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Strongly interacting Higgs bosons

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The sensitivity of present-energy weak interactions to a strongly interacting heavy-Higgs-boson sector is discussed. The gauged nonlinear σ model, which is the limit of the linear model as the Higgs-boson mass goes to infinity, is used to organize and catalogue all possible heavy-Higgs-boson effects. As long as the $SU(2)_L \times SU(2)_R$ symmetry of the Higgs sector is preserved, these effects are found to be small, of the order of the square of the gauge coupling times logarithms (but not powers) of the Higgs-boson mass divided by the W mass. We work in the context of a simplified model with gauge group $SU(2)_L$; the extension to $SU(2)_L \times U(1)$ is briefly discussed.

I. INTRODUCTION

The evidence for a unified theory of weak and electromagnetic interactions, based on a spontaneously broken $SU(2)_L \times U(1)$ gauge theory, is impressive. Unfortunately, although our understanding of gauge theories has continued to develop, we have made very little progress in understanding the origin of spontaneous symmetry breakdown. For the most part, the Higgs mechanism continues to be described by the *ad hoc* introduction into the Lagrangian of elementary, weakly self-coupled scalar fields. In the minimal model,[1] a complex SU(2) doublet is used, providing three Goldstone bosons (longitudinal W bosons) and one physical massive scalar.

Even though the dynamical mechanism underlying symmetry breakdown is not really understood, one can anticipate some qualitative features which should be present if the Higgs quanta are not elementary. The necessary existence of zero-mass Goldstone bosons suggests the presence of strong forces and that, in turn, leads to a natural guess for the mass scale of the physical spectrum in the Higgs sector. A rough estimate can be gotten in a variety of equivalent ways. The relation $\sqrt{2}G_F = \langle \phi \rangle^{-2}$ between the Fermi decay constant and vacuum expectation value of an effective (or elementary) scalar field gives $\langle \phi \rangle \simeq 300$ GeV and so this must be a natural scale for the Higgs sector. If the forces are of unit strength on this scale, then masses on the order of or somewhat more than 300 GeV are to be expected. Alternatively, in terms of the conventional complex-doublet theory, the connection between the scalar-field expansion parameter ($\sim\lambda/\pi^2$) and the Higgs-boson mass M_H is

$$\frac{\lambda}{\pi^2} = \frac{g^2 M_H^2}{8\pi^2 M_W^2} = \frac{1}{\sqrt{2}\pi^2} G_F M_H^2 . \tag{1.1}$$

In the strong-coupling regime, $\lambda/\pi^2 \sim 1$ and, therefore, $M_H \sim 1$ TeV, another indication of the mass scale of the physical bound states or resonances of the Higgs sector. Throughout this paper, the words "low energy" will refer to the region $E \ll 1$ TeV.

If the Higgs sector is heavy and strongly interacting, it is important to look carefully at its impact on the rest of the gauge theory. This paper is an effort to do that as systematically as possible for the low-energy structure of the theory. Currently available center-of-mass energies are considerably below 1 TeV and it will be some time before 1-TeV energies are available in elementary channels such as e^+e^- or $q\bar{q}$. It will be possible to analyze the low-energy structure in a rather model-independent way, without having to specify in detail the dynamical mechanism underlying symmetry breakdown.

The study of heavy-Higgs-boson effects has been going on for some time. References 2–7 comprise a list of some of the papers we are aware of. The main goal of the present paper is to study this problem in a more complete and general way in order to answer the following question: What is the strongest impact that heavy Higgs particles can have on experiments done at $E \ll 1$ TeV, assuming only some rather general properties of the Higgs sector? The answer, in agreement with the various specific computations in the literature, is that at one-loop order $[O(\alpha = g^2/4\pi)$ in the gauge-coupling expansion$]$, the sensitivity is at most logarithmic. In higher orders, the effective expansion parameter becomes $G_F M_H^2/\sqrt{2}\pi^2$, and since this is assumed to be of order one, the expansion breaks down. Nevertheless, these strong blobs will remain shielded[2] from low-energy probes by at least one power of α.

In rough outline, the present analysis goes as follows. It is first argued that the usual Higgs theory with elementary scalar fields provides a good low-energy description of a dynamically generated Higgs mechanism. It also provides a natural cutoff, the Higgs-boson mass M_H, to test for the low-energy sensitivity to the strong 1-TeV Higgs-boson physics that can be generated through quantum corrections. The Higgs theory will be used in much the same spirit that phenomenological chiral Lagrangians[8] are used to describe low-energy hadronic physics. With the assumption of a global $SU(2)_L \times SU(2)_R$ symmetry in the Higgs sector, it will in fact be precisely the usual linear σ model.

A convenient way to search for M_H sensitivity, which will keep the important $SU(2)_L \times SU(2)_R$ symmetry explicit, is to take the $M_H \to \infty$ limit formally at the beginning. The resulting gauged nonlinear σ model is pertubatively nonrenormalizable and thus new, in principle measurable, cutoff dependence can be anticipated at one-loop order and beyond. All such dependence can be searched for by listing the counterterms allowed by the symmetries of the nonlinear theory and then estimating and finally computing the coefficients. The cutoff dependence can be regarded as M_H dependence or, more generally, as the sensitivity of low-energy physics to the 1-TeV Higgs world.

In Sec. II, the program will be outlined in detail. Attention will be restricted throughout the paper to an SU(2) gauge theory although some remarks about the extension to SU(2) × U(1) and other realistic theories will be included in the last section. The important role of the gauged nonlinear σ model will be explained.

In Sec. III, the features of this theory will be examined in more detail. General arguments based on dimensional analysis are used to estimate the dependence on the cutoff of the counterterms generated in each order of the loop expansion. All possible structures allowed by the nonlinear symmetry which can be generated at one-loop order will then be listed.

Section IV will be devoted to an explicit computation of the new Lagrangian terms as functions of the cutoff. Some care is required in dealing with the scalar (Goldstone-boson) sector of the theory, in particular with respect to the interplay of quantum corrections and the choice of parametrization of the model in terms of the Goldstone fields.

In Sec. V, the experimentally measurable effects due to this cutoff dependence are listed, most coming in the form of "corrections to natural relations." Natural relations are constraints among coupling strengths and masses which arise because of the form of the original Lagrangian dictated by the symmetries. The new invariant structures of the nonlinear theory can eliminate these constraints since they can contribute to the same coupling strengths and masses which the original Lagrangian generates. The various measurable corrections can simply be read off from the counterterms of Secs. III and IV.

Our results are summarized in Sec. VI along with some remarks about the SU(2) × U(1) theory.

II. THE HIGGS MODEL AS AN EFFECTIVE LOW-ENERGY THEORY

We proceed by writing down and describing the Higgs model to be analyzed and then discussing the extent to which it can be regarded as the low-energy limit of a gauge theory with some strong interaction driving the spontaneous breakdown.

The starting point is the $SU(2)_L \times SU(2)_R$ σ model which will be taken to describe the Higgs sector. The matrix field

$$M(x) = \sigma(x) + i\vec{\tau}\cdot\vec{\pi}(x) \tag{2.1}$$

transforms from the left and right according to the $(\frac{1}{2},\frac{1}{2})$ representation of $SU(2)_L \times SU(2)_R$. The set $\phi(x) = (\sigma(x), \vec{\pi}(x))$ transforms as a vector under the isomorphic $O(4)$ group and the most general form of the scalar potential is

$$V(\phi) = \tfrac{1}{4}\lambda(\tfrac{1}{2}\,\mathrm{Tr}M^\dagger M - f^2)^2 = \frac{M_H{}^2}{8f^2}(\sigma^2 + \vec{\pi}^2 - f^2)^2 , \tag{2.2}$$

where f is the vacuum expectation value. The gauge field $W_\mu(x) \equiv W_\mu(x)\cdot\vec{\tau}/2i$ will be taken to transform according to the $SU(2)_L$ subgroup. The gauge-invariant Lagrangian is

$$\mathcal{L}_{\mathrm{inv}} = +\tfrac{1}{2}\,\mathrm{Tr}(F_{\mu\nu})^2 + \tfrac{1}{4}\,\mathrm{Tr}D_\mu M(D^\mu M)^\dagger + \frac{M_H{}^2}{8f^2}(\tfrac{1}{2}\,\mathrm{Tr}M^\dagger M - f^2)^2 , \tag{2.3}$$

where $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g[W_\mu, W_\nu]$ and $D_\mu = \partial_\mu + gW_\mu$. The essential ingredients of our analysis will not directly involve any fermions which might be introduced and so we dispense with them for now. They will be included later in order to model some interesting physical processes.

