Υ family of resonances above threshold

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The coupled-channel method developed previously is applied to the $b\bar{b}$ system of resonances above the threshold for Zweig-allowed decays $(2m_B)$. The structure of ΔR is calculated for representative values of $\Delta M = m(4\,{}^3S_1)-2m_B$. Typically the $4\,{}^3S_1$ resonance has a complex structure with a total width 10–40 MeV and ΔR(peak)$\sim$1–3. This structure has a strong ΔM dependence which may allow the determination of m_B from the observed structure of ΔR in the Υ''' region. The contribution of each exclusive channel is shown. Higher $b\bar{b}$ resonances are briefly discussed.

A coupled-channel formalism has been developed by Eichten *et al.*[1] to study heavy-quark–antiquark resonances above the threshold for Zweig-allowed decays, and has been applied by the same authors[2] to the charmonium system of resonances above charm threshold. In this Addendum to those works this coupled-channel formalism is applied to the $b\bar{b}$ system.

The spectrum of $(b\bar{b})$ states before coupling to decay channels can be calculated using a simple potential[2]

$$V(R) = -\frac{K}{R} + R/a^2 . \tag{1}$$

The expectation from quantum chromodynamics is that $V(R)$ should be independent of quark mass for sufficiently heavy quarks. In particular, a should be unchanged from the $c\bar{c}$ system; i.e., $a = 2.34$ GeV^{-1}. The parameter K is used to model the more complicated true behavior at short distance

$$RV(R) \to -\frac{4}{3}\,\frac{12\pi}{(33-2n_f)\ln(1/R^2\Lambda^2)} \text{ as } R\to 0 , \tag{2}$$

where n_f is the number of light flavors $(n_f \approx 3)$. Since the mean square radius of a $(b\bar{b})$ state is smaller than the corresponding $(c\bar{c})$ state, K_Υ would be expected to be smaller than K_ψ (~0.52). As discussed in Ref. 2, fitting the $m(\Upsilon') - m(\Upsilon)$ to the experimental value[3] (~560 MeV) yields $K_\Upsilon = 0.483$ and $m_b = 5.17$ GeV. The resulting spectrum is shown in Fig. 1.

The low-lying mesons carrying the b flavor will be denoted $B^- = 1^1S_0(b\bar{u})$, $B^0 = 1^1S_0(b\bar{d})$, $B^{*-} = 1^3S_1(b\bar{u})$, and $B^{*0} = 1\,{}^3S_1(b\bar{d})$. The systematics of heavy-light systems are well understood.[4] The center of gravity (c.o.g.) of the B mesons may be determined by the relation

$$\frac{3m(B^*)+m(B)}{4} = (m_b - m_c) + \frac{3m(D^*)+m(D)}{4} . \tag{3}$$

The hyperfine splitting has the form $(\vec{J}^2 - \frac{3}{4})\,A/m_H$, where the full dependence on the total angular momentum $(\vec{J})$ and the heavy-quark mass (m_H) has been exhibited. Thus

$$m(B^*) - m(B) = \left(\frac{m_c}{m_b}\right)[m(D^*) - m(D)] . \tag{4}$$

Since m_b and m_c are not measured but only inferred from the $(c\bar{c})$ and $(b\bar{b})$ resonance they are model dependent. There is, however, considerable cancellation of the model dependence in the mass difference and mass ratio; it is reasonable to say $m_b - m_c$ is known to less than 100 MeV while m_c/m_b is known to approximately 10%. Using $m_c = 1.84$ GeV[2] and $m(D^*) - m(D) \simeq 140$ MeV (Ref. 5) we conclude that the c.o.g. for the mesons is at 5.30 GeV while the hyperfine splitting of the B^* and B is 50 MeV.

