Acoplanarity distributions at large transverse momenta

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We consider within quantum chromodynamics the acoplanarity distribution of two hadrons produced with large transverse momenta in hadronic collisions. Our method consists of summing the leading double logarithms arising from soft-gluon bremsstrahlung, taking also into account momentum conservation. With a reasonable choice of parameters we obtain a very good agreement with the available experimental data.

I. INTRODUCTION

The CERN ISR experiments at large transverse momenta (p_T) gave support to the simple model for hard hadronic collisions of Berman, Bjorken, and Kogut (BBK).[1] Indeed, for large p_T ($p_T \geq 6$ GeV), the p_T dependence of the single-hadron inclusive cross section[2] is very close to the canonical one (p_T^{-4}). To obtain a quantitative agreement with experiment we have to appeal to quantum chromodynamics (QCD).[3] More precisely, the introduction of scale breaking in the distribution and fragmentation functions provides an effective p_T dependence p_T^{-n}, with $n = 5$–6 (for present energies), while the introduction of non-Abelian gluons gives a large absolute magnitude of the cross section. The shape and magnitude of the p_T yield are essential features of experimental large-p_T physics and our present understanding of them is due to QCD.

Clearly it is of high interest to reinforce our belief in QCD as a theoretical framework for large-p_T physics by proposing new tests and confronting the theory with more detailed data. We feel that acoplanarity distributions at large transverse momenta serve this purpose. Consider the reaction $p+p \to h_1 + h_2 + X$, where the two hadrons h_1, h_2 are produced with transverse momenta p_{T1}, p_{T2} at azimuthal angles ϕ_1, ϕ_2 and define $\phi = \pi - (\phi_2 - \phi_1)$ (Fig. 1). At the parton-model level (BBK model) we expect two coplanar jets and therefore $\phi = 0$.[4] However, within QCD gluon emission by the scattered constituents (quarks, gluons) will give rise to acoplanar events.

Perturbative calculations for acoplanarity distributions have appeared already.[5] However, these calculations (i.e., $qq \to qqg$ at the Born level) are meaningful only when $p_{\text{out}} \simeq p_T$.[6] In the region $p_{\text{out}} \ll p_T$ we have to resort to resummation techniques, i.e., sum all the double logarithms which would spoil a naive perturbative expansion.[7,8] We present such a calculation in this paper. Our results are very suggestive: we have to replace the δ function $\delta(\phi)$ appearing in the double inclusive cross section (Born term) by a function $\Delta(\phi)$, calculable in QCD, which although peaked at $\phi = 0$ has a rather broad tail. In Sec. II we derive the function $\Delta(\phi)$ and in Sec. III we compare our results with the data of the CCOR (Ref. 9) and AABCS (Ref. 10) collaboration.

II. CHASING THE DOUBLE LOGARITHMS

The Altarelli-Parisi (AP) (Ref. 11) equations for longitudinal momenta have been extended also to transverse momenta.[12,13] In this spirit the probability to find within a quark another quark with longitudinal momentum fraction x and a transverse momentum p_z along the z direction (the direction normal to the beam-trigger plane) is given by[13]:

$$N_{qq}(x, p_z) = \frac{\alpha_s}{2\pi} P_{qq}(x) \frac{1}{p_z} \, dx \, dp_z \ , \tag{1}$$

where $P(x)$ is the usual AP function. If the coupling constant was small (rather $\alpha_s \ln\phi$ small, with $\phi = p_{\text{out}}/p_x$), as in QED, then in quark-quark scattering we could see the $1/\phi$ tail[14]

$$\frac{d\sigma}{d\phi} = \sigma_0 r(\phi) \ ,$$
$$r(\phi) = -\frac{\alpha_s}{\pi} C \left(\frac{\ln|\phi|}{|\phi|}\right)_+ , \tag{2}$$

where $C = 4$, $C_q = \frac{4}{3}$ and the plus sign is defined by[8,15]

$$\int_0^1 d\phi \, [H(\phi)]_+ f(\phi) \equiv \int_0^1 d\phi \, H(\phi) [f(\phi) - f(0)] . \tag{3}$$

