feynman diagrams for the reaction $N_i\bar{N}_j \to \pi^a\pi^b$.

$$g_\rho = 2g_\pi\,,\quad m_\rho^{\,2} = 2m_\pi^{\,2}\,. \tag{55}$$

The amplitude is

$$m(N_i\bar{N}_j \to \pi^a\pi^b) = ig_\pi^{\,2}\,\frac{\tilde{t}G_u + \tilde{u}G_t}{-\tilde{s}}\,m^A\,. \tag{56}$$

Here $\tilde{s} = s - m_\rho^{\,2} = (p_1+p_2)^2 - m_\rho^{\,2}$. From the condition of Eq. (54) we predict a zero direction at

$$\theta_0^* = \frac{\pi}{2} \tag{57}$$

for the process $\bar{p}n \to \pi^-\pi^0$.

If the group factors in process (53) satisfy

$$G_u = -G_t\,, \tag{58}$$

(e.g., $\bar{p}n \to \pi^-\pi^0$) the process has another kind of zero, provided that

$$m_\rho^{\,2} = 2m_\pi^{\,2}\,. \tag{59}$$

Indeed for $\bar{p}n \to \pi^-\pi^0$ we have

$$\overline{\overline{\sum}}\,|m(\bar{p}n \to \pi^-\pi^0)|^2 = 8g^2G_t^{\,2}\,\frac{[(\tilde{u}-m_\pi^{\,2})(\tilde{t}-m_\pi^{\,2}) - m_\pi^{\,2}s]\tilde{t}^2}{(\tilde{s}\tilde{u})^2} \times \left[\left(1+\frac{\tilde{u}}{\tilde{t}}\right)^2 - \eta^2\frac{\tilde{u}}{\tilde{t}}\right]^2. \tag{60}$$

Here

$$\eta \equiv \frac{g_\rho}{g_\pi}\,. \tag{61}$$

We can see that there are zeros when

$$\frac{u}{t} = \left(\frac{\eta^2}{2} - 1\right) \pm \frac{\eta}{2}(\eta^2 - 4)^{1/2} \tag{62}$$

and

$$\eta \geq 2\,. \tag{63}$$

The case $\eta = 2$ is like the one discussed above. It is interesting to note that we have two zeros when $\eta > 2$, and these two angles are mutually complementary, $\theta_1^* = \pi - \theta_2^*$, as we have mentioned in Sec. III. In the limit $\eta \to \infty$, these directions become collinear—forward and backward.

V. REMARKS

Non-Abelian gauge theories have many attractive features. Some of them are universal; some of them may be specific to certain processes. We think that factorization and the existence of zeros in some annihilation or scattering processes belong to the latter. We have mentioned a few candidate processes to check the theory. They are not easy to explore experimentally because quarks, gluons, and W bosons have not been observed and the ρ meson is unstable. In addition, we only calculated in the tree approximation and our tests require high energies. We therefore emphasize the theoretical rather than the experimental significance of our work. To study more processes, perhaps in higher order, could possibly deepen our understanding of non-Abelian gauge theories.

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