In quantizing this theory, it is most convenient to work in Landau gauge. In that way, the Fadeev-Popov ghost couples only to the gauge field, direct transitions of the W into the π are forbidden, and the π field stays massless. The Goldstone-realized global $SU(2)_L \times SU(2)_R$ symmetry of the Higgs sector is then kept manifest throughout.

The question now is what happens when M_H becomes so large that

$$\frac{\lambda}{\pi^2} = \frac{M_H{}^2}{2\pi^2 f^2} \sim 1 . \tag{2.4}$$

A key ingredient in answering this question is the Higgs-sector $SU(2)_L \times SU(2)_R$ symmetry and the low-energy behavior that follows from it. If this symmetry is hidden, as it always is in the generation of Feynman rules for a Higgs theory, the analysis can become exceedingly complicated. It is only through the cancellation of many diagrams that the symmetry is reinstated so that general properties are hard to discover.

The best way to avoid these problems and to continue to keep the $SU(2)_L \times SU(2)_R$ symmetry explicit is to take the limit $M_H \to \infty$ formally at the beginning. As far as the scalar sector is concerned, this takes it from a linear to a nonlinear σ model with the constraint

$$M^\dagger M = MM^\dagger = (2M_W)^2/g^2 = f^2. \tag{2.5}$$

In terms of the $\vec{\pi}$ field as it was introduced in Eq. (2.1), the scalar part of the Lagrangian becomes

$$\mathcal{L}_{\rm scalar} = \tfrac{1}{2}(\partial_\mu \vec{\pi})^2 + \tfrac{1}{2}(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2/(f^2 - \vec{\pi}^2). \tag{2.6}$$

The full theory is then a left-gauged nonlinear σ model. The full invariant Lagrangian is

$$\mathcal{L}_{\rm NL} = +\tfrac{1}{2}\,{\rm Tr}(F_{\mu\nu})^2 + \tfrac{1}{4}D_\mu M(D^\mu M)^\dagger \tag{2.7}$$

with M subject to the constraint of Eq. (2.5). In addition to $\mathcal{L}_{\rm scalar}$, this contains the usual Yang-Mills pieces along with interaction terms containing one W field and $2, 3, 5, 7, \ldots, \pi$ fields. To (2.7), gauge-fixing and ghost terms must be added. In the R_ξ gauges,

$$\mathcal{L}_{\rm GF} = -\tfrac{1}{2}\xi(\partial_\mu W^{\mu a} + gf\pi^a/2\xi)^2, \tag{2.8}$$

with the corresponding Fadeev-Popov ghost term. In the limit $\xi \to \infty$ (Landau gauge), this takes the form

$$\mathcal{L}_{\rm FP} = -\bar{\chi}^a \,\Box\, \chi^a + g\epsilon_{abc}(\partial^\mu \bar{\chi}^a)\chi^b W^c_\mu. \tag{2.9}$$

Since the nonlinear theory is nonrenormalizable, the limit $M_H \to \infty$ does not actually exist in perturbation theory. As a result, computations with the nonlinear theory will lead to new divergences which force the introduction of new counterterms. These terms must respect the $SU(2)_L \times SU(2)_R$ symmetry and it is possible to list all such structures which can be generated at one or two loops. It is only these new structures which can lead to deviations from the predictions of the original theory, and once they are listed, all possible new effects can be read off. The cutoff dependence of these new terms can be computed and the cutoff can then be taken to be M_H of the original linear theory. It is the size of the new terms in the limit (2.4) that is of interest. The details of this program are presented, beginning in Sec. III.

It is worth recalling that a gauge theory coupled to the nonlinear σ model is formally equivalent to a Yang-Mills theory in which a mass term is added by hand, the so-called massive Yang-Mills theory.[9] The easiest way to see this is to go back to the linear theory and quantize in a general R_ξ gauge. If the limit $\xi \to 0$ (instead of $\xi \to \infty$, the Landau gauge) is then taken, the theory goes into unitary gauge. The π is explicitly absorbed by the W and the result is a massive Yang-Mills theory coupled to the Higgs particle described by $\sigma(x)$. The limit $M_\sigma \to \infty$ then formally removes the Higgs particle from the Hilbert space and the result is the massive Yang-Mills theory. While it would certainly be possible to examine the limit (2.4) in the unitary gauge, it would be extremely cumbersome. The Landau gauge is really much more "physical" for this particular problem.

The role being played by the nonlinear σ model here is similar in some ways to its role as a phenomenological Lagrangian describing low-energy pion physics.[8] With the pion as an approximate Goldstone boson, the combined constraints of $SU(2)_L \times SU(2)_R$ current algebra and pole dominance are embodied in the nonlinear Lagrangian. Used in tree approximation, it then reproduces the current-algebra pole-dominance results for soft-pion S-matrix elements.

If it is assumed that the Goldstone symmetry involved in the Higgs mechanism is also $SU(2)_L \times SU(2)_R$, the same nonlinear model, used in a similar way, will be relevant. However, the differences are important. Here the π's are the Higgs ghosts, explicitly present only because of a gauge choice. Because they only appear virtually and because direct $W \to \pi$ transitions are forbidden, they enter only in loops and the momentum flowing through them must be integrated over. Thus the scalar theory (2.6) is being used and tested more seriously than in low-energy pion physics. It is still taken to represent the Higgs sector at low energies and, to the extent that the loop computations are dominated by low energies ($\ll M_H$), it should suffice. The problem, of course, is that the loop computations are not always dominated by low energies. The whole point of the program we have so far outlined and will next present in detail is to see just how far one can go with the nonlinear model. It might be possible to reformulate this program using a current-algebra and pole-dominance language, but that we shall not attempt here.

III. THE STRUCTURE OF COUNTERTERMS IN THE NONLINEAR MODEL

In this section we will examine the structure of the divergences which appear in perturbation-theory computations with the nonlinear Higgs Lagrangian. These divergences contain all the information about the effects of a strong, heavy-Higgs-boson sector on the gauge theory at low energies. Divergences not of the form of the original Lagrangian arise because of the scalar-field self-couplings which have two derivatives and arbitrary powers of the field.

