PHYSICAL REVIEW D VOLUME 22, NUMBER 7 1 OCTOBER 1980

Asymptotically free SU(5) model with three generations

Ngee-Pong Chang
Physics Department, The City College of the City University of New York, New York, New York 10031

Ashok Das
Physics Department, University of Maryland, College Park, Maryland 20742

Juan Perez-Mercader
Physics Department, The City College of the City University of New York, New York, New York 10031
(Received 28 February 1980; revised manuscript received 12 August 1980)

We present a new SU(5) model that needs only three generations of light fermions to preserve asymptotic freedom. This SU(5) model has the same low-energy structure as the standard SU(5) theory. Predictions for proton lifetime and $\sin^2\theta$ are presented.

In a series of papers[1] we reported on a new renormalization program for grand unified theories and applied it extensively to SU(5) theory. The single most important prediction of SU(5) which can be directly tested by experiment is the proton decay lifetime. In principle, SU(5) can give a precise value for τ_p, once α_s, the gluon running coupling constant, is experimentally known. The measurement of τ_p can be used to either confirm or deny SU(5) as a grand unified theory. Early estimates of the proton lifetime have indeed been made[2] and, as they turn out to be within reach of experiments, have sparked considerable interest on the subject.

In the standard SU(5) theory, because of the large number of independent Higgs couplings, there is no definitive statement on the mass-renormalization corrections. In an asymptotically free SU(5) model, however, where all couplings are predetermined by eigenvalue conditions,[3] such corrections can be easily taken into account. In our previous note[4] we reported on one such asymptotically free SU(5) model, in which seven generations of light fermions were needed to preserve asymptotic freedom. The resulting τ_p, including all renormalization effects, was tabulated in accordance with an allowed range of present low-energy input for α_s, between 0.2 and 0.35.

Seven generations, by all accounts, may be excessive. We have therefore continued to work toward a model that needs a smaller number of generations for asymptotic freedom.

In this paper we report on an asymptotically free SU(5) model that needs only *three* generations of light fermions. The Higgs-boson and light-fermion structure in this asymptotically free model is *identical* to the standard SU(5) model, with

$$\underline{5}, \underline{24} \text{ Higgs bosons}, \qquad \underline{24} \text{ gauge bosons}, \qquad \text{three generations of } \underline{5}_R \text{ and } \underline{10}_L \text{ light fermions.} \tag{1}$$

For asymptotic freedom it turns out to be necessary to introduce superheavy fermions, much like the "regulator" fields needed for renormalizability. These superheavy fermions, however, are physical, with mass scale of the same order as the X gauge bosons. For our work we have taken these superheavy fermions to be "supersymmetric" with the Higgs bosons, i.e., we include

$$\underline{5}, \underline{24} \text{ superheavy fermions.} \tag{2}$$

Since at low energies, by the Appelquist-Carrazone theorem,[5] the superheavy fermions effectively decouple, the low-energy structure of this SU(5) model is identical to the usual SU(5) theory.

Our numerical prediction for τ_p, calculated with this $n_f=3$ asymptotically free SU(5) model, is not subject to theoretical uncertainties. Insofar as three generations is acceptable from the point of view of astrophysical data,[6] our prediction is physically relevant.

We shall give the outline for the complete Lagrangian of this model. Before doing so, however, it will be useful to explain what the new ingredient is that made three generations possible, whereas previously seven were needed.

To see this, recall that the Higgs-boson quartic self-couplings λ_i by themselves tend not to respect asymptotic freedom. Under a change in renormalization scale, they behave much like the ordinary QED charge, e. In the presence of fermions that couple to Higgs bosons, the fermion loops add a negative (i.e., asymptotically free) contribution to the renormalization-group equation for λ_i, being proportional to

$$-n_f h^4 . \tag{3}$$

Here n_f denotes the number of fermion generations and h refers to the Yukawa couplings such as

$$-\sqrt{2}\, h\bar{\psi}_{L\alpha\beta}\, \psi_R^{\alpha} H^{\beta} + \text{H.c.} \tag{4}$$

In our earlier work we did not include the SU(5)-

allowed Yukawa coupling[7]

$$-\tfrac{1}{4}h'\epsilon_{\alpha\beta\mu\nu\lambda}\tilde{\psi}_L^{\alpha\beta}C^{-1}\psi_L^{\mu\nu}H^\lambda+\text{H.c.} \tag{5}$$

With this, the fermion-loop contribution to the renormalization-group equation now is of the form

$$-n_f(ah^4+bh'^4)\,. \tag{6}$$

Upon comparing (6) with (3) it is clear that so long as h' is not negligible relative to h, the effect of h' is to reduce the number of generations needed for asymptotic freedom. A detailed calculation of the renormalization-group equations indeed supports this conclusion.

