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Subquark model of leptons and quarks

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Various subquark models so far proposed are briefly reviewed. Classifications of leptons and quarks in the models are discussed and compared. Our spinor-subquark model of leptons and quarks in which leptons and quarks are made of three subquarks of spin 1/2 is discussed in detail. The possibility that gauge bosons and Higgs scalars are also made of a subquark-antisubquark pair is discussed. Exotic states of subquarks such as leptons and quarks of spin 3/2, exotic fermions, and exotic bosons are predicted in our model. Subquark currents and their algebra are proposed. Two unified subquark models of strong and electroweak interactions are discussed. The one is a gauge model and the other is a model of the Nambu–Jona-Lasinio type. A subquark model of gravity and its supergrand unification is proposed. Finally, a speculation is made on "color-space correspondence."

I. INTRODUCTION

Elementary-particle physics is at the moment best described in a word by "lepton-quark physics." There exist at least six leptons including the three charged leptons e, μ, and τ and the three accompanying neutrinos ν_e, ν_μ, and ν_τ. There also exist at least five flavors and three colors of quarks including the two flavors and three colors of quarks of charge $\frac{2}{3}$, u, and c, and the three flavors of quarks of charge $-\frac{1}{3}$, d, s, and b. More leptons and more quarks are expected to be found. In some models,[1] a dozen leptons (six neutrinos and six charged leptons) and a dozen flavors and three colors of quarks (6×3 up quarks and 6×3 down quarks) are predicted. Undoubtedly, the time has come again to convince ourselves of the existence of more elementary particles,[2,3] subquarks, which are building blocks of these leptons and quarks. In this paper, subquark models of leptons and quarks will be discussed in great detail.

A working hypothesis in elementary-particle physics is the gauge principle. The most attractive picture for interactions of elementary particles is given by quantum flavor dynamics, the $\mathrm{SU}(2)_W\times \mathrm{U}(1)$ gauge theory of Glashow, Weinberg, and Salam[4] for the weak and electromagnetic (electroweak) interactions of leptons and quarks, and by quantum chromodynamics,[5] the Yang-Mills gauge theory[6] of color $\mathrm{SU}(3)_c$ (Ref. 7) for the strong interaction of quarks. If this picture is the right one, there must exist the weak vector bosons $W^\pm$ and Z, the physical Higgs scalar η, and the color-octet vector gluons G^a ($a=1-8$). It is well known[8] that pure quantum chromodynamics is asymptotically free but that quantum chromodynamics is not if there exist more than 16 flavors of quarks. Also known is that even if the number of quark flavors is not larger than 16, the asymptotic freedom of quantum chromodynamics may be jeopardized if quark masses are generated by elementary Higgs scalars. Recently, it has been shown[9] that the possible freedom of quantum chromodynamics cannot be asymptotic but can only be temporary due to the mutual interference between the strong and electroweak interactions of quarks. Furthermore, it has also been demonstrated[10] that even if quarks were confined temporarily inside hadrons in quantum chromodynamics, they could be liberated by other interactions of quarks than by the strong one. It has come for the first time when we must take care of the strong and electroweak interactions of quarks simultaneously. Throughout this paper, it is assumed that quarks are not permanently confined. For the last several years, grand unification of all the elementary-particle forces[11] has been extensively studied both in gauge models of the Pati-Salam or Georgi-Glashow type[12] and in fermion models of the Nambu–Jona-Lasinio type.[13,3] In the latter models, the gauge bosons γ, $W^\pm$, Z, and G^a, as well as the physical Higgs scalar η, appear as collective excitations of fermion-antifermion pairs, and, therefore, gauge invariance among other things may be violated at extremely short distances. It cannot be stressed too strongly, however, that there does not yet exist any experimental evidence for the local gauge invariance in the strong and weak interactions. When the weak vector bosons are found, they may appear completely different from what they should in the gauge model.[14] They might even decay into a pair of subquark and antisubquark.[1]

It is a final goal in physics to unify all the four basic forces—the strong, weak, electromagnetic, and gravitational forces. Recently, such a supergrand unification has been attempted in two ways: One is a model of supergravity,[15] the supersymmetric gauge theory[16] of gravity, and the other is a model of "pregeometry,"[17,18,11] the unified spinor theory of gravity. As they stand now, the former approach seems to have a serious problem that none of the models so far proposed can be physical or realistic, while the latter seems to be still incomplete in that no explanation can be made in the models for the "natural cutoff" at around the Planck length. In either approach, a further reduction of the number of fundamental fermions seems to be necessary for the model to be physical and truly unified.[19] It is at this point again that we must believe the existence of more fundamental fermions, the subquarks.

This paper is organized as follows: In Sec. II, various subquark models so far proposed will be reviewed. Also, classifications of leptons and quarks in these models and their comparison will be made. Our spinor-subquark model in which leptons and quarks are made of three subquarks of spin $\frac{1}{2}$ will be discussed in detail. In Sec. III, the possibility that gauge bosons and Higgs scalars are also made of a subquark-antisubquark pair will be discussed. In Sec. IV, exotic states of subquarks such as leptons and quarks of spin $\frac{3}{2}$, exotic fermions, and exotic bosons will be predicted in our model. In Sec. V, subquark currents and their algebra will be proposed. In Sec. VI, two unified subquark models of strong and electroweak interactions will be discussed. One is a gauge model and the other is a model of the Nambu–Jona-Lasinio type. In Sec. VII, a subquark model of gravity and its supergrand unification will be proposed. In Sec. VIII, some concluding remarks, including a speculation on "color-space correspondence," will be made.

II. SUBQUARK MODELS AND CLASSIFICATIONS OF LEPTONS AND QUARKS

Let us briefly review various subquark models and classifications of leptons and quarks in the models in chronological order.

(1) In 1960, Maki, Nakagawa, Ohnuki, and Sakata proposed and Matumoto and Nakagawa[20] discussed the so-called "Nagoya model" in which Sakata's fundamental triplet of baryons $\mathcal{P}$, $\mathcal{N}$, and λ are made of a "B^+ matter" and one of the three leptons ν, e^-, and μ^-:

$$\mathcal{P}=\langle B^+\nu\rangle,\quad \mathcal{N}=\langle B^+e^-\rangle,\quad \lambda=\langle B^+\mu^-\rangle\,. \tag{2.1}$$

In the same year, Taketani and Katayama[21] proposed a somewhat different model in which the two charged leptons e^- and μ^- and the three fundamental baryons $\mathcal{P}$, $\mathcal{N}$, and λ are made of the neutrino (ν), the "ϵ charge (or matter)" (ϵ^-), and/or the "b charge (or matter)" (b^+):

$$\begin{aligned} &e^-=(\nu\epsilon^-),\quad \mu^-=(\nu\epsilon^-)'\,,\\ &\mathcal{P}=(\nu b^+),\quad \mathcal{N}=(\nu\epsilon^-b^+),\quad \lambda=(\nu\epsilon^-b^+)'\,. \end{aligned} \tag{2.2}$$

This is an admixture of the Nagoya model and what Ferreira *et al.* call "São Paulo model" in which e^- and μ^- are originated from ν by "attaching" the electric charge. Although their model is too primitive to be realistic now, it certainly has a point that the strong, weak, and electromagnetic interactions are constructed from the "currents of charge." Two years later in 1962, when the two neutrinos ν_e and ν_μ were found, Katayama, Matumoto, Tanaka, and Yamada, and independently, Maki, Nakagawa, and Sakata[23] modified the Nagoya model into the "new Nagoya model" in which the fourth fundamental baryon (in today's terminology, charmed baryon) V was predicted:

$$\begin{aligned} &\mathcal{P}\equiv\langle B^+\nu_1\rangle,\quad V\equiv\langle B^+\nu_2\rangle\,,\\ &\mathcal{N}\equiv\langle B^+e^-\rangle,\quad \lambda\equiv\langle B^+\mu^-\rangle \end{aligned} \tag{2.3}$$

with $\nu_1\equiv\nu_e\cos\theta+\nu_\mu\sin\theta$ and $\nu_2\equiv-\nu_e\sin\theta+\nu_\mu\cos\theta$, where θ is the weak mixing angle[23] (or Cabibbo angle[24]).

(2) In 1968, Hayashi, Koide, and Ogawa[25] proposed the so-called "Hiroshima model" in which the four quarks $\mathcal{P}$, $\mathcal{N}$, λ, and $\mathcal{P}'$ are made of a B^+ matter, one lepton, and one antilepton of ν_e, e^-, ν_μ, and μ^-:

$$\begin{aligned} &\mathcal{P}=\langle B^+(\nu_e\bar\nu_e\cos\theta+\nu_\mu\bar\nu_\mu\sin\theta)\rangle\,,\\ &\mathcal{P}'=\langle B^+(-\nu_e\bar\nu_e\sin\theta+\nu_\mu\bar\nu_\mu\cos\theta)\rangle\,,\\ &\mathcal{N}=\langle B^+(e^-\bar\nu_e)\rangle,\\ &\lambda=\langle B^+(\mu^-\bar\nu_\mu)\rangle\,. \end{aligned} \tag{2.4}$$

Although this model can explain the famous $\Delta I=\frac{1}{2}$ rule[26] for nonleptonic decays of hadrons as intended, it seems to have some difficulty in the present weak-interaction phenomenology. Three years later in 1971, Senju[27] proposed a model of leptons and quarks which is similar to the Hiroshima model. His model, however, seems to be totally unphysical and unrealistic though it has a point that not only quarks but leptons are made of three subquarks.

