Photon circular polarization as a neutral current effect in bremsstrahlung and pair production

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Neutral-current induced photon circular polarization effects in bremsstrahlung and pair production are calculated to the first order in the weak interaction. The effects are found to be of the same order of magnitude as corresponding lepton-helicity effects. Numerical results are presented for the Weinberg-Salam model. A brief discussion of the pure electromagnetic photon circular polarization effects is given.

I. INTRODUCTION

The existence of neutral currents, which has been established in neutrino scattering[1] and in electron-deuteron scattering,[2] gives rise to parity-violating effects, e.g., correlations between particle helicities and momenta which are not present in pure electromagnetic cross sections. Theoretical studies of parity-violating effects of polarized leptons have been made for deep-inelastic scattering[3] and for pair production.[4] Parity-violating effects involving the photon helicity would be expected to be present in processes involving high-energy photons, and would be expected to be of the same order of magnitude as lepton-helicity parity-violating effects. In the present paper we study the neutral-current produced circular polarization of bremsstrahlung (Sec. II) and the corresponding asymmetry effects in lepton pairs, produced by circularly polarized photons (Sec. III). In Sec. IV numerical results are presented together with a brief discussion of the results. In a recent paper Yokoo *et al.*[5] give pair production asymmetries which are in agreement with our results.

The circular polarization of bremsstrahlung due to neutral currents has been calculated in the static approximation by Jarlskog and Salomonson.[6] Their formulas, valid for low energies (below 50 MeV incoming electron energy), are in agreement with our results in this limit.

We make the usual approximation of neglecting the lepton mass for the high energies and large momentum transfers involved in the processes. We further neglect interactions other than Bethe-Heitler-type electromagnetic and weak interactions. Interactions in which the real photon is emitted or absorbed by the nucleon are expected to give small corrections to the cross sections.[4]

II. CIRCULAR POLARIZATION OF BREMSSTRAHLUNG

The cross section for deep-inelastic bremsstrahlung in lepton-nucleon collisions

$$l(p_1)+N(P)\to l(p_2)+\gamma(k)+X(P')$$

is to the first order in the weak coupling in the Weinberg-Salam model given by

$$d^6\sigma=\frac{\alpha^3}{\pi^2}\frac{d^3k}{2\omega}\frac{d^3p_2}{2E_2}\frac{1}{Q^4}\frac{M}{p_1\cdot P}\times\left(L^{\gamma\gamma}_{\mu\nu}W^{\mu\nu}-\frac{1}{2\sin^2 2\theta_W}\frac{Q^2}{Q^2-M_Z{}^2}L^{\gamma Z}_{\mu\nu}R^{\mu\nu}\right), \tag{1}$$

where $Q=P-P'=k+p_2-p_1$ is the momentum transfer and M the target mass. θ_W is the Weinberg angle and M_Z the mass of the neutral vector boson. The energies and momenta of the particles are defined by $p_1=(E_1,\vec{p}_1)$, $p_2=(E_2,\vec{p}_2)$, and $k=(\omega,\vec{k})$ and we use the metric and conventions of Bjorken and Drell.[7]

The electromagnetic nucleon structure tensor has the form[8]

$$W^{\mu\nu}=-W_1\left(g^{\mu\nu}-\frac{Q^\mu Q^\nu}{Q^2}\right)+\frac{W_2}{M^2}\left(P^\mu-\frac{P\cdot Q}{Q^2}Q^\mu\right)\left(P^\nu-\frac{P\cdot Q}{Q^2}Q^\nu\right).$$

Since $L^{\gamma\gamma}_{\mu\nu}$ is gauge invariant, $Q_\mu L^{\gamma\gamma}_{\mu\nu}=0$, and $W^{\mu\nu}$ is effectively given by

$$W^{\mu\nu}=-g^{\mu\nu}W_1+\frac{P^\mu P^\nu}{M^2}W_2\,. \tag{2}$$

The lepton tensor for the pure electromagnetic term is for a polarized photon with polarization vector e^μ, when we sum and average over electron polarizations,

$$L^{\gamma\gamma}_{\mu\nu}=\tfrac{1}{2}\,\mathrm{Tr}\,\not{p}_2 M^{\sigma}_{\mu}e^*_\sigma\not{p}_1\bar{M}^{\omega}_{\nu}e_\omega \tag{3}$$

