

Effective Hamiltonian for nucleon decay

L. F. Abbott

Department of Physics, Brandeis University, Waltham, Massachusetts 02254

Mark B. Wise

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 9 May 1980)

Renormalization effects for the $SU(3) \otimes SU(2) \otimes U(1)$ -invariant baryon-number-violating operators of lowest dimension are calculated. Linear relations involving these operators are presented and a minimal set is given for nucleon decay processes.

I. INTRODUCTION

Renormalization-group analyses¹ of $SU(3)$, $SU(2)$, and $U(1)$ gauge theories have shown that unification of the strong, weak, and electromagnetic interactions is possible at a mass scale of order 10^{15} GeV. New interactions resulting from this unification may violate baryon-number conservation as in the $SU(5)$ model of Georgi and Glashow.^{2,3} If they do, nucleon decay will provide us with important information about these interactions and could help to determine how the $SU(3)$, $SU(2)$, and $U(1)$ theories are unified. However, the parameters measured in a nucleon decay experiment refer to a mass scale of order the proton mass m_p , whereas the mass scale relevant to grand unified models is 10^{15} GeV. In this paper, we calculate the $SU(3)$, $SU(2)$, and $U(1)$ renormalization effects⁴ which allow one to relate parameters at these two widely different mass scales.

If nucleon decay is governed by a mass of order 10^{15} GeV, then only those baryon-number-violating operators of lowest possible dimension will contribute at an observable rate. Such operators consistent with $SU(3) \otimes SU(2) \otimes U(1)$ and Lorentz symmetry have been enumerated by Weinberg and by Wilczek and Zee.⁵ In the notation of Weinberg, the operators are

$$O_{abcd}^{(1)} = (d_{\alpha a R} u_{\beta b R})(q_{i \gamma c L} l_{j d L}) \epsilon_{\alpha \beta \gamma} \epsilon_{i j}, \quad (1.1)$$

$$O_{abcd}^{(2)} = (q_{i \alpha a L} q_{j \beta b L})(u_{\gamma c R} l_{d R}) \epsilon_{\alpha \beta \gamma} \epsilon_{i j}, \quad (1.2)$$

$$O_{abcd}^{(3)} = (q_{i \alpha a L} q_{j \beta b L})(q_{k \gamma c L} l_{l d L}) \epsilon_{\alpha \beta \gamma} \epsilon_{i j} \epsilon_{k l}, \quad (1.3)$$

$$O_{abcd}^{(4)} = (q_{i \alpha a L} q_{j \beta b L})(q_{k \gamma c L} l_{l d L}) \epsilon_{\alpha \beta \gamma} (\vec{\tau} \epsilon)_{i j} \cdot (\vec{\tau} \epsilon)_{k l}, \quad (1.4)$$

$$O_{abcd}^{(5)} = (d_{\alpha a R} u_{\beta b R})(u_{\gamma c R} l_{d R}) \epsilon_{\alpha \beta \gamma}, \quad (1.5)$$

where α, β, γ are $SU(3)$ color indices; i, j, k, l are $SU(2)$ indices; a, b, c, d refer to generation numbers, and L and R refer to left- and right-handed fields. We have used two-component spinor notation in Eqs. (1.1)–(1.5) with spinor indices contracted as in the Appendix. The correspondence

with four-component spinor notation is given in the Appendix. The operator $O^{(6)}$, which appears in Ref. 5, can be expressed in terms of $O^{(5)}$ by the relation

$$O_{abcd}^{(6)} = (O_{cbad}^{(5)} - O_{cabd}^{(5)}) \quad (1.6)$$

and therefore need not be considered separately.

