PHYSICAL REVIEW D VOLUME 22, NUMBER 8 15 OCTOBER 1980

Dynamical theory of spontaneous breakdown for chiral-invariant quantum chromodynamics: Resolution of the U(1) problem

Adrian N. Patrascioiu and Michael D. Scadron
Physics Department, University of Arizona, Tucson, Arizona 85721
(Received 31 March 1980)

We show how Nambu's dynamical theory of spontaneous breakdown can be applied to the quantum-chromodynamic pion and kaon but not to the U(1) η_0. We therefore suggest that the η_0 is not a Nambu-Goldstone boson.

I. INTRODUCTION

The theory of quantum chromodynamics (QCD) describing the interactions between quarks and gluons is rapidly becoming accepted as the correct theory underlying the strong interactions of the hadrons. While the large-q^2 structure of QCD now has a firm foundation based on parton scaling and renormalization-group scaling violations, the low-q^2 chiral-invariant and chiral-breaking theory of Nambu[1]-Goldstone[2] pseudoscalar bosons in a QCD context is somewhat in disarray. The origin of this confusion is the "U(1) problem": QCD appears to extend the current-algebra–PCAC (partial conservation of arial-vector current) program so successful for $I=1$ pions and $I=\frac{1}{2}$ kaons to the $I=0$ η and η' mesons, where it may not be in good agreement with data. More specifically QCD "naively" appears to require the heavy U(1) singlet $\eta_0(\sim 900)$ to be a Nambu-Goldstone boson, not a welcome result.

In an attempt to resolve this U(1) problem, nonconventional ideas have been tried which have implications for the pion and kaon as well. In particular, nonperturbative instanton effects have been invoked to resolve the U(1) problem[3] and thereafter have been employed to explain quark confinement[4] and spontaneous breakdown for the π and K.[5] But technical problems cast doubt about the role and the effect of instantons in QCD.[6,7] Thus we shall ignore instantons in favor of a more conventional perturbative approach towards spontaneous breakdown—still nonperturbative when one sums over an infinite set of graphs. We have, however, nothing to say about confinement.

Our theory is based upon the original dynamical perturbative summation analysis of Nambu and Jona-Lasinio[1] for the four-fermion chiral-invariant Lagrangian, but stated now in the language of the vector-gluon quark (chiral-invariant) coupling of QCD. First, in Sec. II we consider the dynamical spontaneous breakdown of chiral symmetry for non-Abelian SU(3) flavor currents and QCD color-invariant couplings. Following Ref. 8 we show that the formal Goldstone theorem[2] has a dynamical realization based upon the link between the (nonperturbative) axial Ward identities and quark-antiquark Bethe-Salpeter equations. Then in Sec. III we suggest that this dynamical realization of the Goldstone theorem *cannot* be extended to the U(1) flavor axial-vector current. In the absence of the U(1) anomaly, the $I=0$ axial-vector current would indeed spontaneously generate a massless η_0, just as in the π, K, and η_8 cases. But the AVV U(1) anomaly does exist; it breaks the theorem and is the dynamical origin of the η_0 mass in the chiral limit.

To demonstrate that this dynamical realization of the Goldstone theorem makes sense in *all* cases in the real (chiral-breaking) world, in Sec. IV we review the Hamiltonian formulation of η-η' mixing in the current quark basis.[9] Stated in $\partial\cdot A$ language, this successful phenomenology corresponds to m_{η_0} being generated by the U(1) anomaly and leading to a U(1) decay constant of[10] $f_0 \sim f_\pi$ in the chiral limit, also in agreement with data.

Armed with this theoretical and phenomenological picture of spontaneous breakdown and the non-Goldstone nature of the η_0, in Sec. V we investigate the Goldstone[2] approach to $\partial\cdot A_0$ and the anomaly. We suggest that the dynamical generation of the η_0 mass via the anomaly does not conflict with the general Goldstone theorem, at least not in solvable two-dimensional field-theory models. Finally in Sec. VI we comment upon other aspects of the U(1) problem, including the Brandt-Preparata $\eta_{3\pi}$ problem[11] and the Glashow[12]-Weinberg[12]-Crewther[13] vacuum Ward-identity problem. We then summarize the situation in Sec. VII.

II. DYNAMICAL SPONTANEOUS BREAKDOWN FOR QCD

The general Goldstone theorem related to the charge Q_5 associated with a conserved (axial-vector) current states that if $Q_5|\text{vac}\rangle \neq 0$, then there must exist a massless Goldstone boson in the theory. However, our understanding of non-Abelian field theories such as QCD does not always make this theorem transparent. On the other hand, the alternative dynamical approach of Nambu[1] applied to four-fermion theories can be extended to other chiral-invariant field theories by exploiting the role of the axial Ward identity.[8] In this section we review how dynamical spontaneous breakdown of chiral symmetry works for non-Abelian QCD. The theory is sufficiently complex that it is helpful to introduce the key notions in stages.

(a) *Summation of ladder graphs.* First we suppress all internal-symmetry labels and consider only massless vector-gluon exchanges with dressed quark lines determined by the general inverse fermion propagator

$$S^{-1}(p)=\not p - m_0+\Sigma(p)=C(p^2)+D(p^2)\not p\,. \tag{1}$$

The idea of Nambu and Jona-Lasinio is that *if* the bare quark mass is zero, $m_0=0$, so that the bare Lagrangian is chiral invariant, then there exists a formal identity between ladder summations of pseudoscalar bound-state Bethe-Salpeter graphs as $q\to 0$ and quark dressing graphs. For three-point couplings such a possibility is depicted in Fig. 1; it cannot be achieved.