with $M_{\mu\sigma}$ the gauge-invariant Compton-type amplitude

$$M^{\mu\sigma}=\gamma^\mu\frac{\not{p}_1-\not{k}}{2p_1\cdot k}\gamma^\sigma-\gamma^\sigma\frac{\not{p}_2+\not{k}}{2p_2\cdot k}\gamma^\mu\ , \tag{4}$$

which clearly satisfies $\bar{u}_2M^{\mu\sigma}u_1k_\mu=\bar{u}_2M^{\mu\sigma}u_1Q_\mu=0$.

In the electromagnetic-weak interference term $L^{\gamma Z}_{\mu\nu}R^{\mu\nu}$, the nucleon structure tensor has the form[9]

$$R_{\mu\nu}=-g_{\mu\nu}R_1+\frac{P_\mu P_\nu}{M^2}R_2+i\epsilon_{\mu\nu\alpha\beta}\frac{P^\alpha Q^\beta}{2M^2}R_3\,, \tag{5}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor, antisymmetric in all indices, with $\epsilon_{0123}=1$. As in Eq. (2), we have dropped terms proportional to Q^μ. The corresponding lepton tensor is given by

$$L^{\gamma Z}_{\mu\nu}=\tfrac{1}{2}\,\mathrm{Tr}(v-a\gamma_5)\not{p}_2 M^\sigma_{\ \mu} e^*_\sigma \not{p}_1 \bar{M}^{\omega}_{\ \nu} e_\omega\,, \tag{6}$$

where v and a are the lepton weak-interaction vector and axial-vector coupling constants, respectively. In the Weinberg-Salam theory they are given by

$$v=4\sin^2\theta_W-1,\quad a=-1. \tag{7}$$

When we specialize to circular photon polarization with $\vec{k}$ along the 3 axis, the circular-polarization parts of $L^{\gamma\gamma}_{\mu\nu}$ and $L^{\gamma Z}_{\mu\nu}$ may be separated out by means of the relation[10]

$$e^*_\sigma e_\omega=-\tfrac{1}{2}g_{\sigma\omega}-\tfrac{1}{2}i\xi\epsilon_{0\sigma\omega 3}\,,$$

where ξ is the photon helicity, $\xi=+1$ for right-handed (R) and $\xi=-1$ for left-handed (L) circular polarization. One may also write

$$e^*_\sigma e_\omega=-\tfrac{1}{2}g_{\sigma\omega}+\frac{i}{2\omega}\epsilon_{\sigma\omega\alpha\beta}\xi^\alpha k^\beta\,, \tag{8}$$

where we have introduced a helicity four-vector $\xi^\alpha=(0,\vec{\xi})=(0,0,0,\xi)$. It should be noted that we have used gauge invariance to obtain $g_{\sigma\omega}$ in Eq. (8). In this way we find

$$L^{\gamma\gamma}_{\mu\nu}=-L^s_{\mu\nu}-\xi L^a_{\mu\nu} \tag{9a}$$

and

$$L^{\gamma Z}_{\mu\nu}=-vL^s_{\mu\nu}+aL^{5a}_{\mu\nu}-\xi(vL^a_{\mu\nu}-aL^{5s}_{\mu\nu})\,, \tag{9b}$$

where we define the tensors symmetric in (μ,ν)

$$L^s_{\mu\nu}=\tfrac{1}{4}\,\mathrm{Tr}\not{p}_2 M_{\mu\sigma}\not{p}_1\bar{M}^\sigma_{\ \nu}\,,\qquad L^{5s}_{\mu\nu}=\tfrac{1}{4}i\,\mathrm{Tr}\gamma_5\not{p}_2 M^\sigma_{\ \mu}\not{p}_1\bar{M}^\omega_{\ \nu}\epsilon_{0\sigma\omega 3} \tag{10}$$

and antisymmetric in (μ,ν)

$$L^a_{\mu\nu}=\tfrac{1}{4}i\,\mathrm{Tr}\not{p}_2 M^\sigma_{\ \mu}\not{p}_1\bar{M}^\omega_{\ \nu}\epsilon_{0\sigma\omega 3}\,,\qquad L^{5a}_{\mu\nu}=\tfrac{1}{4}\,\mathrm{Tr}\gamma_5\not{p}_2 M_{\mu\sigma}\not{p}_1\bar{M}^\sigma_{\ \nu}\,. \tag{11}$$