(3) Since the existence of quarks inside hadrons was established by the now classic SLAC-MIT experiments, there have appeared several papers in which quarks are considered as composites. In 1972, Chang[28] proposed that the conventional three flavors and three colors of quarks, $\mathcal{P}_i$, $\mathcal{N}_i$, and λ_i $(i=1,2,3)$ are composed of one of three "electric quarks" of spin $\frac{1}{2}$, $\mathcal{P}$, $\mathcal{N}$, and λ, and one of three "magnetic antiquarks" of spin 0, $\bar{\mathcal{P}}'$, $\bar{\mathcal{N}}'$, and $\bar\lambda'$:

$$\begin{pmatrix} \mathcal{P}_1 & \mathcal{P}_2 & \mathcal{P}_3\\ \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3\\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix}=\begin{pmatrix} \mathcal{P}\\ \mathcal{N}\\ \lambda \end{pmatrix}\otimes(\bar{\mathcal{P}}'\bar{\mathcal{N}}'\bar\lambda')\,. \tag{2.5}$$

Two years later in 1974, Pati and Salam[29] proposed their unified model of strong, weak, and electromagnetic interactions in which lepton number is treated as the fourth color. It is in a footnote of their paper that they considered the mathematical objects $F=(\mathcal{P},\mathfrak{N},\lambda,\chi)$ and $B=(a,b,c,d)$ as fundamental fields and the 16-fold quark field Ψ to be composite:

$$\Psi=\begin{bmatrix}\mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \mathcal{P}_d=\nu\\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & \mathfrak{N}_d=e^-\\ \lambda_a & \lambda_b & \lambda_c & \lambda_d=\mu^-\\ \chi_a & \chi_b & \chi_c & \chi_d=\nu'\end{bmatrix}=\begin{bmatrix}\mathcal{P}\\ \mathfrak{N}\\ \lambda\\ \chi\end{bmatrix}\otimes(a,b,c,d). \tag{2.6}$$

Successively, similar models have been proposed by Matumoto, Greenberg, and Miyazawa,[30] all independently. In the Matumoto model, the color-triplet and flavor-quartet quarks $q_{\alpha i}$ ($\alpha=1,\ldots,4$ and $i=1,2,3$), and the color-octet gluons U_μ^A ($A=1,\ldots,8$) and their possible excited states are considered to be composed of "subhadronic constituents," S_α of spin $\frac{1}{2}$ and b_i of spin 0, as

$$q_{\alpha i}=(S_\alpha b_i)\ \text{ and }\ U^i_{j\mu}=(b_i^*b_j)_\mu\,, \tag{2.7}$$

and the interactions among hadronic constituents are also considered to be generated from the same basis. It is also assumed that the "constructive" interaction among subhadronic constituents is given by

$$iG(\bar{S}_\alpha\gamma_\mu S_\alpha-b_i^*\overleftrightarrow{\partial}_\mu b_i)G^\mu \tag{2.8}$$

and that the "composite dynamics" is again in the framework of field theory. In the Greenberg model, a colored quark nonet is a two-body system $(Q\bar{C})$, Q being a spin-$\frac{1}{2}$ SU(3)-triplet Fermi object carrying the usual SU(3) quantum numbers and C being a spin-0 SU(3)-triplet Bose object carrying the color-SU(3) quantum numbers:

$$q_{\alpha i}=(Q_\alpha\bar{C}_i)\,. \tag{2.9}$$

It is not until they discussed space excitations of subquark states in these models [(2.7) and (2.9)] that subquarks are apparently considered not just as mathematical symbols but as physical objects.

(4) In 1975, Pati, Salam, and Strathdee[2] also proposed another model in which leptons and quarks are composites of three entities, one in the valency quartet $Q=(\mathcal{P}\mathfrak{N}\lambda\chi)$, one in the color quartet $C=(abcd)$, and a neutral singlet fermion S, which can incorporate the "mirror" or "heaviness" quantum number

$$\Psi=(\bar{Q}CS)\,. \tag{2.10}$$

A year later, Koike[31] assumed, as an extension of the Hiroshima model, a very similar model with the "weak-source" structure of quarks,

$$q_{\alpha i}=\langle f_\alpha,\bar{c}_i;b\rangle\,, \tag{2.11}$$

where f_α ($\alpha=1,2,3,4$) and $\bar{c}_i$ ($i=1,2,3$) represent the flavor weak source and the color weak source, respectively.

(5) In 1976, Akama and the present author,[32] later with Chikashige,[3] proposed and discussed[1,11,33-40] a spinor-subquark model of leptons in which leptons and quarks are made of three subquarks of spin $\frac{1}{2}$, w_i ($i=1,2$), h_i ($i=1,2,\ldots,N$), and C_i ($i=0,1,2,3$). The left-handed $w_L=(w_1,w_2)_L$ and the right-handed w_{1R} and w_{2R} are a doublet and singlets of the Glashow-Weinberg-Salam SU(2),[4] respectively. The h_i's form an N-plet of the unknown "horizontal" symmetry (H symmetry).[32] Also, the C_0 and C_i's ($i=1,2,3$) are singlet and triplet under the SU(3) color symmetry. The w, h, and C subquarks are related to the weak and electromagnetic interactions, the horizontal degree of freedom and the color symmetry, respectively. The charge assignment to these subquarks is ambiguous since only a sum of three subquarks is determined by the charge of a lepton or quark (0 and -1 for a lepton or $\frac{2}{3}$ and $-\frac{1}{3}$ for a quark). In general, the charges of subquarks, Q's, can be written in the form

$$\begin{aligned}&Q_{w_1}=Q\,,\quad Q_{w_2}=Q-1\,,\\ &Q_{h_j}=Q_h\quad\text{for } j=1,2,3,\ldots,N\,,\\ &Q_{C_0}=-(Q+Q_h),\quad Q_{C_i}=\tfrac{2}{3}-(Q+Q_h)\quad\text{for } i=1,2,3\,.\end{aligned} \tag{2.12}$$

Once the subquark charges are given as above, however, the desired quantization of the lepton and quark charges will become automatic. Leptons and quarks are expressed in terms of these subquarks as follows:

$$\begin{aligned}&\nu_e=(w_1h_1C_0),\quad \nu_\mu=(w_1h_2C_0),\quad \nu_\tau=(w_1h_3C_0)\,,\ldots,\\ &e=(w_2h_1C_0),\quad \mu=(w_2h_2C_0),\quad \tau=(w_2h_3C_0)\,,\ldots,\end{aligned} \tag{2.13}$$

$$\begin{aligned}&u_i=(w_1h_1C_i),\quad c_i=(w_1h_2C_i),\quad t_i=(w_1h_3C_i),\ldots,\\ &d_i=(w_2h_1C_i),\quad s_i=(w_2h_2C_i),\quad b_i=(w_2h_3C_i),\ldots,\end{aligned}$$

or, more generally,

$$\begin{aligned}&\nu_j=(w_1h_jC_0),\quad u_{ji}=(w_1h_jC_i)\,,\\ &l_j=(w_2h_jC_0),\quad d_{ji}=(w_2h_jC_i)\,,\\ &\text{for } i=1,2,3\ \text{ and } j=1,2,3,\ldots,N\,.\end{aligned} \tag{2.14}$$

Already in the original paper[3,32] we have suggested a picture in which the gauge bosons γ, $W^\pm$, and Z, as well as the physical Higgs scalar η, are also composites of a $w\bar{w}$ pair which behaves as $(\bar{w}Qw)$, $(\bar{w}\tau^\pm w)$ [where τ's are the Pauli isospin SU(2) matrices], $(\bar{w}Rw)$ (where R is orthogonal to the charge Q), and $(\bar{w}w)$, respectively, while the color-octet gluons G^a ($a=1-8$) are those of a $C\overline{C}$ pair which behave as $(\overline{C}\lambda^a C)$ [where λ^a's are the Gell-Mann color-SU(3) matrices]. Also, we have proposed the unified spinor-subquark model of the Nambu–Jona-Lasinio type for all elementary-particle forces including gravity, which is an alternative to the unified lepton-quark model[3] and in which the above described picture can be realized. I shall discuss this picture in much more detail in Secs. III, VI, and VII. Here I only emphasize that this subquark model looks most satisfactory among the models so far proposed since each subquark (w, h, or C) has only a single quantum number (weak isospin, horizontal, or color) and, therefore, a single function (electroweak interaction, heaviness, or strong interaction).

