Perturbative quantum-chromodynamic corrections to the hadronic decay width of the Higgs boson

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(Received 23 June 1980; revised manuscript received 22 July 1980)

Using the operator-product expansion it is shown that perturbative quantum-chromodynamic corrections to the hadronic decay width of the Higgs boson can be calculated without encountering mass singularities. The result is given in terms of the "running quark mass" of the renormalization group and calculable corrections in powers of $1/\ln(M^2/\Lambda^2)$. The next-to-leading-order correction amounts to about 35% for the Higgs-boson mass $M = 50$ GeV.

I. INTRODUCTION

The successful $SU(2) \times U(1)$ model for the weak and electromagnetic interactions[1] as well as other theoretical possibilities of unification make it urgent to explore Higgs bosons experimentally.[2] Recently Braaten and Leveille calculated gluon radiative corrections to the Higgs-boson decay into a quark-antiquark pair.[3] They found mass singularities which invalidate the perturbative calculation. By summing leading logarithms they obtained the decay rate formula in which the quark mass is replaced by the running quark mass evaluated at the Higgs-boson mass.

The purpose of this paper is to show that perturbative quantum-chromodynamics (QCD) corrections for the hadronic decay of the Higgs boson can be calculated unambiguously without encountering mass singularities. I will give a precise formulation in terms of the operator-product expansion and the renormalization-group equation, and present the result of the next-to-leading-order QCD correction. The correction is about 35% for a 50-GeV Higgs boson.

In the next section I formulate the renormalization-group equation for scalar-current correlation functions and present the leading- and the next-to-leading-order perturbative QCD results for the Higgs-boson decay width. Section III is devoted to a brief discussion.

II. RENORMALIZATION-GROUP EQUATION

The standard $SU(2) \times U(1)$ model[1] gives the interaction Lagrangian of the decay of the Higgs boson ϕ into a pair of quarks ψ as

$$L = -g_Y \bar{\psi}\psi\phi \equiv -g_Y J\phi\,, \tag{1}$$

where g_Y and J are the Yukawa coupling and the scalar current. The lowest-order decay width for each quark flavor of mass m is given by

$$\Gamma = \frac{3M}{8\pi} g_Y^2 \left(1 - \frac{4m^2}{M^2}\right)^{3/2}, \tag{2}$$

where M is the Higgs-boson mass and the color factor 3 is included. This formula can be obtained from the imaginary part of the quark loop diagram in Fig. 1, which is very similar to the well-known process $e^+e^- \to q\bar{q}$. One can easily recognize that perturbative QCD corrections to the hadronic decay of the Higgs boson can be treated almost analogously to the e^+e^- annihilation into hadrons using the operator-product expansion.[4-6]

Let us introduce the correlation function of scalar currents with the four-momentum q^μ,

$$\Pi = i\int d^4x\, e^{iqx}\langle 0|T(J(x)J(0))|0\rangle\,, \tag{3}$$

whose imaginary part gives the Higgs-boson decay width

$$\Gamma = \frac{g_Y^2}{M}\,\mathrm{Im}\Pi\Big|_{q^2=M^2}\,. \tag{4}$$

Similar to the vacuum polarization due to the electromagnetic current,[5,6] Π requires subtractive renormalization. It also needs multiplicative renormalization because the scalar current is not conserved in contrast to the electromagnetic current. The renormalized Π in $4-\epsilon$ dimensions is obtained from the bare one π_0,

$$\Pi(q^2, g, m, \mu) = \frac{Z_Y^2}{Z_2^2}\Pi_0(q^2, g_0, m_0, \epsilon) - q^2\mu^{-\epsilon}K(g,\epsilon) - m^2\mu^{-\epsilon}L(g,\epsilon)\,, \tag{5}$$

where μ is the renormalization mass scale and the subtractive counterterms are given as a sum of simple poles in ϵ in the minimal-subtraction scheme[7]

$$K(g,\epsilon) = \sum_{i=1}^{\infty} K^i(g)/\epsilon^i, \quad L(g,\epsilon) = \sum_{i=1}^{\infty} L^i(g)/\epsilon^i\,. \tag{6}$$

The renormalized QCD coupling constant g, quark mass m, quark field ψ, and Yukawa coupling g_Y

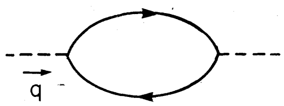

FIG. 1. The lowest-order diagram for the correlation function Π of scalar currents.

are related to bare ones (with subscript 0) by renormalization constants as

$$\mu^{\epsilon/2}g = Z_g^{-1}g_0\,, \quad m = Z_m^{-1}m_0\,, \tag{7}$$

$$\psi = \sqrt{Z_2}\psi_0, \quad \mu^{\epsilon/2}g_Y = g_{Y_0}Z_Y^{-1}Z_2\,. \tag{8}$$

Neglecting nonperturbative effects,[8] Π is nothing but the coefficient function of the unit operator in the operator-product expansion of the T product of scalar currents.