The invariant nonlinear Lagrangian is given by Eq. (2.7). As mentioned above, the choice of Landau gauge has the advantage of keeping the $\vec{\pi}$ field explicitly massless at all stages of the calculation and of eliminating direct coupling of the $\vec{\pi}$ to the ghosts. Since the nonrenormalizability comes only from the $\vec{\pi}$ self-couplings, this gauge choice keeps the nonrenormalizability from infecting graphs with external ghosts—the divergences may be canceled merely by subtracting subgraphs with external $\vec{\pi}$ and $\vec{W}_\mu$ fields only. For this reason, counterterms necessary to cancel divergences are expected to be local, explicitly gauge-invariant functions of $\vec{\pi}$ and $\vec{W}_\mu$. In other gauges, the counterterms would be functions of the ghost fields also, and their structure would have to be determined by using the more general Becchi-Rouet-Stora (BRS) invariance.[10]

There is a fairly simple power-counting argument which determines the structure of possible divergent counterterms that may arise at a given order in the loop expansion. This argument is made most easily by defining dimensionless scalar fields. We first write the M matrix of Eq. (2.1) in terms of a unitary matrix U:

$$M=fU\,. \tag{3.1}$$

U is then parametrized with some set of scalar fields. For example, we may write

$$U=(1-\underline{\vec{\pi}}^2)^{1/2}+i\vec{\tau}\cdot\underline{\vec{\pi}}\,, \tag{3.2}$$

where $\underline{\vec{\pi}}=\vec{\pi}/f$ is dimensionless. The effective Lagrangian then takes the form

$$\mathcal{L}=\tfrac{1}{4}f^2\,\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger]+\mathcal{L}_G\,, \tag{3.3}$$

where

$$\mathcal{L}_G\equiv\tfrac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})+\mathcal{L}_{\mathrm{GF}}+\mathcal{L}_{\mathrm{FP}}\,. \tag{3.4}$$

In the power-counting analysis which follows, we use the fact that there exists a dimensionful, gauge- and chiral-invariant regulator Λ. (In principle, one way to obtain such a regulator would be to perform renormalized calculations in the linear model. The renormalized Higgs-boson mass could then be taken large and identified with Λ.[11]) Now define D as the dimension of a counterterm which appears at L loops. D counts one for every derivative and gauge field; scalar fields are dimensionless and will appear in arbitrary numbers, governed only by gauge and chiral invariance. In addition, let n be the number of powers of f^2 and r be the number of powers of the regulator Λ which accompany this counterterm. Dimensional analysis then implies

$$D+2n+r=4\,. \tag{3.5}$$

The number n is not hard to determine because f^2, which multiplies the first term of the Lagrangian (3.3), acts basically like a loop-counting parameter. If we just use propagators and vertices from this first term, then the usual loop-counting argument gives

$$n=1-L\,. \tag{3.6}$$

If we now allow I_G propagators and V_G vertices from $\mathcal{L}_G$, (3.6) is altered to read

$$n=1-L+b\,, \tag{3.7}$$

where $b\equiv I_G-V_G$. It is not hard to see that, except in zero loops,

$$b\geqslant 0 \tag{3.8}$$

with equality coming only when $I_G=0$ or when $L=1$ and *all* vertices and propagators in the diagram are from $\mathcal{L}_G$. That is, $b=0$ only in diagrams with no internal gauge or ghost lines, or in the pure Yang-Mills one-loop graphs. Putting (3.7) and (3.8) in (3.5) gives

$$D\leqslant 2+2L-r\,, \tag{3.9}$$

with equality under the same conditions as (3.8).

Equation (3.9) is the result we were after. It says that divergent counterterms with the greatest dimension are generated in L loops by the logarithmic ($r=0$) divergences of graphs with no internal gauge or ghost lines (except when $L=1$, when the pure Yang-Mills diagrams are also leading in D—but even there, we will find that the Yang-Mills graphs are not important for the calculation of physically significant large-M_H effects). These "leading" counterterms have

$$D=D_{\max}=2+2L\,. \tag{3.10}$$

In $L+1$ loops, new counterterms with higher D will appear as logarithmic divergences; the counterterms that first appeared at L loops can now appear with quadratically divergent coefficients (and two more powers of f in the denominator) or with logarithmically divergent coefficients from graphs with internal gauge or ghost lines. Similarly, counterterms that first appeared at $L-1$ loops can now appear with quartic divergences, and so on.

We now focus our attention on the one-loop graphs. The form of the divergent counterterms that appear can be rather simply determined by the power-counting arguments given above and by the requirement of gauge and chiral invariance.

Equation (3.9) implies that, at one loop, there may be logarithmic divergences with $D=4$, quadratic divergences with $D=2$, and quartic divergences with $D=0$.

The quartic divergences are easily disposed of. Since $D=0$, they must be made up of $\vec{\pi}$ fields alone, with no gauge fields or derivatives. The only invariant is then

$$\mathrm{Tr}\,UU^{\dagger}=1\,, \tag{3.11}$$

so there will, in fact, be no quartic divergences. Explicit loop computations bear out this formal argument when care is taken to treat the loop expansion in a chirally invariant way.[12]

The quadratic divergences have $D=2$. A little thought convinces one that the only invariant $D=2$ structure is $\mathrm{Tr}D_{\mu}U(D^{\mu}U)^{\dagger}$, which is of the form of the original Lagrangian.[13] Thus, such divergences can be absorbed into redefinitions of the original parameters (wave-function renormalization of the field and redefinition of f) and are physically unimportant.

The only counterterms which are important at one loop are, therefore, the logarithmic divergences with $D=4$. There are several such structures, and it proves convenient, in their enumeration, first to define the dimension-one object

$$V_{\mu}\equiv(D_{\mu}U)U^{\dagger}=gW_{\mu}+(\partial_{\mu}U)U^{\dagger}. \tag{3.12}$$

V_{μ} is an $SU(2)_R$ invariant and transforms covariantly under the gauged $SU(2)_L$ as

$$V_{\mu}\to V'_{\mu}=G_L V_{\mu} G_L^{\dagger}. \tag{3.13}$$

We can take covariant derivatives of V_{μ} with the adjoint representation operator

$$\mathfrak{D}_{\nu}\equiv\partial_{\nu}+g[W_{\nu},\ ]. \tag{3.14}$$

Using the identity

$$(D_{\mu}U)^{\dagger}=-U^{\dagger}(D_{\mu}U)U^{\dagger}, \tag{3.15}$$

we can arrange things so that derivatives act only on U (and not on $U^{\dagger}$). It then becomes clear that one can construct all nontrivial, local-invariant quantities out of traces of strings of V_{μ}'s and their covariant derivatives, and $F_{\mu\nu}$'s and their covariant derivatives. For example, the first term in the original Lagrangian (3.3) is proportional to $\mathrm{Tr}V_{\mu}V^{\mu}$.

The candidates for the $D=4$ invariants are then

$$\begin{aligned}
&\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\,,\\
&\mathrm{Tr}(F_{\mu\nu}\mathfrak{D}^{\mu}V^{\nu})\,,\\
&\mathrm{Tr}(F_{\mu\nu}V^{\mu}V^{\nu})\,,\\
&\mathrm{Tr}[(\mathfrak{D}_{\mu}V^{\mu})(\mathfrak{D}_{\nu}V^{\nu})]\,,\\
&\mathrm{Tr}[(\mathfrak{D}_{\mu}V_{\nu})(\mathfrak{D}^{\mu}V^{\nu})]\,,\\
&\mathrm{Tr}[(\mathfrak{D}_{\mu}V_{\nu})(\mathfrak{D}^{\nu}V^{\mu})]\,,\\
&\mathrm{Tr}(V_{\mu}V_{\nu}\mathfrak{D}^{\mu}V^{\nu})\,,\\
&\mathrm{Tr}(V_{\mu}V_{\nu})\,\mathrm{Tr}(V^{\mu}V^{\nu})\,,\\
&\mathrm{Tr}(V_{\mu}V^{\mu})\,\mathrm{Tr}(V_{\nu}V^{\nu})\,,
\end{aligned} \tag{3.16}$$

where we have eliminated terms that differ by a simple integration by parts, and where we have used the simplicity of the trace in SU(2) to write traces of four V's as products of traces of two V's. The terms in (3.16) are, however, not independent. We can use the antisymmetry of $F_{\mu\nu}$, the identities

$$[\mathfrak{D}_{\mu},\mathfrak{D}_{\nu}]\psi=g[F_{\mu\nu},\psi]\,, \tag{3.17a}$$