It was shown in Ref. 8 that this goal is not quite realized because of the *homogeneous* nature of the Bethe-Salpeter equation associated with the composite wave function $P\gamma_5$. A constant inhomogeneous term is missing in this composite wave function which prevents P from being linked with C of Eq. (1).

(b) *Axial Ward identity* (AWI). The next refinement is to identify the general pseudoscalar composite wave function with the divergence of the axial-vector quark current in the spirit of PCAC,

$$\Psi_q(p)\propto -iq^\mu\Gamma^5_\mu(p;q)=P(p^2)\gamma_5+A(p^2)\not q\gamma_5 + Q(p^2)p\cdot q\not p\gamma_5\,, \tag{2}$$

where we have included only chiral-invariant terms in (2). Thus we reinterpret Fig. 1 as in Fig. 2, again in the ladder approximation. Then indeed the desired result emerges: the 0^- binding equations (PBE) as $q\to 0$ become identical with the dressing equations (DE) for the quark, assuming $m_0=0$. The only constraint on the axial-vector current is that it must satisfy the general Ward identity (again for $m_0=0$)

$$-iq^\mu\Gamma^5_\mu(p;q)=S^{-1}(p+\tfrac{1}{2}q)\gamma_5+\gamma_5S^{-1}(p-\tfrac{1}{2}q)\,. \tag{3}$$

After much algebra, we find that Fig. 2 makes sense, providing the form factors in (1) and (2) are related in the ladder approximation as[8]

$$P(p^2)=2C(p^2),\quad A(p^2)=D(p^2),\quad Q(p^2)=2D'(p^2)\,. \tag{4}$$

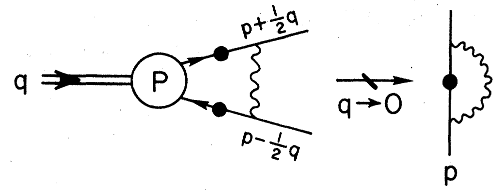

FIG. 1. Inequivalence of Bethe-Salpeter ladder graphs for pseudoscalar coupling and quark dressing diagram.

Now the power of the Ward-identity method is that (3) is valid (still ignoring Abelian anomalies) beyond the ladder approximation and perturbation theory. Thus (4) are presumably general identities, a fact that we now verify.

(c) *Low-energy theorem.* The relations (4) are quite obvious when one makes a low-q expansion of (3), leading to the general form for the axial-vector current

$$-i\Gamma^5_\mu(p;q)=\frac{2C(p^2)q_\mu\gamma_5}{q^2}+D(p^2)\gamma_\mu\gamma_5 + 2D'(p^2)p_\mu\not q\gamma_5+O(q^2)\,. \tag{5}$$

Then contraction of (5) with q^μ and comparison with (2) immediately reproduces (4). At this point we also recognize the characteristic "induced pseudoscalar" structure of the zero-mass 0^- pole in (5), with $-C(p^2)/D(p^2)$ corresponding to the dressed fermion mass in the resulting Goldberger-Treiman relation ($f_\pi g=mg_A$ with $g_A\to 1$).

Thus *if* the quark acquires all its mass via ladder (and nonladder) diagrams such as the right-hand graph of Fig. 2 so that $C(p^2=m^2)\neq 0$ in (5), *then* a zero-mass Nambu-Goldstone pseudoscalar automatically appears in the theory, i.e., in (5). That is, (4) implies that the equations that bind a quark and antiquark in an s wave at $q=0$ (through the first *two* leading orders in q) correspond to the equations that dress the quark and give it a mass

$$\mathrm{PBE}\,\big|_{q\to 0}=\mathrm{DE}\,. \tag{6}$$

This relation (6) is the thrust of the Nambu and Jona-Lasinio approach to spontaneous breakdown.

While this "if-then" proposition and (6) are simply realizations of the Goldstone theorem, our discussion will be very illuminating for the U(1)

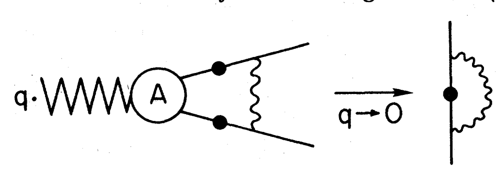

FIG. 2. Equivalence of Bethe-Salpeter ladder graphs for axial-vector current as $q\to 0$ and quark dressing diagram.

problem. As we shall show in a separate section, the Goldstone theorem is subtle to apply for the U(1) axial-vector current. However, the dynamical criterion (6) remains clear and simple in that case too.

(d) *Two-dimensional model field theories.* The axial Ward identity (3) and low-q expansion (5) are valid in any dimension. Thus it is useful to search for a counterexample in solvable two-dimensional model field theories—subject to the Coleman theorem[14] which states that there is no Goldstone pion in two dimensions. In particular, in the Gross-Neveu $1/N$ expansion[15] of the $(\bar{\psi}\psi)^2-(\bar{\psi}\gamma_5\psi)^2$ Lagrangian, it would appear that $(\bar{q}q)_0\neq 0$ in lowest order. The Coleman theorem, however, requires that high-order terms in the $1/N$ expansion again reinstate the $(\bar{q}q)_0=0$ no-pion condition. But then the mean-field approximation still generates an effective fermion mass term proportional to $\langle[\bar{q}(1+i\gamma_5)q\bar{q}(1-i\gamma_5)q]^{1/2}\rangle_0$ in the mean-field Lagrangian—an apparent contradiction to (4), (5) (i.e., $C\neq 0$ but no pion). Not so. Indeed by a boson representation of the theory one could easily show that the Gross-Neveu model contains a decoupled massless Goldstone boson.[16] This massless particle would not show up in any boson Green's functions (other than its own), but could appear in fermion Green's functions such as Γ^5_μ.