Finally, the $B^0 - B^-$ and $B^{*0} - B^{*-}$ mass splittings can be estimated following the method employed by Lane and Weinberg[6] for the charmed mesons. The results are[7]

$$m(B^0) - m(B^-) = 4.4 \text{ MeV} , \tag{5a}$$

$$m(B^{*0}) - m(B^{*-}) \simeq m(B^0) - m(B^-) . \tag{5b}$$

Denoting the $1S_J(b\bar{s})$ mesons by E^0 and E^{*0} for $J = 0$ and 1, respectively, and using the values 2.14 GeV and 2.03 GeV for the F^* and F masses[8] one would expect

$$\frac{3m(E^*)+m(E)}{4} \simeq 120 \text{ MeV} + \frac{3m(B^*)+m(B)}{4} \tag{6a}$$

and

$$m(E^*) - m(E) = 35 \text{ MeV} . \tag{6b}$$

The spectrum of low-lying B and E mesons resulting from the above considerations is shown in Fig. 2. In order to allow the decays of Υ states into the B and B^* mesons above threshold for those Zweig-allowed decays, the states in the $(b\bar{b})$ sector and the $(b\bar{u})$, $(b\bar{d})$, and $(b\bar{s})$ sectors of the theory must be coupled. The method of coupling the two sectors of the Hilbert space was discussed

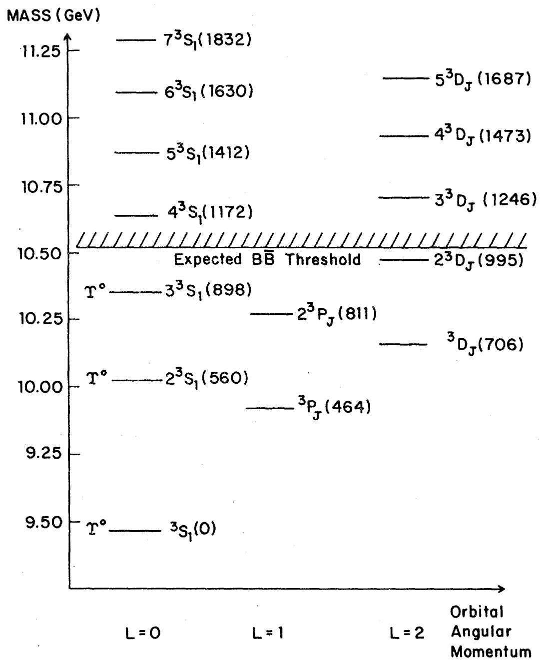

FIG. 1. Spectrum of $(b\bar{b})$ excitations within the Cornell potential model with $K=0.483$, $a=2.34$ GeV^{-1}, and $m_b=5.17$ GeV. The Υ'-Υ mass difference was used as an input. The mass of the B^- meson shown is 5.26 GeV.

in detail in Secs. III B, III C, and III D of Ref. 1. It was assumed that the instantaneous interaction which causes the binding of the $q\bar{q}$ states is also responsible for the decays; that is,

$$H_I = \tfrac{3}{8}\sum_{\alpha=1}^{8}\int :\rho_\alpha(\vec{r})\,V(\vec{r}-\vec{r}\,')\,\rho_\alpha(\vec{r}\,'):\; d^3\vec{r}\; d^3\vec{r}\,' , \tag{7}$$

where $\rho_\alpha(\vec{r}) = \Psi^+(\vec{r})(\lambda_\alpha/2)\Psi(\vec{r})$ is the octet of color densities of the quark field Ψ. In addition to providing an attractive interaction between $b\bar{b}$ pairs,

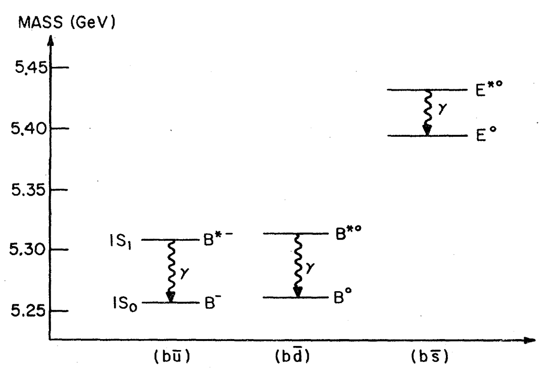

FIG. 2. Spectrum of 1S-state mesons carrying b flavor. The pseudoscalar mesons are $B^-=(b\bar{u})$, $B^0=(b\bar{d})$, and $E^0=(b\bar{s})$; an asterisk denotes the associated vector meson. The radiative transitions $V\to\gamma+P$ are indicated.