An immediate application of our spinor-subquark model has been made by Fujikawa.[41] He has considered six subquarks including the two w_i ($i=1,2$) and the four C_i ($i=0-3$) as the fundamental sextet of the unified SU(6) gauge symmetry of leptons and quarks, which has been proposed by Inoue, Kakuto, and Nakano, by Abud, Buccella, Ruegg, and Savoy, by Lee and Weinberg, and by Yoshimura.[42] The role of h in his model is also the same as in our spinor-subquark model. As a result, he presents the antisymmetric 15-plet in the SU(6) model as

$$\begin{aligned}
\bar{\nu}_{jL} &= \frac{1}{\sqrt{2}}(C_0 w_2 - w_2 C_0, h_j)_R ,\\
\bar{l}_{jL} &= \frac{1}{\sqrt{2}}(C_0 w_1 - w_1 C_0, h_j)_R ,\\
\bar{l}_{jR} &= \frac{1}{\sqrt{2}}(w_1 w_2 - w_2 w_1, h_j)_L ,\\
u_{jiL} &= \frac{1}{\sqrt{2}}(C_i w_1 - w_1 C_i, h_j)_L ,\\
d_{jiL} &= \frac{1}{\sqrt{2}}(C_i w_2 - w_2 C_i, h_j)_L ,\\
\bar{u}_{jiR} &= \frac{1}{\sqrt{2}}\Big(\sum_{k,l=1}^{3} \epsilon_{ikl} C_k C_l, h_j\Big)_L ,\\
d_{jiR} &= \frac{1}{\sqrt{2}}(C_i C_0 - C_0 C_i, h_j)_R .
\end{aligned} \tag{2.15}$$

Almost all the models proposed before and after our spinor-subquark model can be taken as a special case in our universal model. The model of Pati and Salam given in (2.6) [or the model of Matumoto in (2.7)], for example, is a special limit in which w_i ($i=1,2$) and h_i ($i=1,2$) form four flavors of "disubquarks" as

$$\begin{pmatrix} \mathcal{P} & \chi \\ \mathfrak{N} & \lambda \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \otimes (h_1 h_2) . \tag{2.16}$$

On the other hand, the model recently proposed by Yasuè,[43] by Tanikawa and Saito,[44] by Yamanashi and Yasuè,[45] and Ne'eman[46] is another limit. In the Yasuè model, leptons are given by

$$\begin{aligned}
\nu_e &= (L u_1), \quad \nu_\mu = (L' u_1) ,\\
e &= (L u_2), \quad \mu = (L' u_2) ,
\end{aligned} \tag{2.17}$$

where L (L') is a boson carrying the electronic (muonic) lepton number and u_i's ($i=1,2$) are fermions. Also, quarks are

$$\begin{aligned}
&\mathcal{P}_w = (B\cos\theta + B'\sin\theta, u_1), \quad \mathcal{P}'_w = (B'\cos\theta - B\sin\theta, u_1) ,\\
&\mathfrak{N}_w = (B\cos\phi + B'\sin\phi, u_2), \quad \lambda_w = (B'\cos\phi - B\sin\phi, u_2) ,
\end{aligned} \tag{2.18}$$

where q_w's are quarks in the weak eigenstate, B (B') is a boson carrying the baryon number, and $\theta \mp \phi = \pm\theta_C$ for the Cabibbo angle θ_C. He has also suggested that the familiar weak bosons $W^\pm$ and Z may be regarded as bound states of u_1 and u_2. In the model of Tanikawa and Saito, leptons and quarks are all composites of "spinor subquarks" Q_i ($i=1,2$) and "scalar subquarks" $B_\alpha^{(h)}$ ($\alpha=0,1,2,3$ and $h=1,2,3,\ldots$) as follows:

$$\begin{aligned}
&\nu_e = (Q_1 B_0^{(1)}), \quad \nu_\mu = (Q_1 B_0^{(2)}), \quad \nu_\tau = (Q_1 B_0^{(3)}), \ldots ,\\
&e = (Q_2 B_0^{(1)}), \quad \mu = (Q_2 B_0^{(2)}), \quad \tau = (Q_2 B_0^{(3)}), \ldots ,\\
&u_i = (Q_1 B_i^{(1)}), \quad c_i = (Q_1 B_i^{(2)}), \quad t_i = (Q_1 B_i^{(3)}), \ldots ,\\
&d_i = (Q_2 B_i^{(1)}), \quad s_i = (Q_2 B_i^{(2)}), \quad b_i = (Q_2 B_i^{(3)}), \ldots ,
\end{aligned} \tag{2.19}$$

where the $B_i^{(h)}$ ($i=1,2,3$) form an SU(3)-color triplet for each h and carry the baryon number $\frac{1}{3}$ of h kind and the charge $+\frac{2}{3}$ while the $B_0^{(h)}$ is a color singlet and carries the unit lepton number of h kind and the zero charge. They consider "duality diagrams" drawn by "subquark lines" in which the weak vector bosons $W^\pm$, B (carrying both electron and muon numbers), and B' (carrying both lepton and baryon numbers) are s-channel resonances or t-channel Regge poles. In the model of Yamanashi and Yasuè, leptons $q_0^{(i)}$ ($i=1,2,3,\ldots$) and quarks $q_j^{(i)}$ ($j=1,2,3$) are expressed in terms of bosons $B_\alpha^{(i)}$ ($\alpha=1,2,3$) and fermions f_1 and f_2 as

$$\begin{aligned}
u_\alpha^{(i)} &= (B_\alpha^{(i)} f_1) ,\\
d_\alpha^{(i)} &= (B_\alpha^{(i)} f_2) .
\end{aligned} \tag{2.20}$$

Also, in the "primitive-particle model" of Ne'eman, leptons and quarks are made of one set of fundamental fermions $(\alpha^0, \alpha^-)_L$, α_R^0, α_R^- (which are colorless) and two bosons $\beta_B^{2/3}$ and β_s^0. All of these models which are essentially identical to each other can be reduced to our spinor-subquark model if C_i ($i=0,1,2,3$) and h_j ($j=1,2,3,\ldots$) form disubquarks $B_i^{(j)}$,

$$B_i^{(j)} = (C_i h_j) . \tag{2.21}$$

The "heavy-color" model of Susskind[47] and our subquark model have a similarity in that the Higgs scalar ϕ is a composite of a heavy-color quark and heavy-color antiquark:

$$\phi \sim (\overline{T}T) . \tag{2.22}$$

Recently, Harari and, independently, Shupe[49] have considered a scheme in which all leptons and quarks are composites of only two types of fundamental spin-$\frac{1}{2}$ objects (T and V) or (a^+ and a^0), with electric charges $\frac{1}{3}$ and 0. The concepts of color and flavor acquire meaning only at the level of the composite systems.[38] Gauge bosons such as $W^\pm$, Z, and G^a connect composite states and are not fundamental. Their model has, therefore, some part in common with our spinor-subquark model but is very different in its origin of color and flavor.

It has become clear from the above comparison of various subquark models so far proposed that our spinor-subquark model is very universal and works perfectly for classification of leptons and quarks. Therefore, I shall particularly assume the spinor-subquark model hereafter unless otherwise specified. In concluding this section, it should be emphasized that if leptons and quarks are further made of more fundamental particles at all (which seems to have begun to be accepted[48]), the spinor-subquark model given in (2.14) is one of the best possibilities.

III. SUBQUARK MODEL OF GAUGE BOSONS AND HIGGS SCALARS

The possibility that the origin of Higgs scalars is dynamical has been discussed in literature mainly for the following two reasons: (1) It is more economical that the spontaneous breakdown of symmetry be a completely dynamical one, in the manner of Nambu and Jona-Lasinio,[13] and (2) it is well known that existence of elementary Higgs bosons would jeopardize the "asymptotic" freedom of the strong interaction of quarks in quantum chromodynamics. However, suppose that Higgs scalars are composite. Then, why are the gauge bosons such as $W^\pm$ and Z not also composite since they must be mixed with the Higgs scalars in the so-called Higgs mechanism? This logic hopefully gives an additional support to our ansatz that not only Higgs scalars but also all the gauge bosons including γ, $W^\pm$, Z, and G^a are composite. What are they made of, then? It is possible to assume that they are made of lepton-antilepton or quark-antiquark pairs. In fact, as mentioned before, in our unified lepton-quark model of the Nambu–Jona-Lasinio type[13,3] for the strong and electroweak interactions, the photon and the weak vector bosons as well as the Higgs scalars are collective excitations of lepton-antilepton and quark-antiquark pairs while the gluons are those of quark-antiquark pairs. In this unified lepton-quark model, however, composite states of quark-antiquark pair are taken in two-fold ways, in one as mesons such as π and ρ and in the other as the

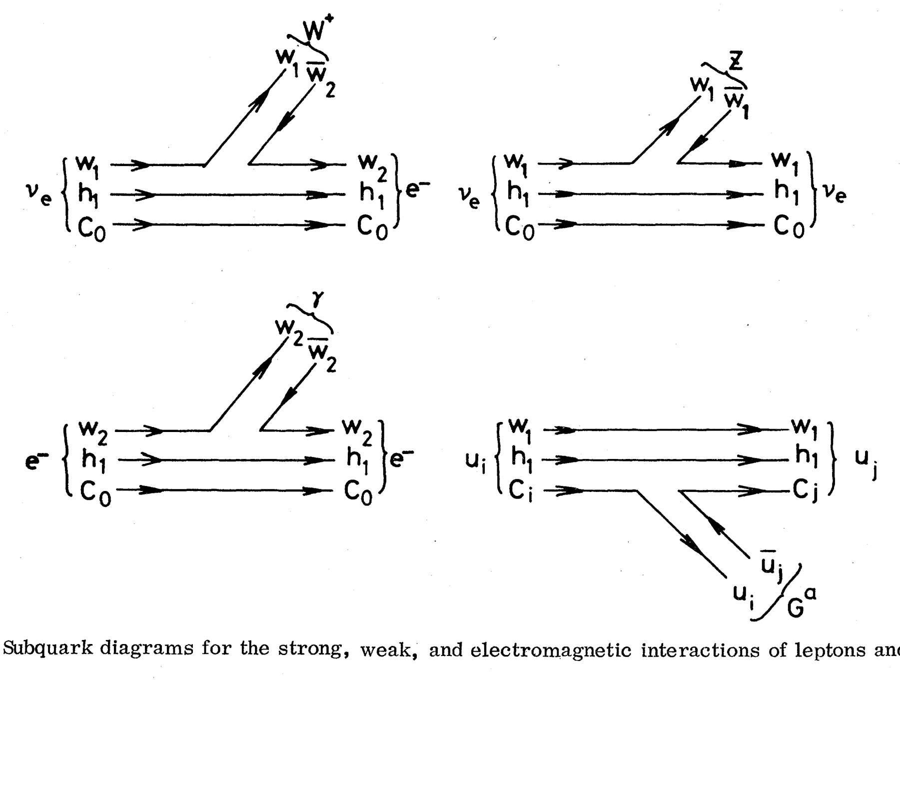

FIG. 1. Subquark diagrams for the strong, weak, and electromagnetic interactions of leptons and quarks.

physical Higgs scalar and the gauge bosons such as γ, $W^\pm$, Z, and G^a. It seems, therefore, more elegant and transparent that the gauge bosons as well as the Higgs scalar are composites of subquark-antisubquark pairs at the same level as leptons and quarks are those of three subquarks. If this is the case, the answer is necessarily the following: The photon γ, the weak vector bosons $W^\pm$ and Z, and the Higgs scalar are bound states of $w\overline{w}$ pairs which behave as $(\overline{w}\gamma_\mu Qw)$, $(\overline{w}\gamma_\mu \tau^\pm w)$, $(\overline{w}\gamma_\mu Rw)$, and $(\overline{w}_L w_R)$, respectively, while the color-octet gluons G^a are those of $C\overline{C}$ pairs which behave as $(\overline{C}\gamma_\mu \lambda^a C)$.