Next consider the two-dimensional Thirring model[17] with Lagrangian $(\bar{\psi}\gamma_\mu\psi)^2$. In this case the fermion remains massless but the pion exists—again an apparent contradiction to (4), (5), but in the opposite fashion as the Gross-Neveu analysis (i.e., $C=0$ but a pion). Now, however, the pion is a free particle and as such does not violate the Coleman nor our low-q theorem (4), (5).

(e) *Non-Abelian* QCD: Since we can find no obvious contradictions to (4), (5) or equivalently (6) in lower dimensions, we proceed to one final generalization by introducing internal *color* symmetry matrices λ^a, $a=1,\ldots,8$ into the chiral-limiting QCD Lagrangian, now in four dimensions,

$$\mathcal{L}=\bar{\psi}(i\partial\!\!\!/+g\tfrac{1}{2}\lambda^a V\!\!\!/^a)\psi-\tfrac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}\,, \tag{7}$$

where V^a_μ are color vector gluons and

$$G^a_{\mu\nu}=\partial_\mu V^a_\nu-\partial_\nu V^a_\mu+gf^{abc}V^b_\mu V^c_\nu \tag{8}$$

is the non-Abelian field tensor. On the other hand, the axial-vector quark current is characterized by the *flavor* symmetry matrices T^i, where for U(3), $i=0,1,\ldots,8$, i.e., $J^i_{\mu 5}=\bar{\psi}T^i i\gamma_\mu\gamma_5\psi$. Working in an axial gauge, the dressed axial-vector quark current still satisfies (3) for the non-Abelian SU(3) flavor configurations, $i=1,\ldots,8$,

$$\begin{aligned}-iq^\mu\Gamma^i_{\mu 5}(p;q)=S^{-1}(p+\tfrac{1}{2}q)\gamma_5 T^i\\ +T^i\gamma_5 S^{-1}(p-\tfrac{1}{2}q)\,.\end{aligned} \tag{9}$$

The important point concerning color vs flavor matrices is that they *commute*. In particular, Fig. 2 applies in the non-Abelian as well as the Abelian case because $\lambda^a T^i\lambda^a=\lambda^a\lambda^a T^i$. Thus Eqs. (4)–(6) remain unchanged for SU(3) flavor currents satisfying (9). Consequently *all* the u, d, s flavor quarks can acquire all their mass in the chiral limit via the self-energy-type dressing relations. Then by (6) there *must* exist a Nambu-Goldstone 0^- SU(3) π, K, and η_8 in the theory.

III. U(1) QCD INCLUDING ANOMALIES

As is well known, the essential difference between $i=0$ U(1) flavor axial-vector currents (and also the axial-vector current in QED) as opposed to the non-Abelian $i\neq 0$ currents is that although the latter currents are conserved in the chiral limit

$$\partial^\mu J^i_{\mu 5}=0\,,\quad i=1,\ldots,8\,, \tag{10}$$

the former currents are not, but instead satisfy the anomalous relations[18,19]

$$\partial^\mu J^{\mathrm{em}}_{\mu 5}=\frac{e^2}{16\pi^2}\epsilon^{\alpha\beta\gamma\delta}F^{\mathrm{em}}_{\alpha\beta}F^{\mathrm{em}}_{\gamma\delta}\,, \tag{11a}$$

$$\partial^\mu J^{i=0}_\mu=\frac{\sqrt{6}g^2}{64\pi^2}\epsilon^{\alpha\beta\gamma\delta}G^a_{\alpha\beta}G^a_{\gamma\delta}\,. \tag{11b}$$

Here $G^a_{\alpha\beta}$ is given by (8) and $F^{\mathrm{em}}_{\mu\nu}=\partial_\mu V^{\mathrm{em}}_\nu-\partial_\nu V^{\mathrm{em}}_\mu$.

Now given that the dressing relations for the flavor quarks are equal to the $q\to 0$ binding equations associated with the $i=1,\ldots,8$ flavor currents as represented by (6), we do not see how it is possible for these *same* dressing relations also to be equal to the *different* U(1) binding equations associated with the $i=0$ flavor current. Put another way, the U(1) axial-vector Ward identity (9) is modified by the anomaly (11b) as depicted in Fig. 3 to leading order in vector-gluon exchange. The left-hand graph is topologically equivalent to the right-hand graph and the latter explicitly contains the triangle anomaly. While this observation is certainly valid for the ladder graphs depicted in Figs. 2 and 3, we suggest that it is also true for nonplanar diagrams as well. That is, it is difficult to imagine the U(1) anomaly not altering (6) as $q\to 0$ because the binding-dressing equations (4) are derived for the leading *two* orders of q in (6).