For renormalization-group calculations it is useful to take into account any relations between the operators being considered. The operators $O^{(3)}$ and $O^{(4)}$ can be written as the symmetric and antisymmetric part (in the first two generation indices) of a single operator. We therefore find it most convenient to define an operator

$$\tilde{O}_{abcd}^{(4)} = (q_{\alpha i a L} q_{\beta j b L})(q_{\gamma k c L} l_{l d L}) \epsilon_{\alpha \beta \gamma} \epsilon_{i j} \epsilon_{k l} \quad (1.7)$$

and note that⁶

$$O_{abcd}^{(3)} = -(\tilde{O}_{abcd}^{(4)} + \tilde{O}_{bacd}^{(4)}) \quad (1.8)$$

and

$$O_{abcd}^{(4)} = -(\tilde{O}_{abcd}^{(4)} - \tilde{O}_{bacd}^{(4)}). \quad (1.9)$$

With the relations (1.8) and (1.9) the effective Hamiltonian for nucleon decay can be expressed in terms of only four types of operators:

$$O_{abcd}^{(1)}, O_{abcd}^{(2)}, \tilde{O}_{abcd}^{(4)}, \text{ and } O_{abcd}^{(5)}.$$

To avoid confusion we will retain the original numbering in Eqs. (1.1)–(1.5). Thus, no $O_{abcd}^{(3)}$ will occur in our analysis. Note that there are still some relations between these remaining operators, namely,⁶

$$O_{abcd}^{(2)} = O_{bacd}^{(2)} \quad (1.10)$$

and

$$\tilde{O}_{abcd}^{(4)} + \tilde{O}_{bacd}^{(4)} - \tilde{O}_{cabd}^{(4)} - \tilde{O}_{cbad}^{(4)} = 0. \quad (1.11)$$

In terms of the original operators $O^{(3)}$ and $O^{(4)}$, Eq. (1.11) becomes

$$O_{abcd}^{(3)} = \frac{1}{2}(O_{cabd}^{(3)} + O_{cabd}^{(4)} + O_{cbad}^{(3)} + O_{cbad}^{(4)}). \quad (1.12)$$

In the following section, we derive the one-loop renormalization factors for the operators $O_{abcd}^{(1)}$, $O_{abcd}^{(2)}$, $\bar{O}_{abcd}^{(4)}$, and $O_{abcd}^{(5)}$ from SU(3), SU(2), and U(1) interactions, and then apply our results to nucleon decay into nonstrange and strange final states. We will ignore the extremely small effects from light-Higgs-boson renormalization of the operators. Our results allow one to include SU(3), SU(2), and U(1) renormalization effects in calculations of nucleon decay rates and branching ratios in grand unified models.⁷ For example, operators of type one and two can occur from vector-boson exchange while operators of type four and five originate from Higgs-boson exchange.⁵

II. RESULTS

To determine renormalization effects in a renormalization-group approach one needs to know the anomalous-dimension matrix for the operators of interest.⁸ This is determined from the renormalization Z factors which relate the bare and renormalized operators. In our calculation of these Z factors, we have used dimensional regularization in $n = 4 - 2\epsilon$ dimensions and minimal subtraction. The calculation was performed both in the Landau gauge and in the Feynman gauge (where external wave-function renormalization must be taken into account). Our results are

$$O_{abcd}^{(1)0} = \left[1 + \frac{\alpha_s}{4\pi\epsilon} (2) + \frac{\alpha_2}{4\pi\epsilon} \left(\frac{9}{4}\right) + \frac{\alpha_1}{4\pi\epsilon} \left(\frac{11}{12}\right) \right] O_{abcd}^{(1)}, \quad (2.1)$$

$$O_{abcd}^{(2)0} = \left[1 + \frac{\alpha_s}{4\pi\epsilon} (2) + \frac{\alpha_2}{4\pi\epsilon} \left(\frac{9}{4}\right) + \frac{\alpha_1}{4\pi\epsilon} \left(\frac{23}{12}\right) \right] O_{abcd}^{(2)}, \quad (2.2)$$

