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Anomalous magnetic moment and limits on fermion substructure

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Experimental constraints on possible lepton and quark substructure are analyzed and expressed in terms of a general formalism for describing composite particles in terms of their constituents. In particular, the measured gyromagnetic ratios may very severely restrict possible internal structure of light leptons (electrons and muons) in some models. Simple expressions for hadronic g values and electromagnetic radii are given in terms of their quark-gluon infinite-momentum-frame wave function. The contribution to the anomalous moment of a fermion due to internal structure is shown to vanish as the mass or inverse-size scale of the internal state becomes infinitely large.

I. INTRODUCTION

Quarks and leptons are presently viewed as pointlike constituents of matter. Direct tests of quantum electrodynamics in high-energy electron-positron collisions at center-of-mass energies up to 32 GeV have confirmed the absence of lepton structure in processes probing distances as small as 2×10^{-16} cm.[1] The behavior of large-momentum-transfer lepton-hadron interactions is also consistent with the interpretation that pointlike quark constituents, as analyzed in perturbative quantum chromodynamics, are the local carriers of the weak and electromagnetic currents within hadrons. However, as the number of generations of quarks and leptons grow, and as the mass ratios between the different generations increases to very large values, for example, $m_\tau/m_e \sim 3600$, the postulate that the quarks and leptons themselves may be composites of a smaller number of more fundamental units becomes theoretically more appealing.[2] Indeed, it would be very attractive on fundamental theoretical grounds to unify quarks with leptons in terms of a small number of common constituents.

In this paper we will be concerned with experimental constraints on lepton and quark substructure, which we will express in terms of a general formalism for describing composite particles. The higher-energy accelerators and storage rings now being built or planned will permit experiments which can probe for evidence of structure at momentum transfers up to $\sim 10^3$ GeV, corresponding to a resolution scale of $\sim 10^{-17}$ cm. However, as we shall show here, the very (almost incredibly) precise measurements of the electron and muon gyromagnetic ratios g_e and g_μ put exceedingly restrictive limits on the possibility of lepton internal structure. The critical point is that the lepton g values are very close to the Dirac value of 2—and there is no *a priori* reason for $g \sim 2$ in the case of composite fermions.[3] The relationship of the anomalous magnetic moment $a = \frac{1}{2}(g-2) = F_2(0)$ of a fermion to its general relativistic composite structure will be discussed in detail in Sec. III.

If the electron or muon is in fact a composite system, it is very different from the familiar picture of a bound state formed of elementary constituents since it must be simultaneously light in mass and small in spatial extension. For a typical nonrelativistic system such as an atom or nucleus, the size R is given roughly by $R \sim (ME_B)^{-1/2} > M^{-1}$, where M is the mass and $E_B < M$ is the binding energy. A simple bag model for nucleons built of elementary quarks leads to a size $R \sim M^{-1}$. However, for the electron we know that the intrinsic size of any constituent structure is limited by $R \lesssim 10^{-16}$ cm, which is much less than its Compton wavelength $m_e^{-1} \sim 4\times 10^{-11}$ cm.

It is a special challenge for a composite model of the electron or muon (and presumably for the quarks, too) to explain why its mass is so light on the scale of its size $1/R \gtrsim 100$ GeV. It is natural to look for a chiral symmetry in the underlying dynamics in order to explain the occurrence of massless fermions or the suppresssion of contributions to their self-energies. As we will see, such dynamical symmetries can have a major effect on the predicted value of the magnetic moment of a composite fermion.

It is simple to think of a fermion as having a very small spatial extension because it is a very tightly bound structure of internal constituents of a much larger mass $m^* \gg m_l$. Let us ignore for the moment the possibility of cancellations or suppression factors due to symmetries in the underlying dynamics that might account for the very small mass m_l of the composite lepton itself. In this case we find that the contribution to the anomalous moment is linear in the mass ratio[4]

$$\delta a \sim O\left(\frac{m_l}{m^*}\right) . \tag{1}$$

This result reflects the fact that the natural scale for the magnetic moment μ is eR, where $R \sim 1/m^*$ is the scale size of the system.[5] In contrast, a quadratic dependence on $R^2 \sim (1/m^*)^2$ is characteristic of vacuum-polarization corrections.

To explore the significance of (1) consider the agreement between theory and experiment for the electron's $g-2$ value. The most precise published experimental value for the anomalous magnetic moment of the electron is[6]

$$a_{e^-}^{\exp} = 1\,159\,652\,200(40) \times 10^{-12} .$$

The prediction of quantum electrodynamics through order $(\alpha/\pi)^3$, including uncertainties in the value of the fine-structure constant and of the numerical integration of the α^3 contributions, together with small, weak, and hadronic corrections, is[7]

$$a_e^{\mathrm{QED}} = 1\,159\,652\,570(150) \times 10^{-12} .$$

Aside from possible eighth-order contributions now under study,[8] the possible extra contribution from an electron internal structure is thus limited to

$$a_e^{\mathrm{QED}} - a_{e^-}^{\exp} = (370 \pm 155) \times 10^{-12} ,$$

i.e.,

$$|\delta a_e| \lesssim 5 \times 10^{-10} .$$

If we assume the linear parametrization of Eq. (1), and define

$$|\delta a_e| = \frac{m_e}{m^*} \equiv m_e R_e , \tag{2}$$

we find

$$m^* \geq 10^6 \text{ GeV} = 10^3 \text{ TeV} ,$$

$$R_e \leq 2 \times 10^{-20} \text{ cm} .$$

This bound is almost four orders of magnitude smaller than the present high-energy limit. Thus, paradoxically, one of the lowest-energy experiments[6] in physics yields the highest-energy bound on elementary-particle substructure. For the muon the bound is comparable, since[9]

$$-20 \times 10^{-9} < a_\mu^{\exp} - a_\mu^{\mathrm{th}} < 26 \times 10^{-9}. \quad (95\% \text{ conf.})$$

