

## Global and pole duality applied to *c* and *b* quarks

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We investigate  $e^+e^-$  duality in heavy-quark systems with application to the *c*- and *b*-quark sectors. An alternative duality formulation permits quark masses to be determined from “bound” quarkonium states alone. The *c*- and *b*-quark masses are found to be  $m_c = 1.45 \pm 0.05$  GeV and  $m_b = 4.58 \pm 0.08$  GeV, respectively.

### INTRODUCTION

Duality in  $e^+e^-$  annihilation is based on the idea that the underlying short-time quark dynamics is revealed by suitable energy smearing of the observed hadronic data.<sup>1-3</sup> Unfortunately, it is not obvious how this smearing should be done. In previous work<sup>4</sup> we have shown that  $e^+e^-$  annihilation duality can be formulated in terms of moments over hadronic data of a given flavor

$$\frac{1}{s_0^{n+1}} \int_{s_0}^{\bar{s}} ds s^n R_{\text{exp}}(s) = \frac{1}{s_0^{n+1}} \int_{s_0}^{\bar{s}} ds s^n R_{q\bar{q}}(s), \quad (1)$$

where

$$R(s) \equiv \sigma(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

and  $R_{\text{exp}}$  and  $R_{q\bar{q}}$  are experimental and dual values for  $R(s)$ ;  $s$  is the square of the total c.m. energy and  $s_0$  is a convenient dimensional factor. From the experimental data in the charm sector we have determined<sup>4</sup> the shape of the dual function  $R_{c\bar{c}}$  and its threshold which defines the charm-quark mass.

In this paper we show that Eq. (1) can be recast into a “pole-duality” form in which only the discrete states appear. In Sec. I we review the derivation of Eq. (1) and discuss the pole formulation of duality. A simplified model solution for heavy quarks is proposed in Sec. II in which the quark-mass solution is half the lowest vector-meson state and the energy gap to the continuum is proportional to  $\Gamma_{e^+e^-}/R_A$ , where  $R_A$  is the asymptotic plateau value of  $R(s)$ . The charm-sector data is used in Sec. III to verify that the discrete states alone yield the same duality function and charm-quark mass as the global-duality relation<sup>4</sup> of Eq. (1). The pole form of duality also accurately predicts the charm production threshold. In Sec. IV we use the sharp  $\Upsilon$  states to determine the *b*-quark mass and the *b*-flavor production threshold. The dual function  $R_{b\bar{b}}(v)$  is found to have the same shape as  $R_{c\bar{c}}(v)$ . Our conclusions are presented in Sec. V, together with some comparisons to the recent work of other authors.

### I. GLOBAL AND POLE DUALITY IN TERMS OF MOMENT SUM RULES

The hadronic cross section  $R(s)$  is just the discontinuity of the suitably normalized photon hadronic vacuum polarization. The vacuum polarization<sup>5</sup>  $\pi(s)$  is analytic in the cut  $s$  plane and vanishes  $\pi(0) = 0$  at  $s = 0$  due to charge renormalization. Integrating the quantity  $s^n[\pi_{\text{exp}}(s) - \pi_{q\bar{q}}(s)]$  around the finite contour of Fig. 1, we obtain the sum rules (1) under the following assumptions:

(i)  $\pi_{\text{exp}}(s) \approx \pi_{q\bar{q}}(s)$  on the circular contour. This is expected to be valid if the radius of the circular contour  $\bar{s}$  is sufficiently large so that  $R(s)$  is smoothly varying (i.e., above the resonances).

(ii)  $\pi_{\text{exp}} \approx \pi_{q\bar{q}}(s)$  near  $s = 0$ ; or, alternatively, the derivatives of  $\pi(s)$  evaluated at  $s = 0$  are equal if evaluated using the experimental  $R_{\text{exp}}(s)$  (and a dispersion relation) or by differentiating the quark quantum-chromodynamics (QCD) result. For heavy quarks the  $s = 0$  point<sup>3</sup> is far from quark thresholds and perturbative methods are expected to apply. If a calculated QCD result is assumed, we can compare its derivatives to the experimental

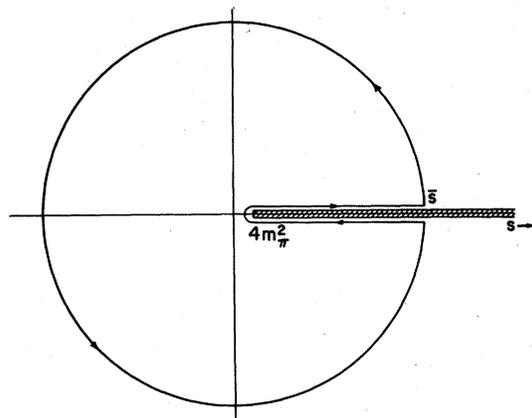


FIG. 1. Contour in the complex  $s$  plane used in deriving Eq. (1). We assume the radius  $\bar{s}$  is large enough so that asymptotic QCD can be used even close to the timelike axis.

