PHYSICAL REVIEW D VOLUME 22, NUMBER 7 1 OCTOBER 1980

Quark-vacuum scattering

John F. Donoghue
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 15 February 1980)

It has been suggested that there are two phases of the quantum-chromodynamic vacuum: the perturbative vacuum and the complex true vacuum. In this framework, I demonstrate a new mechanism for particle production in deep-inelastic reactions: excitation of pions by the phase change of the vacuum induced by separating quark color charges. This process, quark-vacuum scattering, is discussed in general, and a simple model is examined in detail.

I. INTRODUCTION

The most important state in any field theory is the ground state—the vacuum. There are growing indications that the vacuum of quantum chromodynamics (QCD) is nontrivial and that by understanding the vacuum we may explain properties such as confinement, chiral-symmetry breaking, and the particle spectrum. A particularly promising possibility is that there are two phases of vacuum in QCD. It is already clear that the perturbative vacuum, which we know as the ground state of QED, is not the lowest-energy state in QCD.[1] This result is also suggested by the success of the idea of spontaneously broken chiral symmetry, which requires a nontrivial vacuum.[2] The true vacuum is then some configuration, presumably quite complex, with lower energy than the perturbative vacuum. However, it has been suggested that in the presence of strong gluon or quark fields the true vacuum undergoes a phase transition to the perturbative vacuum. The phenomena has been demonstrated within the context of the QCD instanton gas.[3] It is the basis of the MIT bag model description of hadrons.[4,5] The success of the bag model in describing the particle spectrum and hadronic properties is then a positive indicator for the two-vacuum picture.

The vacuum of a theory is that state with zero particle number. For any asymptotic particle α with any momentum $\vec{p}$, the vacuum state satisfies

$$a_\alpha(\vec{p})\,|0\rangle = 0, \tag{1}$$

where $a_\alpha(\vec{p})$ is the destruction operator for that particle. This is a strong constraint. It is known that changes in the vacuum structure generally lead to the violation of Eq. (1), i.e., the state now contains free particles.[4] For example, one can rigorously define the QED vacuum state between two perfectly reflecting mirrors at rest. If those mirrors are then moved, changing the boundary condition on the electromagnetic fields, photons are produced from the vacuum.[6] This property of vacuum emission of particles has also been used in general relativity to study particle production in expanding universes[7] and near black holes.[8] It is the object of this paper to study the particle emissions of a two-phase vacuum, such as that proposed for QCD. The physical picture which I have in mind can be illustrated by considering $e^+e^- \to q\bar{q}$. Before the quark pair was produced the hadronic state was that of the true vacuum. The quark pair is produced from the virtual photon and separate, with a gluon flux (string) connecting the two color charges. In the presence of the quark and gluon fields, the vacuum changes phase in a limited region of space. This phase change leads to the emission of pairs of pions. The quarks produced in the above reaction also "fragment" through more conventional processes[9,10] into hadrons, but some of the observed particles will be due to the vacuum emission stimulated by the quark field, a process which can be called "quark-vacuum scattering."

The idea of quark-vacuum scattering is close in spirit to the flux-tube model of Casher, Neuberger, and Nussinov.[10] These authors note that the gluon flux decays into a quark pair, shielding the original quarks. They build an attractive model for a particle cascade through the repeated quark separation, flux generation, and pair production. The work of the present paper involves a complementary but distinct physical effect of the gluon flux. We will return to discuss this point later in the paper.

The outline of the paper is as follows. In Sec. II, I construct a model for quark-vacuum scattering, and sum a particular set of contributions to pion production. In Sec. III, a specific example is considered in order to obtain an indication of the possible importance of this process. A discussion of the program and of the relation to other models is given in Sec. IV.