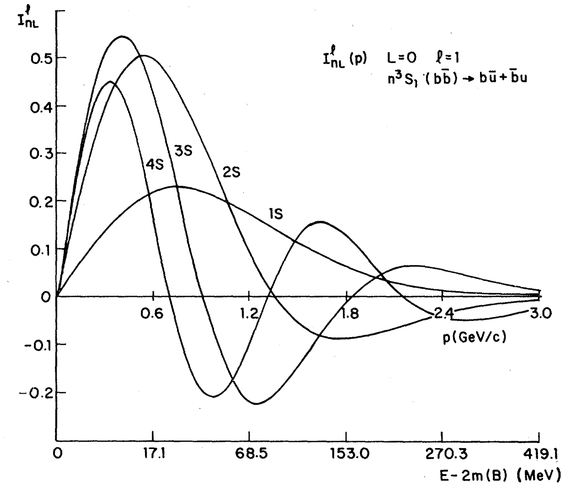

FIG. 3. Sample decay amplitudes $I^l_{nL}(p)$. L is the orbital angular momentum and n the radial quantum number of initial $b\bar{b}$ state, while l denotes the relative orbital angular momentum of the decay products. The lower scale is nonlinear and shows the total kinetic energy of the outgoing pair associated with the momentum on the upper scale.

H_I has pair-creation terms in which a $b\bar{b}$ pair creates or destroys a light-quark pair. These are the terms that describe decay.

The resulting decay amplitudes may be expressed in terms of certain reduced amplitudes. The $n\ ^3L_1(b\bar{b})$ state decaying into $1S(b\bar{u})$ meson (B^- or B^{*-}) and a $1S(\bar{b}u)$ meson (B^+ or B^{*+}) in a relative P wave ($l=1$) is expressible in terms of reduced amplitudes denoted by $I^l_{nL}(\vec{p})$ defined from Eqs. (3.33), (D14), and (D16) appearing in Ref. 1 by simply replacing the c-quark states by b-quark states. The results of the numerical calculations of the decay amplitude $I^l_{nL}(\vec{p})$ in the Υ system is shown in Fig. 3 for $n\ ^3S_1(b\bar{b})$ states ($L=0$; $n=1,2,3,4$) decaying into a $(b\bar{u})+(\bar{b}u)$ state in a relative P wave ($l=1$). Comparing these amplitudes with the corresponding amplitudes in the $c\bar{c}$ system (see, e.g., Ref. 2, Fig. 9), the general structures are found to be very similar as a function of momentum. If, however, the decay amplitudes are shown as a function of the total kinetic energy of the mesons (lower scale in Fig. 3), a very rapid energy variation is observed. This is purely a kinematical effect produced by the large b-quark mass, but has the important consequence that an exclusive channel may become important within a few MeV of its threshold.

The decay amplitudes can be used to calculate ΔR_b (the contribution to R from the b quark) as well as the separate contribution of each of the exclusive channels: B^-B^+, $B^0\bar{B}^0$, $B^-B^{*+}+B^{*-}B^+$, $B^0\bar{B}^{*0}+B^{*0}\bar{B}^0$, $B^{*-}B^{*+}$, and $B^{*0}\bar{B}^{*0}$. The Υ