$$\mathfrak{D}_{\mu}V_{\nu}-\mathfrak{D}_{\nu}V_{\mu}=gF_{\mu\nu}+[V_{\mu},V_{\nu}]\,, \tag{3.17b}$$

and repeated integrations by parts to eliminate all but the following counterterms:

$$\begin{aligned}
&\mathfrak{L}_0=\tfrac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\,,\\
&\mathfrak{L}_1=\alpha_1\mathrm{Tr}(V_{\mu}V^{\mu})\,\mathrm{Tr}(V_{\nu}V^{\nu})\,,\\
&\mathfrak{L}_2=\alpha_2\mathrm{Tr}(V_{\mu}V_{\nu})\,\mathrm{Tr}(V^{\mu}V^{\nu})\,,\\
&\mathfrak{L}_3=\alpha_3 g\,\mathrm{Tr}(F_{\mu\nu}[V^{\mu},V^{\nu}])\,,\\
&\mathfrak{L}_4=\alpha_4\mathrm{Tr}[(\mathfrak{D}_{\mu}V^{\mu})(\mathfrak{D}_{\nu}V^{\nu})]\,.
\end{aligned} \tag{3.18}$$

The structures $\mathfrak{L}_1-\mathfrak{L}_4$, which do not appear in the original Lagrangian, completely determine the new physical effects at one loop. The coefficients $\alpha_1-\alpha_4$ can be at most logarithmically divergent at the one-loop level. (It will turn out, however, that α_4 is not divergent.) These coefficients will be reinterpreted in the linear model as proportional to $\ln M_H$. Wave-function renormalization of the fields inside these counterterms only appears at two loops. Of course, $\mathfrak{L}_0$, since it originally appears at tree level, gets wave-function and charge renormalization in the usual way. We will compute the divergent coefficients of the above quantities in Sec. IV.

IV. THE COMPUTATION OF ONE-LOOP DIVERGENCES

The coefficients $\alpha_1,\ldots,\alpha_4$ of the counterterms $\mathfrak{L}_1,\ldots,\mathfrak{L}_4$ (3.18) will now be computed. A key point is that since our power-counting argument has shown that the only physically significant one-loop divergences are logarithmic, we can use di-

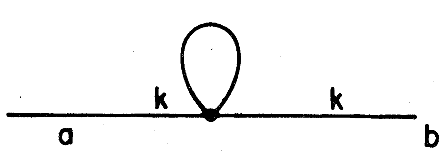


FIG. 1. One-loop π self-energy.

mensional continuation and merely interpret $1/\epsilon$ as $\ln\Lambda$ or $\ln M_H$ at the end. This is important because the only obvious, dimensionful, gauge- and chiral-invariant regulator of the nonlinear model is the renormalized linear model in the large-M_H limit.[14] (See the end of this section for some comments about the failure of a simple cutoff in momentum space.) However, if we were actually forced to compute in the linear model, the whole simplicity of our approach would be lost.

Before proceeding, we recall that $\mathcal{L}_0 = \frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ has internal charge and wave-function renormalization and gives rise to the usual three counterterms (two of which are independent)

$$\begin{aligned} &-\tfrac{1}{4}(Z_3-1)(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2\,,\\ &-(Z_1-1)g\epsilon^{abc}\partial_\mu W_\nu^a W_\mu^b W_\nu^c\,, \qquad (4.1)\\ &-\tfrac{1}{4}(Z_1{}^2/Z_3-1)g^2\epsilon^{abc}\epsilon^{ade}W_\mu^b W_\nu^c W_\mu^d W_\nu^e\,. \end{aligned}$$

We note that Z_1 and Z_3 are, in fact, not equal to the corresponding counterterms of the linear model but contain additional logarithmic divergences that appear only as $M_H\to\infty$.

The quantities $\alpha_1,\ldots,\alpha_4$ and Z_1 and Z_3 can now be calculated. The relevant vertices and propagators are obtained by putting (3.2) into (3.3) and expanding $(1-\vec{\pi}^2)^{1/2}$ in terms of $\vec{\pi}^2$. The counterterms $\mathcal{L}_1,\ldots,\mathcal{L}_4$ may be similarly expanded. The Feynman rules and the contributions of $\mathcal{L}_1,\ldots,\mathcal{L}_4$ to various Green's functions are described in the Appendix and listed in Tables I and II.

It is convenient to determine α_4 first. This is rather easy because only $\mathcal{L}_4$ can contribute to the renormalization of the π self-energy; that is, only $\mathcal{L}_4$ has a term with two $\vec{\pi}$ fields and no gauge fields. There is just one diagram, Fig. 1, for the π self-energy. (Recall that, except for pure Yang-Mills diagrams with no π lines at all, graphs with internal gauge lines can never contribute to the highest-dimension counterterms.) However, inspection of Fig. 1 shows that it has no logarithmic divergences, only quartic and quadratic—or equivalently, that it cannot produce the four powers of momentum present in the counterterm from $\mathcal{L}_4$ (see Table II) because the loop has no dependence on the external momentum. Thus, we have $\alpha_4=0$, and $\mathcal{L}_4$ is eliminated.[15]

We next consider $\mathcal{L}_3$, which is also quite easy to compute. $\mathcal{L}_3$ is the only remaining counterterm that renormalizes graphs with two π's and a W external. The only graph appears in Fig. 2, and its divergent part is

$$\frac{-1}{16\pi^2}\frac{g}{3\epsilon}\,\epsilon^{ab_1b_2}[q_{1\mu}(k\cdot q_2)-q_{2\mu}(k\cdot q_1)]\,, \qquad (4.2)$$

where $\epsilon=4-n$. Comparison with the contribution of $\mathcal{L}_3$ (see Table II) gives

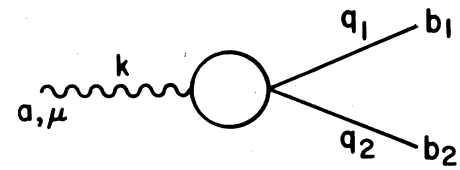


FIG. 2. One-loop $\pi\pi W$ vertex.

$$\alpha_3=\left(\frac{1}{16\pi^2}\right)\frac{1}{12\epsilon}\,. \qquad (4.3)$$

A check on this result can be obtained by calculating the graph of Fig. 3 with two W's and a π external. $\mathcal{L}_3$ is the only contributor to the renormalization and one again arrives at (4.3).

The remaining coefficients α_1 and α_2 may now be computed in a variety of ways. We outline three independent calculations because each is instructive in its own right. First, consider only the graphs describing the interaction of the W's, the physical particles of the present theory. From Table II we see that the $2W$ and $3W$ one-particle-irreducible (1PI) Green's functions can be used to determine Z_3 and Z_1, since α_3 is already known. Then the $4W$ function will determine α_1 and α_2. Now each of these Green's functions receives contributions both from π loops and also from pure Yang-Mills graphs which give the same contribution to Z_1 and Z_3 that they would in an unbroken, pure gauge theory. (The contributions do not depend on M_W, as a simple power-counting argument shows.) Since the latter theory is independently renormalizable, the contributions from the pure Yang-Mills diagrams will be absorbable into Z_3 and Z_1 and will have no effect on the new physically important quantities, the α_i. Thus, we may totally ignore the pure Yang-Mills graphs in calculating α_i. The only remaining graphs are shown in Figs. 4, 5, and 6. Their divergent parts are given by

$$\text{Fig. 4:}\quad -\left(\frac{1}{16\pi^2}\right)\frac{ig^2}{6\epsilon}\,\delta_{ab}[g_{\mu\nu}k^2-k_\mu k_\nu]\,, \qquad (4.4)$$

$$\text{Fig. 5:}\quad -\left(\frac{1}{16\pi^2}\right)\frac{g^3}{12\epsilon}\,\epsilon^{abc}[g_{\mu\nu}(k-q)_\lambda+g_{\nu\lambda}(q-r)_\mu +g_{\lambda\mu}(r-k)_\nu]\,, \qquad (4.5)$$

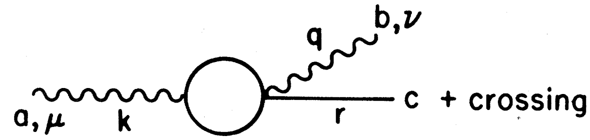


FIG. 3. One-loop πWW vertex.