The above partonic interpretation is maintained in higher orders, if we perform our calculations in the axial gauge.[7] In this gauge, the dominant graphs have a ladder structure and summing the contributions of soft gluons we find[7,13] the Sudakov form factor

$$T(\phi) = \exp\left[-\frac{\alpha_s}{\pi} C \ln^2\phi\right] . \tag{4}$$

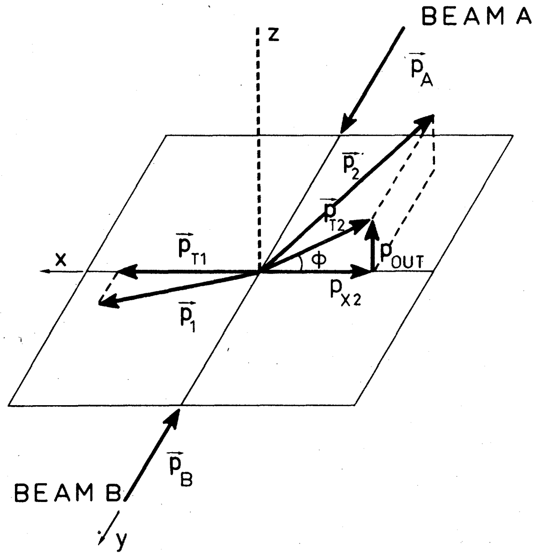

FIG. 1. Kinematics of the large-p_T event.

This derivation ignores momentum conservation. Inserting, in each order of perturbation, the function $\delta(\sum_i p_{z_i} - p_{\text{out}})$ in its integral representation we obtain[16]

$$\Delta(\phi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\phi\sigma} e^{h(\sigma)} d\sigma \qquad (5)$$

$$h(\sigma) = \int_{-1}^{1} d\phi r(\phi) e^{i\phi\sigma} . \qquad (6)$$

To get an approximate expression for $h(\sigma)$, we extend the limits of integration in Eq. (6) to infinity and we find[15]

$$h(\sigma) \simeq -\frac{\alpha_s}{\pi} C[(\ln|\sigma| + \gamma_E)^2 + \text{const}] \qquad (7)$$

Since we are working in the double logarithmic approximation we finally obtain[18,26]

$$\Delta(\phi) = \frac{1}{\pi} \int_0^{\infty} \cos\phi\sigma \exp\left[-\frac{\alpha_s}{\pi} C \ln^2\sigma\right] d\sigma , \qquad (8)$$

Equation (8) constitutes our main result. For quark-gluon and gluon-gluon scattering we have

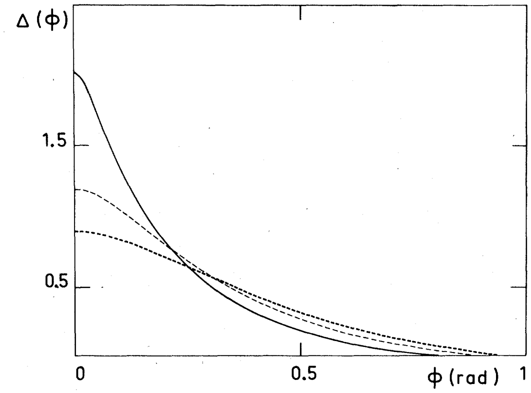

FIG. 2. The acoplanarity distributions $\Delta(\phi)$ for quark-quark scattering (solid line), quark-gluon scattering (thin dashed line), and gluon-gluon scattering (dashed line).

analogous formulas with $C = 4C_q$ replaced by

$$C_{qg} = 2C_q + 2C_g , \quad C_{gg} = 4C_g ,$$

where $C_g = 3$.

In Fig. 2 we plot the functions $\Delta_{qq}(\phi)$, $\Delta_{qg}(\phi)$, $\Delta_{gg}(\phi)$ using $\alpha_s = 0.2$.

The following properties of the function $\Delta(\phi)$ are worthy to be pointed out.

(i) When we switch off the strong interactions ($\alpha_s = 0$), we obtain $\Delta(\phi) = \delta(\phi)$.

(ii) Making an expansion in α_s we obtain

$$\Delta(\phi) = \delta(\phi) - \frac{\alpha_s}{\pi} C \frac{\ln\phi}{\phi} + \cdots .$$

(iii) Changing the variable of integration to $\phi\sigma$, we find for large ϕ (small $\ln\phi$) an expression similar to the DDT form factor.[7]

(iv) At $\phi = 0$ the DDT form factor gives zero,[7] while our function $\Delta(\phi)$ has a finite value. This is a consequence of the momentum conservation we have imposed. For small values of ϕ, the dominant contribution is coming from emission of gluons whose transverse momenta are not small and add up to zero.[8]

(v) Since the gluon is "more colored" than the quark, $\Delta_{gg}(\phi)$ has a broader tail than $\Delta_{qq}(\phi)$.