An immediate consequence of this hypothesis is that the conservation of quantum numbers possessed by leptons and quarks such as weak isospin and color and the universality of strong and electroweak couplings with leptons and quarks are automatic. This is because it is a subquark (w or C) which is contained in a lepton or quark that is transferred between leptons and quarks in strong or electroweak interaction. What is needed is, therefore, the conservation of subquark number. This situation can be understood more clearly in the "subquark diagrams" for lepton and quark reactions illustrated in Fig. 1. It should be noticed, however, that in this picture of the charged weak bosons $W^\pm$, the strangeness-changing weak interaction violates the conservation of individual-subquark number. This problem, relating to the origin of the Cabibbo angle and angles alike, will be discussed later in Sec. V. Also, interactions of these subquarks will be discussed later in Sec. VI.

IV. EXOTIC STATES OF SUBQUARKS

In the previous Secs. II and III, subquark models of leptons and quarks and those of gauge bosons and Higgs scalars have been introduced. A special emphasis has been made on the naturalness of our spinor-subquark model. It has already become clear that the model works perfectly in classification of not only leptons and quarks but the gauge bosons and Higgs scalars needed in quantum electroweak dynamics and chromodynamics. It should be noticed, however, that the familiar leptons, quarks, gauge bosons, and Higgs scalars do not exhaust all that can be made of these subquarks. There are possible exotic bound states of these subquarks whose existence, if found, would give a most certain and exciting evidence for this simple subquark model. In this section, I shall make a list of such exotic states of subquarks.

A. Leptons and quarks of spin $\frac{3}{2}$

In our spinor-subquark model, ordinary leptons and quarks of spin $\frac{1}{2}$ are assigned to bound states of spin $\frac{1}{2}$, $(whC)^{J=1/2}$, containing a w (w_1 or w_2), an h ($h_1, h_2, h_3, \ldots,$ or h_N), and a C ($C_0, C_1, C_2,$ or C_3), all of which have a spin $\frac{1}{2}$. It is then natural to ask whether bound states of spin $\frac{3}{2}$, $(whC)^{J=3/2}$, containing w, h, and C, exist or not. Unless the spin-spin force between these subquarks gives an infinite potential, such leptons and quarks of spin $\frac{3}{2}$ should exist in nature although their masses could be much higher than those of ordinary leptons and quarks. This is in a perfect analogy to the old SU(3) quark model of hadrons[50] in which not only an octet of baryons of spin $\frac{1}{2}$ but also a decuplet of baryons of spin $\frac{3}{2}$ do exist as bound states of three quarks of spin $\frac{1}{2}$. Therefore, possible discovery of a lepton or quark of spin $\frac{3}{2}$ would give a most definite evidence for this spinor-subquark model of leptons and quarks, as the famous discovery of Ω^- did for the SU(3) quark model. The best way to determine the spin of a new lepton or quark seems to be e^+-e^- colliding-beam experiments. The total cross section for one-photon annihilation into lepton-antilepton or quark-antiquark pairs, the angular distributions of produced leptons or quark jets, and the angular distributions of the secondary leptons or hadrons due to the decay of original leptons or quarks all strongly depend on whether the new lepton or quark has spin $\frac{1}{2}$ or $\frac{3}{2}$.

B. Exotic fermions

Suppose there exist a doublet of w's (w_1, w_2), a N-plet of h's $(h_1, h_2, h_3, \ldots, h_N)$, and a quartet of C's (C_0, C_1, C_2, C_3): The total number of subquarks is $6+N$. Therefore, there are ${}_{6+N}C_3 - 8N$ combinations of three subquarks other than $(w_i h_j C_k)$ for $i=1,2$, $j=1,2,3,\ldots,N$, and $k=0,1,2,3$. They include the combinations of (www), (hhh), (CCC), (wwC), etc. Whether these exotic combinations can form bound states strongly depends on the force between the subquarks, which is totally unknown. If they could, there would exist leptons with abnormal charges such as $\frac{3}{2}$, etc., and exotic quarks with abnormal colors such as color sextet, octet, and even decuplet ones.

C. Exotic bosons

There are $(6+N)^2 - 13$ combinations of a subquark and an antisubquark other than $(\overline{w}_i w_j)$ for $i=j=1,2$, and $(\overline{C}_i C_j)$ for $i=j=1,2,3$. Among these, the bound states of $(h_i h_j)$ for $i=j=1,2,3,\ldots,N$ seem to have the best chance to exist. They can be nothing but gauge bosons of the horizontal symmetry of SU(N),[32,51] which has been proposed as the third gauge symmetry after the flavor symmetry of SU(2)$\times$U(1) and the color symmetry of SU(3). Also familiar are the combinations of $(\bar{C}_0 C_i)$ for $i=1,2,3$ and $(\bar{w}_i C_j)$ for $i=1,2,3$ and $j=1,2,3$. They can be nothing but "leptoquark" gauge bosons and Higgs scalars which change a lepton into a quark and vice versa, eventually making the proton unstable, in the unified gauge models of the Pati-Salam or Georgi-Glashow type.[12] In our subquark model, there may exist not only this type of exotic boson consisting of a subquark and an antisubquark but also the other type of exotic boson (or disubquark), consisting of two subquarks such as (ww), (CC), (hh), (wh), (wC), and (hC).

It should be emphasized in concluding this section that possible existence of these exotic states of subquarks other than ordinary leptons and quarks would make lepton-quark physics much more colorful and similar to what hadron physics has been for the last two decades. It should be noticed, however, that the close analogy drawn with the old-fashioned composite models may fail at the subquark level, in which case the existence of these exotic states is not guaranteed.

V. SUBQUARK CURRENTS AND THEIR ALGEBRA

Before the proposal of the quark model, the weak charged hadronic current was written in terms of hadrons as

$$J_\mu^+ = \frac{G^\beta}{G^\mu}\bar{p}\gamma_\mu\left(1-\frac{g_A^\beta}{g_V^\beta}\gamma_5\right)n + \frac{G^\Lambda}{G^\mu}\bar{p}\gamma_\mu\left(1-\frac{g_A^\Lambda}{g_V^\Lambda}\gamma_5\right)\Lambda + \cdots, \tag{5.1}$$

where G^μ, G^β, and G^Λ are the μ-, β-, and Λ-decay coupling constants, and g_A's and g_V's are the axial-vector and vector coupling constants, respectively. In order to describe all the weak interactions of hundreds of hadrons by the weak current written in this way, one must assume hundreds (more precisely, hundreds of hundreds) of terms in the current and, therefore, introduce hundreds of parameters, the weak coupling constants. In the quark model,[50] however, the same weak-charged hadronic current has been approximately written in terms of quarks more simply as

$$J_\mu^+ \cong \cos\theta_C \bar{u}\gamma_\mu(1-\gamma_5)d + \sin\theta_C \bar{u}\gamma_\mu(1-\gamma_5)s + \cdots, \tag{5.2}$$

where θ_C is the Cabibbo angle. The weak current written in this way (which is called the quark current) is no simpler at present and will become more and more complicated as the number of quark flavors increases. Suppose, for example, that there exist a dozen flavors (six up flavors and six down flavors) of quarks, the quark current must then contain 36 terms, in general. It seems, therefore, natural and a "historical necessity" to expect that the weak current can be again rewritten in terms of more fundamental particles, the subquarks. In this section, I shall discuss this expectation of "subquark currents" and their algebra which can describe the weak and electromagnetic interactions of leptons and quarks in the most fundamental way.