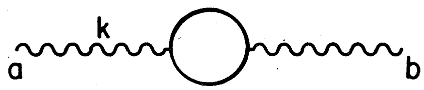


FIG. 4. One-loop W self-energy.

Fig. 6: $+\left(\frac{1}{16\pi^2}\right)\frac{ig^4}{6\epsilon}\,[\delta_{ab}\delta_{cd}+\text{perms}]\,[g_{\mu\nu}g_{\lambda\sigma}$

$$+\text{perms}]\,. \tag{4.6}$$

After dividing (4.6) into the two tensor structures that appear in the $4W$ counterterm, these results may be compared with Table II. The result is four equations in the four unknowns: α_1, α_2, and the $\underline{\pi}$-loop contributions to Z_1 and Z_3. Solving for α_1 and α_2 then gives the values

$$\alpha_1=-\left(\frac{1}{16\pi^2}\right)\frac{1}{12\epsilon}\,,\qquad \alpha_2=-\left(\frac{1}{16\pi^2}\right)\frac{1}{6\epsilon}\,. \tag{4.7}$$

Note that the nonvanishing of α_1 and α_2 and the equivalence, mentioned in Sec. II, between the present theory and massive Yang-Mills theory, immediately show that the latter theory is perturbatively nonrenormalizable. The divergent part

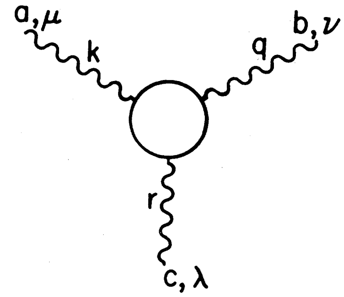


FIG. 5. One-loop $3W$ vertex.

of the $4W$ vertex has a tensor structure (4.6) which is not of the form of the original Lagrangian.[16] It is, in some sense, a simpler structure since the Bose symmetry is found separately in both the internal indices and the Lorentz indices. The $4W$ vertex will be discussed further in Sec. V in assessing the physical consequences of the cutoff dependence.

We now compute α_1 and α_2 in a second way. The logarithmic, $D=4$ divergence in the 4π 1PI Green's function comes only from the graph in Fig. 7. This is a pure nonlinear-σ-model diagram and the coupling to the gauge theory is totally irrelevant here. The divergent part of Fig. 7 takes the form

$$\frac{1}{16\pi^2}\frac{4i}{3\epsilon}\,[\delta_{a_1a_2}\delta_{a_3a_4}(t^2+u^2+ut)+\text{perms}]+\text{terms which vanish when } q_1{}^2=q_2{}^2=q_3{}^2=q_4{}^2=0\,, \tag{4.8}$$

where t,u are the usual Mandelstam variables. We have separated off the terms which vanish when $q_i{}^2=0$ for an important reason: It is known that in the nonlinear σ model the one-loop divergences off mass shell are not invariant under the naive nonlinear symmetry.[17] One way to cope with this problem is to redefine the symmetry—or, equivalently, to redefine the $\underline{\pi}$ field—order by order in the loop expansion. As is usually the case in field theory, redefinitions have no effect on the

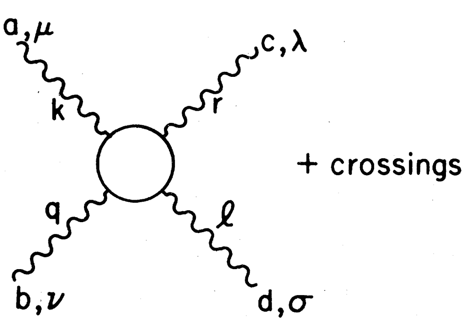


FIG. 6. One-loop $4W$ vertex.

on-mass-shell amplitudes, where the symmetry is manifest. This and other subtleties of the loop expansion in the nonlinear σ model will be discussed more completely in a forthcoming paper.[18]

Even if the reader accepts the fact that it is convenient to work on-shell with manifest symmetry, he may be troubled by the following question: What do we mean by "on mass shell" for the present gauge theory, where the $\underline{\pi}$ is not a physical particle but a Goldstone boson which is absorbed by the Higgs mechanism? The answer is simple in Landau gauge. If we insist on calculating graphs

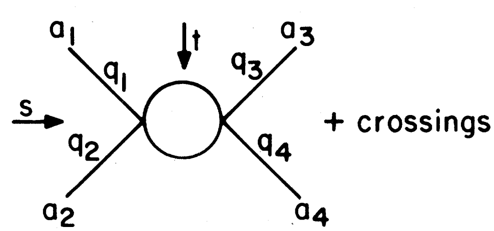


FIG. 7. One-loop 4π vertex.

with external π lines, then "on mass shell" means $q^2=0$, where the π propagator has a pole. (The pole remains at $q^2=0$ even in higher order because Landau gauge is not renormalized,[19] which means that there is always a spurious $q^2=0$ pole in the W propagator. Therefore, the π pole—and the ghost pole—must also remain at $q^2=0$ to cancel it.) At the pole, the graphs will be invariant under redefinitions of the π field and hence the symmetry will be manifest. Of course, one may wish only to calculate graphs with physical external particles. In that case, there will be no source coupled to the π field, and the generating functional $W[J]$ will be at least formally invariant under all redefinitions of the π field. Thus the symmetry will be automatically manifest, as exemplified by the first calculation of α_1 and α_2. We do not have to put the W particles on shell (although the Landau gauge condition will, in general, be necessary—see below).

Returning now to the second calculation of α_1 and α_2, we may set $q_i^2=0$ and then compare (4.8) with the appropriate counterterm in Table II. A little algebra and the fact that $s+t+u=0$ when $q_i^2=0$ then reproduces (4.7). It is important to reiterate here that when $q_i^2\neq 0$, it is impossible to choose α_1 and α_2 to absorb the divergences in (4.7). As described above, the naively invariant counterterms are not sufficient of "mass shell"—a redefinition of the symmetry will, in general, be required. We note that the quantities α_1 and α_2 have been calculated previously in this context of the pure nonlinear σ model.[17,20]

A third computation of α_1 and α_2 provides an additional insight. Consider the Green's function with $3W$'s and one π external. It is given by the graph in Fig. 8, which has divergent part

$$\left(\frac{1}{16\pi^2}\right)\frac{g^3}{6\epsilon}\,[\delta_{ab}\delta_{cd}(4g_{\mu\nu}l_\lambda+g_{\mu\lambda}l_\nu+g_{\nu\lambda}l_\mu)+\text{perms}]$$
$$+\text{terms proportional to } k_\mu,\ q_\nu,\ \text{or } r_\lambda. \qquad (4.9)$$

The terms proportional to k_μ, q_ν, or r_λ will vanish when contracted into the Landau gauge propagators on the external lines. We can then compare (4.9) with the appropriate counterterm in Table II: the result is again (4.7). This shows the importance of the Landau-gauge condition; as emphasized in Sec. III, the counterterms will otherwise not possess simple gauge invariance but only BRS invariance. Here, we would be unable to absorb the divergences of (4.9) into the gauge-invariant counterterms without the Landau gauge condition.