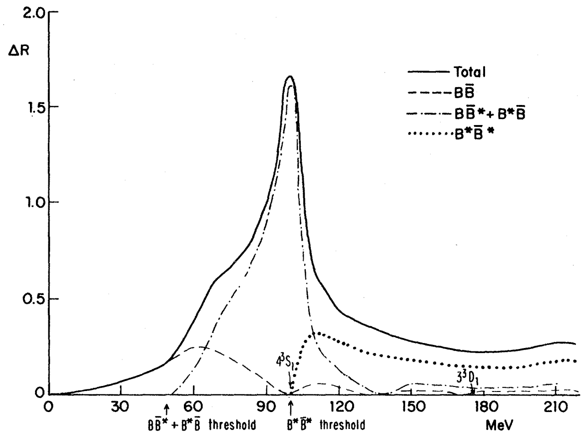

FIG. 4. The contribution of the various exclusive channels and total ΔR_b for $\Delta M \equiv M(4\,^3S_1) - 2M(B) = 100$ MeV.

states above B^-B^+ threshold will appear as resonances in ΔR_b with widths that are determined from the decay amplitudes above. The general method is described in detail in Secs. III E, III F, and III G of Ref. 1. There are no free parameters; m_b, a, and K_Υ are determined in the potential sector, and $m(B)$ and $m(B^*)$ from the considerations above. The $4\,^3S_1(b\bar{b})$ state (Υ''') is expected to be ~100 MeV above B^-B^+ threshold. The threshold region ΔR_b and exclusive channels are shown in Fig. 4. We see a complex structure in the region of the 4S state which results from each exclusive channel's complex energy dependence. This general property was also found in the charm threshold region. The $4\,^3S_1$ state has a ΔR (peak) ≈ 1.5 and a full width at half maximum (FWHM) approximately 25 MeV.[9] The charged and neutral exclusive channels have not been shown separately; very near threshold even these small mass differences can be significant. For example, if the $B^{*-}B^{*+}$ threshold was only 6 MeV lower than the $B^{*0}\bar{B}^{*0}$ threshold, the charged-B-to-neutral-B production ratio would be significantly different from one in this energy region (as shown in Fig. 5).

Because of the uncertainty in the mass difference $m_b - m_c$ the actual position of the c.o.g. of the B-meson system [Eq. (3)] is uncertain. It is therefore important to investigate the dependence of the structure of the 4S resonance on the position of Zweig-allowed $B\bar{B}$ threshold. There are some indications from hadronic production experiments that the B meson may have been observed with a mass of 5.30± 0.02 GeV.[10] This suggests that the threshold might be closer to the 4S resonance than 100 MeV.

Denoting $\Delta M \equiv M(4\,^3S_1) - 2M(B^-)$ the threshold behavior ΔR_b has been investigated as a function of ΔM. Representative curves are shown in Figs. 6–8 for

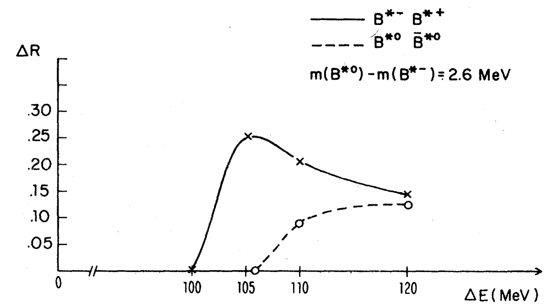

FIG. 5. The effect of a small mass difference between the charged and neutral B^* on the behavior of exclusive channels.

$\Delta M = 20$, 50, and 70 MeV, respectively. The general pattern of the structure of ΔR in the $4\,^3S_1$ region is as follows:

(1) The resonance becomes narrow quite rapidly as ΔM goes to zero. For $\Delta M \lesssim 40$ MeV the FWHM is less than experimental resolution of the Cornell Electron Storage Ring (CESR)[11] at this energy (~10 MeV) and ΔR(peak)$\gtrsim 4$, (see Fig. 6).

