Metric vector and mass spectrum of gauge fields

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We construct a gauge theory of local U(4) inner symmetry in spinor space by treating the γ matrices as a metric vector. Using the covariant expansion to extract the physical fields, we obtain relations among the masses and the coupling constants, respectively, that the gauge fields satisfy. Generalization to the case where the spinor fields have internal symmetry is also discussed.

I. INTRODUCTION

Recently, it has been shown that it is possible to formulate a gauge theory of a noncompact internal-symmetry group by introducing a metric field.[1,2] After a new kind of symmetry breakdown, some of the gauge fields become massive.[1-3] The phenomenological implications of such a gauge theory have also been discussed.[4]

From another perspective, algebraic chromodynamics has been studied.[5,6] Using a covariant expansion in spinor space, one can simultaneously introduce the massive scalar field, vector field, etc., as gauge fields.[7]

This paper shows that a synthesis of these ideas leads to some interesting gauge-theory results. Starting with the spinor space of a single fermion, we extend its U(1) symmetry to local U(4) symmetry and construct a gauge theory by generalizing the ordinary γ matrices to metric vector fields and introducing gauge fields. In Sec. II we formulate a U(4) gauge theory with only one spinor field. The subsequent problem of Lorentz invariance is then solved in Sec. III by finding a local representation of the Lorentz group. After breakdown of the symmetry (i.e., the metric fields return to constant metric matrices), the fields become massive. With the covariant expansion we project the gauge fields into physical vector fields, pseudoscalar fields, etc., then find the relations that the masses and the coupling constants of these gauge fields must observe, respectively; this is done in Sec. IV. In Sec. V we make a further generalization to the spinor field with internal symmetry. We illustrate it by constructing a U(4)×U(n) gauge theory as an example. Section VI gives a brief conclusion.

II. METRIC VECTOR AND U(4) GAUGE THEORY

It is well known that the kinetic energy term of a spinor field possesses U(1)×U(1) symmetry. Namely, the term

$$-K=\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi \tag{1}$$

is invariant under the transformation

$$\psi'=W\psi\,,\qquad W=\exp(i\theta+\chi\gamma_5)\,. \tag{2}$$

Here θ,χ are the group parameters. If we add a mass term $m\bar{\psi}\psi$, only the U(1) symmetry survives.

However, the fermion field has higher symmetry. Obviously, (1) and $m\bar{\psi}\psi$ are invariant under the unitary U(4) transformation

$$\psi'=U\psi\,,\qquad \gamma'_{\mu}=U\gamma_{\mu}U^{-1}\,, \tag{3}$$

and the physics remains the same because it only depends on the structure of the γ algebra, and not its representations. Usually Eq. (3) is not taken into account because the γ's are not regarded as fields.

Actually these γ matrices can be treated as a metric vector. They can further become metric fields when we consider the local symmetry. So we start with the following free Lagrangian of a spinor field:

$$\mathcal{L}_0=\bar{\psi}[i\gamma^{\mu}(x)\partial_{\mu}-m]\psi\,, \tag{4}$$

$$\gamma_{\mu}(x)=U_0(x)\gamma_{\mu}U_0^{-1}(x)\,. \tag{5}$$

Here $U_0(x)$ is an arbitrarily specified transformation of a local unitary group U(4,x). These "local" γ matrices have the same algebraic relations among each other as the "global" ones

$$\{\gamma_{\mu}(x),\gamma_{\nu}(x)\}=2g_{\mu\nu} \tag{6}$$

and the metric tensor $g^{\mu\nu}$ remains unchanged; this result is very important.