This implies by Eq. (2) that

$$m^* \gtrsim 2 \times 10^6 \text{ GeV} ,$$

$$R_\mu \lesssim 10^{-20} \text{ cm} .$$

It should be emphasized that any model of heavy-fermion constituents which leads to Eq. (1) and the above estimates for δa would be expected, on dimensional grounds, to lead to a large first-order contribution to the fermion self-energy; i.e.,

$$\delta m_l \sim O(m^*) . \tag{3}$$

However, the observed lepton masses are very small, effectively vanishing on the scale $m^* \gg m$. Hence we have two choices: Either (3) must be cancelled by a large bare mass—or, more naturally perhaps, (3) itself must be suppressed, either by a chiral symmetry, or another special selection rule of the theory. From this point of view, the challenge of building a composite model of leptons and quarks is to make the contributions to both δa and to δm simultaneously very small.

The simplest possibility[10] for accomplishing this is to introduce a second, and still larger, mass scale, and describe the leptons as bound states of a fermion of mass m_f and a much heavier boson of mass λ; the boson may itself be nothing but a massive state of two bound leptons. In this case $(m_f^2 \ll \lambda^2)$ (see Secs. II and III)[11]

$$\delta a \sim O\left(\frac{m_l m_f}{\lambda^2}\right) . \tag{4}$$

The resulting bound $\lambda \gtrsim (m_f/\lambda) \times 10^6$ GeV for composite electrons is clearly not very restrictive for $m_f^2/\lambda^2 \ll 1$. Choosing the fermion mass m_f small in this model also implies that the lepton mass can be kept small.

A more natural possibility, which we discuss further in Sec. II, is to design the couplings so that both left- and right-handed constituent fermions of large mass m^* appear with equal weight in the state wave function of the composite lepton. This is a chirally invariant model with the property that the symmetry of amplitudes under the transformation $m^* \to -m^*$ removes the linear dependence of Eq. (1); thus we can obtain a small effect, $\delta a \sim (m_l^2/m^{*2})$. Also, the perturbative contribution to the lepton mass vanishes in such a chiral model. The chiral symmetry of such a model requires an effective doubling of the number of constituents and leads to as yet unobserved leptonic states of anomalous parity.

In the following section we consider some very elementary models of composite leptons in order to illustrate the dynamical effects which control the anomalous moment. In Sec. III we give a general analysis of composite system which shows that the above estimates for δa are applicable to the extent that there are specific spin states of the constituents which can couple to leptons with both $S_z = +\frac{1}{2}$ and $S_z = -\frac{1}{2}$. We also show in the Appendix how the sum rule[4] which relates the square of the lepton anomalous moment to polarized photoabsorption cross sections leads to complementary constraints on lepton compositeness.

The message of Eq. (1) is that one proceeds at peril when introducting lepton structure on a mass scale lower than 10^3 TeV. Indeed, if (1) is applicable, it leads to the conclusion that at least for the foreseeable generation of accelerators, which will reach into the ~1 TeV energy range, electrons and muons will behave as elementary point particles. In the following we will explain the basis for Eqs. (1) and (4), which leads to this conclusion.

II. MODELS OF LEPTON SUBSTRUCTURE

We consider first a simple prototype model for a composite lepton—the two-particle system represented in Fig. 1(a), where m_f is the mass of a heavy fermion ($m_f \gg m_l$) which carries the lepton charge and λ is the mass of a heavy boson constituent which we take as vector or pseudoscalar. In particular, this λ boson may be viewed as the bound state formed of two heavy fermions of mass m_f. In this simple model we shall assume a vertex function with simple Dirac structure $\phi(k)\gamma^\alpha$ or $\phi(k)\gamma^5$. In order to insure finite wavefunction normalization, we also assume that the square of the vertex function falls off as some arbitrary power of the boson propagator:

$$\phi^2(k) \propto g_0{}^2\left[\frac{\lambda^2}{\lambda^2-(p-k)^2}\right]^\delta . \quad \delta>0 .$$

We then fix $g_0{}^2$ to normalize the total charge of the bound state to e.