A further remark on the issue of generations is appropriate at this point. While asymptotic freedom places a constraint on the number of generations of light fermions, SU(5) by itself cannot distinguish among the three generations. For our considerations we have simply ignored the difference between the e, μ, and τ and replaced their individual Yukawa couplings by a common, averaged Yukawa coupling. At grand unification energies, the light fermions are in any case massless and ignoring the difference between e, μ, and τ may not be a bad initial approximation. To really understand the difference between e, μ, and τ, we would have to go to an SO(N) or a higher SU(N) grand unified theory.[8]

Having thus settled all these preliminaries, we now write down the complete Lagrangian for our model:

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_\nu-\partial_\nu A_\mu-ig[A_\mu,A_\nu])^2-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu\phi-ig[A_\mu,\phi])^2-|\partial_\mu H-igA_\mu H|^2\\&+\tfrac{1}{2}\mu^2\mathrm{Tr}(\phi^2)-\tfrac{1}{4}\lambda_1[\mathrm{Tr}(\phi^2)]^2-\tfrac{1}{2}\lambda_2\mathrm{Tr}(\phi^4)+\tfrac{1}{2}\nu^2H^\dagger H-\tfrac{1}{4}\lambda_3(H^\dagger H)^2-\tfrac{1}{2}\lambda_4H^\dagger H\,\mathrm{Tr}(\phi^2)-\tfrac{1}{2}\lambda_5H^\dagger\phi^2H\\&-\sum_{\text{generations}}[\bar{\psi}_R\gamma_\mu D_\mu\psi_R+\bar{\psi}_L\gamma_\mu D_\mu\psi_L+(\sqrt{2}\,h\bar{\psi}_{L\alpha\beta}\psi_R^\alpha H^\beta+\tfrac{1}{4}h'\epsilon_{\alpha\beta\mu\nu\lambda}\tilde{\psi}_L^{\alpha\beta}C^{-1}\psi_L^{\mu\nu}H^\lambda+\text{H.c.})]\\&-\bar{\chi}\gamma_\mu D_\mu\chi-\bar{B}\gamma_\mu D_\mu B-(k_4\bar{B}_\beta^\alpha\chi^\beta H_\alpha^\dagger+\text{H.c.})-k_2\bar{\chi}_\alpha\chi^\beta\phi_\beta^\alpha-k_5\bar{B}_\beta^\alpha B_\gamma^\beta\phi_\alpha^\gamma-k_6\bar{B}_\gamma^\beta B_\beta^\alpha\phi_\alpha^\gamma+\mathcal{L}_{\text{gf}}+\mathcal{L}_{\text{ghost}}\,,\end{aligned} \tag{7}$$

where the coupling constants and parameters are all fixed by the eigenvalue conditions:

$$\begin{aligned}n_f=3,\quad &h=-0.254\,406g,\quad k_2=-0.635\,486g,\quad k_4=-0.942\,053g\,,\\&h'=-0.746\,771g,\quad k_5=-0.809\,565g,\quad k_6=0.706\,639g\,,\\&\lambda_1=0.029\,244g^2,\quad \lambda_2=0.457\,611g^2,\quad \lambda_3=1.196\,053g^2,\\&\lambda_4=-0.012\,374g^2,\quad \lambda_5=0.909\,170g^2,\end{aligned} \tag{8}$$

and

$$\nu^2=-0.927\,207\mu^2\,. \tag{9}$$

At the Lagrangian level, these relations involve bare coupling constants. Being, however, eigenvalue solutions to renormalization-group equations, these relations are in fact maintained even after renormalization. As a result of these numerical relationships, our model is truly a one-coupling-constant, one-mass-scale theory with a low-energy structure identical to the standard SU(5) theory.

At the fundamental level, the eigenvalue conditions may be understood as follows. Our Lagrangian is really a phenomenological Lagrangian, much as the Landau-Ginzburg theory is the phenomenological one in relation to a microscopic BCS theory. A fundamental Lagrangian should presumably be free of the ubiquitous Higgs boson.

That fundamental theory will most likely still be a gauge theory and it will have asymptotic freedom. In that theory[9] all Higgs-boson couplings will be induced couplings and are fully calculable. We believe that the result of those calculations will reproduce the eigenvalue conditions that were necessary to preserve asymptotic freedom.