The Lagrangian (4) is invariant under the global U(4) transformation (3). Now we generalize it to local U(4) symmetry. According to the standard formulation, we construct the gauge-covariant derivatives of the spinor field as well as the metric vector field:

$$D_{\mu}\psi\equiv(\partial_{\mu}+B_{\mu})\psi\,, \tag{7}$$

$$D_\mu \gamma_\nu \equiv \partial_\mu \gamma_\nu(x) + [B_\mu, \gamma_\nu(x)] \,. \tag{8}$$

The B_μ is a matrix gauge field; its strength is defined by

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu] \,. \tag{9}$$

We can thereby construct a gauge theory which possesses local U(4) symmetry in the spinor space

$$\mathcal{L} = \tfrac{1}{2} i\bar{\psi}\gamma^\mu D_\mu \psi - \tfrac{1}{2} i(D_\mu \psi)^\dagger \gamma_0 \gamma^\mu \psi - m\bar{\psi}\psi - \frac{1}{16g^2}\mathrm{tr}\,\bar{B}^{\mu\nu} B_{\mu\nu} + \frac{3m_p^2}{128g^2}\mathrm{tr}\,\overline{D_\mu \gamma_\nu} D^\mu \gamma^\nu \,. \tag{10}$$

Here the adjoint conjugation of any matrix X is defined by

$$\bar{X} = \gamma_0(x) X^\dagger \gamma_0(x) \,. \tag{11}$$

Under $U(4,x)$ the field quantities undergo the following transformations:

$$\begin{aligned} \psi' &= U(x)\psi \,, \\ \gamma'_\mu(x) &= U(x)\gamma_\mu(x)U^{-1}(x) \,, \\ B'_\mu &= U(B_\mu - U^{-1}\partial_\mu U)U^{-1} \,; \end{aligned} \tag{12}$$

$$\begin{aligned} B'_{\mu\nu} &= U B_{\mu\nu} U^{-1} \,, \\ (D_\mu \psi)' &= U D_\mu \psi \,, \\ (D_\mu \gamma_\nu)' &= U(D_\mu \gamma_\nu)U^{-1} \,. \end{aligned} \tag{13}$$

III. LORENTZ INVARIANCE

We usually prove the Lorentz invariance of a spinor field theory by finding a representation of the Lorentz group in spinor space:

$$\Lambda = \exp(-\tfrac{1}{4} i\sigma_{\mu\nu}\,\omega^{\mu\nu}) \,. \tag{14}$$

The $\omega^{\mu\nu}$ here are the group parameters. Can we find an appropriate representation that guarantees the invariance of (10)? The answer is "yes," and the form of the representation is the same as in (14), only now it becomes a local representation of the Lorentz group:

$$\Lambda(x) = \exp[-\tfrac{1}{4} i\sigma_{\mu\nu}(x)\omega^{\mu\nu}] \,, \tag{15}$$

$$\sigma_{\mu\nu}(x) = \tfrac{1}{2} i[\gamma_\mu(x), \gamma_\nu(x)] \,. \tag{16}$$

It is easy to verify that the Lagrangian (10) is invariant under the following Lorentz transformations:

$$\begin{aligned} x'_\mu &= a_{\mu\nu} x^\nu \,, \\ \psi'(x') &= \Lambda(x)\psi(x) \,, \\ \gamma'_\mu(x') &= a_{\mu\nu}\Lambda(x)\gamma^\nu(x)\Lambda^{-1}(x) \,, \\ B'_\mu(x') &= a_{\mu\nu}\Lambda[B^\nu(x) - \Lambda^{-1}\partial^\nu \Lambda]\Lambda^{-1} \,. \end{aligned} \tag{17}$$

The other induced transformations are

$$\begin{aligned} \bar{\psi}'(x') &= \bar{\psi}(x)\Lambda^{-1}(x) \,, \\ \bar{B}'_\mu(x') &= a_{\mu\nu}\Lambda[\bar{B}^\nu + \Lambda^{-1}\gamma_0(\partial^\nu \Lambda^{\dagger -1})\gamma_0]\Lambda^{-1} \,; \end{aligned} \tag{18}$$

$$\begin{aligned} B'_{\mu\nu}(x') &= a_{\mu\sigma}a_{\nu\rho}\Lambda B^{\sigma\rho}(x)\Lambda^{-1} \,, \\ \bar{B}'_{\mu\nu}(x') &= a_{\mu\sigma}a_{\nu\rho}\Lambda \bar{B}^{\sigma\rho}\Lambda^{-1} \,. \end{aligned} \tag{19}$$