A. Subquark currents

In our spinor-subquark model discussed in Secs. II and III, the weak charged current, if it is of the $V-A$ type, can be written in terms of w's most simply as

$$J_\mu^+ = \bar{w}_1\gamma_\mu(1-\gamma_5)w_2\,. \tag{5.3}$$

Similarly, the neutral current is written in terms of w's (and, in general, also h's or C's if $Q_h \neq 0$ or $Q_C \neq 0$) as

$$\begin{aligned} J_\mu^0 = {} & \alpha_1\bar{w}_1\gamma_\mu(1-\rho_1\gamma_5)w_1 + \alpha_2\bar{w}_2\gamma_\mu(1-\rho_2\gamma_5)w_2 \\ & + \sum_{i,j=1}^{N}\beta_{ij}\bar{h}_i\gamma_\mu(1-\kappa_{ij}\gamma_5)h_j \\ & + \delta_0\bar{C}_0\gamma_\mu(1-\xi_0\gamma_5)C_0 \\ & + \delta_1\bar{C}_i\gamma_\mu(1-\xi_1\gamma_5)C_i\,, \end{aligned} \tag{5.4}$$

where α_i, β_{ij}, δ_i, ρ_i, κ_{ij}, and ξ_i are constants. These constant parameters are to be fixed in a dynamical model, which will be discussed in Sec. VI. The electromagnetic current is unambiguously written in terms of subquarks as

$$\begin{aligned} J_\mu^{\text{e.m.}} = {} & Q\bar{w}_1\gamma_\mu w_1 + (Q-1)w_2\gamma_\mu w_2 \\ & + Q_h\sum_{i=1}^{N}\bar{h}_i\gamma_\mu h_i - (Q+Q_h)\bar{C}_0\gamma_\mu C_0 \\ & + [\tfrac{2}{3}-(Q+Q_h)]\sum_{i=1}\bar{C}_i\gamma_\mu C_i\,. \end{aligned} \tag{5.5}$$

Let us restrict ourselves to the weak charged subquark current (5.3) in the rest of this section. Remember that the parameters in the weak-charged hadronic current (5.1) such as G^β/G^μ, G^Λ/G^μ, g_A^β/g_V^β, g_A^Λ/g_V^Λ, etc., can be defined by the matrix element of the quark current (5.2) between a proton and a neutron state and that between a proton and a Λ state:

$$\cos\theta_C\langle p|\bar{u}\gamma_\mu(1-\gamma_5)d|n\rangle = \frac{G^\beta}{G^\mu}\bar{p}\gamma_\mu\left(1-\frac{g_A^\beta}{g_V^\beta}\gamma_5\right)n + \cdots$$

and (5.6)

$$\sin\theta_C\langle p|\bar{u}\gamma_\mu(1-\gamma_5)s|\Lambda\rangle = \frac{G^\Lambda}{G^\mu}\bar{p}\gamma_\mu\left(1-\frac{g_A^\Lambda}{g_V^\Lambda}\gamma_5\right)\Lambda + \cdots\,.$$

Remember also that these definitions together with the CVC (conserved vector current) and PCAC (partially conserved axial-vector current) hypotheses have led to the approximate relations

$$\frac{G^{\beta}}{G^{\mu}}=\cos\theta_C\,,\quad \frac{G^{\Lambda}}{G^{\mu}}=\sin\theta_C\,,\quad \frac{g_A^{\beta}}{g_V^{\beta}}\cong 1\,,\quad \text{and}\,\frac{g_A^{\Lambda}}{g_V^{\Lambda}}\cong 1\,. \tag{5.7}$$

It is then natural to expect that the parameters in the weak-charged quark current (5.2) such as the Cabibbo angle θ_C can be defined by the matrix element of the subquark current (5.3) between an up-quark and a down-quark state and that between an up-quark and a strange-quark state[35,36]:

$$\langle u|\bar{w}_1\gamma_\mu(1-\gamma_5)w_2|d\rangle\cong\cos\theta_C\bar{u}\gamma_\mu(1-\gamma_5)d+\cdots$$

and (5.8)

$$\langle u|\bar{w}_1\gamma_\mu(1-\gamma_5)w_2|s\rangle\cong\sin\theta_C\bar{u}\gamma_\mu(1-\gamma_5)s+\cdots\,.$$

An immediate consequence of this picture of subquark current is that the Cabibbo angle given by

$$\theta_C=\tan^{-1}\frac{\langle u|\bar{w}_1\gamma_\mu w_2|s\rangle}{\langle u|\bar{w}_1\gamma_\mu w_2|d\rangle} \tag{5.9}$$

may depend on momentum transfers between the quarks as the parameters such as G^{β}/G^{μ}, G^{Λ}/G^{μ}, g_A^{β}/g_V^{β}, and $g_A^{\Lambda}/g_V^{\Lambda}$ depend on those between the baryons. I therefore predict in the subquark model that the Cabibbo angle and similar angles would vary at higher momentum transfers where the subquark structure of quarks may become relevant. This effect would be observed by measuring the ratio of strangeness-changing to nonchanging events in future high-energy neutrino or lepton scattering experiments. In this picture of Cabibbo mixing, all the h's are assumed to be identical and quarks (and leptons also) of different generations are different only dynamically. The nonexistence or extreme smallness of lepton mixing then remains to be explained.

As discussed in Sec. III, the universality of electroweak couplings with leptons and quarks is automatic in the subquark model of gauge bosons. It should be emphasized here that the universality becomes even clearer in this subquark model of currents. It is simply because the electroweak currents are made of the subquarks which are universally contained in every lepton or quark. Also automatic is the "vector-meson dominance"[52] of these subquark currents in the subquark model of gauge bosons since both the currents and the gauge bosons which couple to the currents are made of the same subquark-antisubquark pairs. Furthermore, even the "field-current identity"[53] holds exactly for the gauge bosons and the subquark currents in a unified subquark model of strong and electroweak interactions of leptons and quarks, which will be discussed in Sec. VI.

Let us make the CVC hypothesis[54] and the PCAC hypothesis[55] on these subquark currents as on the quark currents. An immediate consequence of these hypothesis is that the renormalized coupling strength of a quark-vector (axial-vector) current is equal (approximately equal) to that of a lepton-vector (axial-vector) current despite possible strong-interaction effects on the former current and that the $V-A$ form of the charged quark current is preserved at least approximately, given the same form of the charged subquark current. The PCAC hypothesis on the subquark current together with the Higgs-scalar pole dominance further indicates a Goldberger-Treiman relation[56] at lepton-quark level, e.g.,

$$(g_A/g_V)_{ud}(m_u+m_d)=G_{ud\phi^+}f_{\phi^+}\,, \tag{5.10}$$

where $(g_A/g_V)_{ud}$ is the axial-vector-to-vector ratio of the charged current of the up and down quarks, m's are the quark masses, $G_{ud\phi^+}$ is the Yukawa coupling constant of the charged Higgs scalar ϕ^+ and the quarks, and f_ϕ is the "decay constant of ϕ^+" defined by $\partial^\mu J_\mu^+=m_\phi^2 f_\phi\phi^+$. This type of relation is essentially the same as the familiar one due to the spontaneous symmetry breaking in the Weinberg-Salam model, e.g.,

$$m_u=G_{uu\phi^0}\langle\phi^0\rangle\,, \tag{5.11}$$

where $G_{uu\phi^0}$ is the Yukawa coupling constant and $\langle\phi^0\rangle$ is the vacuum expectation value of the neutral Higgs scalar ϕ^0.

There remain two difficult problems in the weak interaction: the $\Delta I=\frac{1}{2}$ rule and CP violation. The origin of $\Delta I=\frac{1}{2}$ rule is nonleptonic decays must be nothing but a dynamical one since the subquark currents have no hadronic isospins any more. Also, the origin of CP violation cannot be found anywhere but in an unknown role of the h currents or the horizontal symmetry[32,51] since there is no other room to contain the necessary degree of freedom. How to solve these problems in the subquark model is certainly one of the most important subjects to be illuminated.

B. Subquark current algebra

Let us consider the subquark vector and axial-vector currents of weak interaction:

$$V_\mu^{(i)}=\bar{w}\gamma_\mu\tfrac{1}{2}\tau_i w\ \text{ and }\ A_\mu^{(i)}=\bar{w}\gamma_\mu\gamma_5\tfrac{1}{2}\tau_i w\ \text{ for } i=1,2,3\,. \tag{5.12}$$

As the familiar quark currents do,[57] these subquark currents form an algebra of $SU(2)\times SU(2)$ at equal time:

$$\delta(x_0-y_0)[V_0^{(i)}(x), V_0^{(j)}(y)] = i\epsilon_{ijk}\delta(x-y)V_0^{(k)}(x)\,,$$
$$\delta(x_0-y_0)[V_0^{(i)}(x), A_0^{(j)}(y)] = i\epsilon_{ijk}\delta(x-y)A_0^{(k)}(x)\,, \quad (5.13)$$
$$\delta(x_0-y_0)[A_0^{(i)}(x), A_0^{(j)}(y)] = i\epsilon_{ijk}\delta(x-y)V_0^{(k)}(x)\,.$$

The subquark vector and axial-vector currents of strong interaction,

$$v_\mu^{(a)} = \bar{C}\gamma_\mu \tfrac{1}{2}\lambda^a C \text{ and } a_\mu^{(a)} = \bar{C}\gamma_\mu\gamma_5\tfrac{1}{2}\lambda^a C \text{ for } a=1,2,\ldots,8\,, \quad (5.14)$$

also form an algebra of color SU(3)×SU(3) as

$$\delta(x_0-y_0)[v_0^{(a)}(x), v_0^{(b)}(y)] = if_{abc}\delta(x-y)v_0^{(c)}(x)\,,$$
$$\delta(x_0-y_0)[v_0^{(a)}(x), a_0^{(b)}(y)] = if_{abc}\delta(x-y)a_0^{(c)}(x)\,, \quad (5.15)$$
$$\delta(x_0-y_0)[a_0^{(a)}(x), a_0^{(b)}(y)] = if_{abc}\delta(x-y)v_0^{(c)}(x)\,,$$

where f_{abc}'s are the SU(3) structure constants. Furthermore, the subquark vector and axial-vector currents of possible horizontal symmetry, if relevant,

$$V_\mu^{(l)} = \bar{h}\gamma_\mu\tfrac{1}{2}\Lambda^{(l)}h \text{ and } A_\mu^{(l)} = \bar{h}\gamma_\mu\gamma_5\tfrac{1}{2}\Lambda^{(l)}h \quad \text{for } l=1,2,\ldots,N^2-1\,, \quad (5.16)$$

may form an algebra of, for example, SU(N) × SU(N) as

$$\delta(x_0-y_0)[V_0^{(l)}(x), V_0^{(m)}(y)] = ig_{lmn}\delta(x-y)V_0^{(n)}(x)\,,$$
$$\delta(x_0-y_0)[V_0^{(l)}(x), A_0^{(m)}(y)] = ig_{lmn}\delta(x-y)A_0^{(n)}(x)\,, \quad (5.17)$$
$$\delta(x_0-y_0)[A_0^{(l)}(x), A_0^{(m)}(y)] = ig_{lmn}\delta(x-y)V_0^{(n)}(x)\,,$$

where $\Lambda^{(l)}$'s are the generalized Gell-Mann matrices for SU(N) and g_{lmn}'s are the SU(N) structure constants. If exotic currents such as $\bar{w}_i\gamma_\mu C_j$, $\bar{w}_i\gamma_\mu h_k$, and $\bar{C}_j\gamma_\mu h_k$ ($i=1,2$; $j=0,1,2,3$; $k=1,2,\ldots,N$) are included, an algebra of SU(6+N) × SU(6+N) can be formed in principle.