The standard calculation of the lepton vertex

$$\mathfrak{M}_\mu = \bar{u}(p+q)\left\{\gamma_\mu F_1(q^2) + \frac{1}{4m_l}[\not{q},\gamma_\mu]F_2(q^2)\right\}u(p)$$

from Fig. 1(b) then gives integrals of the form

$$F_i(q^2=0) \propto g_0{}^2\int_0^1 dz(1-z)z^\delta \int_0^\infty d\kappa^2\kappa^2 \frac{(\lambda^2)^\delta}{[\kappa^2+c(z)]^{3+\delta}} N_i ,$$

where, in the limit $(m_l/m_f)\to 0$,

$$c(z) = z\lambda^2 + (1-z)m_f{}^2 ,$$

$$N_1 = m_f{}^2 + \tfrac{1}{2}\kappa^2 , \quad N_2 = 4m_l m_f z \quad \text{(vector)} ,$$

$$N_1 = m_f{}^2 + \tfrac{1}{2}\kappa^2 , \quad N_2 = -2m_l m_f(1-z) \quad \text{(pseudoscalar)} .$$

Thus we immediately have

$$a = \frac{F_2(0)}{F_1(0)} \propto \left(\frac{m_l m_f}{m_f{}^2+\bar{\kappa}^2}\right) , \tag{5}$$

where $\bar{\kappa}^2 = (1/\delta)c(\bar{z})$ is the mean value of the intrinsic momentum. Equation (5) indicates the linear relation as in (1) for a massive internal fermion. For example, for $\delta=1$ and $\lambda^2=m_f{}^2$, $a=m_l/m_f$ for the vector case and $a=-\frac{1}{2}m_l/m_f$ for the pseudoscalar. Note that for very large internal momenta, or for a very massive boson, such that $\bar{\kappa}^2 > m_f{}^2$ the anomaly a vanishes quadratically rather than linearly, as in Eq. (4). Similar results are obtained if the boson has nonzero charge.

Let us next enhance this prototype model by including two equal amplitudes in the lepton wave function, one containing a meson of mass m_f produced in a state of positive chirality $(1+\gamma_5)u(k)$ and the other with negative chirality $(1-\gamma_5)u(k)$ as illustrated by the graphs of Fig. 2. Since, in this model, the transformation $m_f \to -m_f$ is an invariance operation, the numerators N_2 in Eq. (5) vanish and there is no contribution to a that is linear in m_l/m^*. The absence of linear mass terms in such a model also implies that the lepton bound state will be massless.

III. THE FORM FACTORS OF GENERAL COMPOSITE SYSTEMS

In order to analyze the consequences of lepton substructure in greater generality, we will describe the lepton wave function and its electromagnetic form factors using the light-cone (infinite-momentum-frame) Fock-space description.[12,13] We choose light-cone coordinates with the incident lepton directed along the z direction ($p^\pm \equiv p^0 \pm p^3$) (see Ref. 14):

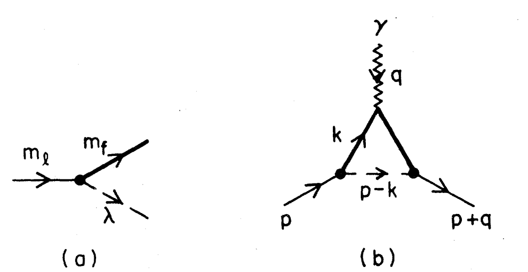


FIG. 1. (a) Simple composite model for lepton with a charged fermion and neutral boson constituent structure. (b) Calculation of electromagnetic form factors.

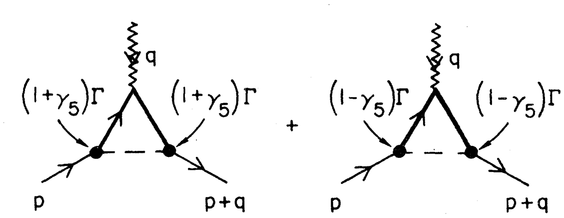


FIG. 2. Chiral model for lepton constituent structure. The cancelling contributions of the left-handed and right-handed fermion constituents eliminates anomalous moment contributions linear in the internal fermion mass.

$$p^\mu \equiv (p^+, p^-, \vec{p}_\perp) = \left(p^+, \frac{M^2}{p^+}, \vec{0}_\perp\right),$$
$$q = \left(0, \frac{2q\cdot p}{p^+}, \vec{q}_\perp\right), \tag{6}$$

where $q^2 = -2q\cdot p = -q_\perp^2$ and $M = m_l$ is the mass of the composite system. The Dirac and Pauli form factors can be identified[13] from the spin-conserving and spin-flip current matrix elements ($J^+ = J^0 + J^3$):

$$\mathfrak{M}^+_{\uparrow\uparrow} = \left\langle p+q, \uparrow \left| \frac{J^+(0)}{p^+} \right| p, \uparrow \right\rangle = 2F_1(q^2), \tag{7}$$

$$\mathfrak{M}^+_{\uparrow\downarrow} = \left\langle p+q, \uparrow \left| \frac{J^+(0)}{p^+} \right| p, \downarrow \right\rangle = -2(q_1 - iq_2)\frac{F_2(q^2)}{2M}, \tag{8}$$

where $\uparrow$ corresponds to positive spin projection $S_z = +\frac{1}{2}$ along the $\hat{z}$ axis.