(2) $40 \lesssim \Delta M \lesssim 60$ MeV. In this range the 4S resonance is near the opening of the $B\bar{B}^* + B^*\bar{B}$

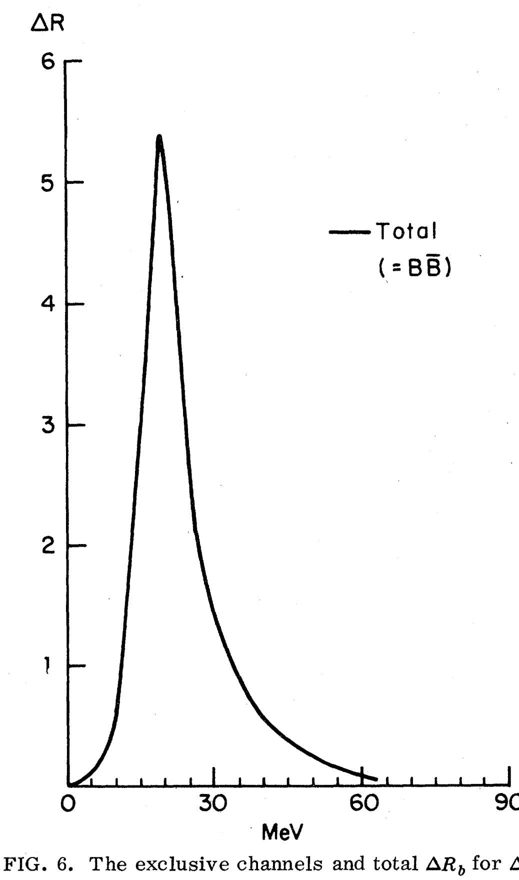

FIG. 6. The exclusive channels and total ΔR_b for ΔM = 20 MeV.

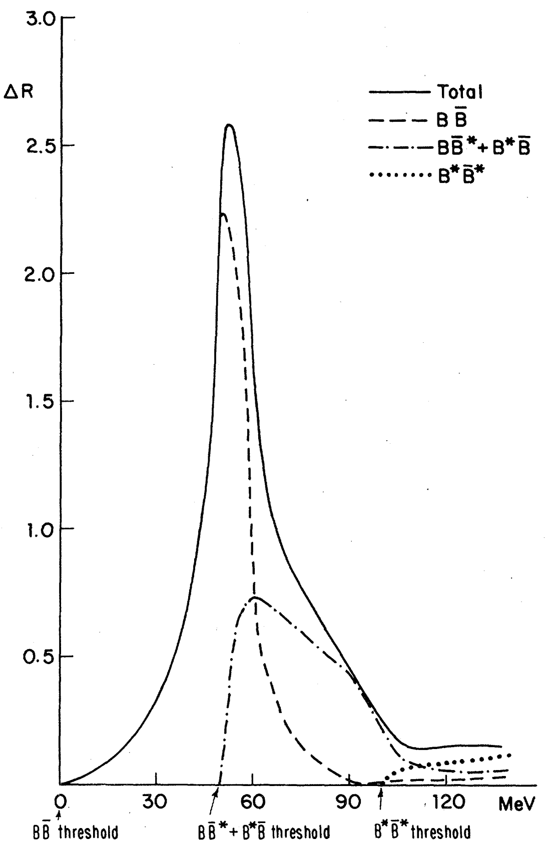

FIG. 7. The exclusive channels and total ΔR_b for $\Delta M = 50$ MeV.

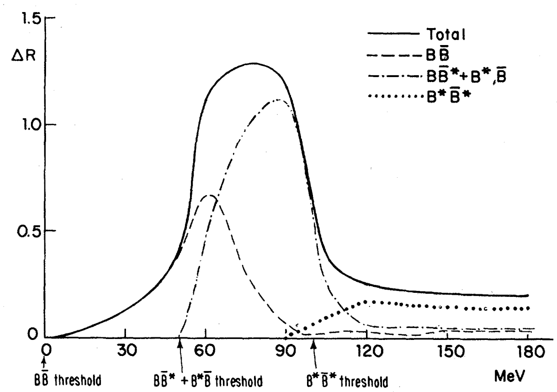

FIG. 8. The exclusive channels and total ΔR_b for $\Delta M = 70$ MeV.

exclusive channel and these final states play a significant role in the structure of ΔR_b. The opening of the new exclusive channel broadens the structure above the peak in ΔR_b shape is characteristic of this range of ΔM. The values of ΔR (peak) and FWHM vary smoothly from a ΔR (peak) ~4 and FWHM ~ 10 MeV at $\Delta M = 40$ MeV to ΔR (peak) ~ 1.5 and FWHM ~ 25 MeV at $\Delta M = 60$ Mev. The other characteristic of this range is the appearance of a strong signal in 50 MeV photons from the decays $B^* \to B + \gamma$ at energies just above $\bar{B}B^* + \bar{B}^*B$ threshold.