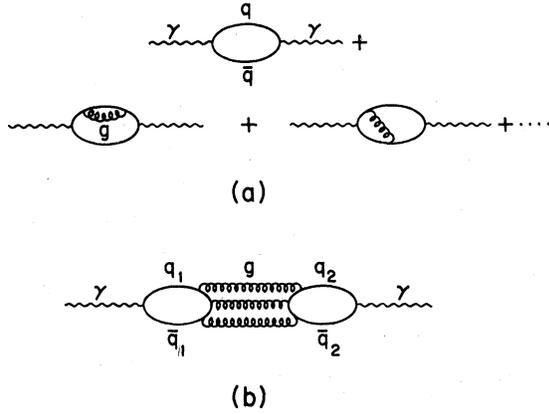


FIG. 2. (a) Perturbation contributions to  $\pi_{q\bar{q}}$  with one quark flavor. (b) Flavor-mixing contributions to  $\pi_{q\bar{q}}$ .

derivatives as advocated by Novikov *et al.*<sup>3</sup> We prefer to reverse the argument assuming that the residue of  $s^n[\pi_{\text{exp}}(s) - \pi_{q\bar{q}}(s)]$  is negligible, thus defining a dual vacuum polarization  $\pi_{q\bar{q}}(s)$ .

The dominant vacuum-polarization diagrams involve quarks of only one flavor as shown in Fig. 2(a). The polarization can then be grouped into a sum of flavor loops and thus the sum rule (1) holds for each flavor. Flavor leakage occurs through violation of the Okubo-Zweig-Iizuka rule,<sup>6</sup> as illustrated in Fig. 2(b). The lowest-order leakage diagram is a factor  $\alpha_s^3$  down compared to the flavor-conserving loop. This *flavor separation* is an essential simplification in our work.

For a given flavor, Eq. (1) ensures the equality of each experimental and theoretical moment  $M_n$  of  $R(s)$ ,

$$M_n^{\text{exp}} = M_n^{q\bar{q}} \quad (2)$$

defined by

$$M_n^{\text{exp}} = \frac{1}{S_0^{n+1}} \int_{4m_q^2}^{\infty} ds s^n R_{\text{exp}}(s),$$

$$M_n^{q\bar{q}} = \frac{1}{S_0^{n+1}} \int_{4m_q^2}^{\infty} ds s^n R_{q\bar{q}}(s)$$

(global duality). The contributions to  $M_n^{\text{exp}}$  can sometimes be divided into the discrete poles

$$M_n^{\text{pole}} = \frac{9\pi}{\alpha^2} \sum_{\text{poles}} \left( \frac{m_V^2}{S_0} \right)^{n+1} \frac{\Gamma_{q\bar{q}}}{m_V} \quad (3)$$

and the continuum starting at  $s = s_t$ ,

$$M_n^{\text{cont}} = \frac{1}{S_0^{n+1}} \int_{s_t}^{\infty} ds s^n R_{\text{exp}}(s). \quad (4)$$

This being the case, Eq. (2) can be cast into another form by dividing the integration range at

$s = s_t$ , yielding

$$M_n^{\text{pole}} = \frac{1}{S_0^{n+1}} \int_{4m_q^2}^{s_t} ds s^n R_{q\bar{q}}(s) + \frac{1}{S_0^{n+1}} \int_{s_t}^{\infty} ds s^n [R_{q\bar{q}}(s) - R_{\text{exp}}(s)]. \quad (5)$$

Assuming  $R_{q\bar{q}}$  is known to be dual to  $R_{\text{exp}}$  in the continuum range, the second integral can be neglected and Eq. (5) becomes

$$M_n^{\text{pole}} = \frac{1}{S_0^{n+1}} \int_{4m_q^2}^{s_t} ds s^n R_{q\bar{q}}(s) \quad (6)$$

(pole duality). This is the "pole" form of duality since  $M_n^{\text{pole}}$ , defined by Eq. (3), depends only on the discrete quarkonium states. This form of duality is expected to be useful only for heavy quarkonia where the continuum threshold is above the quark production threshold and the separation between discrete and continuum regions is clean.