$$\begin{aligned} \bar{O}_{abcd}^{(4)0} &= \left[1 + \frac{\alpha_s}{4\pi\epsilon} (2) + \frac{\alpha_2}{4\pi\epsilon} \left(\frac{3}{2}\right) + \frac{\alpha_1}{4\pi\epsilon} \left(\frac{1}{6}\right) \right] \bar{O}_{abcd}^{(4)} \\ &+ \frac{\alpha_2}{4\pi\epsilon} (2) (\bar{O}_{bacd}^{(4)} + \bar{O}_{cbad}^{(4)} + \bar{O}_{acbd}^{(4)}), \end{aligned} \quad (2.3)$$

$$O_{abcd}^{(5)0} = \left[1 + \frac{\alpha_s}{4\pi\epsilon} (2) + \frac{\alpha_1}{4\pi\epsilon} (1) \right] O_{abcd}^{(5)} + \frac{\alpha_1}{4\pi\epsilon} \left(\frac{10}{3}\right) O_{acbd}^{(5)}. \quad (2.4)$$

Here a superscript 0 refers to a bare operator, α_s is the SU(3) coupling constant, which at some large mass M is given by

$$\alpha_s(M) = \frac{4\pi}{\beta_0^{(3)} \ln(M^2/\Lambda^2)}, \quad (2.5)$$

and α_1 and α_2 are the SU(2) and U(1) couplings related to the electromagnetic coupling α_{EM} at the W -boson mass M_W by

$$\alpha_1(M_W) = \alpha_{EM}(M_W) / \cos^2 \theta_W \quad (2.6)$$

and

$$\alpha_2(M_W) = \alpha_{EM}(M_W) / \sin^2 \theta_W. \quad (2.7)$$

Note that under renormalization $\bar{O}_{abcd}^{(4)}$ mixes with $\bar{O}_{bacd}^{(4)}$, $\bar{O}_{cbad}^{(4)}$, and $\bar{O}_{acbd}^{(4)}$ and $O_{abcd}^{(5)}$ mix with $O_{acbd}^{(5)}$.

We apply these results to two relevant cases.

First consider $a = b = c = d = 1$ so that we have operators relevant (apart from Cabibbo-suppressed modes) to nucleon decay into nonstrange final states.⁹ In this case there are only four linearly independent operators, which we denote by

$$Q_1 = O_{1111}^{(1)} = (d_{\alpha R} u_{\beta R})(u_{\gamma L} e_L - d_{\gamma L} \nu_L^e) \epsilon_{\alpha\beta\gamma}, \quad (2.8)$$

$$Q_2 = -\frac{1}{2} O_{1111}^{(2)} = (d_{\alpha L} u_{\beta L})(u_{\gamma R} e_R) \epsilon_{\alpha\beta\gamma}, \quad (2.9)$$

$$Q_3 = \bar{O}_{1111}^{(4)} = (d_{\alpha L} u_{\beta L})(u_{\gamma L} e_L - d_{\gamma L} \nu_L^e) \epsilon_{\alpha\beta\gamma}, \quad (2.10)$$

and

$$Q_4 = O_{1111}^{(5)} = (d_{\alpha R} u_{\beta R})(u_{\gamma R} e_R) \epsilon_{\alpha\beta\gamma}. \quad (2.11)$$

These do not mix under renormalization. If $A_1^{\text{GUM}}, \dots, A_4^{\text{GUM}}$ are the tree-level coefficients of Q_1, \dots, Q_4 in a grand unified model, then the corresponding coefficients at a mass scale of order the proton mass $\mu \simeq m_p$ are given by

$$A_1(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{9/4\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{11/12 C^2 \beta_0^{(1)}} A_1^{\text{GUM}}, \quad (2.12)$$

$$A_2(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{9/4\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{23/12 C^2 \beta_0^{(1)}} A_2^{\text{GUM}}, \quad (2.13)$$

$$A_3(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{15/2\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{1/8 C^2 \beta_0^{(1)}} A_3^{\text{GUM}}, \quad (2.14)$$

and

$$A_4(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{13/3 C^2 \beta_0^{(1)}} A_4^{\text{GUM}}. \quad (2.15)$$