The cross section for circularly polarized bremsstrahlung is then obtained as

$$\begin{aligned}d^6\sigma=&\frac{1}{2}\frac{\alpha^3}{\pi^2}\frac{d^3k}{2\omega}\frac{d^3p_2}{2E_2}\frac{1}{Q^4}\frac{M}{(p_1\cdot P)}\frac{1}{(p_1\cdot k)(p_2\cdot k)}\\&\times\Big[L_1W_1-L_2W_2-\frac{1}{2\sin^2 2\theta_W}\frac{Q^2}{Q^2-M_Z{}^2}\Big(vL_1R_1-vL_2R_2-\frac{a}{2M}L_3R_3\Big)\\&+\xi\frac{1}{2\sin^2 2\theta_W}\frac{Q^2}{Q^2-M_Z{}^2}\Big(aL^5_1R_1-aL^5_2R_2-\frac{v}{2M}L^5_3R_3\Big)\Big].\end{aligned} \tag{12}$$

Here the functions L_1 and L_2 were already obtained for pair production by Drell and Walecka[11] and L_3 by Mikaelian and Oakes,[4] while the functions L^5_1, L^5_2, L^5_3 giving the photon circular polarization are new terms.

According to Eqs. (1), (2), (5), and (9) we find

$$\begin{aligned}&L_1=2(p_1\cdot k)(p_2\cdot k)L^{s\mu}_{\ \mu},\quad L_2=2(p_1\cdot k)(p_2\cdot k)\frac{P^\mu P^\nu}{M^2}L^s_{\mu\nu},\\&L_3=-\frac{2i}{M}(p_1\cdot k)(p_2\cdot k)\epsilon_{\mu\nu\alpha\beta}L^{5a\mu\nu}P^\alpha Q^\beta\,,\\&L^5_1=2(p_1\cdot k)(p_2\cdot k)L^{5s\mu}_{\ \mu},\quad L^5_2=2(p_1\cdot k)(p_2\cdot k)\frac{P^\mu P^\nu}{M^2}L^{5s}_{\mu\nu},\\&L^5_3=-\frac{2i}{M}(p_1\cdot k)(p_2\cdot k)\epsilon_{\mu\nu\alpha\beta}L^{a\mu\nu}P^\alpha Q^\beta\,.\end{aligned} \tag{13}$$

For convenience we keep the definition of L_1, L_2, and L_3 of Mikaelian and Oakes[4] and we find for bremsstrahlung

$$L_1=4[(p_1\cdot Q)^2+(p_2\cdot Q)^2],\quad L_1^5=4[(p_1\cdot Q)^2-(p_2\cdot Q)^2],$$

$$L_2=\frac{1}{2}L_1+\frac{2}{M^2}\{Q^2[(p_1\cdot P)^2+(p_2\cdot P)^2]-2(p\cdot Q)[(p_1\cdot P)(p_1\cdot Q)+(p_2\cdot P)(p_2\cdot Q)]\},$$

$$L_2^5=\frac{1}{2}L_1^5+\frac{2}{M^2}\{Q^2[(p_1\cdot P)^2-(p_2\cdot P)^2]-2(P\cdot Q)[(p_1\cdot P)(p_1\cdot Q)-(p_2\cdot P)(p_2\cdot Q)]\},\tag{14}$$

$$L_3=\frac{P\cdot Q}{M}L_1^5-\frac{4Q^2}{M}[(p_1\cdot P)(p_1\cdot Q)-(p_2\cdot P)(p_2\cdot Q)],$$

$$L_3^5=\frac{P\cdot Q}{M}L_1-\frac{4Q^2}{M}[(p_1\cdot P)(p_1\cdot Q)+(p_2\cdot P)(p_2\cdot Q)].$$