The upshot of this discussion is that given the validity of (6), a similar U(1) relation cannot be simultaneously valid, i.e.,

$$\mathrm{U(1)\ PBE}\,|_{q\to 0}\neq \mathrm{DE}\,. \tag{12}$$

Thus we conclude that *because* the π, K, and η_8 are dynamically generated Nambu-Goldstone bosons by (6), the U(1) η_0 *cannot* be a Nambu-Gold-

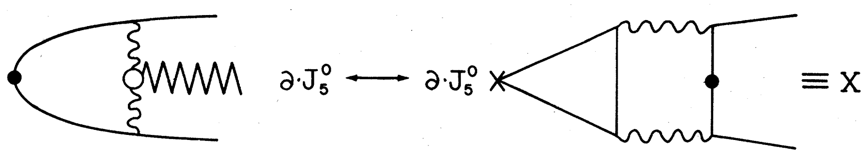

FIG. 3. Additional term in the U(1) Bethe-Salpeter axial-vector-current divergence equation generated by the U(1) anomaly.

stone boson due to (12). Moreover, given X from Fig. 3 (in lowest order), the η_0 mass in the chiral limit is generated via

$$\langle 0|\partial\cdot J_5^0|\eta_0\rangle = f_0(m_{\eta_0}^{\rm CL})^2 \quad (13a)$$

$$= \langle 0|X|\eta_0\rangle . \quad (13b)$$

That is, only in the absence of the anomaly would η_0 be a Nambu-Goldstone boson, satisfying a relation similar to (10). We cannot, however, turn off the U(1) anomaly in QCD and so we have not unequivocably demonstrated the existence of the η_0—except to suggest in (13) that η_0 alone prevents the U(1) axial-vector-current matrix elements from being conserved in the chiral limit. The recent work by Witten[7] on the $1/N$ expansion helps to justify the existence of the η_0. Alternatively, we may turn to quark-gluon QCD phenomenology to reaffirm the existence and compute the mass of η_0.

IV. QCD PHENOMENOLOGY AND THE U(1) PROBLEM

While our insight into the non-Goldstone nature of the U(1) pseudoscalar meson was based upon dynamical ladder graphs, real world phenomenology presumably corresponds to summations over all QCD graphs. We now demonstrate that phenomenology also suggests that η_0 is not a Nambu-Goldstone boson and that m_{η_0} is generated by the U(1) anomaly.

In Hamiltonian language the current quark mass matrix

$$\mathcal{H}' = \bar{q}\mathfrak{M}q = m_{0u}\bar{u}u + m_{0d}\bar{d}d + m_{0s}\bar{s}s + \cdots \quad (14)$$

is the origin of the Nambu-Goldstone π and K masses

$$\langle\pi|\mathcal{H}'|\pi\rangle = m_\pi^2 = \xi 2\hat{m}_0^n , \quad (15a)$$

$$\langle K|\mathcal{H}'|K\rangle = m_K^2 = \xi(m_{0s}^n + \hat{m}_0^n) , \quad (15b)$$

where $\hat{m}_0 = \frac{1}{2}(m_{0u}+m_{0d})$. Here $\hat{m}_0$, m_{0s} are renormalized chiral-breaking quark masses which vanish along with m_π and m_K in the chiral limit. Also ξ is a flavor-independent scale factor and n is the power of quark mass that enters the hadron mass matrix. While ξ and n are model dependent,[20] our considerations of (15) will be model independent.

Now for the $I=0$ pseudoscalar mesons we follow the analysis of Ref. 9 and references therein to write

$$\langle\eta_{NS}|\mathcal{H}'|\eta_{NS}\rangle = \xi 2\hat{m}_0^n + 2\beta_P , \quad (16a)$$

$$\langle\eta_S|\mathcal{H}'|\eta_S\rangle = \xi 2\hat{m}_{0s}^n + \beta_P , \quad (16b)$$

where $\eta_{NS} = (\bar{u}u + \bar{d}d)/\sqrt{2}$, $\eta_S = \bar{s}s$, and β_P is the strength of the quark-gluon annihilation graph of Fig. 4 which couples to all $\bar{q}q$ channels. In principle, the two gluons exchanged in Fig. 4 represent the sum of even numbers of gluons coupling to 0^{-+} mesons. Next we eliminate the unknown $\hat{m}_0^n$, m_{0s}^n factors in (16) in favor of the π and K masses from (15) and then rediagonalize the resulting $I=0$ pseudoscalar mass matrix as

$$\begin{pmatrix} m_\pi^2 + 2\beta_P & \sqrt{2}\beta_P \\ \sqrt{2}\beta_P & 2m_K^2 - m_\pi^2 + \beta_P \end{pmatrix} \to \begin{pmatrix} m_\eta^2 & 0 \\ 0 & m_{\eta'}^2 \end{pmatrix} . \quad (17)$$

If the quark-gluon graph of Fig. 4 makes sense, then the *two* constraints on β_P in (17) must lead to the *same* value for β_P. Indeed this is almost so, for the trace of (17) requires $\beta_P \approx 12.7 m_\pi^2$ while the determinant of (17) implies $\beta_P \approx 14.5 m_\pi^2$. A slight bit of multiplicative SU(3) breaking (due to the fact that $m_{0s} \gg \hat{m}_0$) in (17) of the form β_P, $x\beta_P$, $x^2\beta_P$ in the NS, NS-S, and S elements[9,21] then leads to a unique value of β_P and the corresponding 1-8 pseudoscalar mixing angle of

$$\beta_P \approx 14.7\ m_\pi^2, \quad \theta_P \approx -13° \quad (18)$$

along with $x \approx 0.8$. The mixing angle in (18) is reasonably consistent with all data.[9]