III. COMPARISON WITH EXPERIMENT

There is a wealth of experimental data concerning acoplanarity distributions.[2] In the most recent experiments the hadrons observed have large transverse momenta and their full azimuthal dependence has been explored. This is an interesting development since large values of p_T, p_{out}, render from a theoretical point of view the perturbative approach reliable, while from an experimental point of view make sure that the jet structure is not contaminated by spectator quarks from the beam fragments.

Following the analysis of the previous section, the invariant double inclusive cross section (two hadrons h_1, h_2 in opposite sides) will be given by

$$E_1E_2\frac{d^2\sigma}{d^3p_1d^3p_2}=\frac{4}{\pi s x_{T1}{}^2x_{T2}{}^2}\sum_{ij}\int\frac{dx_a}{x_a}z^2F_{i/p}(x_a)F_{j/p}(x_b)\frac{d\sigma_{ij}}{d\hat{t}}G_{h_1/i}\left(\frac{x_{T1}}{z}\right)G_{h_2/j}\left(\frac{x_{T2}}{z}\right)\Delta_{ij}(\phi)\ . \qquad (9)$$

We have simply replaced the delta function $\delta(\phi)$ appearing in the double inclusive cross section (Born term) by $\Delta(\phi)$. The rest of the kinematics is identical and the details can be found in Ref. 19 and 20. The sum in Eq. (9) is over the proton's constituents (quarks, gluons).

The distribution and fragmentation functions we are using are effective ones and they do not contain scaling breaking.[18] Since the momentum scale controlling our process[7] is p_{out} and not p_T, these distribution functions correspond to an effective Q_0 (the reference momentum) not very large. For the quark distributions we use those of Ref. 21 at $Q_0=3.5$ GeV. The shape of the gluon distribution function is poorly known.[22] The only constraint is the momentum sum rule. We use

$$F_{g/p}(x)=\tfrac{1}{2}(n+1)(1-x)^n \qquad (10)$$

with $n=8$. For the quark and gluon fragmentation functions we have taken those of Ref. 23 at $Q_0=3.5$ GeV. We use also $\alpha_s=0.2$. This choice is not in disagreement with the arguments of Parisi and Petronzio[24] for a frozen α_s at moderate values of Q^2.

As has been repeatedly emphasized,[20] part of the acoplanarity can be accounted for as originating from the hadronization of quarks and gluons (soft component). In Ref. 19 this component was parametrized as

$$D(\phi)=\frac{1}{2}\frac{1}{\sqrt{\pi z}}\exp\left[-\frac{\phi^2}{4z}\right], \qquad (11)$$

$$z=(p_{T1}{}^2+p_{T2}{}^2)\pi d^2/(4p_{T1}{}^2p_{T2}{}^2)\ . \qquad (12)$$

The most accepted value for the parameter d is $d=0.3$ GeV, corresponding to the usual transverse-momentum cutoff observed in soft hadronic processes. The above function $D(\phi)$ in the σ space is equivalent to $\exp[-z\sigma^2]$. Therefore in our calculations we take

$$\Delta(\phi)=\frac{1}{\pi}\int_0^\infty\cos\phi\sigma\exp\left[-\frac{\alpha_s}{\pi}C\ln^2\sigma-z\sigma^2\right]d\sigma\ . \qquad (13)$$

It is clear that the contribution of the soft component at large values of p_{T1}, p_{T2} is negligible.

With these reasonable choices we can confront now the experimental data. The normalized acoplanarity distribution to be considered (number of tracks per unit azimuth per event)[19] is

$$\frac{dn}{d\phi}=p_{T2}\int dy_2E_1E_2\frac{d^2\sigma}{d^3p_1d^3p_2}\bigg/E_1\frac{d\sigma}{d^3p_1}\ . \qquad (14)$$

The CCOR experiment[9] used as a trigger a neutral particle with $11<p_{T1}<7$ GeV and $|y_1|<0.5$. To simplify the calculation we have taken $p_{T1}=8.0$ GeV and $y_1=0.0$. This approximation permits us also to use for the single inclusive cross section the experimental value[25] which we took $4.5\ 10^{-35}$ cm^2 GeV^{-2}. The rapidity of the opposite-side hadron has been integrated over ($|y_2|<0.7$). In Fig. 3 we compare our results with the CCOR data for two values of p_{T2}, 4.5 GeV [Fig. 3(a)] and 3.5 GeV [Fig. 3(b)]. The dashed line corresponds to the hard component [Eq. (8)], while the solid line includes the soft component also [Eq. (13)]. Both the absolute magnitude and the shape of the distributions are accounted for very well.