Each Fock-state wave function $|n\rangle$ of the incident lepton is represented by the functions $\psi^{(n)}_{p,S_z}(x_i, \vec{k}_{\perp i}, S_i)$, where

$$k^\mu \equiv (k^+, k^-, \vec{k}_\perp) = \left(xp^+, \frac{k_\perp^2 + m^2}{xp^+}, \vec{k}_\perp\right)$$

specifies the light-cone momentum coordinates of each constituent $i = 1, \ldots, n$, and S_i specifies its spin projection S_z^i. Momentum conservation on the light cone requires

$$\sum_{i=1}^n k_{\perp i} = 0, \quad \sum_{i=1}^n x_i = 1,$$

and thus $0 < x_i < 1$. The amplitude to find n (on-mass-shell) constituents in the lepton is then $\psi^{(n)}$ multiplied by the spinor factors $u_{S_i}(k_i)/(k_i^+)^{1/2}$ or $v_{S_i}(k_i)/(k_i^+)^{1/2}$ for each constituent fermion or antifermion.[15] The Fock state is off the "energy shell":

$$\left(p^- - \sum_{i=1}^n k_i^-\right)p^+ = \sum_{i=1}^n \left(\frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right).$$

The quantity $(\vec{k}_{\perp i}^{\,2} + m_i^2)/x_i$ is the relativistic analog of the kinetic energy $\vec{p}_i^{\,2}/2m_i$ in the Schrödinger formalism.

The wave function for the lepton directed along the final direction $p+q$ in the current matrix element is then

$$\psi^{(n)}_{p+q,S_z'}(x_i, \vec{k}'_{\perp i}, S_i'),$$

where [see Fig. 3(a)][16]

$$\vec{k}'_{\perp j} = \vec{k}_{\perp j} + (1 - x_j)\vec{q}_\perp$$

for the struck constituent and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$$

for each spectator ($i \neq j$). The $\vec{k}'_\perp$ are transverse to the $p+q$ direction with

$$\sum_{i=1}^n \vec{k}'_{\perp i} = 0.$$

The interaction of the current $J^+(0)$ conserves the spin projection of the struck constituent fermion $(\bar{u}_{S'}\gamma^+ u_S)/k_+ = 2\delta_{SS'}$. Thus, from Eqs. (7) and (8)

$$F_1(q^2) = \tfrac{1}{2}\mathfrak{M}^+_{\uparrow\uparrow} = \sum_j e_j \int [dx][d^2\vec{k}_\perp]\psi^{*(n)}_{p+q,\uparrow}(x, \vec{k}'_\perp, S)\psi^{(n)}_{p,\uparrow}(x, \vec{k}_\perp, S), \tag{9}$$

and

$$-\left(\frac{q_1 - iq_2}{2M}\right)F_2(q^2) = \tfrac{1}{2}\mathfrak{M}^+_{\uparrow\downarrow} = \sum_j e_j \int [dx][d^2\vec{k}_\perp]\psi^{*(n)}_{p+q,\uparrow}(x, \vec{k}'_\perp, S)\psi^{(n)}_{p,\downarrow}(x, \vec{k}_\perp, S), \tag{10}$$

where e_j is the fractional charge of each constituent. [A summation of all possible Fock states (n) and spins (S) is assumed.] The phase-space integration is

$$[dx] \equiv \delta\left(1 - \sum x_i\right)\prod_{i=1}^n dx_i, \tag{11}$$

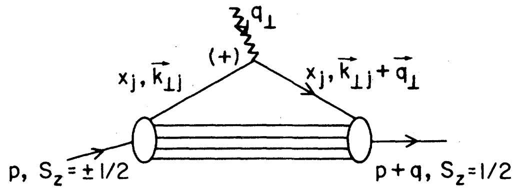
and

$$[d^2k_\perp] \equiv 16\pi^3\delta^{(2)}\left(\sum k_{\perp i}\right)\prod_{i=1}^n \frac{d^2k_\perp}{16\pi^3}. \tag{12}$$

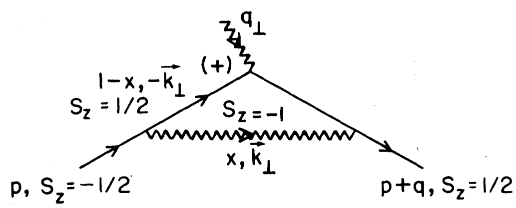
Equation (9) evaluated at $q^2 = 0$ with $F_1(0) = 1$ is equivalent to wave-function normalization. The anomalous moment $a = F_2(0)/F_1(0)$ can be determined from the coefficient linear in $q_1 - iq_2$ from the coefficient linear in $q_1 - iq_2$ from ψ^*_{p+q} in Eq. (10). In fact, since[17]

$$\frac{\partial}{\partial\vec{q}_\perp}\psi^*_{p+q} \equiv -\sum_{i\neq j} x_i \frac{\partial}{\partial\vec{k}_{\perp i}}\psi^*_{p+q} \tag{13}$$

(summed over spectators), we can, after integration by parts, write explicitly



(a)



(b)

FIG. 3. (a) Calculation of the electromagnetic vertex for a general composite system in light-cone (infinite-momentum frame) perturbation theory. (b) Calculation of the $\alpha/2\pi$ contribution to the electron anomalous moment in light-cone perturbation theory.

$$\frac{a}{M}=-\sum_j e_j\int[dx]\int[d^2k_\perp]\times\sum_{i\neq j}\psi^*_{p\uparrow}x_i\left(\frac{\partial}{\partial k_{1i}}+i\frac{\partial}{\partial k_{2i}}\right)\psi_{p\downarrow}\,. \tag{14}$$

The wave-function normalization is

$$\int[dx]\int[d^2k_\perp]\psi^*_{p\uparrow}\psi_{p\uparrow}=\int[dx]\int d^2k_\perp\psi^*_{p\downarrow}\psi_{p\downarrow}=1\,. \tag{15}$$

A sum over all contributing Fock states is assumed in Eqs. (14) and (15).