Under the gauge transformation U(4), the Lorentz transformation (15) keeps the same form

$$\begin{aligned} \Lambda'(x) &= U\exp[-\tfrac{1}{4} i\sigma_{\mu\nu}(x)\omega^{\mu\nu}]\,U^{-1} \\ &= \exp(-\tfrac{1}{4} i\sigma'_{\mu\nu}\omega^{\mu\nu}) \,. \end{aligned} \tag{20}$$

Thus the Lagrangian (10) is indeed Lorentz invariant.

IV. MASS SPECTRUM

From (5) and (12) one can see that the local γ matrices can always be "gauged back" to global ones, i.e.,

$$\gamma_\mu = U_1(x)\gamma_\mu(x)U_1^{-1}(x), \quad U_1 \subset U(4,x) \,. \tag{21}$$

When we choose such a special transformation as a gauge transformation, the Lagrangian has the same form, but the γ's return to constant matrices. Considering this we obtain a Lagrangian with reduced symmetry:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \tfrac{1}{2} i\,\bar{\psi}(\gamma^\mu B_\mu - \bar{B}_\mu\gamma^\mu)\psi - \frac{1}{16g^2}\mathrm{tr}\,\bar{B}^{\mu\nu}B_{\mu\nu} - \frac{3m_p^2}{64g^2}\mathrm{tr}\,(\bar{B}_\mu\gamma_\nu B^\mu\gamma^\nu - 4\bar{B}_\mu B^\mu). \tag{22}$$

As a consequence of this symmetry breaking, the kinetic energy term of the metric field generates a mass term of the gauge field B_μ.

From B_μ one can read off 16 gauge vector fields in terms of infinitesimal operators of U(4) or the covariant expansion in spinor space can be used to extract other kinds of gauge fields. The latter is physically more interesting.

The gauge field B_μ can be trace-independently expanded as

$$B_\mu = E'_\mu + \gamma_5 A'_\mu + \sigma_{\mu\nu}V'^\nu + \gamma_5\gamma_\mu P' + \gamma_\mu S' + \epsilon_{\mu\nu\sigma\rho}\gamma^\nu T'^{\sigma\rho} + \epsilon_{\mu\nu\sigma\rho}\gamma_5\gamma^\nu Q'^{\sigma\rho} \,. \tag{23}$$

Then we have

$$\begin{aligned} \gamma^\mu B_\mu - \bar{B}_\mu\gamma^\mu &= \gamma^\mu(E'_\mu - E'^*_\mu) + \gamma^\mu\gamma_5(A'_\mu - A'^*_\mu) + 4(S' - S'^*) \\ &\quad + 3i\gamma_\nu(V'^\nu + V'^{*\nu}) - 4\gamma_5(P' + P'^*) \\ &\quad + \epsilon_{\mu\nu\sigma\rho}\gamma^\mu\gamma^\nu(T'^{\sigma\rho} + T'^{*\sigma\rho}) \\ &\quad - \epsilon_{\mu\nu\sigma\rho}\gamma_5\gamma^\mu\gamma^\nu(Q'^{\sigma\rho} - Q'^{*\sigma\rho}) \,. \end{aligned} \tag{24}$$