$\alpha_1(M_W)$ and $\alpha_2(M_W)$ are given by Eqs. (2.6) and (2.7), $\alpha_s(\mu)$ is the SU(3) running coupling constant evaluated at the renormalization point $\mu \simeq m_p$, and α_{GUM} is the grand unified coupling. C is the normalization factor between the U(1) of SU(2) \otimes U(1) and the U(1) subgroup of the grand unified gauge group. In SU(5), $C^2 = \frac{5}{3}$.¹⁰ Renormalization effects due to the electromagnetic interactions have been neglected between the W -boson mass and the renormalization-point mass since $\alpha_{\text{EM}} \ln(M_W^2/\mu^2)$ is a small number. The β functions are given by¹¹

$$\beta_0^{(3)} = 11 - \frac{2}{3}N_f, \quad (2.16)$$

$$\beta_0^{(2)} = \frac{22}{3} - \frac{2}{3}N_f - \frac{1}{6}, \quad (2.17)$$

and

$$\beta_0^{(1)} = -\frac{2}{3}N_f - \frac{1}{10}, \quad (2.18)$$

when one light Higgs doublet is included. The last terms in Eqs. (2.17) and (2.18) are the contributions of the light Higgs doublet. N_f is the number of quark flavors.

Now consider the case of nucleon decay into strange final states.⁹ The linearly independent set of operators relevant to this case are

$$Q'_1 = O_{2111}^{(1)}, \quad (2.19)$$

$$Q'_2 = O_{1121}^{(1)}, \quad (2.20)$$

$$Q'_3 = O_{2111}^{(2)}, \quad (2.21)$$

$$Q'_4 = (2\bar{O}_{2111}^{(4)} + \bar{O}_{1211}^{(4)}), \quad (2.22)$$

$$Q'_5 = (\bar{O}_{2111}^{(4)} - \bar{O}_{1211}^{(4)}), \quad (2.23)$$

and

$$Q'_6 = O_{2111}^{(5)}. \quad (2.24)$$

Q'_1, \dots, Q'_6 have been defined in such a way that they do not mix under renormalization. Denoting the tree-level coefficients of these operators in a grand unified model (i.e., the values of the coefficients at the superheavy mass scale) by $A_1^{\text{GUM}}, \dots, A_6^{\text{GUM}}$, it follows from Eqs. (2.1)–(2.5) that the coefficients determined at $\mu \simeq m_p$ are

$$A_1'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{9/4\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{11/12C^2\beta_0^{(1)}} A_1^{\text{GUM}}, \quad (2.25)$$

$$A_2'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{9/4\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{11/12C^2\beta_0^{(1)}} A_2^{\text{GUM}}, \quad (2.26)$$

$$A_3'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{9/4\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{23/12C^2\beta_0^{(1)}} A_3^{\text{GUM}}, \quad (2.27)$$

$$A_4'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{15/2\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{1/6C^2\beta_0^{(1)}} A_4^{\text{GUM}}, \quad (2.28)$$

$$A_5'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{\alpha_2(M_W)}{\alpha_{\text{GUM}}} \right]^{3/2\beta_0^{(2)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{1/6C^2\beta_0^{(1)}} A_5^{\text{GUM}}, \quad (2.29)$$

and

$$A_6'(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_{\text{GUM}}} \right]^{2/\beta_0^{(3)}} \left[\frac{C^2 \alpha_1(M_W)}{\alpha_{\text{GUM}}} \right]^{13/3C^2\beta_0^{(1)}} A_6^{\text{GUM}}. \quad (2.30)$$