## II. A SIMPLE DUAL MODEL FOR HEAVY QUARKONIA

In this section we consider a very simple model satisfying the duality sum rules. We will see that all of the gross features of the numerical solutions are present and that a heavy-quark mass is exactly equal to half the  $^3S_1$  ground-state energy. We assume the hadronic  $R_{\text{exp}}(s)$  consists of one state of mass  $m_V$  and a step function continuum plateau as shown in Fig. 3(a). The dual  $R_{q\bar{q}}(s)$  will be assumed to have the step form illustrated in Fig. 3(b). By Eqs. (2) and (3) the moments are

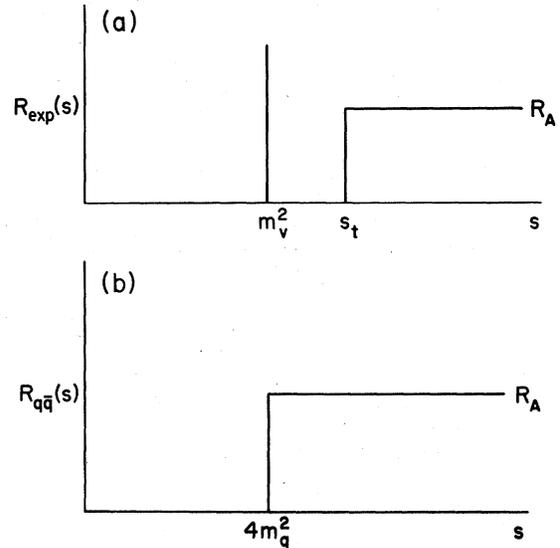


FIG. 3. (a) Simple dual-model "data." (b) Simple dual-model quark production.

given by

$$M_n^{\text{exp}} = \frac{9\pi}{\alpha^2} \left( \frac{m_V^2}{s_0} \right)^{n+1} \frac{\Gamma_{ee}}{m_V} + \frac{R_A}{s_0^{n+1}} [(\bar{s})^{n+1} - s_t^{n+1}], \quad (7)$$

$$M_n^{q\bar{q}} = \frac{R_A}{n+1} [(\bar{s})^{n+1} - (4m_q^2)^{n+1}].$$

Equating the moments yields the relation

$$\frac{9\pi}{\alpha^2} (m_V^2)^{n+1} \frac{\Gamma_{ee}}{M_V} = \frac{R_A}{n+1} [s_t^{n+1} - (4m_q^2)^{n+1}]. \quad (8)$$

If we now assume that  $s_t - 4m_q^2$  is much smaller than  $4m_q^2$ , the right-hand side of Eq. (8) can be expanded to yield

$$\frac{1}{n+1} [s_t^{n+1} - (4m_q^2)^{n+1}] \approx (4m_q^2)^{n+1} \left( \frac{s_t - 4m_q^2}{4m_q^2} \right). \quad (9)$$

The above assumption will be justified by our final result when the quark mass is large. Combining Eqs. (8) and (9) we obtain

$$\frac{9\pi}{\alpha^2} (m_V^2)^{n+1} \frac{\Gamma_{ee}}{m_V R_A} \approx (4m_q^2)^{n+1} \left( \frac{s_t - 4m_q^2}{4m_q^2} \right). \quad (10)$$

We conclude from Eq. (10) that

$$m_q = \frac{1}{2} m_V \quad (11)$$

and

$$s_t = 4m_q^2 + \frac{9\pi}{\alpha^2} \frac{m_V \Gamma_{ee}}{R_A}. \quad (12)$$

The approximation needed for the expansion in Eq. (9) requires that

$$\frac{s_t - 4m_q^2}{4m_q^2} = \frac{9\pi}{\alpha^2} \frac{\Gamma_{ee}}{m_V R_A} \ll 1. \quad (13)$$

It is well known<sup>7</sup> that the ratio  $\Gamma_{ee}/R_A$  is a universal constant for all known quarkonia ground states and that

$$\Gamma_{ee}/R_A \approx 3.6 \text{ keV}. \quad (14)$$

The inequality of Eq. (13) is then equivalent to

$$m_V \gg \frac{9\pi}{\alpha^2} \frac{\Gamma_{ee}}{R_A} \approx 1.9 \text{ GeV}. \quad (15)$$

Even in the case of charmonium, where  $m_V \approx 3.1$  GeV, the continuum threshold is expected to be, from Eq. (12),

$$\sqrt{s_t} \approx 3.9 \text{ GeV} \quad (16)$$

which is a reasonable estimate to the actual  $D\bar{D}$  threshold of 3.7 GeV, considering the simplicity of the model.