We are now ready to apply our $n_f=3$ model to the discussion of proton decay. As we indicated earlier,[4] the tree diagram for this process is given by

$$G\equiv\frac{g_x{}^2}{8\tilde{M}^2}\,. \tag{10}$$

In Eq. (10), $\tilde{M}$ is the effective mass of X, introduced in the previous paper,[1] while g_x is its coupling to fermions. In this asymptotically free SU(5) model the two apparently independent mass scales μ^2 and ν^2 are related. This is not demanded by the eigenvalue conditions imposed on the coupling constants. However, upon a study of the renormalization-group equations for the masses μ^2 and ν^2, and after substituting the eigenvalues, we find[10]

$$16\pi^2 \frac{d\mu^2}{dt} = 13.419\,956\bar{g}^2\mu^2 + 0.847\,298\bar{g}^2\nu^2 ,$$
$$16\pi^2 \frac{d\nu^2}{dt} = 8.134\,063\bar{g}^2\mu^2 + 21.406\,988\bar{g}^2\nu^2 . \quad (11)$$

This set of coupled equations, in general, has solutions which in the limit $t\to\infty$ become proportional to each other:

$$\nu^2(t) \xrightarrow{t\to\infty} 10.353\,678\mu^2(t) . \quad (12)$$

Such behavior would destory the $SU(3)_c$ and the SU(2) vacuum symmetry at high energies. There is, however, a special solution to these equations which relates ν^2 and μ^2 and under which the $SU(3)_c$ and SU(2) symmetries of the vacuum at high energies are preserved.

With our parameters and mass scales so fixed, we can directly calculate the equation for the effective mass of the X boson,

$$16\pi^2 \frac{d\tilde{M}^2}{dt} = (\tfrac{8}{3} n_f + 4.634\,335)\bar{g}^2\tilde{M}^2 , \quad (13)$$

so that we find, as before,[4] a *suppression* of the effective mass at 1 GeV compared with the effective mass at 10^{15} GeV ($\equiv M_X$),

$$\frac{\tilde{M}^2(1\text{ GeV})}{\tilde{M}^2(10^{15}\text{ GeV})} = \frac{1}{4.1} , \quad (14)$$

which shortens the lifetime by a factor of ~16. There is also a compensating suppression of g_X at low energies, given by its renormalization-group-equation analysis,[1] that in this case turns out to be

$$\frac{g_X{}^4(1\text{ GeV})}{g_X{}^4(10^{15}\text{ GeV})} = \frac{1}{7.2} . \quad (15)$$

The results of our investigations according to the program that was discussed earlier[4] are summarized in the following table, where we have used $\alpha = 1/137.036$ at m_e scale, and the quoted values of $\alpha_s(m)$ correspond to a scale of $m = 6$ GeV (chosen to be between the end of the charmonium threshold and the onset of the Υ threshold) ($n_f = 3$) (Ref. 11):

$\alpha_s(m)$	$\sin^2\theta(m)$	M_X (GeV)	τ_p (yr)
0.2	0.214	5.77×10^{14}	1.0×10^{30}
0.25	0.210	1.24×10^{15}	1.6×10^{31}
0.3	0.207	2.05×10^{15}	9.0×10^{31}
0.35	0.205	2.95×10^{15}	2.4×10^{32}

In this paper we have reported on what we believe to be the most realistic asymptotically free SU(5) model with a low-energy structure that is *identical* to the standard SU(5) theory. We need only three generations of light fermions in this asymptotically free model.

The advantage of an asymptotically free theory is that the theory, in spite of its Higgs structure, is truly a one-coupling-constant *and* one-mass-scale theory. As a result all physically measured quantities can be reliably calculated.[1] Our prediction for proton lifetime is thus to be compared with the proton lifetime calculated in a standard model where uncertainties persist due to the complicated Higgs potential with five arbitrary couplings.

One of us (J.P.-M.) would like to thank Dr. J. Uson of the Princeton University Gravitation Research Group for several discussions on Astrophysical Data and the number of light neutrinos. The work of N.-P.C. was supported in part by NSF Grant No. 77-01350. The work of N.-P.C. and J.P.-M. was supported in part by the Research Foundation of CUNY under PSC-BHE Award No. 12336. The work of A. D. was supported by the National Science Foundation.