What is the use of these algebras? Recalling the celebrated history of the now classic algebra of currents, one can easily write down the Adler sum rule[58] for inelastic structure functions of a lepton or a quark as

$$\int_0^1 \frac{dx}{x}[\overline{F_2}(x,q^2) - F_2(x,q^2)] = 4\langle I_3\rangle = \begin{cases} 2 & \text{for } \nu, u \\ -2 & \text{for } l, d \end{cases} \quad (5.18)$$

and the Adler-Weisberger relation[59] for the g_A/g_V ratio of the lepton or quark current and the total cross sections for Higgs-scalar–lepton or Higgs-scalar–quark scatterings as, e.g.,

$$\left(\frac{g_A}{g_V}\right)_{ud}^2 = \left(1 + \frac{(m_u+m_d)^2}{\pi G_{ud\phi^+}{}^2}\int_{m_{\phi^+}}^\infty \frac{d\nu}{\nu^2}\, k[\sigma_{\phi^- u}(\nu) - \sigma_{\phi^+ u}(\nu)]\right)^{-1}, \quad (5.19)$$

which is totally unphysical. To test the sum rule (5.18) seems to be difficult at present, if relevant at all. It should be emphasized, however, that the g_A/g_V ratio of a lepton or a quark may be slightly different from unity, reflecting the possible structure of leptons and quarks. Whether or not the ratio, especially for the electron and muon, conflicts with the experimental data depends on the unknown subquark dynamics and is a serious question to be answered.

VI. UNIFIED SUBQUARK MODEL OF STRONG AND ELECTROWEAK INTERACTIONS

As previously mentioned in Secs. II and III, there are two pictures in which strong and electroweak interactions are described in terms of subquarks. One is a unified gauge model in which gauge bosons as well as Higgs scalars are elementary, while the other is a unified fermion model in which they are composites of subquark-antisubquark pairs. In this section, I shall present these two pictures and their results in some detail.

A. Unified gauge model

Let us suppose that the electroweak interaction of subquark is described by a $SU(2)_W \times U(1)$ gauge model of the Glashow-Weinberg-Salam type and that the strong interaction of C's is described by the Yang-Mills gauge theory of color $SU(3)_c$. The left-handed w's form a doublet of $SU(2)_W$, $w_L = (w_1, w_2)_L$, and all the other subquarks including w_{1R}, w_{2R}, C_0, C_i ($i=1,2,3$), and h_j ($j=1,2,3,\ldots,N$) are singlets. Their weak hypercharges are given by

$$Y_{w_L} = 2Q-1\,,\quad Y_{w_{1R}} = 2Q\,,\quad Y_{w_{2R}} = 2(Q-1)\,,$$
$$Y_{C_0} = -2(Q+Q_h)\,,\quad Y_{C_i} = 2[\tfrac{2}{3} - (Q+Q_h)]\,, \quad (6.1)$$

and

$$Y_h = 2Q_h\,,$$

respectively, The three C's, of course, form a triplet of color $SU(3)_c$, $C = (C_1, C_2, C_3)$. The gauge fields of U(1), $SU(2)_W$, and $SU(3)_c$ are denoted by B_μ, $\vec{A}_\mu$, and G^a ($a = 1,2,3,\ldots,8$), respectively, and the $SU(2)_W$-doublet Higgs scalar by $\phi = (\phi^+, \phi^0)$. We are then ready to write down the Lagrangian for the strong and electroweak interactions of subquarks as

$$L = \overline{w}_L i\gamma^\mu (\partial_\mu + ig'\tfrac{1}{2}Y_{w_L}B_\mu - ig\tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu)w_L + \overline{w}_{1R} i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{w_{1R}}B_\mu)w_{1R} + \overline{w}_{2R}i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{w_{2R}}B_\mu)w_{2R}$$
$$- G_{w_1}(\overline{w}_L\phi^G w_{1R} + \overline{w}_{1R}\phi^{G+}w_L) - G_{w_2}(\overline{w}_L\phi w_{2R} + \overline{w}_{2R}\phi^+ w_L) + \overline{C}_0 i\gamma_\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{C_0}B_\mu)C_0$$
$$+ \overline{C}i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{C_i}B_\mu - if\tfrac{1}{2}\lambda^a G^a_\mu)C + \overline{h}i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_h B_\mu)h$$
$$-\tfrac{1}{4}(B_{\mu\nu})^2 - \tfrac{1}{4}(\vec{A}_{\mu\nu})^2 - \tfrac{1}{4}(G^a_{\mu\nu})^2 + |D_\mu\phi|^2 - \mu^2|\phi|^2 - \lambda(|\phi|^2)^2, \qquad (6.2)$$

where g, g', f, G_{w_1}, G_{w_2}, μ^2, and λ are all constants,

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu ,$$
$$\vec{A}_{\mu\nu} = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu ,$$
$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + ff^{abc}G^b_\mu G^c_\nu , \qquad (6.3)$$
$$D_\mu\phi = (\partial_\mu + ig'\tfrac{1}{2}B_\mu - ig\tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu)\phi ,$$

and

$$\phi^G = i\tau_2\phi^* .$$

The Higgs potential in the Lagrangian L produces the familiar spontaneous breakdown of $SU(2)_W \times U(1)$ gauge symmetry of the vacuum if

$$-\mu^2 > 0 . \qquad (6.4)$$

The Higgs scalar ϕ will then acquire the nonvanishing vacuum expectation value

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \quad \text{with } v = \left(\frac{-\mu^2}{\lambda}\right)^{1/2} . \qquad (6.5)$$

This vacuum expectation value generates, through the Yukawa interactions in L, the masses of w's given by

$$m_{w_i} = \frac{1}{\sqrt{2}} G_{w_i} v \quad \text{for } i = 1, 2 . \qquad (6.6)$$

It also generates the masses of the charged and neutral weak vector bosons $W^\pm$ and Z, given by

$$m_{W^\pm} = \tfrac{1}{2}gv \quad \text{for } W^\pm_\mu = \frac{1}{\sqrt{2}}(A_\mu{}^1 \pm iA_\mu{}^2) \qquad (6.7)$$

and

$$m_Z = \frac{m_{W^\pm}}{\cos\theta_W} \quad \text{for } Z_\mu = \cos\theta_W A_\mu{}^3 + \sin\theta_W B_\mu , \qquad (6.8)$$

but leaves the photon field

$$A_\mu = -\sin\theta_W A^3 + \cos\theta_W B \qquad (6.9)$$

massless, where the Weinberg-Salam angle θ_W is defined by

$$\tan\theta_W = g'/g . \qquad (6.10)$$

The Lagrangian for the electromagnetic and weak interactions of subquarks then takes a form of

$$L_{\text{electroweak}} = -eA_\mu\left[(\overline{w}_1,\overline{w}_2)\gamma^\mu\begin{bmatrix}Q & 0\\ 0 & Q-1\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix} + Q_{C_0}\overline{C}_0\gamma^\mu C_0 + Q_{C_i}\overline{C}\gamma^\mu C + Q_h\overline{h}\gamma^\mu h\right]$$
$$+\left(\frac{G_F}{\sqrt{2}}\right)^{1/2} m_{W^\pm}(\overline{w}_1,\overline{w}_2)\gamma^\mu(1-\gamma_5)(\tau^+W^+_\mu + \tau^-W^-_\mu)\begin{bmatrix}w_1\\ w_2\end{bmatrix}$$
$$+\left(\frac{2G_F}{\sqrt{2}}\right)^{1/2} m_Z Z_\mu\left\{(\overline{w}_1,\overline{w}_2)\gamma^\mu\left[(1-\gamma_5)\tfrac{1}{2}\tau^3 - 2\sin^2\theta_W\begin{bmatrix}Q & 0\\ 0 & Q-1\end{bmatrix}\right]\begin{bmatrix}w_1\\ w_2\end{bmatrix}\right.$$
$$\left. - 2\sin^2\theta_W(Q_{C_0}\overline{C}_0\gamma^\mu C_0 + Q_{C_i}\overline{C}\gamma^\mu C + Q_h\overline{h}\gamma^\mu h)\right\} , \qquad (6.11)$$

where the electromagnetic and Fermi coupling constants e ($\alpha \equiv e^2/4\pi \cong 1/137.036$) and G_F ($G_F m_p{}^2 \cong 1.026\times 10^{-5}$) are defined by

$$e = g\sin\theta_W = g'\cos\theta_W \qquad (6.12)$$

and

$$G_F/\sqrt{2} = -g^2/8m_{W^\pm}{}^2 . \qquad (6.13)$$

Notice that the masses of the weak vector bosons are predicted, as in the Weinberg-Salam model, to be

$$m_{W^\pm} = \left[\frac{\sqrt{2}\ \pi\alpha}{2G_F\sin^2\theta_W}\right]^{1/2}$$
$$\cong \frac{37.3\ \text{GeV}}{\sin\theta_W} \cong 80\ \text{GeV} \quad \text{for } \sin^2\theta_W = 0.22 \qquad (6.14)$$

and

$$m_Z = m_{W^\pm}/\cos\theta_W \cong 90 \text{ GeV} \quad \text{for } \sin^2\theta_W = 0.22\,. \tag{6.15}$$