The circular polarization of bremsstrahlung to the first order in the weak interaction is then

$$P_{\mathrm{ph}}=\frac{d^6\sigma_R-d^6\sigma_L}{d^6\sigma_R+d^6\sigma_L}$$

$$=\frac{(1/2\sin^2 2\theta_W)[Q^2/(Q^2-M_Z^2)][aL_1^5R_1-aL_2^5R_2-(v/2M)L_3^5R_3]}{L_1W_1-L_2W_2-(1/2\sin^2 2\theta_W)[Q^2/(Q^2-M_Z^2)][vL_1R_1-vL_2R_2-(a/2M)L_3R_3]}.\tag{15}$$

For momentum transfers such that $|Q^2|\ll M_Z^2$, we get the simple practical formula

$$P_{\mathrm{ph}}=-\frac{G_F}{\sqrt{2}}\frac{|Q^2|}{4\pi\alpha}\frac{L_1^5R_1-L_2^5R_2+(v/2M)L_3^5R_3}{L_1W_1-L_2W_2},\tag{16}$$

where we have used $a=-1$ and

$$\frac{Q^2}{2\sin^2 2\theta_W M_Z^2}=\frac{G_F}{\sqrt{2}}\frac{Q^2}{4\pi\alpha},$$

where[12] $G_F=1.166\times10^{-5}\ \mathrm{GeV}^{-2}$. Actually since $v=4\sin^2\theta_W-1$ is known to be small[1] the term proportional to v represents usually a small correction. Equation (16) shows that in the bremsstrahlung process the photon circular polarization due to neutral currents is of the same order of magnitude $P_{\mathrm{ph}}\sim-10^{-4}[Q\ (\mathrm{GeV})]^2$ as the corresponding lepton longitudinal polarization.[4]

III. ASYMMETRY EFFECTS IN PAIR PRODUCTION

For pair production the cross section to first order in the weak interaction corresponding to Eq. (12) is obtained in the usual manner by replacing p_1, p_2, k, ξ by $-p_+$, p_-, $-k$, $-\xi$, respectively, and

$$(d^3k/2\omega)(d^3p_2/2E_2)M/(p_1\cdot P)\ \text{by}\ (d^3p_+/2E_+)(d^3p_-/2E_-)M/(k\cdot P).$$

The result is

$$d^6\sigma=\frac{\alpha^3}{\pi^2}\frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\frac{1}{Q^4}\frac{M}{(k\cdot P)}\frac{1}{(k\cdot p_+)(k\cdot p_-)}\left[L_1W_1-L_2W_2-\frac{1}{2\sin^2 2\theta_W}\frac{Q^2}{Q^2-M_Z^2}\left(vL_1R_1-vL_2R_2-\frac{a}{2M}L_3R_3\right)\right.$$
$$\left.-\xi P_{\mathrm{ph}}\frac{1}{2\sin^2 2\theta_W}\frac{Q^2}{Q^2-M_Z^2}\left(aL_1^5R_1-aL_2^5R_2-\frac{v}{2M}L_3^5R_3\right)\right],\tag{17}$$

where P_{ph} is the magnitude of the photon polarization. It should be noted that a factor $\frac{1}{2}$ in Eq. (12) is missing in Eq. (17) since in pair production the circularly polarized photon is in the initial state.

The lepton functions of Eq. (17) corresponding to Eq. (14) are for pair production

$$L_1 = 4[(p_+ \cdot Q)^2 + (p_- \cdot Q)^2], \quad L_1^5 = 4[(p_+ \cdot Q)^2 - (p_- \cdot Q)^2],$$

$$L_2 = \frac{1}{2}L_1 + \frac{2}{M^2}\{Q^2[(p_+ \cdot P)^2 + (p_- \cdot P)^2] - 2(P \cdot Q)[(p_+ \cdot P)(p_+ \cdot Q) + (p_- \cdot P)(p_- \cdot Q)]\},$$

$$L_2^5 = \frac{1}{2}L_1^5 + \frac{2}{M^2}\{Q^2[(p_+ \cdot P)^2 - (p_- \cdot P)^2] - 2(P \cdot Q)[(p_+ \cdot P)(p_+ \cdot Q) - (p_- \cdot P)(p_- \cdot Q)]\}, \tag{18}$$

$$L_3 = \frac{P \cdot Q}{M}L_1^5 - \frac{4Q^2}{M}[(p_+ \cdot P)(p_+ \cdot Q) - (p_- \cdot P)(p_- \cdot Q)],$$

$$L_3^5 = \frac{P \cdot Q}{M}L_1 - \frac{4Q^2}{M}[(p_+ \cdot P)(p_+ \cdot Q) + (p_- \cdot P)(p_- \cdot Q)].$$