II. FORMALISM

A. The model

The phenomena of two vacua can be simply illustrated by considering the σ model,[11] which consists of massless nucleons, pions, and σ field with the mesons interacting through a potential

$$V(\sigma,\vec{\pi})=\lambda(\sigma^2+\vec{\pi}\cdot\vec{\pi}-F_\pi^2)\,. \tag{2}$$

The perturbative vacuum is the one where the fields are quantized around $\sigma=0$, $\vec{\pi}=0$. However, this is not the lowest-energy state. The minimum of V occurs when $\langle\sigma^2+\vec{\pi}\cdot\vec{\pi}\rangle=F_\pi^2$, which can be chosen to correspond to $\langle\sigma\rangle=-F_\pi$. Writing $\sigma=\langle\sigma\rangle+\tilde{\sigma}$, and quantizing around $\tilde{\sigma}=0$, $\vec{\pi}=0$, one finds a massless pion, massive σ, and massive nucleon, with the Goldberger-Treiman relation $m_N=g_{\pi NN}F_\pi/g_A$. The ground state has an energy per unit volume λF_π^4 lower than the perturbative vacuum.

The properties of chiral-symmetry breaking and of an energy difference between vacua are the most important aspects of the suggested QCD structure. As the σ model provides a reasonable description of the properties of the pion, I will use this model to describe the pionic behavior in the two phases. For this application, the pions are treated as elementary particles. The σ field represents the vacuum properties with $\sigma=-F_\pi+\tilde{\sigma}$ and

$$\begin{aligned}\tilde{\sigma}(x)&=0\,,\quad \text{true vacuum}\,,\\ \tilde{\sigma}(x)&=F_\pi,\quad \text{perturbative vacuum}\,,\end{aligned} \tag{3}$$

or simply

$$\tilde{\sigma}(x)=F_\pi\theta(V)\,,$$

where V is the space of perturbative vacuum. The quark or gluon fields which drive the phase change are treated as external variables whose effect is manifest only through the value of $\tilde{\sigma}(x)$.

The coupling constants in the theory are known from phenomenological considerations. Use of the Goldberger-Treiman relation yields $F_\pi=93$ MeV. The energy per unit volume difference is well known from the bag model to be given by the bag constant $B\approx(130\text{ MeV})^4$. The effect of the phase change on the potential is then

$$V(\sigma=0,\vec{\pi})-V(\sigma=-F_\pi,\vec{\pi})=B-\frac{2B}{F_\pi^2}\vec{\pi}\cdot\vec{\pi}\,, \tag{4}$$

and the effective pion coupling given by the phase change is (for convenience constants are absorbed into σ^2)

$$\mathcal{L}_{\text{int}}(x)=\sigma^2(x)\vec{\pi}\cdot\vec{\pi}\,, \tag{5}$$

where

$$\sigma^2(x)=\frac{2B}{F_\pi^2}\theta(V)\,. \tag{6}$$

This, then, is our model for the vacuum interaction.

B. First-order scattering

We wish to calculate the particle production due to changes in the phase of the vacuum. An exact solution has not been found but it is possible to consider the process in a perturbation theory using the interaction of Eq. (5). To do so, consider the time evolution operator from $t=-\infty$ to $t=+\infty$,

$$U=T\exp\left[-i\int d^4x\,\mathcal{H}_{\text{int}}(x)\right]\,. \tag{7}$$

If we start with a pure true vacuum in the remote past, the final state obtained is

$$|\text{out}\rangle=U\,|0\rangle\,. \tag{8}$$

The number of particles produced can be measured by considering the number operator

$$N=\sum_i\int d^3k\,a_i^\dagger(k)a_i(k)\,, \tag{9}$$

with the result that

$$\begin{aligned}N_{\text{out}}&\equiv\langle\text{out}|N|\text{out}\rangle\\ &=\sum_i\int d^3k\langle 0|U^\dagger a_i^\dagger(k)a_i(k)U|0\rangle\,.\end{aligned} \tag{10}$$

For no interaction, $U=1$, this of course yields $N_{\text{out}}=0$. However, for a general interaction N_{out} will be nonzero.