The kinetic energy terms of gauge fields are

$$-\frac{1}{16g^2}\,\mathrm{tr}\,\bar{B}^{\mu\nu}B_{\mu\nu}=-\frac{1}{4g^2}(\partial_\mu E_\nu'^*-\partial_\nu E_\mu'^*)(\partial^\mu E'^\nu-\partial^\nu E'^\mu)-\frac{1}{2g^2}(\partial_\mu V_\nu'^*-\partial_\nu V_\mu'^*)(\partial^\mu V'^\nu-\partial^\nu V'^\mu)$$
$$+\frac{3}{2g^2}\partial_\mu P'^*\partial^\mu P'+\frac{1}{3g^2}T'^*_{\mu\alpha\beta}T'^{\mu\alpha\beta}$$
$$-\left[-\frac{1}{4g^2}(\partial_\mu A_\nu'^*-\partial_\nu A_\mu'^*)(\partial^\mu A'^\nu-\partial^\nu A'^\mu)+\frac{3}{2g^2}\partial_\mu S'^*\partial^\mu S'-\frac{1}{3g^2}Q'^*_{\mu\alpha\beta}Q'^{\mu\alpha\beta}\right]+\cdots \tag{25}$$

Here we have used the following definition of $T'_{\mu\alpha\beta}$:

$$T'_{\mu\alpha\beta}\equiv\partial_\mu T'_{\alpha\beta}+\partial_\beta T'_{\mu\alpha}+\partial_\alpha T'_{\beta\mu}\,. \tag{26}$$

The gauge-field mass terms are

$$-\frac{3m_p^2}{64g^2}\mathrm{tr}\,(\bar{B}_\mu\gamma^\nu B^\mu\gamma_\nu-4\bar{B}_\mu B^\mu)=-\frac{3m_p^2}{2g^2}(-\tfrac{3}{2}V_\mu'^*V'^\mu+P'^*P'+3T'^*_{\sigma\rho}T'^{\sigma\rho}+A_\mu'^*A'^\mu-3S'^*S'-Q'^*_{\sigma\rho}Q^{\sigma\rho})\,. \tag{27}$$

From Eq. (25) we see that the fields A'_μ, S', and $Q'^{\sigma\rho}$ are not acceptable in this scheme because we require the Hamiltonian to be positive-definite and the field E_μ is unavoidable.[7] Furthermore, we restrict our attention to the part of the gauge field which is interacting with the fermion. Finally, we need to normalize the kinetic energy terms of (25) by rescaling the fields, i.e.,

$$E'_\mu=igE_\mu\,,\quad V'_\mu=\frac{1}{\sqrt{2}}gV_\mu\,,\quad P'=\frac{1}{\sqrt{3}}gP\,,\quad T'_{\sigma\rho}=\tfrac{1}{2}gT_{\sigma\rho}\,. \tag{28}$$

So we must expand B_μ as follows:

$$B_\mu=g\left(iE_\mu+\frac{1}{\sqrt{2}}\sigma_{\mu\nu}V^\nu+\frac{1}{\sqrt{3}}\gamma_5\gamma_\mu P+\tfrac{1}{2}\epsilon_{\mu\nu\sigma\rho}\gamma^\nu T^{\sigma\rho}\right) \tag{29}$$

and all fields are now real.

We write down the final Lagrangian (imposing the condition $\partial_\mu V^\mu=0$)