The important point of this analysis for the U(1) problem is that β_P does exist and indeed generates the bulk of the physical η and η' masses. In par-

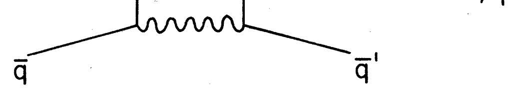

FIG. 4. Quark-antiquark annihilation graph contribution to the $I=0$ pseudoscalar mesons.

ticular, the determined parameters (18) imply[9] that β_P corresponds to 26% of $m_\eta{}^2$ and 71% of $m_{\eta'}{}^2$. Moreover, in the chiral limit with m_0, $m_{0s} \to 0$ in (16) along with $m_\eta{}^2 + m_{\eta'}{}^2 = m_{\eta_0}{}^2 + m_{\eta_8}{}^2$, the chiral-limiting η_0 mass is

$$(m_{\eta_0}^{\mathrm{CL}})^2 = 3\beta_P, \quad m_{\eta_0}^{\mathrm{CL}} \sim 900 \text{ MeV}. \tag{19}$$

Thus we see that QCD phenomenology implies that m_{η_0} is not zero in the chiral limit, i.e., that η_0 is not a Nambu-Goldstone boson.

To reinforce this result, we link up the phenomenological quark-gluon annihilation graphs of Fig. 4 with the dynamical U(1) anomaly of Fig. 3 and Eq. (13). "Tying together" one $\bar{q}q$ pair in Fig. 4 indeed corresponds to multiplication by f_0 times $m_{\eta_0}{}^2$; i.e., f_0 times quark-annihilation graph = anomalous divergence. Following Ref. 10 we further note that a factor of $m_{\eta_0}{}^2$ also appears in the evaluation of the right-hand side "diamond diagram" of (13), with the result that $m_{\eta_0}{}^2$ cancels out of the calculation [but $m_{\eta_0}^{\mathrm{CL}} \neq 0$ by (19)]. The resulting analysis, including an asymptotic-freedom QCD cutoff of the relevant Feynman integrals, then leads to the scale[10]

$$f_0 \sim f_\pi \tag{20}$$

in the chiral limit. The conclusion (20) is also reasonably consistent with the data, the latter giving[9] $f_\eta/f_\pi = 1.18 \pm 0.09$ and $f_{\eta'}/f_\pi = 1.05 \pm 0.23$.

V. U(1) PROBLEM AND GOLDSTONE'S THEOREM

The standard lore concerning the U(1) problem for QCD is that the anomaly is "soft," i.e., contains the divergence operation, so that it does not circumvent the Goldstone theorem. Stated in a more quantitative manner, define the U(1) current[22]

$$\tilde{J}^0_{\mu 5} \equiv J^0_{\mu 5} - K_\mu, \tag{21a}$$

$$K_\mu \equiv \frac{\sqrt{6}g^2}{64\pi^2}\epsilon_\mu{}^{\alpha\beta\gamma} V^a_\alpha (\partial_\beta V^a_\gamma + \tfrac{1}{3} g f^{abc} V^b_\beta V^c_\gamma), \tag{21b}$$

so that $\partial \cdot K$ is the anomaly term in (12b). Then in the chiral limit, $\tilde{J}^0_5$ is divergenceless,

$$\partial \cdot \tilde{J}^0_5 = 0, \tag{22}$$

for which there "ought" to exist a Goldstone boson, the η_0. As is well known, the possible loophole in this argument is that K_μ of (21) is gauge dependent (because V_μ is gauge dependent, while $G_{\mu\nu}$ is not). Clearly one does not want the existence of a U(1) Goldstone boson to depend upon a specific gauge.

To try to get around this problem, Kogut and Susskind[23] noticed that the current $\tilde{J}^0_{5\mu} \propto (\tilde{\rho}_5, \vec{\tilde{j}}_5)$ with

$$\tilde{\rho}_5 = \rho_5 + \frac{g^2}{4\pi^2}\vec{B}\cdot\vec{A} = \frac{g^2}{4\pi^2}\vec{B}\cdot\int d^3r\, \vec{U}(\vec{r}-\vec{x}) \times \vec{B}(\vec{r}), \tag{23a}$$

$$\begin{aligned}\vec{\tilde{j}}_5 = \vec{j}_5 &+ \frac{g^2}{4\pi^2}\vec{A}\times\vec{E} + \frac{g^2}{4\pi^2}\int \vec{U}(\vec{r}-\vec{x})\vec{E}(\vec{x})\cdot\vec{B}(\vec{r})d^3r \\ &- \frac{g^2}{4\pi^2}\int \vec{E}(\vec{x})\cdot\vec{U}(\vec{r}-\vec{x})\vec{B}(\vec{r})d^3r \\ &+ \frac{g^2}{4\pi^2}\vec{B}(\vec{x})\int \vec{U}(\vec{r}-\vec{x})\cdot\vec{E}(\vec{r})d^3r\end{aligned} \tag{23b}$$

[with U defined by their Eq. (3.19)] is both gauge invariant and locally conserved. Applying Goldstone's theorem[2] to (23), one concludes that if $\tilde{Q}^0_5|0\rangle \neq 0$, then the theory requires the existence of a Goldstone boson. Kogut and Susskind argued that since

$$\tilde{Q}^0_5 = Q^0_5 \tag{24a}$$

and one certainly hopes that

$$Q^0_5|0\rangle \neq 0 \tag{24b}$$

(as an $I=0$ analogy to the $I=1, \frac{1}{2}$ cases of Sec. III), it must be that $\tilde{Q}^0_5|0\rangle \neq 0$. Hence one would expect η_0 to be massless in the chiral limit. Since this conclusion is opposite to our dynamical (Nambu) generation of the η_0 mass via the anomaly, we are obliged to investigate this problem further.