Let us first consider N_{out} to lowest order in the interaction, corresponding to the production of a single pair, as schematically illustrated in Fig. 1. The figure is meant to represent an overlap of the out state with itself, with the hatched re-

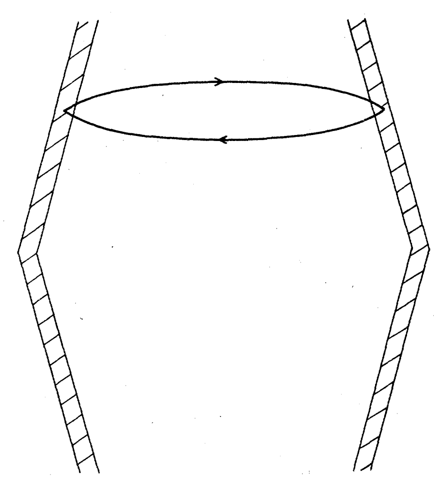

FIG. 1. The diagram for first-order scattering, representing the overlap with itself of an out state containing one pair. The hatched regions are the areas of perturbative vacuum, and the solid lines are pions.

gions denoting regions of false vacuum which can emit pions (the solid lines). N_{out} counts the number of pions in the central region. To calculate N_{out} it is useful to consider the expansion of the pion field

$$\pi^i(x)=\int \frac{d^3q}{(2\pi)^{3/2}(2q_0)^{1/2}}[a_i(q)\,e^{-iq\cdot x}+a_i^\dagger(q)\,e^{+iq\cdot x}]. \tag{11}$$

To first order in the interaction N_{out} becomes

$$N_{\text{out}}=\int d^4x\,d^4y\,\sigma^2(x)\sigma^2(y)\int\frac{d^3k}{(2\pi)^6}\int\frac{d^3q\,d^3q'\,d^3p\,d^3p'}{(16q_0q_0'p_0p_0')^{1/2}}\,e^{i(q+q')\cdot x}\,e^{-i(p+p')\cdot y}$$
$$\times\langle 0\,|a_j(p)a_j(p')a_i^\dagger(k)a_i(k)a_l^\dagger(q)a_l^\dagger(q')\,|0\rangle\,, \tag{12}$$

or

$$N_{\text{out}}=12\int\frac{d^3k}{(2\pi)^3 2k_0}\frac{d^3k'}{(2\pi)^3 2k_0'}\left|\int d^4x\,\sigma^2(x)\,e^{i(k+k')\cdot x}\right|^2. \tag{13}$$

Note that this is a relativistically invariant expression; we have not reintroduced the ether. The static limit also makes sense. If $\sigma^2(x)$ is independent of time, the time integral yields $\delta(k_0+k_0')$ which produces $N_{\text{out}}=0$. This is important since the usual hadrons are thought to be static regions of false vacuum. Likewise a bubble of false vacuum moving with a constant velocity yields $N_{\text{out}}=0$. A nonzero N_{out} is produced by time dependent changes in the size or shape of $\sigma^2(x)$.

C. Higher orders

Higher-order corrections to this basic amplitude are illustrated in Fig. 2. The diagrams become arbitrarily complicated at high enough order. However, a class of these diagrams may be calculated. These are the "bubble" diagrams of Fig. 2(a)–2(c), where the pions always act in pairs. These may be summed to all orders, as is demonstrated below.

Consider first the graphs with only real pion production, as in Fig. 2(a). For a graph with n pairs there is a factor $(1/n!)^2$ from the expansion of $U^\dagger$ to the nth order. There are $n!$ ways to combine the pairs. The number operator counts $2n$ particles, and each pair has a weight $6K$, where

$$K\equiv\int\frac{d^3k}{(2\pi)^3 2k_0}\frac{d^3k'}{(2\pi)^3 2k_0'}\left|\int d^4x\,\sigma^2(x)e^{i(k+k')\cdot x}\right|^2. \tag{14}$$

The nth-order contribution is

$$N_n=\frac{2n}{(n!)^2}n!\,(6K)^n=12K\frac{(6K)^{n-1}}{(n-1)!}\,. \tag{15}$$

When summed, these higher-order corrections would exponentiate.