We thus can express the anomalous moment in terms of a local matrix element at zero momentum transfer. It should be emphasized that Eq. (14) is exact; it is valid for the anomalous moment of any spin-$\frac{1}{2}$ system.

As an example, in the case of the electron's anomalous moment to order α in QED,[18] the contributing intermediate Fock states [see Fig. 3(b)] are the electron-photon states with spins $|-\frac{1}{2},1\rangle$ and $|\frac{1}{2},-1\rangle$. The wave functions are ($\vec{k}_\perp$ and x are the momentum coordinates of the photon):

$$\psi_{p\downarrow}=\frac{e/\sqrt{x}}{M^2-(k_\perp^{\,2}+\lambda^2)/x-(k_\perp^{\,2}+\hat{m}^2)/(1-x)}\times\begin{cases}\sqrt{2}\,\dfrac{(k_1-ik_2)}{x} & (|-\tfrac{1}{2}\rangle\to|-\tfrac{1}{2},1\rangle)\\[2ex] \sqrt{2}\,\dfrac{M(1-x)-\hat{m}}{1-x} & (|-\tfrac{1}{2}\rangle\to|\tfrac{1}{2},-1\rangle)\end{cases} \tag{16}$$

and

$$\psi^*_{p\uparrow}=\frac{e/\sqrt{x}}{M^2-(k_\perp^{\,2}+\lambda^2)/x-(k_\perp^{\,2}+\hat{m}^2)/(1-x)}\times\begin{cases}-\sqrt{2}\,\dfrac{M(1-x)-\hat{m}}{1-x} & (|-\tfrac{1}{2},1\rangle\to|\tfrac{1}{2}\rangle)\\[2ex] -\sqrt{2}\,\dfrac{(k_1-ik_2)}{x} & (|\tfrac{1}{2},-1\rangle\to|\tfrac{1}{2}\rangle)\,.\end{cases} \tag{17}$$

The quantities to the left of the curly bracket in Eqs. (16) and (17) are the matrix elements of

$$\frac{\overline{u}}{(p^+-k^+)^{1/2}}\,\gamma\cdot\epsilon^*\,\frac{u}{(p^+)^{1/2}}\quad\text{and}$$

$$\frac{\overline{u}}{(p^+)^{1/2}}\,\gamma\cdot\epsilon\,\frac{u}{(p^+-k^+)^{1/2}}\,,$$

respectively, where $\hat{\epsilon}=\hat{\epsilon}_{\uparrow(\downarrow)}=\pm(1/\sqrt{2})(\hat{x}\pm i\hat{y})$, $\epsilon\cdot k=0$, $\epsilon^+=0$ in the light-cone gauge for vector spin projection $S_z=\pm1$ (see Refs. 12 and 13). For the sake of generality, we let the intermediate lepton and vector boson have mass $\hat{m}$ and λ, respectively.

Substituting (16) and (17) into Eq. (14), one finds that only the $|-\frac{1}{2},1\rangle$ intermediate state actually contributes to a, since terms which involve differentiation of the denominator of $\psi_{p\downarrow}$ cancel. We thus have

$$a=4Me^2\int\frac{d^2k_\perp}{16\pi^3}\int_0^1 dx\,\frac{[\hat{m}-(1-x)M]/x(1-x)}{[M^2-(k_\perp^{\,2}+\hat{m}^2)/(1-x)-(k_\perp^{\,2}+\lambda^2)/x]^2}\,, \tag{18}$$

or

$$a=\frac{\alpha}{\pi}\int_0^1 dx\,\frac{M[\hat{m}-M(1-x)]x(1-x)}{\hat{m}^2x+\lambda^2(1-x)-M^2x(1-x)}\,, \tag{19}$$

which, in the case of QED ($\hat{m}=M$, $\lambda=0$) gives the Schwinger result $a=\alpha/2\pi$.

The general result (14) can also be written in matrix form:

$$\frac{a}{2M}=-\sum_j e_j\int[dx][d^2k_\perp]\psi^*\vec{S}_\perp\cdot\vec{L}_\perp\psi\,, \tag{20}$$

where S is the spin operator for the total system and $\vec{L}_\perp$ is the generator of "Galilean" transverse boosts[12,13] on the light cone, i.e., $\vec{S}_\perp\cdot\vec{L}_\perp=(S_+L_-+S_-L_+)/2$ where $S_\pm=(S_1\pm iS_2)$ is the spin-ladder operator and

$$L_\pm=\sum_{i\neq j}x_i\left(\frac{\partial}{\partial k_{1i}}\mp i\frac{\partial}{\partial k_{2i}}\right) \tag{21}$$

(summed over spectators) is the analog of the angular momentum operator $\vec{p}\times\vec{r}$. Equation (14) can also be written simply as an expectation value in impact space.