This unified gauge model of subquark interactions is expected to reproduce every feature of the Weinberg-Salam $SU(2)_W \times U(1)$ gauge model for the electroweak interaction of leptons and quarks and of the Yang-Mills gauge theory of color $SU(3)_c$ for the strong interaction of quarks as long as the gauge fields couple with all subquarks inside a lepton or a quark coherently. For example, the asymptotic ratio of the total cross section for $e^+ + e^- \to \gamma^* \to$ all to that for $e^+ + e^- \to \gamma^* \to \mu^+ + \mu^-$ is expected to be

$$R = \frac{\sigma(e^+ + e^- \to \gamma^* \to \text{all})}{\sigma_0(e^+ + e^- \to \gamma^* \to \mu^+ + \mu^-)} = \sum (Q_w + Q_C + Q_h)^2 = \tfrac{8}{3}N \quad \text{with } \sigma_0 \equiv \frac{4\pi\alpha^2}{3s} \tag{6.16}$$

in the energy region where the subquark structure of leptons and quarks is irrelevant. At even higher energies, however, it may show a sudden change into

$$R = \sum Q_w^2 + \sum Q_C^2 + \sum Q_h^2 = Q^2 + (Q-1)^2 + (Q+Q_h)^2 + 3[\tfrac{2}{3} - (Q+Q_h)]^2 + NQ_h^2 \geq \tfrac{5}{6}\,. \tag{6.17}$$

Notice also that this gauge model of subquarks is free from the axial-vector anomaly[60] and, therefore, renormalizable only if the charges of w's are specified[61] as

$$Q = \tfrac{1}{2}, \text{ i.e., } (Q_{w_1}, Q_{w_2}) = (\tfrac{1}{2}, -\tfrac{1}{2})\,. \tag{6.18}$$

A further unification of the strong and electroweak interactions of subquarks can be made in a unified gauge model of the Georgi-Glashow type.[12] A possible candidate for such a grand unification group is $SU(6+N)$ since the total number of subquarks $6+N$.[35] According to the general line of reasoning of Georgi, Quinn, and Weinberg,[12] in such a unified model, the ratio of the $SU(3)_c$ gauge coupling constant to the $SU(2)_W$ one is given by

$$\frac{f^2}{g^2} = \frac{\text{the number of isodoublets}}{\text{the number of color triplets}} = \frac{1}{2}\,, \tag{6.19}$$

and the Weinberg-Salam angle is determined by the weak isospins and charges of fundamental fermion fields, the subquarks, as

$$\sin^2\theta_W = \frac{\sum (I_3)^2}{\sum Q^2} = \frac{\frac{1}{4}}{Q^2 + (Q-1)^2 + (Q+Q_h)^2 + 3[\frac{2}{3} - (Q+Q_h)]^2 + NQ_h^2} \leq \frac{3}{10}\,. \tag{6.20}$$

The last equality in (6.20) holds only if

$$Q = \tfrac{1}{2} \text{ and } Q_h = 0\,, \tag{6.21}$$

which completely fixes the charge assignment of all subquarks as

$$Q_{w_1} = \tfrac{1}{2}, \quad Q_{w_2} = -\tfrac{1}{2}, \quad Q_h = 0, \quad Q_{C_0} = -\tfrac{1}{2}, \quad Q_{C_i} = \tfrac{1}{6} \text{ for } i = 1,\ 2,\ 3. \tag{6.22}$$

This particular choice of w charges coincides with the one previously mentioned in (6.18) for renormalizability. Notice also that not only the sum of w charges but also that of C charges vanishes in this charge assignment. In view of these properties, I suggest that the charge assignment of (6.22), which satisfies the Nakano–Nishijima–Gell-Mann rule $Q = I_3 + (B-L)/2$, is preferable.

B. Unified fermion model

In a unified fermion model of the Nambu–Jona-Lasinio type for the strong and electroweak interactions of subquarks, the gauge bosons γ, $W^\pm$, Z, and G^a as well as the physical Higgs scalars appear as composites of subquark-antisubquark pairs as discussed in Sec. III. The nonlinear fermion Lagrangian is given by

$$\begin{aligned} L = {} & \bar{w}_L i\partial\!\!\!/ w_L + \bar{w}_{1R} i\partial\!\!\!/ w_{1R} + \bar{w}_{2R} i\partial\!\!\!/ w_{2R} + \bar{C}_0 i\partial\!\!\!/ C_0 + \bar{C} i\partial\!\!\!/ C + \bar{h} i\partial\!\!\!/ h \\ & + F_1 (Y_{w_1} \bar{w}_L \gamma_\mu w_L + Y_{w_{1R}} \bar{w}_{1R} \gamma_\mu w_{1R} + Y w_{2R} \bar{w}_{2R} \gamma_\mu w_{2R} + Y_{C_0} \bar{C}_0 \gamma_\mu C_0 + Y_{C_i} \bar{C} \gamma_\mu C + Y_h \bar{h} \gamma_\mu h)^2 \\ & + F_2 (\bar{w}_L \gamma_\mu \vec{\tau} w_L)^2 + F_3 (\bar{C} \gamma_\mu \lambda^a C)^2 + F_4 (-a_1 w_R^{\bar{G}} w_{1L}^c + a_2 \bar{w}_L w_{2R})(-a_1 \bar{w}_{1L}^c w_R^G + a_2 \bar{w}_{2R} w_L), \end{aligned} \tag{6.23}$$

where F's and a's are constants, and c and G denote the charge-conjugate and G-parity-conjugate states. Obviously, this Lagrangian is invariant under the global $SU(3)_C \times SU(2)_W \times U(1)$ symmetry.

To analyze this nonlinear fermion Lagrangian, we use the Kikkawa algorithm,[62] which has been discussed in detail in Ref. 3 for our unified lepton-quark model of the Nambu–Jona-Lasinio type. Instead of repeating the rather long procedure, we shall simply present the results of such analysis in the following. By imposing the masslessness condition of Bjorken,[63] we can show that the original Lagrangian (6.23) becomes effectively equivalent to the Lagrangian (6.2) for the unified gauge model. The difference between these two models, however, lies in the fact that the coupling constants f, g, g', λ, G_{w_1}, and G_{w_2}, and the Higgs mass parameter μ^2 are arbitrary in the unified gauge model, whereas in the unified fermion model they are completely fixed by the quantum numbers of subquarks (the isospins I's and the charges Q's etc.), the cutoff momentum (Λ), and the coupling constants (F's and a's) in the original Lagrangian. A typical example of them is that the fine-structure constant α is determined by the sum of the charge squared of subquarks and by the cutoff momentum as

$$\alpha = 3\pi \Big/ \left(\sum Q^2\right) \ln(\Lambda^2/m^2) , \tag{6.24}$$

where m is the geometric average mass of charged subquarks defined by

$$m = \prod m^{Q^2/\Sigma Q^2} . \tag{6.25}$$

This result (6.24) is essentially the old one of Gell-Mann and Low.[64]

By combining the obtained relations of this type to eliminate the original coupling constants (F's and a's) and the cutoff momentum, we can derive the relations (6.19) and (6.20) of Georgi and Glashow.[12] In addition, we obtain the following two sum rules for the masses of the physical Higgs scalar η, the weak vector bosons $W^\pm$ and Z, and w_1 and w_2:

$$m_\eta = 2\left(\frac{m_{w_1}^4 + m_{w_2}^4}{m_{w_1}^2 + m_{w_2}^2}\right)^{1/2} \tag{6.26}$$

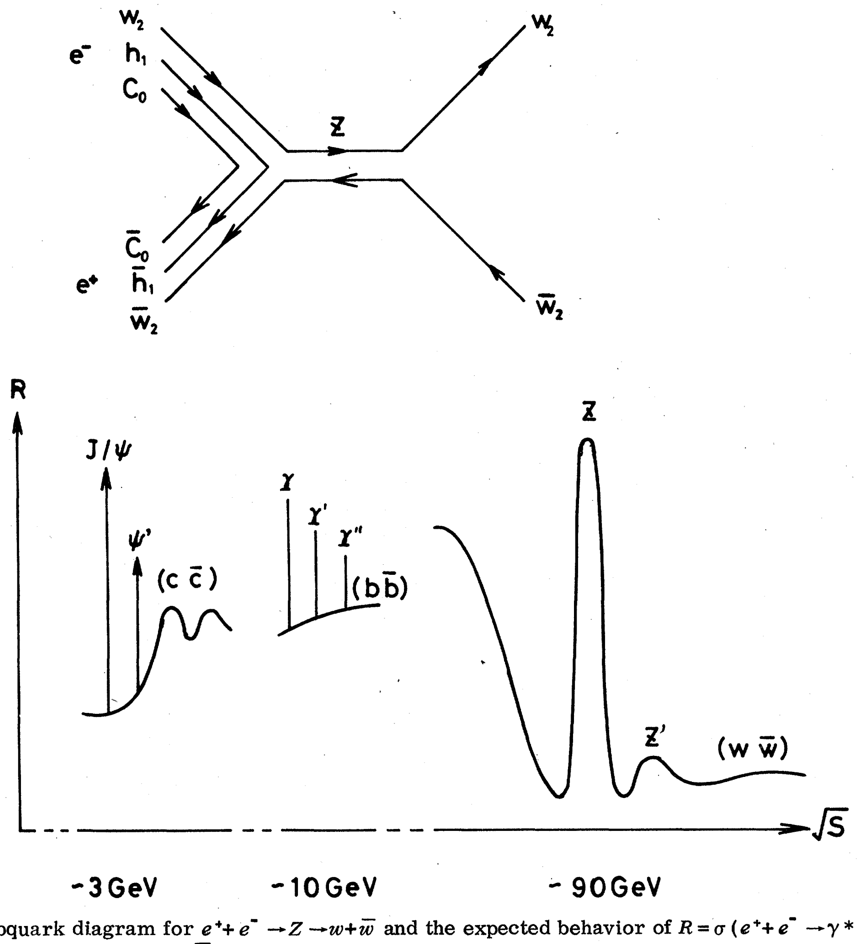

FIG. 2. The subquark diagram for $e^+ + e^- \to Z \to w + \bar{w}$ and the expected behavior of $R = \sigma(e^+ + e^- \to \gamma^*$ or $Z \to \text{all})/\sigma_0(e^+ + e^- \to \gamma^* \to \mu^+ + \mu^-)$ as a function of $\sqrt{s}$ (the total c.m. energy).