While in the absence of any computations it is difficult to prove or disprove the Kogut-Susskind conjecture, we offer the following comments:

(a) To derive (24a) from (23a), one discards a surface term. We question this procedure in an operator equation. In particular, consider the two-dimensional Schwinger or Thirring models where one writes $j_\mu = \epsilon_{\mu\nu}\partial^\nu\phi$, so that

$$Q = \int dx\, \partial_1\phi. \tag{25}$$

While $[Q,\phi] = [Q,\dot{\phi}] = 0$, one obviously has for charged fermion fields

$$[Q,\psi] \propto \psi \neq 0. \tag{26}$$

The analogy of (26) for the Kogut-Susskind model is that charge-operator relations could differ from charge-matrix element relations such that $\tilde{Q}^0_5 \neq Q^0_5$ as an operator relation. The Goldstone alternative for η_0 would then be avoided.

(b) As another possibility, we suppose that $\tilde{Q}^0_5|0\rangle \neq 0$ is indeed valid. Then one way to evade giving η_0 zero mass is to have the resulting Goldstone boson have a free field. Recall that this circumstance occurs in the Thirring and Neveu-Gross models, where the vacuum breaks chiral invariance, yet there is a free-field Goldstone boson. To be more specific, the state $\tilde{Q}^0_5|0\rangle$ could be a free massless particle that decouples from the η_0 which perhaps could be $Q_5^0|0\rangle$.

(c) Kogut and Susskind argue that long-range "vacuum seizing" gives η_0 the nonzero mass. They make the analogy with the two-dimensional QED (Schwinger) model. However, that case could also be regarded as an example of the Higgs mechanism: the massless photon absorbs the would-be Goldstone boson and becomes massive. Of course, it so happens that in two dimensions the Coulomb potential confines. Therefore it is not clear whether in the Schwinger model the evasion of Goldstone's theorem is just another case of the Higgs mechanism or instead is really due to confinement. If confinement were indeed responsible for the nonzero η_0 mass (in four dimensions), one would wonder what it may do to the other eight SU(3) Nambu-Goldstone bosons.

(d) As a concrete counterexample to the conjecture that $\tilde{Q}_5^0|0\rangle = Q_5^0|0\rangle \neq 0$ implies the existence of a Goldstone η_0, we offer our dynamical and phenomenological analysis of Secs. II–IV in terms of which $\partial\cdot J_5^{1-8}=0$ generates eight SU(3) Nambu-Goldstone bosons but $\partial\cdot J_5^0\neq 0$ keeps η_0 massive even in the chiral limit.

In summary, in spite of the softness of the U(1) anomaly, the associated gauge dependence of the axial charges obscures the application of the Goldstone theorem to the U(1) pseudoscalar meson.

VI. OTHER ASPECTS OF THE U(1) PROBLEM

Any puzzle associated with η and η'—their decay modes and relative positions in the pseudoscalar mass spectrum—is often blamed on the U(1) problem. Since we have disposed of the latter in the previous sections, we feel obliged to comment upon these other problems as well.

(a) *Radiative decays.* The recently measured 2γ decay rates of η and η' yielding[24,25] $\Gamma_{\eta\gamma\gamma}=324+46$ eV and $\Gamma_{\eta'\gamma\gamma}=5.9+1.6$ keV are reasonably consistent with SU(3)-invariant decay amplitudes, the $\pi^0\gamma\gamma$ rate of 7.9 eV, and our deduced mixing angle of $\theta_P\approx -13°$. As previously noted, in terms of the em triangle anomaly these rates predict $f_{\eta'}\sim f_\eta \sim f_\pi$, consistent with (13) and (20).

(b) $\eta'\to\eta\pi\pi$ *decay.* The $\delta(980)$ meson controls this process[26] via $\eta'\to\delta\pi$ and $\delta\to\eta\pi$, leading to an acceptable rate.

(c) *Sutherland* $\eta_{3\pi}$ *puzzle.* The $\eta_{3\pi}$ amplitude vanishes for[27] $\mathcal{H}_{em}=\mathcal{H}_{JJ}$. Electromagnetic mass splittings, however, require a u_3 tadpole[28] to exist in $\mathcal{H}_{em}=\mathcal{H}_{JJ}+\epsilon_3 u_3$. Then the $\eta_{3\pi}$ amplitude does not vanish.

(d) PCAC *and rapidly varying* $\eta_{3\pi}$ *amplitude.* Even with a u_3 tadpole in $\mathcal{H}_{em}$, the soft-pion limit for π^+ or π^0 in η_{+-0} leads to different amplitudes, suggesting again the Sutherland puzzle.[27] The PCAC hypothesis, however, should only be applied to smoothly varying amplitudes. Separating out the rapidly varying π, η, and η' poles for the transitions $\langle\pi|\mathcal{H}_{em}|\eta,\eta'\rangle$ and applying pion PCAC to the $\eta_{3\pi}$ background amplitude then leads to the *same* on-shell amplitude no matter which pion becomes soft[29] with

$$M_{\eta_{3\pi}}^{on}=M_P^{on}+M_{cc}-M_P(q_\pi^i\to 0)\,, \tag{27}$$

where $M_{cc}=if_\pi^{-1}\langle\pi^j\pi^k|[Q_5^i,\mathcal{H}_{em}]|\eta\rangle$. Reducing in a second pion in a similar fashion leads to the same total on-shell amplitude as obtained using nonlinear Lagrangian techniques in the tree approximation.[30]