III. CONCLUSIONS

In the previous section we gave a minimal set of SU(3) \otimes SU(2) \otimes U(1)-invariant operators which enter the effective Hamiltonian for nucleon decay into strange and nonstrange final states. It is important for both theoretical and experimental analyses to have a minimal set of such operators. We then calculated the relationship between the coefficients of these operators at the grand unified and proton mass scales. A given grand unified theory predicts most directly (i.e., from tree-level) values of these coefficients at the grand unified mass scale. However, quark-model-type estimates for the matrix elements⁷ of the operators can be expected to be valid at the proton mass

scale. Thus one must make use of Eqs. (2.12)–(2.15) and (2.25)–(2.30) in order to make predictions concerning proton decay from a grand unified theory or to extract from future experiments information on the physics occurring at the grand unified mass scale. In general, the effect of the SU(3) \otimes SU(2) \otimes U(1) renormalization factors is to enhance nucleon decay. For example, in SU(5) with six flavors and $\Lambda = 0.1$ GeV we have the following numerical values for the coefficients appearing in Eqs. (2.12)–(2.15) for operators contributing to nonstrange decays:

$$A_1(m_p) = (2.2)(1.4)(1.1)A_1^{\text{GUM}} = (3.4)A_1^{\text{GUM}}, \quad (3.1)$$

$$A_2(m_p) = (2.2)(1.4)(1.1)A_2^{\text{GUM}} = (3.4)A_2^{\text{GUM}}, \quad (3.2)$$

$$A_3(m_p) = (2.2)(3.0)(1.01)A_3^{\text{GUM}} = (6.7)A_3^{\text{GUM}}, \quad (3.3)$$

$$A_4(m_p) = (2.2)(1.0)(1.3)A_4^{\text{GUM}} = (2.9)A_4^{\text{GUM}}, \quad (3.4)$$

where the numerical factors give the SU(3), SU(2), and U(1) renormalizations, respectively. A large part of the enhancement comes from the common factor of 2.2 from strong interaction effects. However, it is interesting to note that Q_3 receives a significant enhancement from SU(2) interactions as well, yielding a large overall factor of 6.7 for this operator.

Finally, it is worth noting that our analysis is only valid in the simplest possible scenario, where there exist only two relevant mass scales. It is possible that new physics exists at intermediate mass scales. The grand unified group G could break down to SU(3) \otimes SU(2) \otimes U(1) in a series of steps $G \supset G' \supset \dots \supset \text{SU}(3) \otimes \text{SU}(2) \otimes \text{U}(1)$, in which case one must also calculate, for example, the renormalization of the operators due to a G' gauge theory in order to relate the coefficients of operators at the grand unified and proton mass scales.

ACKNOWLEDGMENTS

M.B.W. thanks H. Schnitzer, the Brandeis Physics Department, and Cathy Abbott for their hospitality during a visit when much of this work was done. This research was supported by Department of Energy Contracts DE-AC03-76SF00515 and E(11-1)3230 (L.F.A.) and by the Natural Sciences and Engineering Research Council of Canada (M.B.W.).

APPENDIX

Two-component spinors come in two varieties which, for fields that annihilate particles, we refer to as left- and right-handed. These two transform under SL(2C) according to representations which are complex conjugates of each other. To distinguish these representations we denote left-handed fields with an undotted index and right-handed fields with a dotted index. Under complex conjugation of the fields undotted indices become dotted and vice versa. Either dotted or undotted indices may be lowered, raised, or contracted by the antisymmetric ϵ tensors $\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = \epsilon^{\dot{\alpha}\dot{\beta}}$.