Following the standard procedure,[6] the renormalization-group equation for Π is obtained from the μ independence of Π_0 for fixed g_0, m_0, and ϵ:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} + 2\gamma\right)\Pi(q^2, g, m, \mu) = q^2\overline{K}(g) + m^2\overline{L}(g)\,, \tag{9}$$

where the β function and anomalous dimensions are given, as usual,

$$\beta = -g\mu\frac{\partial}{\partial\mu}\ln Z_g\,,$$
$$\gamma_m = -\mu\frac{\partial}{\partial\mu}\ln Z_m\,, \tag{10}$$
$$\gamma = -\mu\frac{\partial}{\partial\mu}\ln(Z_Y/Z_2)\,,$$

and inhomogeneous terms are given in terms of $1/\epsilon$ pole terms of the subtractive counterterms in Eq. (6),

$$\overline{K}(g) = \frac{\partial g^2K^1(g)}{\partial g^2}\,, \quad \overline{L}(g) = \frac{\partial g^2L^1(g)}{\partial g^2}\,. \tag{11}$$

Using dimensional analysis and the solution of the renormalization-group equation, Π at q^2 can be related to Π at $\hat{q}^2 \equiv q^2e^{-2t}$:

$$\Pi(\hat{q}^2e^{2t}, g, m, \mu) = e^{2t}\Pi(\hat{q}^2, g, me^{-t}, \mu e^{-t}) = e^{2t}\left\{\Pi(\hat{q}^2, \overline{g}(t), \overline{m}(t)e^{-t}, \mu)A(t) - \int_0^t dt'[\hat{q}^2\overline{K}(\overline{g}(t')) + \overline{m}^2(t')e^{-2t'}\overline{L}(\overline{g}(t'))]A(t')\right\}\,, \tag{12}$$

where

$$t = \tfrac{1}{2}\ln(q^2/\hat{q}^2)\,, \tag{13}$$

$$\frac{\partial\overline{g}(t)}{\partial t} = \beta(\overline{g}(t))\,, \quad \overline{g}(0) = g\,, \tag{14}$$

$$\frac{\partial\overline{m}(t)}{\partial t} = \overline{m}(t)\gamma_m(\overline{g}(t))\,, \quad \overline{m}(0) = m\,, \tag{15}$$

$$A(t) = \exp\left(2\int_0^t \gamma(\overline{g}(t'))dt'\right)\,. \tag{16}$$

The running quark mass $m(M)$ at M is defined by the solution of Eq. (15) for $t = \ln M/\mu$,

$$m(M) = \overline{m}(t = \ln M/\mu) = m\exp\left(\int_0^{\ln M/\mu}\gamma_m(\overline{g}(t'))dt'\right)\,. \tag{17}$$

Since I am interested in the lowest order in g_Y^2 but to all orders in g, I obtain

$$Z_Y/Z_2 = Z_m, \quad \gamma = \gamma_m\,. \tag{18}$$

To evaluate the asymptotic behavior of π, I use the perturbative expansion

$$\beta(\overline{g}) = -\frac{\overline{g}^3}{16\pi^2}\beta_0 - \frac{\overline{g}^5}{(16\pi^2)^2}\beta_1 + \cdots\,,$$
$$\gamma_m(\overline{g}) = \gamma(\overline{g}) = \frac{\overline{g}^2}{16\pi^2}\gamma_0 + \left(\frac{\overline{g}^2}{16\pi^2}\right)^2\gamma_1 + \cdots\,, \tag{19}$$
$$\overline{K}(\overline{g}) = K_0 + \frac{\overline{g}^2}{16\pi^2}K_1 + \cdots\,,$$
$$\overline{L}(g) = L_0 + \frac{\overline{g}^2}{16\pi^2}L_1 + \cdots\,.$$

Let us evaluate the asymptotic behavior for $q^2 \to -\infty$ fixing $\hat{q}^2 = -\mu^2$. Leading contributions come from the inhomogeneous terms K_0 and L_0 similar to the e^+e^- case,

$$\Pi(q^2, g, m, \mu) \to -q^2\left(\frac{\beta_0 g^2}{16\pi^2}2t\right)^{\gamma_0/\beta_0}\frac{K_0t}{1+\gamma_0/\beta_0} - m^2\left(\frac{\beta_0 g^2}{16\pi^2}2t\right)^{2\gamma_0/\beta_0}\frac{L_0t}{1+2\gamma_0/\beta_0}\,. \tag{20}$$