$$\mathcal{L}=\bar{\psi}(i\gamma^\mu\partial_\mu-m)\psi-\tfrac{1}{4}(\partial_\mu E_\nu-\partial_\nu E_\mu)(\partial^\mu E^\nu-\partial^\nu E^\mu)-\tfrac{1}{4}(\partial^\mu V^\nu-\partial^\nu V^\mu)(\partial_\mu V_\nu-\partial_\nu V_\mu)+\tfrac{1}{2}(\tfrac{3}{2}m_p)^2V^\mu V_\mu-3g^2V_\mu V^\mu V_\nu V^\nu$$
$$+\tfrac{1}{2}\partial_\mu P\partial^\mu P-\tfrac{1}{2}m_p^2P^2-\tfrac{4}{3}g^2P^4+\tfrac{1}{12}T_{\mu\alpha\beta}T^{\mu\alpha\beta}-\frac{1}{4}\left(\frac{3m_p}{\sqrt{2}}\right)^2T_{\mu\nu}T^{\mu\nu}-\tfrac{1}{4}g^2[(T_{\mu\nu}T^{\mu\nu})^2-(T_{\mu\nu}T^{\nu\alpha}T_{\alpha\beta}T^{\beta\mu})]$$
$$-g\bar{\psi}\gamma^\mu\psi E_\mu+\frac{3}{\sqrt{2}}g\bar{\psi}\gamma^\mu\psi V_\mu+i\frac{4}{\sqrt{3}}g\bar{\psi}\gamma_5\psi P+g\bar{\psi}(\tfrac{1}{2}\epsilon_{\mu\nu\sigma\rho}\sigma^{\mu\nu})\psi T^{\sigma\rho}+g^2(V_\mu T^{\mu\alpha}T_{\alpha\beta}V^\beta)-2g^2T_{\mu\nu}T^{\mu\nu}P^2+3g^2V_\mu V^\mu P^2\,. \tag{30}$$

From (30) we conclude that there are certain relations among the coupling constants of gauge fields with the fermion, as well as their masses:

$$g_E:g_P:g_V:g_T=1:\frac{4}{\sqrt{3}}:\frac{3}{\sqrt{2}}:1\,, \tag{31}$$

$$m_E:m_P:m_V:m_T=0:1:\frac{3}{2}:\frac{3}{\sqrt{2}}\,. \tag{32}$$

The presence of these ratios is a consequence of the fact that there are only two independent free parameters in this theory. The numerical values of the ratios are determined by the structure of the γ algebra.

V. GENERALIZATION TO INTERNAL SYMMETRY

The above theory can be straightforwardly generalized to the case in which several spinor fields form a multiplet. Similar to Eq. (4), in notation of matrices, we start with the following free fermion Lagrangian:

$$\mathcal{L}_1=\bar{\psi}_i[i\gamma^\mu(x)\partial_\mu-m]\psi_i\equiv\bar{\psi}(i\gamma^\mu\partial_\mu-m)\psi\,. \tag{33}$$

Here index i runs from 1 to n. Using the same procedure we can construct a U(4)×U(n) algebraic gauge theory. U(4) is the internal-symmetry group of spinor space. U(n) is the internal-symmetry group among fermions. In this terminology the previous theory could be called U(4)×U(1) algebraic gauge theory. We write down the Lagrangian which is U(4)×U(n) gauge invariant, Lorentz invariant, and Hermitian,

$$\mathcal{L}=\tfrac{1}{2}i\bar{\psi}\gamma^\mu D_\mu\psi-\tfrac{1}{2}i(D_\mu\psi)^\dagger\gamma_0\gamma^\mu\psi-m\bar{\psi}\psi$$
$$-\frac{1}{8g^2}\,\mathrm{Tr}\,\bar{B}^{\mu\nu}B_{\mu\nu}+\frac{3m_p^2}{64g^2}\,\mathrm{Tr}\,\overline{D_\mu\gamma_\nu}\,D^\mu\gamma^\nu\,. \tag{34}$$

The definitions of $D_\mu \psi$, $D_\mu \gamma_\nu$, B_μ, and $B_{\mu\nu}$ are the same as in Sec. II. However, the algebraic gauge field B_μ is now a matrix in $U(4)\times U(n)$ space. We have used the notation Tr in Eq. (34) to represent the trace in both U(4) and U(n) space. Under U(4) and the Lorentz transformation various quantities undergo the same form of transformations as before. Using U(n) element G instead of U(4) element U, one can get the corresponding transformation laws of fields under U(n).