We can easily express our simple model in a pole-duality form. Because of our chosen forms for  $R_{\text{exp}}$  and  $R_{q\bar{q}}$  depicted in Fig. 3, the second integral in Eq. (5) is identically zero and pole duality of Eq. (6) is exactly valid.

### III. CHARM SECTOR

The global-duality relations of Eq. (2) have been previously analyzed<sup>4</sup> for charm production. In this section we will show that pole duality gives equivalent results. We begin by reviewing the global results.

Using the measured  $\psi$  and  $\psi'$  masses and leptonic widths<sup>8</sup> along with the continuum  $R(s)$  measurements of the SLAC group<sup>9</sup> the moments  $M_n^{\text{exp}}$  were computed.<sup>4</sup> With  $\bar{s} = 25 \text{ GeV}^2$  taken to lie above the resonance region and  $s_0 = 15 \text{ GeV}^2$  arbitrarily chosen such that  $M_n$  is comparable to  $M_{-n}$ , we plot these moments in Fig. 4. Assuming that the dual function  $R_{c\bar{c}}$  depends only on the quark velocity, we tried the four dual functions listed in Table I and illustrated in Fig. 4. These examples differ chiefly in the manner that the quark threshold is approached. As indicated in Table I and Fig. 4, the best agreement to the experimental moments occurs for the  $R_{c\bar{c}}(v)$  which rise close to threshold.<sup>10</sup> From the two best fits we find  $m_c \approx 1.47 \text{ GeV}$  and we note that all of the examples approach asymptotic limits  $R_A$  between 1.3 and 1.6.

We now repeat the above analysis using the pole

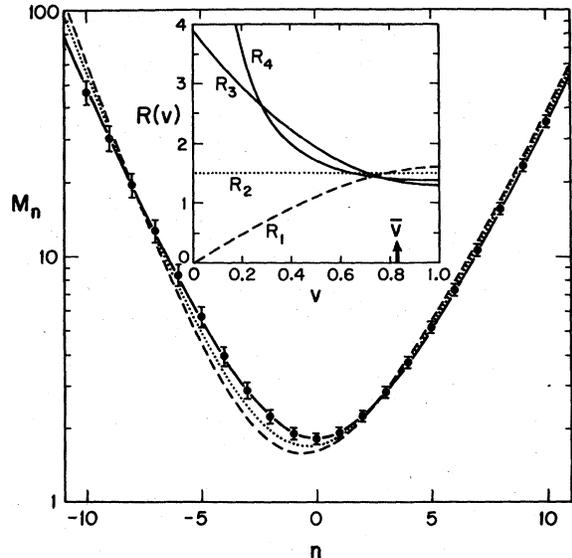


FIG. 4. Moments  $M_n$  of  $R_{\text{exp}}(s)$  as defined in Eq. (2) with  $s_0 = 15 \text{ GeV}^2$  and  $\bar{s} = 25 \text{ GeV}^2$ . The pole contribution (Ref. 8) of Eq. (3) uses  $\Gamma_{e^+e^-} = 4.8 \pm 0.6 \text{ keV}$  for the  $\psi$  (3.095) and  $\Gamma_{e^+e^-} = 2.1 \pm 0.3 \text{ keV}$  for the  $\psi'$  (3.684). The continuum part of Eq. (4) is based on the data of Ref. 9. The interpolating curves between the moments correspond to the various forms of  $R_{c\bar{c}}(v)$  in Table I. These forms are plotted in the insert graph as a function of quark velocity. The quantity  $\bar{v}$  corresponds to the cut-off  $\bar{s}$  by  $\bar{v} = (1 - 4m_c^2/\bar{s})^{1/2}$ .