(e) *Relation of* $\eta_{3\pi}$ *amplitude to vanishing* $\langle\pi\pi|\partial\cdot A^{NS}|\eta\rangle$. It has been noted[11,13,31] that the u_3 tadpole in $\mathcal{H}_{em}$ implies for $q_{\pi^0}\to 0$

$$M_{cc}\propto\langle\pi\pi|\sqrt{2}v_0+v_8|\eta\rangle\propto\langle\pi\pi|\partial\cdot A^{NS}|\eta\rangle\to 0\,, \tag{28}$$

which vanishes by momentum conservation. It turns out, however, that this is *irrelevant* for the *on-shell* $\eta_{3\pi}$ amplitude in (27) because $M_{cc}-M_P(q_\pi\to 0)$ vanishes identically whether M_{cc} does or not.[29] In all cases, the *physical* amplitude is the sum of on-shell pole amplitudes

$$M_{\eta 3\pi}^{on}=M_{\pi P}^{on}+M_{\eta P}^{on}+M_{\eta' P}^{on}\,. \tag{29}$$

Thus (28) is *not* an example of the U(1) problem.

(f) *Magnitude of* $\eta_{3\pi}$ *rate.* Assuming $\eta=\eta_8$ along with the naive scale $\langle\pi^0|\mathcal{H}_{em}|\eta_8\rangle=(\Delta m_K{}^2-\Delta m_\pi{}^2)/\sqrt{3}$, the pole amplitudes (29) lead to $\Gamma_{+-0}^{pole}\approx 70$ eV. The combined effects of $\eta-\eta'$ mixing with $\theta_P\approx -13°$ (due to quark-gluon-*photon* annihilation graphs for the $\langle\pi^0|\mathcal{H}_{em}|\eta\rangle$ transition) leads to the predicted rate[9]

$$\Gamma_{+-0}\approx 126\text{ eV}\,, \tag{30}$$

only $2\frac{1}{2}$ standard deviations below the measured value of 201 ± 29 eV. Thus the $\eta_{3\pi}$ rate does not appear to be a manifestation of the U(1) problem.

(g) U(1) *Vacuum Ward identity.* Glashow,[12] Weinberg,[12] and Crewther[13] have noted a possible inconsistency in the vacuum matrix elements of chiral-*breaking* Ward identities involving the σ-term operators $[Q_5^i, i\partial\cdot A^i]$ for $i=3, NS$. We refer the reader to Ref. 13 where the following $U_2(1)$ relation is derived:

$$m_\pi{}^2 f_\pi{}^2=4\langle\langle\nu^2\rangle\rangle+O(m_\pi{}^4)\,, \tag{31}$$

with $\langle\langle\nu^2\rangle\rangle$ the average of the square of "topological charge ν." Crewther then uses the WKB approximation to evaluate

$$\langle\langle\nu^2\rangle\rangle_{WKB}=O(\hat{m}_0{}^2)\,, \tag{32}$$

where $\hat{m}_0$ is the nonstrange current quark mass. He then assumes the "strong PCAC"[32] chiral-breaking dependence $m_\pi^2 = O(\hat{m}_0)$, which is obviously inconsistent with (31) and (32) since $f_\pi = O(\hat{m}_0{}^0)$.

Two comments come to mind:

(i) Assuming instead "neutral PCAC"[20,33,34] with $m_\pi^2 = O(\hat{m}_0{}^2)$, (31) and (32) are in fact consistent with one another.[35] So perhaps this is an indication that strong PCAC is not applicable in the real world.

(ii) There are serious doubts as to the validity of the WKB approximation stemming from the boundary conditions used to evaluate the fermion determinant.[6]

In passing, we also note that the additional Weinberg[12] problem of a light $I=0$ (unphysical) Goldstone boson of mass $m_L \lesssim \sqrt{3} m_\pi$ never arises for us because the η_0 not being a Goldstone boson eliminates an additional and unwanted $f_0 m_{\eta_0}{}^2$ term in (31).

In any event it is our contention that the sum rule (31) is *not* a U(1) problem. Rather, this $U_2(1)$ construction should be used as a *guide* to the correct chiral-*breaking* scheme. But this occurs, logically speaking, well after one understands spontaneous breakdown in the chiral limit and the non-Goldstone origin of the η_0 mass.

VII. CONCLUSION

In this paper we have attempted to resolve all aspects of the U(1) problem. Our major conclusions are the following:

(i) Dynamical spontaneous breakdown of chiral symmetry for non-Abelian QCD corresponds to the quark dressing equations with $m_0 = 0$ being identical to the 0^- $\bar{q}q$ binding equations at zero four-momentum for $i = 1, \ldots, 8$ SU(3) flavors. This dynamical realization of the Goldstone theorem is linked with π, K, and η_8 being Nambu-Goldstone bosons.

(ii) The *AVV* anomaly for $i=0$ prevents the U(1) binding equations from being the same as the quark dressing equations for $m_0 = 0$. Thus the 0^- η_0 is *not* a Nambu-Goldstone boson; instead, its mass is solely generated in the chiral limit by the U(1) anomaly.