APPENDIX

The complete renormalization-group equations for our model are given by

$$16\pi^2 \frac{dg}{dt} = -\left(\tfrac{55}{3} - 1 - \tfrac{4}{3} n_f - \tfrac{22}{3} n_F\right) g^3 ,$$

$$16\pi^2 \frac{dh}{dt} = h\left(-18g^2 + \tfrac{48}{5} n_F k_4{}^2 - \tfrac{9}{2} h'^2 + 3n_f h'^2\right) + h^3(3 + 4n_f) ,$$

$$16\pi^2 \frac{dh'}{dt} = h'\left[(4n_f - 6)h^2 + \tfrac{48}{5} n_F k_4{}^2 - \tfrac{108}{5} g^2\right] + h'^3(3 + 3n_f) ,$$

$$16\pi^2 \frac{dk_2}{dt} = k_2\left[\tfrac{24}{5} k_4{}^2 + \tfrac{46}{5} n_F (k_5{}^2 + k_6{}^2) - \tfrac{8}{5} n_F k_5 k_6 - \tfrac{72}{5} g^2\right] + k_4{}^2\left(-\tfrac{4}{5} k_5 + \tfrac{46}{5} k_6\right) + k_2{}^3\left(\tfrac{22}{5} + 2n_F\right) ,$$

$$16\pi^2 \frac{dk_4}{dt} = k_4\left[4n_f h^2 + 3n_f h'^2 - \tfrac{4}{5} k_2 k_5 + \tfrac{46}{5} k_2 k_6 + \tfrac{12}{5} k_2{}^2 + \tfrac{23}{10} (k_5{}^2 + k_6{}^2) - \tfrac{2}{5} k_5 k_6 - \tfrac{111}{5} g^2\right] + k_4{}^3\left(\tfrac{29}{10} + \tfrac{48}{5} n_F\right) ,$$

$$16\pi^2\frac{dk_5}{dt}=k_5(2n_Fk_2^2+\tfrac{46}{5}n_Fk_6^2+\tfrac{63}{5}k_6^2+k_4^2-30g^2)+k_5^2(-\tfrac{12}{5}k_6-\tfrac{8}{5}n_Fk_6)+k_5^3(\tfrac{21}{5}+\tfrac{46}{5}n_F)-\tfrac{4}{5}k_6^3,$$

$$16\pi^2\frac{dk_6}{dt}=2k_4^2k_2-\tfrac{4}{5}k_5^3+k_6(2n_Fk_2^2+\tfrac{46}{5}n_Fk_5^2+13k_5^2+k_4^2-30g^2)+k_6^2(-2k_5-\tfrac{8}{5}k_5n_F)+k_6^3(\tfrac{21}{5}+\tfrac{46}{5}n_F),$$

$$16\pi^2\frac{d\lambda_1}{dt}=64\lambda_1^2+\lambda_1[\tfrac{376}{5}\lambda_2+8n_Fk_2^2+\tfrac{184}{5}n_F(k_5^2+k_6^2)-\tfrac{32}{5}n_Fk_5k_6-60g^2]+\tfrac{672}{25}\lambda_2^2+5\lambda_4^2+2\lambda_4\lambda_5+9g^4$$
$$-\tfrac{16}{25}n_Fk_5^4-\tfrac{64}{25}n_Fk_5^3k_6-\tfrac{1296}{25}n_Fk_5^2k_6^2-\tfrac{64}{25}n_Fk_5k_6^3-\tfrac{16}{25}n_Fk_6^4,$$

$$16\pi^2\frac{d\lambda_2}{dt}=\tfrac{128}{5}\lambda_2^2+\lambda_2[24\lambda_1-60g^2+8n_Fk_2^2+\tfrac{184}{5}n_F(k_5^2+k_6^2)-\tfrac{32}{5}n_Fk_5k_6]$$
$$+\tfrac{1}{2}\lambda_5^2+\tfrac{15}{2}g^4-4n_Fk_2^4-\tfrac{84}{5}n_Fk_5^4+\tfrac{64}{5}n_Fk_5^3k_6+\tfrac{96}{5}n_Fk_5^2k_6^2+\tfrac{64}{5}n_Fk_5k_6^3-\tfrac{84}{5}n_Fk_6^4,$$

$$16\pi^2\frac{d\lambda_3}{dt}=9\lambda_3^2+\lambda_3(-\tfrac{144}{5}g^2+16n_fh^2+12n_fh'^2+\tfrac{192}{5}n_Fk_4^2)$$
$$+48\lambda_4^2+\tfrac{96}{5}\lambda_4\lambda_5+\tfrac{132}{25}\lambda_5^2+\tfrac{396}{25}g^4-\tfrac{1856}{25}n_Fk_4^4-32n_fh^4-24n_fh'^4,$$