TABLE I. Global-duality charm analysis; four quark production functions  $R_{c\bar{c}}(v)$  and their associated parameters  $R_A$ ,  $r$ , and  $m_c$  obtained by fitting Eq. (2) to the experimental moments shown in Fig. 4.

$R_{c\bar{c}}$	$R_A$	$r$	$m_c$ (GeV)	$\chi^2$
$R_1 \equiv R_A \frac{1}{2} v (3 - v^2)$	1.6		1.33	65.7
$R_2 \equiv R_A$	1.5		1.40	25.6
$R_3 \equiv R_A + r(1 - v)^2$	1.3	2.6	1.46	0.7
$R_4 \equiv R_A + \frac{r}{v} (1 - v)^2$	1.4	0.7	1.49	0.3

duality of Eq. (6). In this case the pole contribution in Eq. (3) comes from the  $\psi(3.095)$  and  $\psi'(3.684)$  states alone. These moments are depicted in Fig. 5. In this figure we note that the larger negative moments are identical to the global moments in Fig. 4, as expected.

The right-hand side of Eq. (6) is evaluated with two  $R_{c\bar{c}}$  forms.

(i)  $R_{c\bar{c}} = R_A \frac{1}{2} v (3 - v^2) \theta(s - 4m_c^2)$ ; this is just the one-loop result normalized to an asymptotic plateau of  $R_A$ . The quark velocity is given by  $v = (1 - 4m_c^2/s)^{1/2}$ ;

(ii)  $R_{c\bar{c}} = R_A [1 + r(4m_c^2/s)^2] \theta(s - 4m_c^2)$ . In this case  $R_{c\bar{c}}$  can rise or fall near threshold depending on the sign of  $r$ .

We adjust the parameters  $m_c$ ,  $r$ , and  $s_t$  for an optimal fit to the moments as summarized in Table II and plotted in Fig. 5. The plateau value  $R_A$  is poorly determined and has been fixed at  $R_A = 1.44$ , appropriate for a charge- $\frac{2}{3}$  quark and a small gluon correction. We see that the one-loop form does not fit well, as in the global case, and the corresponding quark mass  $m_c = 1.33$  GeV is the same as before.

The alternative dual function represented by the solid curves in Fig. 5 fits the moments well and corresponds to a quark mass  $m_c$  and continuum threshold of

$$\begin{aligned} m_c &= 1.44 \pm 0.02 \text{ GeV}, \\ \sqrt{s_t} &= 3.78 \pm 0.01 \text{ GeV}. \end{aligned} \quad (17)$$

TABLE II. Pole-duality charm analysis; two quark production functions  $R_{c\bar{c}}$  are compared to the experimental moments of Fig. 5.

$R_{c\bar{c}}$	$R_A$ (fixed)	$r$	$m_c$ (GeV)	$\sqrt{s_t}$ (GeV)	$\chi^2$
$R_A \frac{1}{2} v (3 - v^2)$	1.44		1.32	$3.88 \pm 0.02$	87
$R_A [1 + r(4m_c^2/s)^2]$	1.44	$0.66 \pm 0.15$	$1.44 \pm 0.02$	$3.78 \pm 0.02$	1.5

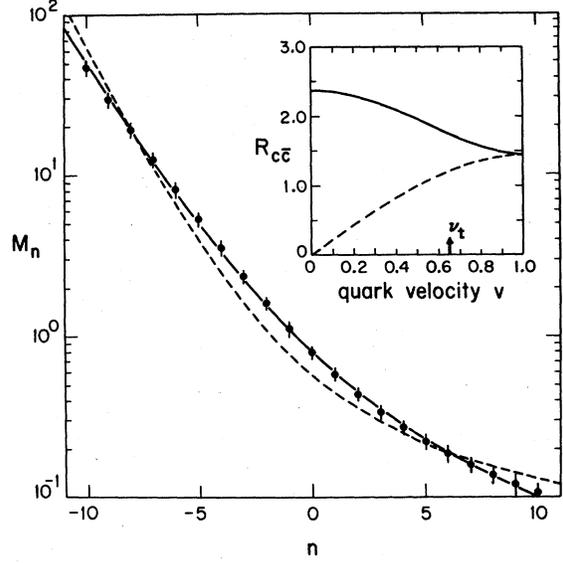


FIG. 5. Charm pole moments (see Fig. 4 caption) and two examples of dual functions  $R_{c\bar{c}}$ . The fitted parameters are given in Table II. The quantity  $v_t = (1 - 4m_c^2/s_t)^{1/2}$ .