(iii) The phenomenology of quark-gluon annihilation diagrams resolves the η'-η mixing problem and reaffirms that the U(1) anomaly is responsible for the chiral-limiting η_0 mass.

(iv) The "soft" structure of the anomaly does not necessarily void the above results (ii) and (iii) via the Goldstone theorem. Two-dimensional field theory models provide guidance in this respect.

(v) The U(1) vacuum Ward identity is *not* a U(1) problem. Instead it provides a severe restriction on the correct scheme of chiral-symmetry *breaking*. If the WKB approximation is correct, then "neutral PCAC" is favored over "strong PCAC."

(vi) The $\eta_{3\pi}$ PCAC structure and rate are *not* U(1) problems once one accounts for the rapidly varying poles in $M_{\eta 3\pi}$ and incorporates quark-gluon-photon annihilation graphs in the evaluation of $\langle \pi^0 | u_3 | \eta \rangle$.

ACKNOWLEDGMENTS

The authors wish to thank R. Delbourgo for many helpful insights on this subject. They also acknowledge support from the National Science Foundation.

[1] Y. Nambu, Phys. Rev. Lett. 4, 380 (1960); Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961).

[2] J. Goldstone, Nuovo Cimento 19, 154 (1961).

[3] G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976).

[4] C. Callan, R. Dashen, and D. Gross, Phys. Rev. D 17, 2717 (1978); 19, 2391 (1979).

[5] R. D. Carlitz and D. B. Creamer, Ann. Phys. (N.Y.) 118, 429 (1979).

[6] E. Gildener and A. Patrascioiu, Phys. Rev. D 19, 2962 (1979); A. Patrascioiu, Erice Summer School lecture, 1979 (unpublished); E. Seiler, Max-Planck Institute report, 1980 (unpublished).

[7] E. Witten, Nucl. Phys. B149, 285 (1978); B156, 269 (1979); R. Weder, Phys. Lett. 85B, 269 (1979).

[8] R. Delbourgo and M. D. Scadron, J. Phys. G 5, 1621 (1979); University of Tasmania report, 1979 (unpublished).

[9] H. F. Jones and M. D. Scadron, Nucl. Phys. B155, 409 (1979).

[10] R. Delbourgo and M. D. Scadron, University of Tasmania report, 1979 (unpublished).

[11] R. Brandt and G. Preparata, Ann. Phys. (N.Y.) 61, 119 (1970).

[12] S. L. Glashow, in *Hadrons and Their Interactions*, edited by A. Zichichi (Academic, New York, 1968), p. 83.

[13] R. Crewther, Phys. Lett. 70B, 349 (1977); CERN Report No. TH. 2548, 1978 (unpublished); Riv. Nuovo Cimento 2, 63 (1979).

[14] S. Coleman, Commun. Math. Phys. 31, 259 (1973).

[15] D. J. Gross and A. Neveu, Phys. Rev. D 10, 3235 (1974).

[16] R. Dashen (private communication).

[17] W. Thirring, Ann. Phys. (N.Y.) 3, 91 (1958).

[18] S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60, 47 (1969).

[19] H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973); R. N. Mohapatra and J. C. Pati, Phys. Rev. D 8, 4212 (1973).

[20] J. F. Gunion, D. C. McNamee, and M. D. Scadron, Phys. Lett. 63B, 81 (1976); Nucl. Phys. B123, 445

(1977).
[21]H. Fritzsch and J. D. Jackson, Phys. Lett. 66B, 365 (1977).
[22]W. A. Bardeen, Nucl. Phys. B75, 246 (1974).
[23]J. Kogut and L. Susskind, Phys. Rev. D 10, 3468 (1974).
[24]A. Browman *et al.*, Phys. Rev. Lett. 32, 1067 (1974).
[25]D. M. Binnie *et al.*, Phys. Lett. 83B, 141 (1979); G. S. Abrams *et al.*, Phys. Rev. Lett. 43, 477 (1979).
[26]C. Singh and J. Pasupathy, Phys. Rev. Lett. 35, 1193 (1975); N. G. Deshpande and T. N. Truong, Phys. Rev. Lett. 41, 1579 (1978).
[27]D. G. Sutherland, Phys. Lett. 23, 384 (1966); Nucl. Phys. B2, 47 (1967).
[28]S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964).
[29]P. C. McNamee and M. D. Scadron, Phys. Rev. D 11, 226 (1975); Nuovo Cimento 30A, 287 (1975).
[30]N. Cabibbo and L. Maiani, in *Evolution of Particle Physics*, edited by M. Conversi (Academic, New York, 1970), p. 50.
[31]J. Kogut and L. Susskind, Phys. Rev. D 11, 3594 (1975).
[32]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2145 (1968); S. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968); S. Weinberg, in *A Festschrift for I. I. Rabi*, edited by L. Motz (N.Y. Academy of Sciences, New York, 1977).
[33]H. F. Jones and M. D. Scadron, Phys. Rev. D 10, 967 (1974); 11, 174 (1975); H. Sazdjian and J. Stern, Nucl. Phys. B94, 163 (1975); N. H. Fuchs, Phys. Rev. D 14, 1709 (1976); N. H. Fuchs and H. Sazdjian, *ibid.* 18, 889 (1978).
[34]N. H. Fuchs and M. D. Scadron, Phys. Rev. D 20, 2421 (1979); M. D. Scadron, University of Melbourne report, 1979 (unpublished); Phys. Lett. 91B, 265 (1980).
[35]N. H. Fuchs, Purdue University report, 1979 (unpublished).