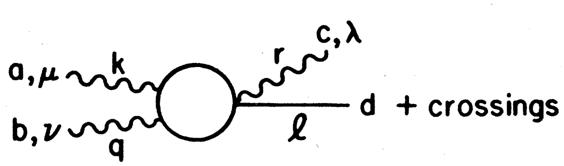


FIG. 8. One-loop πWWW vertex.

Perhaps a few final remarks are in order on the questions of quadratic divergences. In Sec. III we argued that these divergences are always of the form of the $D=2$ part $[\mathrm{Tr}D_\mu U(D_\mu U)^\dagger]$ of the original Lagrangian. However, if a computation is attempted with a simple momentum cutoff on the Feynmann integrals of the nonlinear theory, problems soon arise. For example, the quadratic divergences in the π self-energy and in the 2π-W vertex have relative strengths which are not compatible with the counterterm. (It would seem that a Ward identify is being violated.) The difficulty here, however, is not in the theory but in the method of regularization. The momentum-space cutoff seems to be akin to a π mass term and hence to be chirally noninvariant. An invariant cutoff (for example, returning to the linear model and taking $M_H\to\infty$) gives results compatible with the invariant counterterm. We will discuss this question further in Ref. 18.

V. EXPERIMENTAL EFFECTS

All possible one-loop sensitivity to a heavy-Higgs-boson sector can be read off from the counterterms $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ of Eq. (3.18). The coefficients α_1, α_2, and α_3 are given by Eqs. (4.7) and (4.8) and the dimensional continuation parameter $1/\epsilon$ may be reinterpreted in the linear model by the replacement

$$\frac{1}{\epsilon}\equiv\frac{1}{4-n}\to\ln\frac{M_H}{M_W}\ , \qquad (5.1)$$

where M_W represents any low-energy scale entering the computation. The convention adopted so far is that $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ are counterterms, that is, each is the negative of a cutoff-dependent structure generated by the nonlinear theory. Thus, each $\mathcal{L}_i$ must be multiplied by -1 in listing measurable effects.

The model described so far contains no fermions and we first discuss the in-principle measurable effects there. We then make some remarks about the inclusion of fermions. This brings the model closer to reality but does not change the essentials of the analysis. The further extension to the $SU(2)\times U(1)$ theory is briefly described in Sec. VI.

Without fermions, the only light physical particles are the W bosons so all the new physical effects can be extracted from $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ by setting $\vec{\pi}=0$, that is, $M=fI$ and $V_\mu=gW_\mu$. $\mathcal{L}_1$ and $\mathcal{L}_2$ then produce $4W$ vertices and $\mathcal{L}_3$ produces a $3W$ and $4W$ vertex. Their explicit forms are shown in the third and fourth entries of Table II along with the conventional counterterms arising from rescalings in $\mathcal{L}_0$. Inspection of these vertices shows that $\mathcal{L}_3$ contributes tensor structures which are identical to those coming from $\mathcal{L}_0$. It is only through the introduction of additional particles such as fermions that α_3 can be isolated and that will be considered shortly.

$\mathcal{L}_1$ and $\mathcal{L}_2$ give rise to $4W$ vertices with different tensor structures as shown in Table II. Explicit computation (4.7) has shown that $\alpha_2=2\alpha_1=-(1/16\pi^2)1/6\epsilon$, corresponding to the cutoff-dependent vertex (4.6). The tensor structure in (4.6) corresponds to the Bose symmetry existing independently in the internal and Lorentz indices and is to be contrasted with the rather different Bose-symmetric structure of the original $4W$ vertex (Table I). It is useful to discuss the $4W$ vertex in terms of these two structures (the "new" one and the "old" one), although any linear combinations of the two would do equally as well.

The "old" structure can be used to define a renormalized direct $4W$ coupling. Choosing the $2W$ and $3W$ counterterms (Table II) to cancel the π-loop (along with the usual gauge particle loop) contributions to these Green's functions, the coupling g is *defined* to be the renormalized $3W$ coupling strength. Now consider the $4W$ counterterms of Table II. The quantity $(Z_1^2/Z_3-1)+2\alpha_3$ appearing in both pieces has already been fixed and the π-loop contribution to it is, in fact, zero. That is just as expected since the π-loop contribution to the $4W$ vertex (4.6) has no "old" part at all; it is purely "new." The result of all this is that the renormalized $4W$ coupling constant g_{4W} can be taken to be g^2 up to possible M_H-independent corrections. Thus, with our prescription for g_{4W}, the tree-graph relation $g_{4W}=g_{3W}^2$ is preserved by the one-loop corrections in the nonlinear model, i.e., it is not sensitive to M_H as $M_H\to\infty$.

The new cutoff dependence is found completely in the "new" structure (4.6). It is summarized by the effective interaction

$$-\mathcal{L}_1-\mathcal{L}_2=\frac{1}{16\pi^2}\frac{1}{12\epsilon}(\mathrm{Tr}V_\mu V^\mu\,\mathrm{Tr}V_\nu V^\nu+2\,\mathrm{Tr}V_\mu V_\nu\,\mathrm{Tr}V^\mu V^\nu)\,,\quad(5.2)$$

where $1/\epsilon$ can be replaced by $\ln M_H/M_W$. The corresponding contribution to the S matrix for WW scattering is formed by multiplying (4.6) by polarization vectors. An appropriate set of measurements would then isolate the interaction (5.2) which is logarithmically sensitive to M_H.

Suppose next that fermions are introduced. The mass generation mechanism will involve the coupling of these fermions to the Higgs sector[21] but if the mass is small compared to M_W, it is reasonable to neglect it to first approximation. We shall, therefore, restrict our attention to a massless $SU(2)_L$ doublet coupled only to the gauge field. There are no new invariant, one-loop-generated structures involving the fermion field so the list (3.18) remains unchanged.

The fermion, however, plays the important role of a probe, allowing the isolation and measurement of

$$\mathcal{L}_3=\alpha_3 g\,\mathrm{Tr}F_{\mu\nu}[V^\mu,V^\nu]\,.$$

We compare the $3W$ coupling g_{3W} and the fermion-W coupling g_{ffW} through the one-loop level. They are, of course, identical at the tree level. As mentioned above, the $3W$ counterterm of Table II can be (and has been) arranged so that $g_{3W}\equiv g$ through one loop. However, the corresponding counterterm for the fermion-W vertex does not contain $\alpha_3 g^2$. It contains only a term corresponding to the (Z_1-1) piece of the $3W$ counterterm. It follows that

$$\frac{g_{3W}}{g_{ffW}}=1-\alpha_3 g^2+\cdots=1-\frac{g^2}{16\pi^2}\frac{1}{12}\ln\frac{M_H}{M_W}+\cdots\,.\quad(5.3)$$

Equation (5.3) describes a one-loop deviation (depending logarithmically on M_H) from the natural relation $g_{3W}=g_{ffW}$. This is possible because of the new interaction $\mathcal{L}_3$ in the list (3.18) which contributes to one coupling strength but not the other. Since g_{3W} and g_{ffW} can be independently measured, the relation (5.3) is experimentally meaningful.