The results given in Eqs. (9), (10), and (14) may also be convenient for calculating the anomalous moments and form factors of hadrons in quantum chromodynamics directly from the quark and gluon wave functions $\psi(\vec{k}_{\perp i}, x, S)$. These wave functions can also be used to construct the structure functions and distribution amplitudes which control large momentum transfer inclusive and exclusive processes.[13,19] The charge radius of a composite system can also be written in the form of a local, forward matrix element[20]:

$$\left.\frac{\partial F_1(q^2)}{\partial q^2}\right|_{q^2=0} = -\sum_j e_j \int [dx][d^2k_\perp]\psi^*_{p,\uparrow} \times \left(\sum_{i\neq j} x_i \frac{\partial}{\partial \vec{k}_{\perp i}}\right)^2 \psi_{p,\uparrow}\,. \qquad (22)$$

We thus find that, in general, any Fock state $|n\rangle$ which couples to both $\psi^*_\uparrow$ and $\psi_\downarrow$ will give a contribution to the anomalous moment. Notice that because of rotational symmetry in the $\hat{x}, \hat{y}$ direction, the contribution to $a = F_2(0)$ in (14) always involves the form $(a, b = 1 \cdots n)$

$$M\psi^*_\uparrow \sum_{i\neq j} x_i \frac{\partial}{\partial k_{\perp i}} \psi_\downarrow \sim \mu M\, \rho(\vec{k}^a_\perp \cdot \vec{k}^b_\perp)\,, \qquad (23)$$

compared to the integral (15) for wave-function normalization which has terms of order

$$\psi^*_\uparrow \psi_\uparrow \sim \vec{k}^a_\perp \cdot \vec{k}^b_\perp \rho(\vec{k}^a_\perp \cdot \vec{k}^b_\perp)$$

and

$$\mu^2 \rho(\vec{k}^a_\perp \cdot k^b_\perp)\,. \qquad (24)$$

Here ρ is a rotationally invariant function of the transverse momenta and μ is a constant with dimensions of mass. Thus, in order of magnitude

$$a = O\left(\frac{\mu M}{\mu^2 + \langle \vec{k}_\perp^{\,2}\rangle}\right) \qquad (25)$$

summed and weighted over the Fock states. In the case of a renormalizable theory, the only parameters μ with the dimension of mass are fermion masses. In super-renormalizable theories, μ can be proportional to a coupling constant g with dimension of mass.[21]

In the case where all the mass-scale parameters of the composite state are of the same order of magnitude, we obtain $a = O(MR)$ as in Eqs. (11) and (12), where $R = \langle k_\perp^2\rangle^{-1/2}$ is the characteristic size[22] of the Fock state. On the other hand, in theories where $\mu^2 \ll \langle k_\perp^2\rangle$, we obtain the quadratic relation $a = O(\mu M R^2)$ as in Eq. (4).

Thus composite models for leptons can avoid conflict with the high-precision measurements in several ways.

(a) There can be strong cancellations between the contributions of different Fock states. An example of this is the chiral model of Sec. II.

(b) The parameter μ can be minimized. For example, in a renormalizable theory this can be accomplished by having the bound state of light fermions and heavy bosons. Since $\mu \geq M$, we then have $a \geq O(M^2R^2)$.

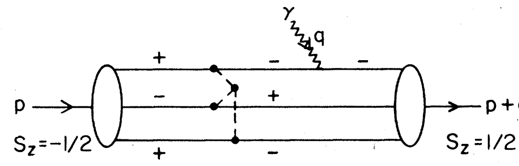


FIG. 4. Example of a contribution to the anomalous moment of the nucleon in the quark model if a $g\phi^3$ coupling of scalars is present. The ± indicate the spin projection S_z of the quarks.