The relationship between the four-component spinor notation and the two-component form is as follows. If we write

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (A1)$$

and

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad (A2)$$

with

$$\sigma^\mu = (1, \vec{\sigma}) = (\sigma^\mu)_{\alpha\dot{\alpha}} \quad (A3)$$

$$\bar{\sigma}^\mu = (1, -\vec{\sigma}) = (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}, \quad (A4)$$

then a four-component spinor can be written as

$$\psi = \begin{pmatrix} L_\alpha \\ R^{\dot{\alpha}} \end{pmatrix}, \quad (A5)$$

$$\bar{\psi} = (\bar{R}^\alpha \bar{L}_{\dot{\alpha}}), \quad (A6)$$

where a bar on a two-component spinor indicates complex conjugation. Familiar bilinears are

$$\bar{\psi}\psi = \bar{R}L + \bar{L}R \equiv \bar{R}^\alpha L_\alpha + \bar{L}_{\dot{\alpha}} R^{\dot{\alpha}} \quad (A7)$$

and

$$\bar{\psi}\gamma_\mu\psi = \bar{R}\sigma_\mu R + \bar{L}\bar{\sigma}_\mu L \equiv \bar{R}^\alpha (\sigma_\mu)_{\alpha\dot{\alpha}} R^{\dot{\alpha}} + \bar{L}_{\dot{\alpha}} (\bar{\sigma}_\mu)^{\dot{\alpha}\alpha} L_\alpha. \quad (A8)$$

The charge-conjugate ψ field is

$$\psi^c = \begin{pmatrix} \bar{R}_\alpha \\ \bar{L}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\psi}^c = (L^\alpha R_\alpha), \quad (A9)$$

where

$$\bar{R}_\alpha = \bar{R}^\beta \epsilon_{\beta\alpha}, \quad \bar{L}^{\dot{\alpha}} = \bar{L}_{\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}}, \quad L^\alpha = \epsilon^{\alpha\beta} L_\beta, \quad R_{\dot{\alpha}} = R^{\dot{\beta}} \epsilon_{\dot{\beta}\dot{\alpha}}, \quad (A10)$$

so that, for example,

$$\bar{\psi}^c\psi = L^2 + R^2 \equiv L^\alpha L_\alpha + R_{\dot{\alpha}} R^{\dot{\alpha}} \quad (A11)$$

and

$$\bar{\psi}^c\gamma_5\psi = -L^2 + R^2 \equiv -L^\alpha L_\alpha + R_{\dot{\alpha}} R^{\dot{\alpha}}. \quad (A12)$$

These formulas can be used to express the operators defined in Eqs. (1.1)–(1.6) in four-component form.

In order to prove some of the identities involving the operators (1.1)–(1.5) and to calculate the renormalization effects, one needs the Fierz-transformation rules

$$(A_R B_R)(C_L D_L) = -\frac{1}{2}(A_R \bar{\sigma}_\mu D_L)(C_L \sigma^\mu B_R), \quad (A13)$$

$$(A_R B_R)(C_R D_R) = -\frac{1}{2}[(A_R D_R)(C_R B_R) + \frac{1}{4}(A_R \bar{\sigma}_{\mu\nu} D_R)(C_R \bar{\sigma}^{\mu\nu} B_R)], \quad (A14)$$

$$(A_L B_L)(C_L D_L) = -\frac{1}{2}[(A_L D_L)(C_L B_L) + \frac{1}{4}(A_L \sigma_{\mu\nu} D_L)(C_L \sigma^{\mu\nu} B_L)], \quad (A15)$$

where

$$\sigma_{\mu\nu} = \frac{i}{2}(\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu) \quad (\text{A16})$$

and

$$\bar{\sigma}_{\mu\nu} = \frac{i}{2}(\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu). \quad (\text{A17})$$

It is also useful to note that

$$(A_R \bar{\sigma}_{\mu\nu} B_R) = -(B_R \bar{\sigma}_{\mu\nu} A_R), \quad (\text{A18})$$

$$(A_L \sigma_{\mu\nu} B_L) = -(B_L \sigma_{\mu\nu} A_L), \quad (\text{A19})$$

$$(A_L \sigma^\mu B_R) = -(B_R \bar{\sigma}^\mu A_L), \quad (\text{A20})$$

and

$$(A_R \bar{\sigma}_{\mu\nu} B_R)(C_L \sigma^{\mu\nu} D_L) = 0. \quad (\text{A21})$$

Equations (A13)–(A21) can be used to write all $SU(3) \otimes SU(2) \otimes U(1)$ -invariant baryon-number-violating nucleon decay operators (of lowest possible dimension) as linear combinations of those in Eqs. (1.1)–(1.5).