This, however, is modified by the occurrence of virtual pion emission and absorption as in Figs. 2(b) and 2(c). The virtual graphs involve the combination

$$L\equiv(-i)^2\int d^4x\,d^4y\langle 0\,|T(\mathcal{H}_{\text{int}}(x)\mathcal{H}_{\text{int}}(y))\,|0\rangle$$
$$=-6\int d^4x\,\sigma^2(x)\int d^4y\,\sigma^2(y)[i\Delta_F(x-y)]^2$$
$$=-\frac{-6}{(2\pi)^8}\int\frac{d^4k}{k^2-m^2+i\epsilon}\frac{d^4k'}{k^2-m^2+i\epsilon}$$
$$\times\left|\int d^4x\,\sigma^2(x)\,e^{i(k+k')\cdot x}\right|^2. \tag{16}$$

The energy integrals may be explicitly done, with the result that

$$L=-6K\,. \tag{17}$$

The weight of the virtual graphs is the same as that of the graphs with real pions, and therefore they must also be included. The general bubble graph of Fig. 2(c) contains m (m') vertices on the right-hand (left-hand) side, and involves n real pairs. The expansion of $U^\dagger$ and U yields a factor of $(1/m!)(1/m'!)$. There are

$$\frac{1}{n!}\frac{m!}{(m-n)!}\frac{m'!}{(m'-n)!}$$

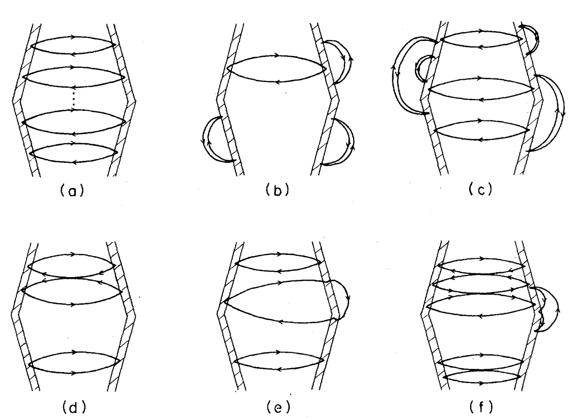

FIG. 2. Several diagrams for higher-order pion production: (a), (b), and (c) are representative of the bubble diagrams which are summed in the text, while (d), (e), and (f) are more complicated processes.

ways to combine n real pairs, and

$$(m-n-1)!!(m'-n-1)!!$$

combinations of the virtual pairs. Each virtual pair contributes a factor $(-6K)$ while a real pair yields $(+6K)$, The number operator counts $2n$, leading to the result

$$\begin{aligned}N_{n,m,m'} &= \frac{2n}{n!}\frac{(m-n-1)!!}{(m-n)!}\frac{(m'-n-1)!!}{(m'-n)!}\\ &\quad\times(+6K)^n(-6K)^{(m-n)/2}(-6K)^{(m'-n)/2}\\ &= 12K\frac{(6K)^{n-1}}{(n-1)!}\frac{1}{l!}\left(\frac{-6K}{2}\right)^{l}\frac{1}{l'!}\left(\frac{-6K}{2}\right)^{l'},\end{aligned} \tag{18}$$

where $2l=m-n$, $2l'=m'-n$. The result for n real pairs and any number of virtual pairs is then modified from Eq. (15) to

$$N_n=12K\frac{(6K)^{n-1}}{(n-1)!}e^{-6K}, \tag{19}$$

and the total multiplicity is given simply by

$$\begin{aligned}N_{\text{out}} &= \sum_{n=1}^{\infty}N_n\\ &= 12K\,e^{6K}\,e^{-6K}\\ &= 12K.\end{aligned} \tag{20}$$

Remarkably, the higher-order corrections to the lowest-order calculation cancel between real and virtual graphs, leaving the lowest-order result unchanged. This is similar to what happens in the photon multiplicity in QED.[12]

Equation (20) was derived by considering bubble graphs. More complicated graphs are possible. However, the individual graphs are small compared to Eq. (20) for both (a) small perturbation on the vacuum and (b) the example in the next section. In the latter case the contribution of nonbubble graphs does not grow linearly with the length of the tube.