and

$$m_{W^\pm} = m_Z \cos\theta_W = \sqrt{3}\left(\frac{m_{w_1}{}^2 + m_{w_2}{}^2}{2}\right)^{1/2} . \quad (6.27)$$

From these two relations we also obtain a remarkable inequality of

$$m_\eta \geq (2/\sqrt{3})\, m_{W^\pm} . \quad (6.28)$$

Furthermore, from (6.13), (6.27), and (6.28), we can predict the average mass of w's and a lower bound on the mass of the physical Higgs scalar as

$$(\langle m_w{}^2\rangle)^{1/2} \cong 46 \text{ GeV and } m_\eta \geq 92 \text{ GeV for } \sin^2\theta_W \cong 0.22. \quad (6.29)$$

This prediction indicates that the threshold for production of $w\bar{w}$ pair, if any, is higher than but very close to the mass of Z boson. This situation is very similar to those in J/ψ and Υ productions where productions of $c\bar{c}$ and $b\bar{b}$ pairs have started just above the ψ' and Υ' peaks, respectively. In view of this anticipation and the previous ones of (6.16) and (6.17), we can sketch a rough picture of the possible future observation in e^+e^- experiments, which is illustrated in Fig. 2.

VII. MODEL OF GRAVITY AND SUPERGRAND UNIFICATION

There are at least two ways known to unify gravity with strong and electroweak interactions. One is a model of supergravity[15] and the other is a model of pregeometry.[17, 18, 11] It is well known[15] that although the unified supergravity model of SO(8) has the maximum capacity for lepton, quark, gauge-boson, and Higgs-scalar multiplets, it is not large enough to contain all the desired or needed multiplets. It has been suggested by Salam[19] that reduction of the total number of fundamental fermions by introducing subquarks may be promising for making a realistic unified model of supergravity. Very recently, Curtright and Freund[65] have introduced an SU(8) group for such supergrand unification. In their model, the fundamental octet of SU(8) consists of a doublet of w's, a triplet of h's and a triplet of C's as

$$\underline{8} = (w_1, w_2) + (h_1, h_2, h_3) + (C_1, C_2, C_3) . \quad (7.1)$$

If this is indeed the physical case, it will certainly indicate the success of the concept.

In our model of subquark pregeometry, on the other hand, the graviton appears as a collective excitation of subquark-antisubquark pairs. The simplest example of Lagrangian for $6+N$ subquarks (w's, h's and C's) which are simply denoted by ψ takes a form of

$$L = \bar{\psi}\, i\tfrac{1}{2}\overleftrightarrow{\partial\!\!\!/}\psi + F_0(T_{\mu\nu})^2$$

with

$$T_{\mu\nu} = \bar{\psi} i\tfrac{1}{4}(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu)\psi , \quad (7.2)$$

where F_0 is a coupling constant. By analyzing this nonlinear fermion Lagrangian as in Ref. 18 and by imposing again the masslessness condition of Bjorken,[63] we can show that this model Lagrangian for pregeometry correctly reproduces the familiar Newtonian gravitational potential if

$$G = 4\pi/\kappa_0 N_0 \Lambda^2 , \quad (7.3)$$

where G is the Newtonian gravitational constant ($=6.67\times10^{-8}$ cm^3 g^{-1}sec^{-2}), $\kappa_0 = \frac{2}{3}$ and $\frac{1}{3}$ depending on the invariant and Pauli-Villars cutoff procedures, and N_0 is the total number of subquarks ($=6+N$). It can also be shown that a more sophisticated model reproduces the Einstein-Weyl theory of general relativity in a generally covariant way.[18] Notice that the cutoff momentum Λ is determined by the condition (7.3) to be around the Planck mass ($G^{-1/2} = 1.221\times10^{19}$ GeV).

In this picture, it is easy to unify gravity with the strong and electroweak interactions of subquarks. All we need is to combine the Lagrangians (6.23) and (7.2). As a result of such supergrand unification, we obtain, by combining (6.24) and (7.3), the relation between the fine-structure constant and the Newtonian gravitational constant:

$$\alpha = \frac{3\pi}{\sum Q^2 \ln(4\pi/\kappa_0 N_0 G m^2)} . \quad (7.4)$$

A relation of this type was conjectured by Landau[66] in 1955. Since the total number of h's is not known and the charges of subquarks are ambiguous, it is hard to decide whether this relation holds experimentally. It is, however, clear that the relation (7.4) and the particular charge assignment of subquarks (6.22) would not coexist since the right-hand side of (7.4) would then become larger than 0.13 for $N \geq 3$ and $m = 10$–100 GeV while the left-hand side is about $\frac{1}{137}$.

In either of these two pictures, a possible supergrand unification of all fundamental forces including gravity seems to be very promising when it is written in terms of subquarks.

VIII. CONCLUDING REMARKS

In our spinor-subquark model, each one of the subquarks (w's, h's, and C's) has a single function of specifying isospin, horizontal-spin, or color-spin quantum number. These subquarks, if they exist, may therefore be ultimate forms of matter which cannot be or need not be further decomposed. It is still tempting to ask further why these quantum numbers (or, in other words, the charge, mass, and color) exist in nature. Obviously, it is not easy to give an answer to this almost metaphysical question. Let us change this question into the following easier ones: "Why do the two isospin states (or w_i for $i=1, 2, \ldots, N$) exist?" "Why do the N horizontal degrees of freedom (h_i for $i=1, 2, \ldots, N$) exist?" "Why do the four color states (C_i for $i=0, 1, 2, 3$) exist?" I do not yet have any ideas about the first two questions. However, I would like to make a conjecture on the last one. It is the "color-space correspondence and their possible symmetry."

The point is that there is some similarity between the space-time and the color space. There exist one dimension in time ($t \equiv x_0$) and three dimensions in space (x_i for $i=1, 2, 3$). There seems to exist, on the other hand, "lepton-quark correspondence,"[67] the one-to-one correspondence between a lepton (l) and a quark flavor (q), and each one flavor of quark has three color degrees of freedom (q_i for $i=1, 2, 3$). More simply, there may exist C_0 and a triplet of C_i for $i=1, 2, 3$. The O(3) symmetry of space is exact. So seems to be the $SU(3)_C$ symmetry of color. It is, therefore, very tempting to make an ansatz that there exists the following color-space correspondence:

$$\begin{array}{cccc} x_0 & x_1 & x_2 & x_3 \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ C_0 & C_1 & C_2 & C_3 \end{array} \quad . \tag{8.1}$$

The nature appreciates the Lorentz symmetry of $O(3,1)$ or $SL(2,C)$ in space-time which keeps, for example, $x_0^2 - x_1^2 - x_2^2 - x_3^2$ invariant. Correspondingly, something like $\overline{C}_0C_0 - \overline{C}_1C_1 - \overline{C}_2C_2 - \overline{C}_3C_3$, which can be related with lepton-quark mass difference, may behave as an invariant. One can even imagine "special relativity" and "general relativity" in the color-space. At this point, it seems to be closely related with the "strong-gravity" model of Isham, Salam, and Strathdee[68] in which the $SL(6,C)$ gauge group, containing $SU(3)_C \times SL(2,C)$, is proposed for the strong interaction. This color-space correspondence may become a clue to a possible relation between fields (or matter) and space-time but its real relevance is obscure at present.

What is more urgent at present is to investigate whether the subquark model discussed in this paper is consistent with all existing experimental data on the short-distance behavior of leptons and quarks. By assuming the possible structure of quarks, Chanowitz and Drell[69] and Tajima and Matumoto[70] have analyzed the observed breaking of the Bjorken scaling in deep-inelastic lepton-nucleon scatterings, and pointed out that quarks may have the structure which becomes relevant at a distance of order 10^{-15} cm. The result of recent extensive analyses[71] indicates, however, that the observed scale breaking is perfectly consistent with the prediction of quantum chromodynamics, showing no sign yet for the intrinsic structure of quarks. As far as the possible structure of leptons is concerned, the strongest constraint comes from comparison of theory and measurement of the muon anomalous moment, which gives a lower limit of about 0.5 GeV for the deviation parameter in the electron propagator.[72] In any case, it seems quite certain that the possible existence of subquarks with the mass of order 50 GeV or even higher does not contradict any presently available experimental data. Whether their existence is real or not will be checked by future high-energy experiments in the 1980's we hope.

In the recent papers by Glück and Lipkin,[73] it is emphasized that obtaining the observed Dirac magnetic moments of the electron and muon is particularly difficult in a composite model of leptons. Although whether our model passes his test strongly depends on the unknown subquark dynamics, how to overcome this difficulty is certainly an important subject for future investigations.

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