¹H. Georgi, H. R. Quinn, and S. Weinberg, *Phys. Rev. Lett.* **33**, 451 (1974); W. J. Marciano, *Phys. Rev. D* **20**, 274 (1979); D. A. Ross, *Nucl. Phys.* **B140**, 1 (1978); T. J. Goldman and D. A. Ross, *Phys. Lett.* **84B**, 208 (1979); L. Hall, Harvard University report (unpublished).

²H. Georgi and S. L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974).

³For other grand unified models see J. C. Pati and A. Salam, *Phys. Rev. D* **8**, 1240 (1973); H. Georgi, in *Particles and Fields, 1979*—Proceedings of the Williamsburg meeting of the Division of Particles and Fields of the American Physical Society, edited by C. E. Carlson, AIP Conference Proceedings No. 23 (A.I.P., New York, 1975); H. Fritzsch and L. Minkowski, *Ann. Phys. (N.Y.)* **93**, 193 (1975); H. Georgi and D. V. Nanopoulos, *Phys. Lett.* **82B**, 392 (1979); *Nucl. Phys.* **B155**, 52 (1979); F. Gursev, P. Ramond and P. Sikivie, *Phys. Lett.* **60B**, 177 (1976); F. Gursev and P. Sikivie, *Phys. Rev. Lett.* **36**, 775 (1976); P. Ramond, *Nucl. Phys.* **B110**, 214 (1976); P. Sikivie and F. Gursev, *Phys. Rev. D* **16**, 816 (1977).

⁴The renormalization effects for $O^{(1)}$ and $O^{(2)}$ have already been calculated. See A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, *Nucl. Phys.* **B135**, 66 (1978); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, *Phys. Lett.* **88B**, 320 (1979); F. Wilczek and A. Zee, *Phys. Rev. Lett.* **43**, 1571 (1979).

⁵S. Weinberg, *Phys. Rev. Lett.* **43**, 1566 (1979); F. Wilczek and A. Zee, *ibid.* **43**, 1571 (1979).

⁶These relations come from using the identities in the Appendix and from the identity

$$\epsilon_{ij} \epsilon_{kl} - \epsilon_{kj} \epsilon_{il} + \epsilon_{ik} \epsilon_{lj} = 0.$$

This can be proved by noting that the left-hand side is completely antisymmetric on its four indices i, j, k, l and thus is zero since each index only takes on two values. We thank B. Ovrut for pointing out this simple proof.

⁷M. Machacek, *Nucl. Phys.* **B159**, 37 (1979); J. F. Donoghue, *Phys. Lett.* **92B**, 99 (1980); C. Jarlskog and F. Yndurain, *Nucl. Phys.* **B149**, 29 (1979).

⁸For an excellent review see D. Gross, in *Methods in Field Theory*, proceedings of Les Houches Summer School, 1975, edited by C. R. Balian and J. Zimm-Justin (North-Holland, New York, 1976).

⁹We have written the operators for a positron or electron antineutrino in the final state. Our results hold equally well for $d=2$, in which case there is an anti-muon or muon antineutrino in the final state. Note also that the quark fields d_L and s_L are weak eigenstates and related to the mass eigenstates d'_L, s'_L by the rotation

$$\begin{pmatrix} d_L \\ s_L \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d'_L \\ s'_L \end{pmatrix}.$$

¹⁰It is possible that such a C factor could enter into the relations of the $SU(3)$ or $SU(2)$ couplings as well. These could be accommodated in a similar manner.

¹¹H. D. Politzer, *Phys. Rev. Lett.* **30**, 1346 (1973); D. J. Gross and F. Wilczek, *ibid.* **30**, 1343 (1973); A. J. Buras *et al.*, Ref. 4.