The quark mass is consistent with the best-fit global value given in Table I and the continuum threshold is in excellent agreement with the observed  $D\bar{D}$  threshold<sup>8</sup>

$$\sqrt{s_{D\bar{D}}} \approx 3.74 \text{ GeV}. \quad (18)$$

Our conclusions for the charm sector are that pole and global duality give nearly identical results and that the pole-duality form accurately predicts the open charm threshold position. Both duality formulations yield a quark mass about  $m_c \approx 1.45$  GeV and a duality function which rises somewhat near quark threshold.

#### IV. $b$ -QUARK SECTOR

The pole-duality method was specifically designed for  $q\bar{q}$  systems in which only the sharp states have been measured. This is at present the case for the  $b\bar{b}$   $\Upsilon$  states found at Fermilab,<sup>11</sup> verified at DORIS,<sup>12</sup> and now more completely investigated at CESR.<sup>13</sup> Using the combined DORIS

TABLE III. The  $\Upsilon$  states obtained by combining the data of DORIS (Ref. 12) and CESR (Ref. 13) experiments.

$\Upsilon$ state	Mass (GeV)	Leptonic width (keV)
$\Upsilon$	9.46	$1.28 \pm 0.27$
$\Upsilon'$	10.02	$0.56 \pm 0.18$
$\Upsilon''$	10.32	$0.45 \pm 0.14$
$\Upsilon'''$	10.59	$0.26 \pm 0.15$

and CESR results we assume the following set of  $\Upsilon$  states given in Table III. The last state  $\Upsilon'''$  was recently reported<sup>13</sup> to have a width larger than instrumental, thus lying above  $B\bar{B}$  threshold.

We can now repeat the pole-duality analysis of Section III for the  $b$ -quark case. Using the  $\Upsilon$ ,  $\Upsilon'$ , and  $\Upsilon''$  parameters from Table III, we compute the pole moments of Eq. (3) which are plotted in Fig. 6. For these moments we have chosen the dimensional parameter  $s_0 = 120 \text{ GeV}^2$ . As in the charm case, we try two quark production functions in the right-hand side of Eq. (6):

$$(i) R_{b\bar{b}} = R_A \frac{1}{2} v (3 - v^2) \theta(s - 4m_b^2),$$

(ii)  $R_{b\bar{b}} = R_A [1 + r(4m_b^2/s)^2] \theta(s - 4m_b^2)$ , where in this case the  $b$ -quark velocity  $v = (1 - 4m_b^2/s)^{1/2}$ . The plateau value  $R_A$  has been fixed at  $R_A = 0.37$ , a reasonable value for a charge- $\frac{1}{3}$  quark and small gluon correction. The parameters  $m_b$ ,  $r$ , and  $s_t$  are varied to optimize the fit to the moments and the results given in Table 4. The  $b$ -quark mass and continuum threshold are found to be

$$\begin{aligned} m_b &= 4.58 \pm 0.08 \text{ GeV}, \\ \sqrt{s_t} &= 10.44 \pm 0.10 \text{ GeV}. \end{aligned} \quad (19)$$

The quark mass should be compared to half the  $\Upsilon$  mass which is 4.73 GeV. Just as in the charm case the quark mass lies slightly lower than half the ground-state energy. The predicted continuum threshold  $\sqrt{s_t}$  lies between the sharp  $\Upsilon''$  (10.32) and the  $\Upsilon'''$  (10.59) which is above  $B\bar{B}$  threshold.

## V. CONCLUSIONS

Our conclusions can be enumerated as follows:

(i) Pole and global duality give equivalent re-

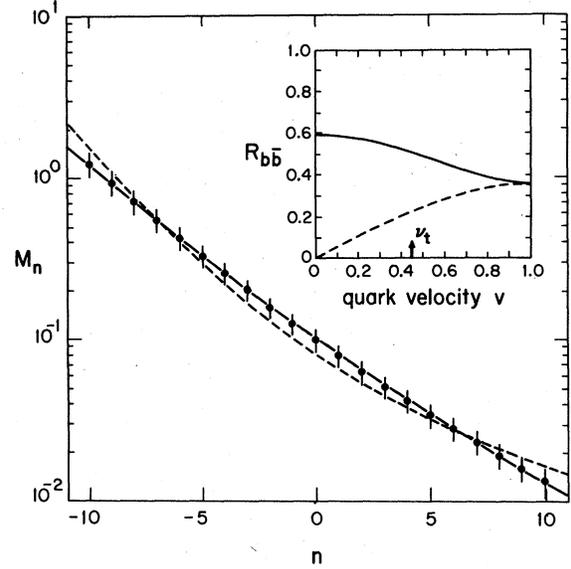


FIG. 6.  $b$ -flavor pole moments using Eq. (3) and the  $\Upsilon$  masses and leptonic widths of Table III. The fitted parameters are given in Table IV. The quantity  $v_t$  corresponds to the continuum threshold quark velocity  $v_t = (1 - 4m_b^2/s_t)^{1/2}$ .

sults for charmonium where complete measurements have been made

- (ii) Pole duality accurately predicts the continuum threshold for both  $c$ - and  $b$ -flavor production.
- (iii) The  $c$ - and  $b$ -quark masses lie slightly below one half the ground state  $^3S_1$  energy.
- (iv) The quark production function  $R_{q\bar{q}}$  seems to gently rise as threshold is approached from above. The shapes of  $R_{c\bar{c}}$  and  $R_{b\bar{b}}$  are consistent.

This last point is somewhat surprising, but it can be verified in a crude way by expressing the sharp states as boxes extending in energy to the next state. This is shown in Fig. 7 for the  $c\bar{c}$  and  $b\bar{b}$  bound states. We see from this figure a definite rise in the average  $R$  as  $s$  decreases for both the  $c\bar{c}$  and  $b\bar{b}$  states. The curves correspond to the best fits in Tables II and IV.

We would like to emphasize that our analysis does not assume anything from QCD other than the existence of a nearly flat asymptotic region

TABLE IV. Pole-duality  $b$ -quark analysis; two quark production functions  $R_{b\bar{b}}$  are compared to the experimental moments of Fig. 6.

$R_{b\bar{b}}$	$R_A$ (fixed)	$r$	$m_b$ (GeV)	$\sqrt{s_t}$ (GeV)	$\chi^2$
$R_A \frac{1}{2} v (3 - v^2)$	0.37		$4.25 \pm 0.02$	$10.77 \pm 0.05$	16.0
$R_A [1 + r(4m_b^2/s)^2]$	0.37	$0.43 \pm 0.4$	$4.58 \pm 0.08$	$10.44 \pm 0.10$	1.5

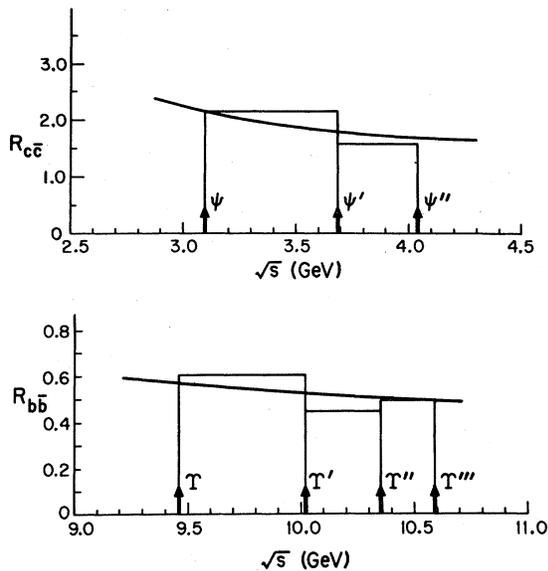


FIG. 7. Smeared pole contributions to  $R(s)$  for charmonium and  $T$ . The curves represent the best fits to the moments given in Tables II and IV.

above the resonances. In this sense our analysis is quite general in that no relation between the asymptotic and threshold regions is required. This generality has been achieved by using a range of moment sum rules. In the recent works by Hagiwara and Sanda, Gounaris, and Greco *et al.*,<sup>14</sup> specific QCD properties of the dual amplitude were assumed. In particular, the first order in  $\alpha_s$  expression for the dual  $R$  has been used. The fact that some of these authors obtain similar values for the charm-quark mass perhaps reflects the basic correctness of the theory, although it should be pointed out that none of the above authors<sup>14</sup> take into account the absorptive part of  $\alpha_s$  beginning at  $s=0$  or the Landau ghost problem inherent in the first-order QCD result.

#### ACKNOWLEDGMENTS

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