All the information contained in $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ has now been extracted. Since these terms contain all the one-loop sensitivity to M_H, there can be no further sensitivity in the models considered. There will be new structures generated in higher orders, however, as well as corrections to the one-loop structures. Since the loop expansion parameter is

$$\lambda/\pi^2=g^2M_H^2/8\pi^2M_W^2=M_H^2/2\pi^2f^2\,,$$

the higher-order corrections will not be small if M_H is so large that $\lambda/\pi^2\to O(1)$. These corrections correspond to the quadratic and higher-power divergences of the nonlinear theory, described in Sec. III. In the $\lambda/\pi^2\sim 1$ regime, relations like (5.3) can at most be taken as rough guides: The correction is of order $(g^2/192\pi^2)\ln M_H/M_W$ times a number of order one. If, on the other hand, λ/π^2 is somewhat less than one, Eq. (5.3) can be taken more seriously.[22]

VI. SUMMARY

It has been shown that the low-energy sensitivity to a heavy-($\simeq$1 TeV)-Higgs-boson sector with an $SU(2)_L \times SU(2)_R$ symmetry can be completely characterized to any order in the loop expansion. By utilizing a nonlinear σ model to describe the low-energy Goldstone bosons of the Higgs sector, the sensitivity to the heavy physical Higgs particles can be summarized in terms of the cutoff dependence generated by the nonlinear theory. This dependence can be written in the form of new Lagrangian terms which must be both gauge invariant and chiral invariant. A complete list of the new terms which can arise at one loop is straightforward to assemble and once that is done, all possible physical effects can be read off.

For the SU(2) model considered above, the list consists of $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ (3.18). We repeat the list here incorporating the calculated values of the coefficients α_1, α_2, and α_3 and a factor of -1 to compensate for the counterterm convention used in Eq. (3.18):

$$\begin{aligned} -\mathcal{L}_1 &= \left(\frac{1}{16\pi^2}\frac{1}{12}\ln\frac{M_H}{M_W}\right)\mathrm{Tr}V_\mu V^\mu \,\mathrm{Tr}V_\nu V^\nu\,, \\ -\mathcal{L}_2 &= \left(\frac{1}{16\pi^2}\frac{1}{6}\ln\frac{M_H}{M_W}\right)\mathrm{Tr}V_\mu V_\nu \,\mathrm{Tr}V^\mu V^\nu\,, \qquad (6.1) \\ -\mathcal{L}_3 &= \left(-\frac{1}{16\pi^2}\frac{1}{12}\ln\frac{M_H}{M_W}\right)g\;\mathrm{Tr}F_{\mu\nu}[V^\mu, V^\nu]\,, \end{aligned}$$

where V_μ is given by Eq. (3.12). Physical processes involve only external W's and are, therefore, described by setting $V_\mu = gW_\mu$. This leads to the two measurable effects described in Sec. V. They are of order $(g^2/16\pi^2)\frac{1}{12}\ln M_H/M_W$, a small effect even in the limit (2.4). Since the list (6.1) is complete, there can be no further one-loop sensitivity to a heavy, strongly interacting Higgs sector.

In higher orders, new structures can be generated and there can be corrections to the one-loop structures. The analysis of Sec. III has shown that when a new, higher-dimensional structure first appears in the loop expansion, it will be at most logarithmically sensitive to the cutoff. This is essentially because all the available powers of $1/f^2$, the expansion parameter of the nonlinear σ model, must compensate the higher dimension of the generated structure. There are none left over to compensate possible powers of M_H^2. The expansion parameter for higher-order corrections to structures of a given dimension will be

$$g^2 M_H^2/8\pi^2 M_W^2 = M_H^2/2\pi^2 f^2$$

and, in the limit (2.4), the expansion will break down.

Nevertheless, the following important conclusion can be drawn: In any measurable effect, there will be at least one power of the weak coupling strength $g^2/16\pi^2$, which is uncompensated by a power of M_H^2/M_W^2. The effect is at most $(g^2/192\pi^2)\ln M_H^2/M_W^2$ times a factor which cannot be reliably computed in the loop expansion but should be, at most, of order unity. The physics of the heavy, strongly interacting Higgs sector is shielded from the low-energy probing of the light particles by at least one power of the coupling constant—except for possible logarithms.

This result relies critically on the $SU(2)_L \times SU(2)_R$ symmetry of the Higgs sector. If the symmetry is relaxed, it is possible to generate a power-law dependence on the heavy-Higgs-boson mass. An example of this has been provided by Toussiant,[23] who used two complex doublets of scalar fields. By an appropriate choice of parameters not respecting the $SU(2)_L \times SU(2)_R$ symmetry, a measurable one-loop effect can be generated, which depends quadratically on one of the Higgs-boson masses. It is easy to see that the effect disappears if the symmetry is reimposed. There are, of course, many ways to break the $SU(2)_L \times SU(2)_R$ symmetry (e.g., a mass term for the $\vec{\pi}$ field) and it would be useful to study the heavy-Higgs-boson effects that result in various cases.

In examining the one-loop corrections to the gauged nonlinear σ model, a number of subtleties were encountered. The most troublesome was the fact that the invariant counterterms (6.1) are not always sufficient to remove the one-loop logarithmic divergences off mass shell. This is already a feature of the usual (nongauged) nonlinear σ model and can be seen, for example, in the 4π amplitude. The only relevant counterterms are $\mathcal{L}_1$ and $\mathcal{L}_2$ (6.1) and while they render finite the on-shell 4π amplitude, off-shell divergences remain. This fact has been observed before[17] and will be discussed more extensively in a future paper.[18]

Finally, we emphasize that the kind of analysis developed here can be applied to any gauge theory. The simplest realistic one is the Weinberg-Salam $SU(2)_L \times U(1)_R$ theory, which is currently being examined.[24] We include here only a few preliminary remarks about this work.

As pointed out in Sec. V, the one-loop heavy-Higgs-boson effects can often be regarded as corrections to natural relations. The best known, and only experimentally tested, natural relation in the Weinberg-Salam model involves M_W, M_Z, and $\cos\theta_W$ (defined by $\sin\theta_W = e/g$, where g is the $W^\pm$ coupling strength). At the tree level,

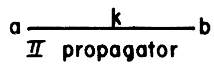
$$\left.\frac{M_Z^2}{M_W^2}\cos^2\theta_W\right|_{\mathrm{tree}} = 1\,. \qquad (6.2)$$

This prediction, which follows from the $SU(2)_L \times SU(2)_R$ symmetry of the Higgs sector,[25] has been checked by low-energy neutrino scattering (where M_W and M_Z are defined to be zero momentum inverse propagators). It is found that[26]

$$\frac{M_W^2}{M_Z^2\cos^2\theta_W}\bigg|_{\text{exp}} = 0.981 \pm 0.037\,, \tag{6.3}$$

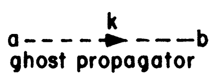
in good agreement with Eq. (6.1). The prediction (6.1) is, however, corrected at one loop by a term of order $(\alpha/\pi)\ln M_H/M_W$.[24] The logarithmic sensitivity to M_H can again be understood in terms of dimensional analysis of the gauged nonlinear σ model. The $SU(2)_L \times SU(2)_R$ symmetry of the Higgs sector continues to be the key ingredient; an explicit example[23] has shown that without it there

TABLE I. Partial list of Feynman rules for the Lagrangian (3.3). (All momenta flow out of vertices.)