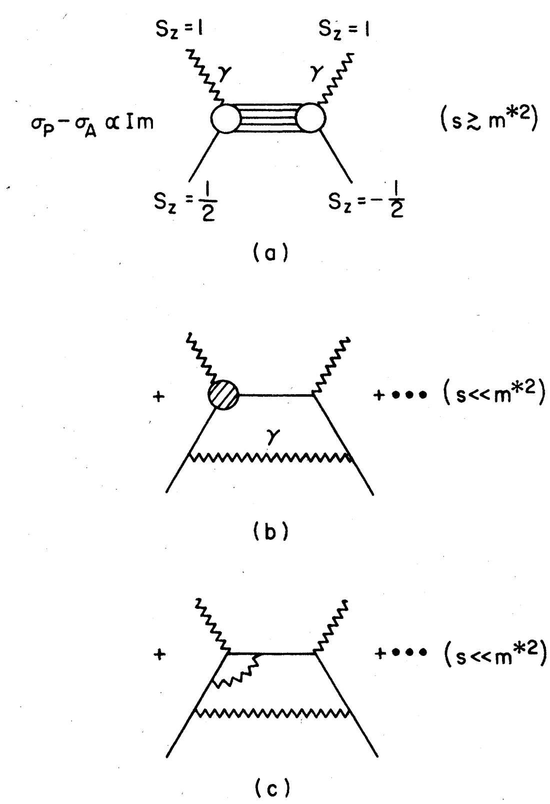


FIG. 5. Calculation of the anomalous moment squared $(\delta a)^2$ from the DHG sum rule. (a) Contribution $(\delta a^{\text{non-QED}})^2$ from internal structure $s \gg m^{*2}$: $\Delta\sigma \sim O((\pi\alpha)\pi/m^{*2})$. (b) Interference contribution $(2\,\delta a^{\text{non-QED}}\,\delta a^{\text{QED}})$ due to internal-structure corrections to the QED calculation. (c) QED contribution $(\delta a^{\text{QED}})^2$ from $\Delta\sigma \sim (\pi\alpha^2/s)(\alpha/2\pi)$.

(c) If the parameter μ is of the same order as the other mass scales in the composite state, then we have the linear condition $a = O(MR)$ and the strong constraints of Sec. I must be satisfied.

IV. CONCLUSION

We have seen that the $g-2$ value poses a constraint on the form of possible models of composite structure for leptons and quarks. In particular, the contribution of a massive charged constituent with spin $\frac{1}{2}$ will be of order (m_l/m^*) unless suppressed by a selection rule such as chiral invariance of the theory or by a large ratio of constituent boson to fermion masses.[10] In each case the self-energy corrections are also suppressed. For a chirally invariant theory there arises the problem of parity doubling of the leptons. Other possible models are considered in Ref. 2. The simplest alternative may be that the leptons are in fact pointlike "elementary particles."

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APPENDIX: SUM-RULE ANALYSIS OF ANOMALOUS MOMENTS

An alternative, but equivalent formulation of the analysis of a particle's anomalous moment (for any spin) can be based on the Drell-Hearn-Gerasimov (DHG) sum rule.[23] For spin-$\frac{1}{2}$ systems,

$$a^2 = \frac{M^2}{2\pi^2\alpha}\int_{s_{\rm th}}^{\infty}\frac{ds}{s}[\sigma_P(s) - \sigma_A(s)]\,, \tag{A1}$$

where $\sigma_{P(A)}$ is the total photoabsorption cross section with parallel (antiparallel) photon and target spins. This sum rule follows from the low energy theorem and the existence of an unsubtracted dispersion relation for the forward spin-flip Compton amplitude. If the lepton has a substructure at short distances, then there will be new resonance or continuum contributions to σ_P and σ_A beyond a new threshold $s_{\rm th} = m^{*2}$ associated with the mass scale of this substructure. Barring special cancellations, we thus have

$$\sigma_P - \sigma_A \sim e^2\frac{\pi}{m^{*2}}f(m^{*2}/s)\,.$$

The contribution to the sum rule from the region $s \gtrsim m^{*2}$ then yields a contribution to the anomalous moment $(\delta a^{\text{non-QED}})^2 \sim (M^2/m^{*2})$ in agreement with Eq. (1). Notice that the contribution to $\sigma_P - \sigma_A$ from the lepton and photon final states at $s \ll m^{*2}$ yield the standard contribution[23] $(\delta a^{\rm QED})^2 = (\alpha/2\pi + \cdots)^2$. In addition, as illustrated in Fig. 5, the interference between QED and non-QED amplitudes yield the expected $2(\delta a^{\rm QED})(\delta a^{\text{non-QED}})$ contributions. Thus the QED and composite structure contributions to the anomalous moment are additive.

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[1] A summary of recent results from the PETRA storage ring is given in G. Wolf, Report No. DESY 80/13, 1980 (unpublished).

[2] Models of composite leptons and/or quarks include the following: J. G. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); J. C. Pati, A. Salam, and J. Strathdee, Phys. Lett. 58B, 265 (1975); H. Terazawa, Phys. Rev. D 22, 184 (1980); O. W. Greenberg and C. A. Nelson, Phys. Rev. D 10, 2567 (1974); O. W. Greenberg, Phys. Rev. Lett. 35, 1120 (1975); J. D. Bjorken (unpublished); G. R. Kalbfleish and B. C. Fowler, Nuovo Cimento 19A, 173 (1974); E. Novak, J. Sucher, and C. H. Woo, Phys. Rev. D 16, 2874 (1977); M. Veltman, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energy, Fermilab*, edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1979); G. 't Hooft, Lecture given at the Cargese Summer Institute, 1979 (unpublished); H. Harari, Phys. Lett. 86B, 83 (1979); M. A. Shupe, *ibid.* 86B, 87 (1979); J. G. Taylor, *ibid.* 88B, 291 (1979); R. Raitio, Helsinki Report No. HU-TFT-79-39, 1979 (unpublished); V. Visnjic-Triantafillou, Fermilab Report No. Pub-80/15, 1980 (unpublished); E. Derman, University of Colorado Report No. COLO-HEP-19, 1980 (unpublished); J. G. Pati, University of Maryland report, 1980 (unpublished); S. Dimopoulos, S. Raby, and L. Susskind, Stanford University Report No. ITP-662, 1980 (unpublished).