The asymmetry in pair production due to the reversal of the photon polarization is to first order in the weak interaction

$$A = \frac{d^6\sigma_R - d^6\sigma_L}{d^6\sigma_R + d^6\sigma_L}$$

$$= -P_{\rm ph}\,\frac{(1/2\sin^2 2\theta_W)[Q^2/(Q^2 - M_Z^2)][aL_1^5R_1 - aL_2^5R_2 - (v/2M)L_3^5R_3]}{L_1W_1 - L_2W_2 - (1/2\sin^2 2\theta_W)[Q^2/(Q^2 - M_Z^2)][vL_1R_1 - vL_2R_2 - (a/2M)L_3R_3]}\,. \tag{19}$$

As for bremsstrahlung for $|Q^2| \ll M_Z^2$, the formula simplifies to

$$A = P_{\rm ph}\frac{G_F}{\sqrt{2}}\frac{|Q^2|}{4\pi\alpha}\frac{L_1^5R_1 - L_2^5R_2 + (v/2M)L_3^5R_3}{L_1W_1 - L_2W_2}. \tag{20}$$

IV. DISCUSSION: NUMERICAL RESULTS

In evaluating the bremsstrahlung polarization Eqs. (15), (16) and the pair production asymmetry Eqs. (19), (20) we use the value of $\sin^2\theta_W = 0.230 \pm 0.015$[1] which gives $v = -0.08$. We further use the quark description of the structure functions as given by Barger and Phillips[13] and Mikaelian and Oakes.[4] Numerical results are presented in Figs. 1–3 for a proton target.

We finally discuss the pure electromagnetic photon circular polarization effects in bremsstrahlung and pair production. For unpolarized leptons and target particle the only possible dependence on the photon circular polarization is through a term $\vec{\xi} \cdot \vec{p}_1 \times \vec{p}_2$ for pure electromagnetic parity conserving bremsstrahlung. The cross section is thus of the form

$$d^6\sigma_{\rm em} = d^6\sigma_0[1 + \alpha f(p_1, p_2, k, P)\vec{\xi} \cdot \hat{\vec{p}}_1 \times \hat{\vec{p}}_2],$$

where $d^6\sigma_0$ is the electromagnetic cross section for unpolarized photons. We have here taken into account that $\vec{\xi} \cdot \vec{p}_1 \times \vec{p}_2$ is odd under time reversal, thus time-reversal invariance requires αf to be proportional to the imaginary part of the bremsstrahlung amplitude, and accordingly the contribution comes from two-photon exchange which accounts for the factor α. The photon polarization

$$P_{\rm ph,\,em} = \alpha f(p_1, p_2, k, P)\hat{\vec{k}} \cdot \hat{\vec{p}}_1 \times \hat{\vec{p}}_2 \tag{21}$$

has an angular dependence radically different from that of the neutral-current induced photon