To obtain the Higgs-boson decay width I take the imaginary part of analytically continued Π at $q^2 = M^2$ (Higgs-boson mass),

$$\mathrm{Im}\Pi(q^2=M^2,g,m,\mu)=\left(\frac{\beta_0 g^2}{16\pi^2}\ln\frac{M^2}{\mu^2}\right)^{\gamma_0/\beta_0}\times\frac{\pi}{2}\left[M^2K_0+m^2\left(\frac{\beta_0 g^2}{16\pi^2}\ln\frac{M^2}{\mu^2}\right)^{\gamma_0/\beta_0}L_0\right]. \quad (21)$$

The γ_0/β_0 power of $\ln M^2/\mu^2$ can be absorbed into the running quark mass [Eq. (17)] which becomes, in the leading order,

$$m(M)\simeq m\left(\frac{\beta_0 g^2}{16\pi^2}\ln\frac{M^2}{\mu^2}\right)^{\gamma_0/2\beta_0}. \quad (22)$$

In fact the Yukawa coupling in Eq. (4) is given by the Fermi constant G_F and the quark mass in the standard SU(2) × U(1) model,[1]

$$g_Y=(\sqrt{2}G_F)^{1/2}m. \quad (23)$$

Inserting Eqs. (21)–(23) into Eq. (4) I obtain the Higgs-boson decay width in the leading order,

$$\Gamma=\frac{\pi K_0}{2}M\sqrt{2}G_F[m(M)]^2\left\{1+\frac{L_0}{K_0}\left[\frac{m(M)}{M}\right]^2\right\}=\frac{3M}{8\pi}\sqrt{2}G_F[m(M)]^2\left\{1-6\left[\frac{m(M)}{M}\right]^2\right\}, \quad (24)$$

where the leading inhomogeneous terms K_0 and L_0 are calculated from the quark-loop diagram in Fig. 1 using Eqs. (6), (11), and (19),

$$K_0=\frac{3}{4\pi^2},\qquad L_0=-\frac{9}{2\pi^2}. \quad (25)$$

The leading-order result in Eq. (24) is the same as the Born term Eq. (2) (without gluon corrections) except: (i) The quark mass m (of the Yukawa coupling) in the Born term is replaced by the running quark mass $m(M)$ evaluated at the Higgs-boson mass,[3] and (ii) the quark mass in the phase-space factor $(1-4m^2/M^2)^{3/2}$ is replaced by the running quark mass $m(M)$ and, up to the first term in the $[m(M)/M]^2$ power expansion, is retained. Terms of order $[m(M)/M]^{2n}$, $n\geq 2$ are contained in the first term $\Pi(\hat{q}^2,\bar{g}(t),\overline{m}(t)e^{-t},\mu)A(t)$ in Eq. (12) and should correspond to contributions from operators $m^{2n}\times 1$ in the operator-product expansion.

The renormalization-group analysis allows precise predictions to any desired order in the running coupling constant $\bar{g}$. As an example I will present the next-to-leading-order correction. The β function[9] and the anomalous dimension[10] γ_m in Eq. (19) have been calculated up to the next to leading order in the minimal-subtraction scheme

$$\beta_0=11-\tfrac{2}{3}N_f,\quad \beta_1=102-\tfrac{38}{3}N_f,\qquad \gamma_0=-8,\quad \gamma_1=-108+\tfrac{40}{9}N_f, \quad (26)$$

where N_f is the number of quark flavors. Two-loop diagrams in Fig. 2 are calculated to obtain

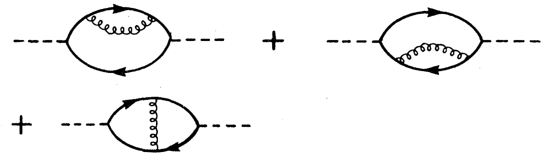

FIG. 2. The order-g^2 diagrams for the correlation function π of scalar currents.

the next-to-leading inhomogeneous term K_1 in Eq. (19) [for simplicity the mass in the loop is neglected, i.e., $\overline{L}(g)=0$],

$$K_1=5/\pi^2. \quad (27)$$

The finite (nonlogarithmic) part of Π is also needed:

$$\Pi(q^2=-\mu^2,g=0,m=0,\mu)\equiv(-\mu^2)K_0B,\qquad B=\begin{cases}\frac{1}{2}(\ln 4\pi-\gamma_E)+1, & \text{for MS scheme}\\ 1, & \text{for }\overline{\text{MS}}\text{ scheme}\end{cases} \quad (28)$$

where the $\ln 4\pi-\gamma_E$ term in the minimal-subtraction (MS) scheme is absorbed into the redefinition of the QCD scale parameter Λ in the modified minimal-subtraction ($\overline{\text{MS}}$) scheme of Ref. 11. Other quantities in Eqs. (26) and (27) are unchanged in the $\overline{\text{MS}}$ scheme. The solution (12) of the renormalization-group equation can now be expanded in powers of $(\ln M^2/\Lambda^2)^{-1}$, leading to the Higgs-boson decay width in the next to leading order,

$$\Gamma=\frac{3M}{8\pi}\sqrt{2}G_F[m(M)]^2\left[1+\frac{1}{\beta_0\ln\frac{M^2}{\Lambda^2}}\left(\frac{K_1}{K_0}-2\gamma_0B\right)\right]. \quad (29)$$