$$16\pi^2\frac{d\lambda_4}{dt}=4\lambda_4^2+\lambda_4[6\lambda_3+52\lambda_1+\tfrac{188}{5}\lambda_2-\tfrac{222}{5}g^2+4n_Fk_2^2+\tfrac{92}{5}n_F(k_5^2+k_6^2)-\tfrac{16}{5}n_Fk_5k_6+8n_fh^2+6n_fh'^2+\tfrac{96}{5}n_Fk_4^2]$$
$$+\lambda_3\lambda_5+\lambda_5^2+\tfrac{48}{5}\lambda_1\lambda_5+\tfrac{112}{25}\lambda_2\lambda_5+3g^4-16n_Fk_4^2k_2^2-16n_Fk_4^2k_2k_6$$
$$-\tfrac{16}{25}n_Fk_4^2k_5^2-\tfrac{416}{25}n_Fk_4^2k_6^2-\tfrac{32}{25}n_Fk_4^2k_5k_6,$$

$$16\pi^2\frac{d\lambda_5}{dt}=\tfrac{21}{5}\lambda_5^2+\lambda_5[\lambda_3+4\lambda_1+\tfrac{76}{5}\lambda_2+8\lambda_4-\tfrac{222}{5}g^2+8n_fh^2+6n_fh'^2+4n_Fk_2^2$$
$$+\tfrac{92}{5}n_F(k_5^2+k_6^2)-\tfrac{16}{5}n_Fk_5k_6+\tfrac{96}{5}n_Fk_4^2]$$
$$+15g^4+\tfrac{16}{5}n_Fk_2^2k_4^2+\tfrac{32}{5}n_Fk_4^2k_2k_5+\tfrac{32}{5}n_Fk_4^2k_2k_6-\tfrac{352}{5}n_Fk_4^2k_5^2+\tfrac{96}{5}n_Fk_4^2k_5k_6+\tfrac{48}{5}n_Fk_4^2k_6^2.$$

---

[1]N. P. Chang, A. Das, and J. Perez-Mercader, Phys. Rev. D 22, 1414 (1980); 22, 1429 (1980); Phys. Lett. 93B, 137 (1980); Report No. CCNY-HEP-79/25 (unpublished).

[2]For example, H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974); D. A. Ross, Nucl. Phys. B140, 1 (1978).

[3]For a review see N. P. Chang, in *Proceedings of the 1978 International Meeting on Frontier of Physics*, edited by K. K. Phua, C. K. Chew, and Y. K. Lim (Singapore National Academy of Science, 1978).

[4]See N. P. Chang, A. Das, and J. Perez-Mercader, Phys. Lett. 93B, 137 (1980).

[5]T. Appelquist and J. Carrazone, Phys. Rev. D 11, 2856 (1975).

[6]G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. 66B, 202 (1977); J. Yang, D. N. Schramm, G. Steigman, and R. T. Rood, Astrophys. J. 227, 697 (1979). There is a very good review by G. Steigman and R. Wagoner in Proceedings of the Summer School on Physical Cosmology, Les Houches, France, 1979 (unpublished).

[7]J. Ellis *et al.*, Phys. Lett. 89B, 320 (1979).

[8]H. Georgi, Nucl. Phys. B156, 126 (1979); P. Frampton and S. Nandi, Phys. Rev. Lett. 43, 1460 (1979); J. P. Chakraborti, R. N. Mohapatra, and M. Popovic, Report No. CCNY-HEP-79/12 (unpublished).

[9]See, for instance, K. Huang, in *Proceedings of the 1980 Canton Conference on Theoretical Particle Physics* (Academia Sinica, 1980) and references quoted therein.

[10]The equations for $d\mu^2/dt$ and $d\nu^2/dt$ may be obtained from Eqs. (2.23) and (2.24) of Report No. CCNY-HEP-79/25 written earlier by us, with, however, the addition of $+6n_fh'^2\nu^2$ to the $d\nu^2/dt$ equation (unpublished).

[11]Our τ_p is based on the formula given by C. Jarlskog and F. Yndurain, Nucl. Phys. B149, 29 (1979), $\tau_{0}(p)=2.84\times10^{-31}$ yr$/G^2A(f)$, with $G^2A(f)$ (theirs) $=G_F^2$ (ours). Our effective G_F at m_p scale is computed according to the program described in Report No. CCNY-HEP-79/25 (unpublished).