III. A SPECIFIC EXAMPLE: THE LONG TUBE

In the previous section a general formula for particle production, Eq. (20), was derived. Here I will apply the technique to a simple example designed to have some resemblance to quark jets in e^+e^- annihilation. The initial space-time description of two quark jets in this model involves the quarks moving back to back in the center of mass. Each quark has a color charge and by Gauss's law there must be a gluonic electric flux from one charge to another. Whenever the gluon electric field is strong the vacuum changes phase. For separations large compared to $B^{-1/4}$ the flux will be contained in a tube running from one quark to the other. This physical picture is similar to the stringlike states of high angular momentum studied in the bag model by Johnson and Thorn.[13]

The above description is only valid until the two charges are screened by the production of a quark-antiquark pair. However, the subsequent development of the system will still involve further separation of color charges and the growth of regions of false vacuum between them. This process will keep repeating until the final hadrons are developed. The exact space-time behavior of the cascade is not certain, but it is clear that a sizable region of perturbative vacuum must be produced, as each of the hadrons generated is itself a bubble of the false vacuum. A simple model mimicking this situation is then a long tube of radius ρ, expanding (with $\beta=1$) in the $\pm z$ directions from $t=0$ to $t=\tau$, where ρ and τ will be estimated later. Essentially, the quark jet is drilling a hole in the vacuum. This situation corresponds to

$$\sigma^2(x)=\frac{2B}{F_\pi^2}\theta(t-|z|)\theta(t)\theta(\tau-t)\theta(\rho^2-x^2-y^2). \tag{21}$$

The reader who is not interested in the details of the analysis may skip to the end of the section where the parameters ρ and τ are estimated and the analysis is reviewed.

The source function $S(q)$,

$$S(q)\equiv\int d^4x\,\sigma^2(x)\,e^{iq\cdot x} \tag{22}$$

is related to the scattering number K by

$$K=\int\frac{d^3k}{(2\pi)^3 2k_0}\frac{d^3k'}{(2\pi)^3 2k_0'}|S(k+k')|^2. \tag{23}$$

For the long tube

$$S(q)=\left(\frac{2B}{F_\pi^2}\right)2\pi\rho^2\frac{J_1(q_\perp\rho)}{q_\perp\rho}\frac{\tau}{2q}e^{iq\tau(1-\cos\theta)}J(q\tau,\cos\theta), \tag{24}$$

where

$$J(a,x)\equiv\frac{1}{x}[j_0(a(1-x)/2)-e^{iax}j_0(a(1+x)/2)], \tag{25}$$

and with

$$\begin{aligned}&q\equiv q_0,\\ &q\cos\theta\equiv\vec{q}\cdot\hat{z},\\ &q_\perp\equiv[(\vec{q}\cdot\hat{x})^2+(\vec{q}\cdot\hat{y})^2]^{1/2}.\end{aligned} \tag{26}$$

Here θ is not a physical angle in the problem, but is just a parameter. Since we do not want $q^2=0$, $q_\perp\neq q\sin\theta$. For $a=q\tau$ large, $J(a,x)$ is $O(1/a)$ everywhere except near the points $x=-1,0,1$, where it is sharply peaked and $O(1)$. The peaks at $x=\pm 1$ correspond to shock waves with both pions moving directly forward or backward. However, these cases do not contribute to K because there is no phase space for this in subsequent integrations over k and k'. Near $x=0$ (pions produced with equal and opposite z component of momentum)

$$J(a,x)=-2\,e^{ia(1+x)}\frac{\sin ax}{ax}+O(1/a)\,. \tag{27}$$

This can be simplified by use of the representation of the Dirac δ function

$$\lim_{a\to\infty}\frac{\sin ax}{\pi x}=\delta(x) \tag{28}$$

to

$$J(a,x)\underset{a\to\infty}{\approx}-\frac{2\pi}{a}\,e^{i(a/2)(1+x)}\delta(x)\,. \tag{29}$$