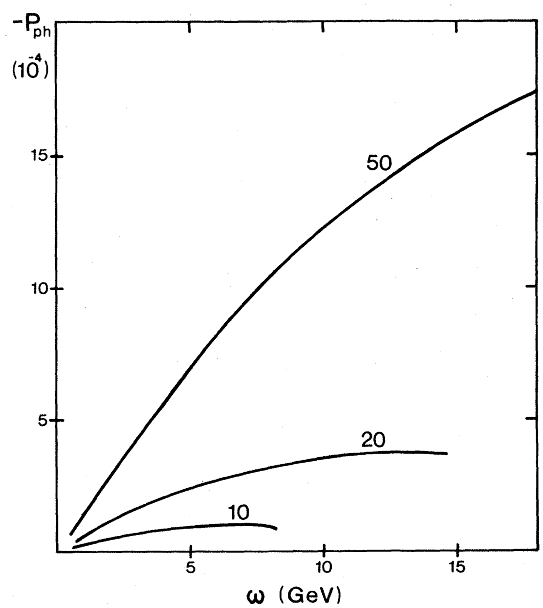

FIG. 1. Bremsstrahlung photon circular polarization $P_{\rm ph}$ in units of 10^{-4} for a proton target as a function of the photon energy ω for $\theta_1 = 10°$, $\theta_2 = 20°$ and the angle between the $\vec{p}_1, \vec{k}$ and $\vec{p}_2, \vec{k}$ planes, $\phi = 0°$. Numbers attached to the curves give the initial energies, E_1 in GeV. $E_2 = E_1/10$.

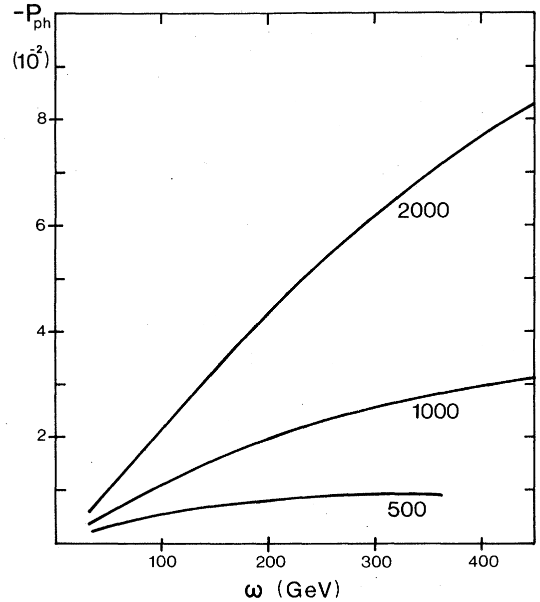

FIG. 2. Same as Fig. 1 except that $\theta_1 = 2°$, $\theta_2 = 4°$, and that P_{ph} is in units of 10^{-2}.

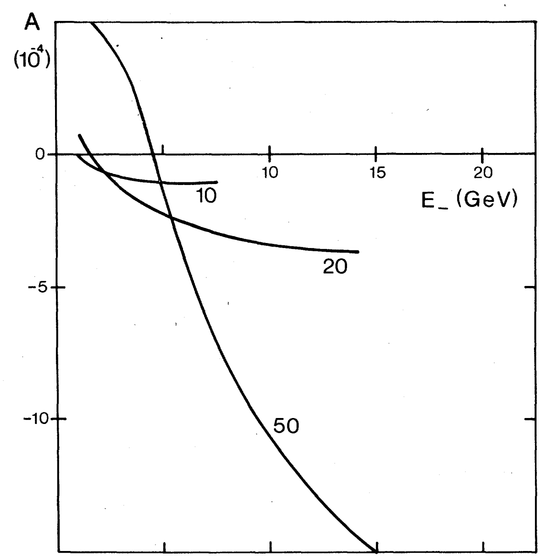

FIG. 3. Lepton-pair production asymmetry A in units of 10^{-4} for a proton target as a function of the electron energy E_- for $\theta_+ = \theta_- = 10°$ and $\phi = 180°$. Numbers attached to the curves give the photon energies ω in GeV $E_+ = \omega/10$.

polarization Eq. (15). The magnitude of f may be estimated from calculations of pair production in a static potential[14] which gives $f = [Q^2/(Q^2 + \omega^2)]F(\theta)$, where $F(\theta)$ is a function of the angles involved of the order 1. A corresponding result is obtained for pair production,

$$A_{em} = P_{ph}\alpha \frac{Q^2}{Q^2 + \omega^2} F(\theta) \hat{\vec{k}} \cdot \hat{\vec{p}}_+ x \hat{\vec{p}}_- . \qquad (22)$$

On the basis of these estimates we conclude that for very high energies and large momentum transfers, the neutral-current effects dominate over the electromagnetic effects. For values of the momentum transfer such that $|Q^2| \ll M_Z^2$ on the other hand the electromagnetic effects may be very large compared to the neutral-current effects. Because of their different angular and energy dependences, it seems, however, reasonable to assume that also in this case the two effects can be separated experimentally.

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