An additional feature occurs in the expansion of the gauge field B_μ. Now we need to first expand the B_μ in the U(n) space:

$$B_\mu = T^a B_\mu^a \, . \tag{35}$$

Here the T^a $(a = 0, 1, 2, \ldots, n^2-1)$ are the Hermitian infinitesimal operators of U(n). Other relevant definitions are as follows:

$$\begin{aligned} &[T^a, T^b] = i f_{abc} T^c \, , \\ &\{T^a, T^b\} = d_{abc} T^c \, , \\ &T^a T^b = q_{abc} T^c \, , \\ &q_{abc} = \tfrac{1}{2}(d_{abc} + i f_{abc}) \, , \\ &\operatorname{tr} T^a T^b = \tfrac{1}{2}\delta^{ab} \, , \\ &\operatorname{tr} T^a = \left(\frac{n}{2}\right)^{1/2} \delta^{a0} \, . \end{aligned} \tag{36}$$

In the same way we expand the curvature:

$$B_{\mu\nu} = T^a B_{\mu\nu}^a \, , \tag{37}$$

$$\begin{aligned} B_{\mu\nu}^a &= \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + \tfrac{1}{2} i f_{abc}\{B_\mu^b, B_\nu^c\} + \tfrac{1}{2} d_{abc}[B_\mu^b, B_\nu^c] \\ &= \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + q_{abc}(B_\mu^b B_\nu^c - B_\nu^b B_\mu^c) \, . \end{aligned} \tag{38}$$

With B_μ^a, the Lagrangian (34) can be written as

$$\begin{aligned} \mathcal{L} = &\tfrac{1}{2} i(\bar\psi\gamma^\mu \partial_\mu \psi - \overline{\partial_\mu \psi}\gamma^\mu \psi) - m\bar\psi\psi \\ &+ \tfrac{1}{2} i\bar\psi T^a(\gamma^\mu B_\mu^a - \bar B_\mu^a \gamma^\mu)\psi - \frac{1}{16g^2}\operatorname{tr} \bar B^{a\mu\nu} B_{\mu\nu}^a \\ &+ \frac{3m_p^{\,2}}{128 g^2}\operatorname{tr}\overline{(D_\mu\gamma_\nu)^a}(D^\mu\gamma^\nu)^a \, . \end{aligned} \tag{39}$$

Here

$$(D_\mu \gamma_\nu)^a \equiv \sqrt{2n}\, \partial_\mu \gamma_\nu \delta^{ao} + [B_\mu^a, \gamma_\nu] \, . \tag{40}$$

Under U(n), B_μ^a, $B_{\mu\nu}^a$, and $(D_\mu\gamma_\nu)^a$ undergo the following transformations:

$$\begin{aligned} &\delta B_\mu^a = f_{abc}\, \theta^b B_\mu^c + i\partial_\mu \theta^a \, , \\ &\delta B_{\mu\nu}^a = f_{abc}\, \theta^b B_{\mu\nu}^c \, , \\ &\delta (D_\mu\gamma_\nu)^a = f_{abc}\, \theta^b (D_\mu \gamma_\nu)^c \, , \end{aligned} \tag{41}$$

where the θ's are the infinitesimal parameters of U(n).

We can also break the U(4) symmetry by "gauging back" the γ-metric fields to constant matrices. Using the covariant expansion we achieve the same conclusion as in Sec. IV, namely the exclusion of scalar, axial-vector, and pseudotensor fields; the same mass spectrum and the same coupling-constant ratios of the gauge fields E_μ^a, P^a, V_μ^a, and $T_{\mu\nu}^a$.

To conclude this section, we explicitly express a Lagrangian which results from the above mechanism and contains massless vector, massive vector, and massive pseudo-scalar fields as gauge fields:

$$B_\mu^a = g\left(iE_\mu^a + \frac{1}{\sqrt3}\gamma_5\gamma_\mu P^a + \frac{1}{\sqrt2}\,\sigma_{\mu\nu} V^{a\nu}\right) , \tag{42}$$

$$E_{\mu\nu}^a = \partial_\mu E_\nu^a - \partial_\nu E_\mu^a - g f_{abc} E_\mu^b E_\nu^c \, , \tag{43}$$