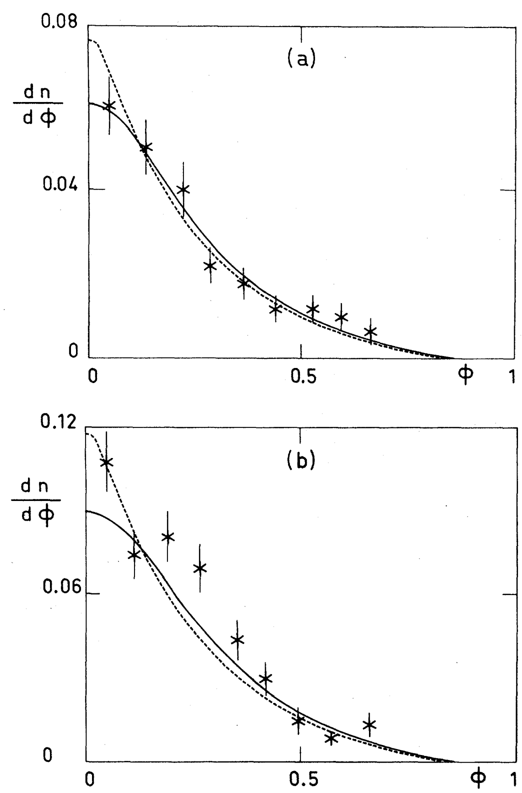

FIG. 3. Acoplanarity distributions. Dashed line corresponds to gluonic emissions only, while solid line includes also the nonperturbative component [Eq. (13)]. Data from Ref. 9. In (a) $p_{T2}=4.5$ GeV and in (b) $p_{T2}=3.5$ GeV.

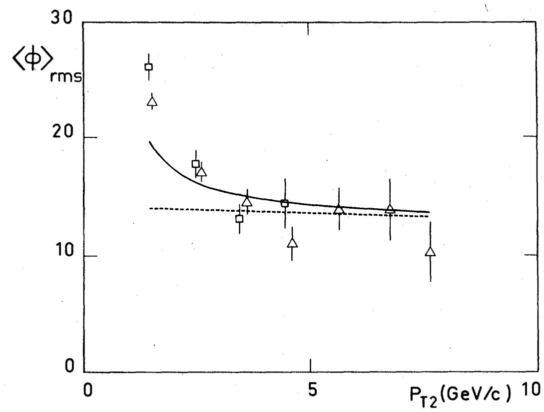

FIG. 4. $\langle\phi\rangle_{rms}$ as a function of p_{T2}, for p_{T1} in the range $9 \leq p_{T1} \leq 11$ GeV. Data from Ref. 10. Lines as in Fig. 3.

If we attempt to fit the distributions with smaller p_{T2} ($p_{T2} < 2.5$ GeV) we find that our curves fall below the experimental ones. This is not disturbing, since it is well known that at such small-p_T spectator partons contaminate the data.[19,20]

With a large-p_T trigger ($9 \leq p_{T1} \leq 11$ GeV) the AABCS collaboration[10] has measured $\langle\phi\rangle_{rms}$ as a function of p_{T2} (Fig. 4). If only one subprocess was operating (say qq scattering), then within our approach we would expect, at large p_{T1}, p_{T2} where the soft component is negligible, a constant $\langle\phi\rangle$ ($=\langle\phi\rangle^{qq}$). Actually we have three subprocesses operating, each one with its own ϕ distribution. Using Eq. (8) we find

$$\langle\phi\rangle_{rms}^{qq} = 12.37°, \quad \langle\phi\rangle_{rms}^{qg} = 16.79°, \quad \langle\phi\rangle_{rms}^{gg} = 19.78° . \qquad (15)$$

The actual value of $\langle\phi\rangle_{rms}$ will depend on which subprocess dominates in a given kinematical configuration. At large p_{T1}, p_{T2} the qq subprocess dominates, since the quark distributions are much stronger at large x and we expect $\langle\phi\rangle_{rms}$ to be close to $\langle\phi\rangle_{rms}^{qq}$. The AABCS collaboration finds[10] a value close to 13°. In Fig. 4 we compare the experimental data with our predictions. The agreement we obtain is satisfactory.

We would like to emphasize that in comparing with the data we did not seek optimal parameters. The conclusion of the above comparisons (Figs. 3 and 4) is that the observed acoplanar events is a manifestation of gluonic emission. We can apply also our formalism to processes where one of the participating constituents is colorless (e.g., a photon). Acoplanarity distributions with a photon trigger will provide further tests of quantum chromodynamics and work along these lines will be reported elsewhere.

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