Taking Higgs-boson mass $M=50$ GeV, $N_f=6$, and $\Lambda=0.5$ GeV for the $\overline{\text{MS}}$ scheme, +35% correction (the second term in the large square brackets) is obtained. The running quark mass $m(M)$ defined in Eq. (17) can also be expanded in powers of $(\ln M^2/\Lambda^2)^{-1}$:

$$m(M)=\tilde{m}F(M), \quad (30)$$

$$F(M)=\left(\frac{M^2}{\Lambda^2}\right)^{\gamma_0/2\beta_0}\times\left[1+\frac{1}{\beta_0\ln\frac{M^2}{\Lambda^2}}\left(\frac{\gamma_0\beta_1}{2\beta_0^2}\ln\ln\frac{M^2}{\Lambda^2}+\frac{\gamma_0\beta_1-\gamma_1\beta_0}{2\beta_0^2}\right)\right], \quad (31)$$

$$\tilde{m}=m/F(\mu). \quad (32)$$

In this expansion I obtain the Higgs-boson decay width

$$\Gamma=\frac{3M}{8\pi}\sqrt{2}G_F\left[\tilde{m}\left(\ln\frac{M^2}{\Lambda^2}\right)^{\gamma_0/2\beta_0}\right]^2C, \quad (33)$$

$$C = 1 + \frac{1}{\beta_0 \ln\frac{M^2}{\Lambda^2}}\left(\frac{K_1}{K_0} - 2\gamma_0 B + \frac{\gamma_0\beta_1}{\beta_0^2}\ln\ln\frac{M^2}{\Lambda^2} + \frac{\gamma_0\beta_1-\gamma_1\beta_0}{\beta_0^2}\right).$$

The correction factor C becomes 1.32 for the same parameters ($M=50$ GeV, $N_f=6$, $\Lambda=0.5$ GeV, $\overline{\mathrm{MS}}$ scheme).

III. DISCUSSION

I have shown how to calculate perturbative QCD corrections to the Higgs decay without encountering mass singularities at all. In practical applications, however, there remain two problems: (i) Since the Higgs boson decays preferentially to heavy quark, it may be important to incorporate the phase-space kinematical factor $(1-4m^2/M^2)^{3/2}$ in Eq. (2), and (ii) the magnitude of quark masses is not accurately known.

As for point (i), I was able to reproduce explicitly the phase-space factor for the running quark mass up to the first term in the $[m(M)/M]^2$ power expansion. I expect that higher-order terms can also be reproduced by using the operator-product expansion. Therefore it seems most reasonable to use the phase-space factor $\{1-[2m(M)/M]^2\}$ for the running quark mass $m(M)$ together with the perturbative QCD correction factors such as Eq. (33).

As for point (ii), one should probably turn the argument around: Our formula can be employed to deduce from the Higgs-boson decay width the running quark mass defined in Eq. (17) which can be used, e.g., in the discussion of grand unified theories.[10,12]

One can treat Higgs bosons with γ_5 coupling or charged Higgs bosons analogously. One can calculate perturbative QCD corrections to inclusive hadron distributions from the Higgs decay using similar renormalization-group analyses with the cut-vertex formalism[13] instead of the operator-product expansion.

While this paper was being typed, I received a report by Inami and Kubota[14] which discussed the Higgs-boson decay from a similar point of view and gave the next-to-leading-order QCD correction in agreement with my result (33), but did not work out the $[m(M)/M]^2$ term [in my Eq. (24)].

Note added in proof. Recently R. Tarrach has recalculated the two-loop anomalous dimension γ_1 for the running quark mass and found a different result[15] than in Ref. 10. His result replaces γ_1 quoted in my Eq. (26) by $\gamma_1 = -\frac{404}{3} + \frac{40}{9}N_f$. This changes my numerical result very little: The correction factor C in Eq. (33) becomes 1.38 instead of 1.32 for $M=50$ GeV.

ACKNOWLEDGMENTS

I would like to thank W. A. Bardeen and A. J. Buras for valuable discussions and reading of the manuscript. I would also like to thank W. Celmaster, M. Einhorn, and P. Q. Hung for useful discussions. I would like to thank T. Inami and T. Kubota for many useful correspondence and for pointing out a numerical error.

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*On leave of absence from Department of Physics, Tohoku University, Sendai 980, Japan.
†Operated by Universities Research Association Inc. under contract with the United States Department of Energy.

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