$$\begin{aligned} \mathcal{L} = &\bar\psi(i\gamma^\mu\partial_\mu - m)\psi - \tfrac{1}{4}E_{\mu\nu}^a E^{a\mu\nu} - \tfrac{1}{4}(\partial_\mu V_\nu^a - \partial_\nu V_\mu^a)(\partial^\mu V^{a\nu} - \partial^\nu V^{a\mu}) + \tfrac{1}{2}(\tfrac{3}{2}m_p)^2 V^{a\mu} V_\mu^a \\ &- \tfrac{1}{4}g^2[(\tfrac{5}{4} f_{abc} f_{ab'c'} + 2 d_{abc} d_{ab'c'}) V_\mu^b V^{b'\mu} V_\nu^c V^{c'\nu} + d_{abc} d_{ab'c'} V_\mu^b V^{c\mu} V_\nu^{b'} V^{c'\nu}] + \tfrac{1}{2}\partial_\mu P^a \partial^\mu P^a - \tfrac{1}{2} m_p^{\,2} P^a P^a \\ &- \tfrac{1}{3} d_{abc} d_{ab'c'} P^b P^c P^{b'} P^{c'} - g\bar\psi\gamma^\mu T^a \psi E_\mu^a + (3/\sqrt2) g\bar\psi\gamma^\mu T^a\psi V_\mu^a + i(4/\sqrt3) g\bar\psi\gamma_5 T^a \psi P^a \\ &- \tfrac{1}{2} g f_{abc}[(\partial_\mu V_\nu^a V^{b\mu} - V_\nu^a \partial_\mu V^{b\nu}) E^{c\mu} + V_\nu^a V^{b\mu}\partial_\mu E^{c\nu}] + \tfrac{1}{2} g f_{abc}(\partial_\mu P_\nu^a P^b - P^a \partial_\mu P^b) E^{c\mu} \\ &- \tfrac{1}{4} g^2 f_{abc} f_{ab'c'}(E_\mu^b E^{b'\mu} V^{c\nu} V_\nu^{c'} - E_\mu^b V^{c\mu} E_\nu^{b'} V^{c'\nu}) + \tfrac{1}{2} g^2 f_{abc} f_{ab'c'} E_\mu^b E^{b'\mu} P^c P^{c'} \\ &+ \tfrac{1}{4} g^2 (4 f_{abc} f_{ab'c'} + d_{abc} d_{ab'c'}) V_\mu^b V^{b'\mu} P^c P^{c'} + \tfrac{1}{2} g^2 d_{abc} d_{ab'c'} V_\mu^b V^{c\mu} P^{b'} P^{c'} \, . \end{aligned} \tag{44}$$

The E_μ^a are the conventional gauge fields. V_μ^a and P^a are new ingredients. The fully symmetric structure constants d_{abc} enter the theory. This is a characteristic feature of the algebraic gauge theory.

VI. CONCLUSIONS

The theory discussed here has some promising characteristics. The concept of a metric vector field and the breakdown of symmetry cause the mass terms of gauge fields to appear naturally. The covariant expansion in spinor space provides a bridge between the algebraic gauge fields and the physical fields. The uniqueness of the starting point of the theory implies the gauge field of a definite mass spectrum and specified coupling constants. Investigations of the phenomenological implications of such a theory should be interesting.

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[1] Kevin Cahill, Phys. Rev. D 18, 2930 (1978).
[2] Kevin Cahill, Phys. Rev. D 20, 2636 (1979).
[3] A. Zee, University of Pennsylvania Report No. UPR-0131T, 1979 (unpublished).
[4] Jihn E. Kim and A. Zee, University of Pennsylvania Report No. UPR-0132T, 1979 (unpublished).
[5] Stephen L. Adler, Phys. Rev. D 17, 3212 (1978).
[6] Stephen L. Adler, Phys. Lett. 86B, 203 (1979).
[7] Zhu Dongpei, University of Wisconsin—Madison report, 1980 (unpublished).