[3] However, we prove here the theorem: if m^* (the mass or inverse-size scale of the internal state) $\to \infty$, the internal structure contribution to the anomalous moment $a = \frac{1}{2}(g-2) \to 0$. Notice that this is contrary to nonrelativistic additivity $\vec{\mu} = \sum_{i=1}^{n}\vec{\mu}_i$, which would predict $g \to 0$. For a discussion of nonrelativistic models, see H. J. Lipkin, Fermilab Report Phys. Lett. 89B, 358 (1980); M. Glück, *ibid.* 87B, 247 (1979).

[4] A derivation of Eq. (1) from the point of view of the Drell-Hearn-Gerasimov sum rule (S. D. Drell and A. C. Hearn Phys. Rev. Lett. 16, 908 (1966); S. B. Gerasimov, Yad. Fiz. 2, 598 (1965) [Sov. Journal Nucl. Phys. 2, 430 (1966)]) was given in S. J. Brodsky, Report No. SLAC-PUB-1699, published in the LAMPF Users Group Newsletter, 8-1 (1976) (Los Alamos), and S. J. Brodsky and G. P. Lepage, Report No. SLAC-PUB-

1966, published in the *Proceedings of the 4th International Colloquium on Advanced Computing Methods in Theoretical Physics, Saint-Maximin, France, 1977*, edited by A. Visconti (CNRS, Marseille, 1977). This result has also been recently derived using sidewise dispersion relations by G. L. Shaw, D. Silverman, and R. Slansky, Los Alamos Report No. LA-UR-80-588, 1980 (unpublished).

[5]This linear dependence on the composite-system size is analogous to results for the nuclear-polarization contribution to the hyperfine splitting of hydrogen which is linear in the nuclear size. See C. Zemach, Phys. Rev. 104, 1771 (1956).

[6]R. S. Van Dyck, Jr., Bull. Am. Phys. Soc. 24, 758 (1979).

[7]For a recent review, see T. Kinoshita, Cornell Report No. CLNS-70/437, 1979 (unpublished).

[8]T. Kinoshita and W. B. Lindquist, Cornell Report Nos. CLNS-424 and CLNS-426, 1979 (unpublished).

[9]J. Bailey *et al.*, Nucl. Phys. B150, 1 (1979); F. J. M. Farley and E. Picasso, Ann. Rev. Nucl. Part. Sci. 29, 243 (1979).

[10]See also S. Dimopoulos *et al.*, Ref. 2. A discussion of the possibility of obtaining massless bound states in models with zero-mass fermion constituents is given in this reference.

[11]The form of this result agrees with the contribution $\delta a \sim O(G_F(m_l m_F))$ due to weak interactions which involve the couplings of the electron or muon to heavy fermions. See K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972); G. L. Shaw *et al.*, Ref. 4.

[12]See J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D 3, 1382 (1971), and references therein. See also Refs. 13–16 and 18 below.

[13]A summary of light-cone perturbation-theory calculation rules for gauge theories is given in G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980). We follow the notation of this reference here.

[14]This is the light-cone analog of the infinite-momentum frame introduced in S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. Lett. 22, 744 (1969). See also S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D 6, 177 (1972).

[15]The polarization of each vector-boson constituent is specified by the helicity index S_i in ψ as in Eq. (16). The spinor representations are given in Ref. 12.

[16]S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 181 (1970).

[17]We use momentum conservation to eliminate the dependence of ψ on $\vec{k}_{\perp j}$, where j is the struck quark. Note that the results (9), (10), and (14) are independent of the charge of the constituents.

[18]Related calculations in the infinite-momentum frame are given in S. J. Chang and S. K. Ma, Phys. Rev. 180, 1506 (1969); Bjorken *et al.*, Ref. 12; D. Foerster, Ph.D. thesis, University of Sussex, 1972; and S. J. Brodsky, R. Roskies, and R. Suaya, Phys. Rev. D 8, 4574 (1973). The infinite-momentum-frame calculation of the order-α^2 contribution to the anomalous moment of the electron is also given in the last reference.

[19]S. J. Brodsky, T. Huang, and G. P. Lepage, Report No. SLAC-PUB-2540 (unpublished).

[20]Parton-model expressions for other definitions of the charge radius are given in F. E. Close, F. Halzen, and D. M. Scott, Phys. Lett. 68B, 447 (1977).

[21]For example, the contribution of Fig. 4 to the nucleon anomalous moment in the quark model gives a contribution $\delta a \propto g M_N / \langle k_\perp^2 \rangle$ if there is a $g\phi^3$ trilinear coupling of scalars. It is thus possible to obtain a contribution to the anomalous moment of a fermion which is linear in its mass even if all of its constituent fermions are massless.

[22]A better estimate (see Ref. 19) is $R^2 = \langle S \rangle^{-1}$, where $S = \sum_{i=1}^{n} [(k_\perp^2 + m^2)/x]_i$.

[23]S. D. Drell and A. C. Hearn, Ref. 4; S. B. Gerasimov, Ref. 4. A discussion of the DHG sum rule for composite systems is given in S. J. Brodsky and J. R. Primack, Ann. Phys. (N.Y.) 52, 315 (1969). A calculation of δa^{QED} using the DHG sum rule is given in G. Altarelli, N. Cabbibo, and L. Maiani, Phys. Lett. 40B, 415 (1972).