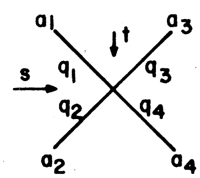


$$\frac{i\delta_{ab}}{f^2(k^2+i\epsilon)}$$

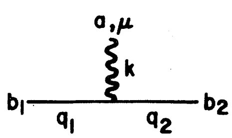
$$\frac{-i\delta_{ab}}{(k^2-M_W^2+i\epsilon)}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)$$



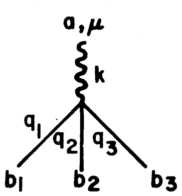
$$\frac{i\delta_{ab}}{k^2+i\epsilon}$$



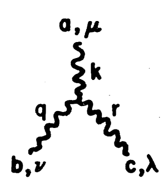
$$if^2\,(\delta_{a_1a_2}\delta_{a_3a_4}s+\delta_{a_1a_3}\delta_{a_2a_4}t+\delta_{a_1a_4}\delta_{a_2a_3}u)$$



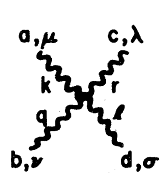
$$\frac{f^2g}{2}\epsilon^{ab_1b_2}(q_1-q_2)_\mu$$



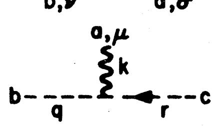
$$-\frac{f^2g}{2}[\delta_{ab_1}\delta_{b_2b_3}(k+2q_1)_\mu+\delta_{ab_2}\delta_{b_1b_3}(k+2q_2)_\mu+\delta_{ab_3}\delta_{b_1b_2}(k+2q_3)_\mu]$$



$$-g\epsilon^{abc}[g_{\mu\nu}(k-q)_\lambda+g_{\nu\lambda}(q-r)_\mu+g_{\lambda\mu}(r-k)_\nu]$$



$$-ig^2[\delta_{ab}\delta_{cd}(2g_{\mu\nu}g_{\lambda\sigma}-g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda})+\text{perms}]$$

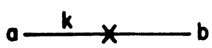
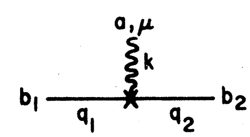
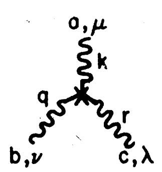
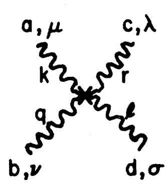
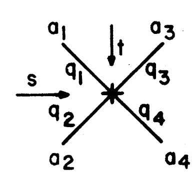
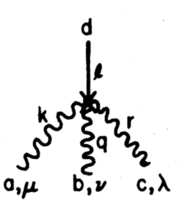


$$\epsilon^{abc}gq_\mu$$

can be a one-loop correction to Eq. (6.2) depending quadratically on M_H.

It is not hard to see that the list (6.1) will not suffice to explain the one-loop correction to Eq. (6.2). The basic process by which the quantity $(M_Z^2/M_W^2)\cos^2\theta_W$ is measured is single W or Z exchange between a lepton and a quark. The one-loop corrections to this process, which can involve the Higgs sector, can only be W and Z propagator insertions, since the fermion mass is being neglected. Since $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ (6.1) give rise only to $3W$ and $4W$ vertices, they clearly cannot produce corrections to Eq. (6.2). However, since the total symmetry of the theory is only $SU(2)_L\times U(1)_R$, there are many new structures which are now allowed. A one-loop example is $[\mathrm{Tr}(D_\mu M)\tau_3 M^\dagger]^2$ where M is given by Eq. (2.1) and D_μ is the covariant derivative appropriate to both the $SU(2)_L$ and $U(1)_R$ gauge groups. In the gauge sector, this term produces a Z-mass contribution which effects the type of deviation from Eq. (6.2) discussed above. A complete analysis of such terms, as well as those that will be relevant above W and Z production threshold, is being carried out.

TABLE II. Contributions of the counterterms $L_1,\ldots,L_4$ to various Green's functions. (All momenta flow out of vertices.)

	$-4i\alpha_4\delta_{ab}k^4$
	$4\alpha_3 g\epsilon^{ab_1b_2}[q_{1\mu}(k\cdot q_2)-q_{2\mu}(k\cdot q_1)]$
	$-i(Z_3-1)\delta_{ab}[g_{\mu\nu}k^2-k_\mu k_\nu]$
	$-g[(Z_1-1)+\alpha_3 g^2]\epsilon^{abc}[g_{\mu\nu}(k-q)_\lambda+g_{\nu\lambda}(q-r)_\mu+g_{\lambda\mu}(r-k)_\nu]$
	$-2ig^2\left[\left(\frac{Z_1^2}{Z_3}-1\right)+(2\alpha_3-\alpha_1)g^2\right][\delta_{ab}\delta_{cd}g_{\mu\nu}g_{\lambda\sigma}+\text{perms}]$ $+ig^2\left[\left(\frac{Z_1^2}{Z_3}-1\right)+(2\alpha_3+\alpha_2)g^2\right][\delta_{ab}\delta_{cd}(g_{\mu\lambda}g_{\nu\sigma}+g_{\nu\lambda})+\text{perms}]$
	$8i\alpha_1[\delta_{a_1a_2}\delta_{a_3a_4}(-st-su+(q_1^2+q_2^2)(q_3^2+q_4^2)+\text{perms}]$ $+4i\alpha_2[\delta_{a_1a_2}\delta_{a_3a_4}(-2ut-st-su+2(q_1^2q_2^2+q_3^2q_4^2)+(q_1^2+q_2^2)(q_3^2+q_4^2)$ $+\text{perms}]$
	$4(\alpha_1-\alpha_3)g^3[\delta_{ab}\delta_{cd}g_{\mu\nu}l_\lambda+\text{perms}]$ $+2(\alpha_2+\alpha_3)g^3[\delta_{ab}\delta_{cd}(g_{\mu\lambda}l_\nu+g_{\nu\lambda}l_\mu)+\text{perms}]$

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APPENDIX

Here we state the Feynman rules for the nonlinear SU(2) gauge theory. We also write down the contributions of the counterterms $\mathfrak{L}_1, \ldots, \mathfrak{L}_4$ to various Green's functions with low numbers of external lines.

The full Lagrangian for the theory, obtained by putting (3.2) into (3.3), is

$$\mathfrak{L} = \tfrac{1}{2}\,\mathrm{Tr}(F_{\mu\nu})^2 + \frac{M_W{}^2}{2}\,\vec{W}^2 + \frac{f^2}{2}\left[(\partial_\mu \underline{\vec{\pi}})^2 + \frac{(\underline{\vec{\pi}}\cdot\partial_\mu\underline{\vec{\pi}})^2}{(1-\underline{\pi}^2)^{1/2}}\right]$$
$$- \frac{f^2 g}{2}\, W^a_\mu [\epsilon^{abc}(\partial_\mu \underline{\pi}^b)\underline{\pi}^c + (\partial_\mu \underline{\pi}^a)(1-\underline{\vec{\pi}}^2)^{1/2} - \underline{\pi}^a \partial_\mu (1-\underline{\vec{\pi}}^2)^{1/2}] + \mathfrak{L}_{\mathrm{GF}} + \mathfrak{L}_{\mathrm{FP}}\,, \qquad \text{(A1)}$$

where $\mathfrak{L}_{\mathrm{GF}}$ and $\mathfrak{L}_{\mathrm{FP}}$ are given by (2.8) and (2.9) with Landau gauge ($\xi \to \infty$) implied. As usual, a term from $\mathfrak{L}_{\mathrm{GF}}$ will cancel the π-W mixing term in (A1). From (A1) we can easily arrive at the Feynman rules. (With dimensional regularization, the naive Matthew theorem is correct even in derivative coupled theories[27] and the Feynman rules follow directly from the Lagrangian.) In Table I, we give the propagators and also those vertices which are necessary for the calculations of Sec. IV. We emphasize that there are no "seagull" vertices involving two W's and two π's, in contrast to the linear model. This is because $\vec{\pi}^2 + \sigma^2$ has fixed magnitude. Interactions not involving the $\vec{\pi}$ field are, of course, the same as in the usual gauge theory; they are included here for completeness.

The contributions of $\mathfrak{L}_1, \ldots, \mathfrak{L}_4$ to various Green's functions are easily determined by inserting the definition of V_μ (3.12) and the parametrization of U (3.2) into (3.18). The resulting counterterms for the Green's functions computed in Sec. IV are listed in Table II. Except in the $\underline{\pi}$ self-energy, we assume that α_4 has already been found to vanish and, therefore, we do not include the $\mathfrak{L}_4$ contributions. Note that all these counterterms have $D=4$, counting one for every power of momentum and one for every gauge field.

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