(3) 60 MeV $\lesssim \Delta M \lesssim$ 90 MeV. In this range the 4S resonance lies roughly midway between the $B\bar{B}^* + B^*\bar{B}$ threshold. This produces a much broader structure ~30–40 MeV (FWHM) without a pronounced peak and a $\Delta R_b \sim 1.2$ (see Fig. 8).

(4) 90 $\lesssim \Delta M \lesssim$ 120 MeV. Similar to the $40 \lesssim \Delta M \lesssim 60$ MeV range as the 4S resonances is near the opening of the $B^*\bar{B}^*$ exclusive channel here. Two differences, however, are (a) the resonance is somewhat broader than the corresponding mass in range (2) [i.e., $\Delta M \sim 50$ MeV] and has a smaller ΔR_b at the peak (see Fig. 4 and compare with Fig. 7), and (b) There should be a signal in 50 MeV photons that roughly doubles as $B^*\bar{B}^*$ threshold is crossed.

(5) $\Delta M \gtrsim 120$ MeV. Since the $3S(b\bar{b})$ state is observed as a narrow state it must be below or very near ($\lesssim$ 40 MeV) threshold. Thus $\Delta M \lesssim 250$ MeV and the 4S state is below the next two-body threshold ($E^0\bar{E}^0$). The structure of ΔR_b has a complicated dependence on ΔM in this region. The FWHM is $\gtrsim$ 30 MeV in the whole region and $\Delta R \lesssim 2$.

There is preliminary evidence from the CLEO and Columbia University–Stony Brook (CUSB) groups[11], at CESR, of structure in ΔR in the region of the expected 4S state. The detailed structure is not yet measured but the ΔR (peak) ~ 1–2 and FWHM ~ 15–20 MeV. Within the context of the coupled-channel calculations presented here this suggests that the ΔM is near 50 or 100 MeV. These two cases can be distinguished by the variation of the signal of 50 MeV photon over the energy range of the resonance.

Finally, some comments on higher S state resonances and the possibility of D-wave states being directly produced in e^+e^- annihilation. The energy region above threshold over which distinct resonances would be expected to be observed should be comparable to this region for charm threshold, i.e., ~700 MeV. Thus both the 5S and 6S resonances should be observable. A significant fraction of the decays of the 5S state may be into E mesons while the 6S will predominantly decay into final states containing one P-wave state "excited B" meson and one S-wave state meson (B or B^*). The $n\,^3D_1(b\bar{b})$ states are unlikely to be produced directly in e^+e^- since the tensor forces mixings in the $(b\bar{b})$ system are expected to be weaker[5] than in the $(c\bar{c})$ and also the mixing induced by the virtual coupling to decay channels will decrease as it must vanish as $[m(B^*) - m(B)]$ approaches zero. These considerations lead to the conclusion that no $n\ ^3D_1(b\bar{b})$ state will appear in ΔR_b with a $\Delta R_b(\text{peak}) > \frac{1}{4}$.

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[8]The masses of the F mesons are taken from G. Flügge, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978,* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), pp. 793–810.

[9]The results presented here for ΔR_b in the region of the $4S(b\bar{b})$ state are not given simply from the naive decay widths $4\,^3S_1 \rightarrow (b\bar{u} + \bar{b}u)$ shown in Fig. 3 because of the complicated energy dependence of these amplitudes. In particular, they disagree with the results of S. Ono, Phys. Rev. D (to be published).

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