When squared

$$\delta^2(x)=\delta(x)\frac{\sin ax}{\pi x}=\frac{a}{\pi}\delta(x)\,, \tag{30}$$

so that

$$|J(a,x)|^2=\frac{4\pi}{a}\delta(x)\,. \tag{31}$$

This isolates the piece of the squared source function which grows linearly with τ,

$$|S(q)|^2=\left(\frac{2B}{F_\pi^{\,2}}\right)^2 4\pi^3\rho^2\frac{J_1^{\,2}(q_\perp\rho)}{q_\perp^{\,2}}\,\frac{\delta(\cos\theta)}{q^3}\,\tau\,. \tag{32}$$

Let us choose $k_\perp$ to be in the x direction and define the differential distribution

$$\frac{dK}{dk\,d\cos\theta_k}=\frac{2\pi k^2}{(2\pi)^3 2k_0}\int\frac{d^3k'}{(2\pi)^3 2k_0'}\,|S(k+k')|^2\,. \tag{33}$$

For simplicity I will treat the pions as massless. Substituting $k'^2=k^2(\cos^2\theta_k+z^2\sin^2\theta_k)$, one obtains

$$\frac{dK}{dk\,d\cos\theta_k}=\frac{1}{32\pi^2}\left(\frac{2B}{F_\pi^{\,2}}\right)^2\frac{\rho^2\tau}{k^2}M(k\rho,\theta_k)\,, \tag{34}$$

with

$$M(x,\theta)\equiv\int_0^\infty\frac{z\,dz}{(\cos^2\theta+z^2\sin^2\theta)^{1/2}}\,\frac{1}{[1+(\cos^2\theta+z^2\sin^2\theta)^{1/2}]^2}\int_0^{2\pi}d\phi\,\frac{J_1^2[x\sin\theta(1+z^2+2z\cos\phi)^{1/2}]}{(1+z^2+2z\cos\phi)}\,. \tag{35}$$

For small x,

$$M(x,\theta)\sim\frac{\pi x^2}{2(1+|\cos\theta|)} \tag{36}$$

and the distribution is constant in k,

$$\frac{dK}{dk\,d\cos\theta_k}\underset{k\to 0}{\longrightarrow}\frac{1}{64\pi}\left(\frac{2B}{F_\pi^{\,2}}\right)^2\frac{\rho^4\tau}{(1+|\cos\theta_k|)}\,. \tag{37}$$

When x is large, the integrand in $M(x,\theta)$ peaks very sharply near $z=1$ and $\cos\phi=-1$, where the Bessel function J_1 is $O(x)$ [elsewhere it is $O(1/\sqrt{x})$]. The angular integral can be evaluated for x large,

$$M(x,\theta)=\int_0^\infty\frac{z\,dz}{(\cos^2\theta_1+z^2\sin^2\theta)^{1/2}}\,\frac{1}{[1+(\cos^2\theta+z^2\sin^2\theta)^{1/2}]^2}\,\frac{1}{\sqrt{z}}\,\frac{H_1(2x\sin\theta(1-z))}{x\sin\theta(1-z)^2} \tag{38}$$

plus terms which vanish for large x. Above, $H_1(z)$ is the Struve function of order 1. One can show that

$$\lim_{a\to\infty}\frac{H_1(2a(1-z))}{a(1-z)^2}=\pi\delta(1-z)\,. \tag{39}$$

This leads to an asymptotic value

$$M(x,\theta)\underset{k\to\infty}{\longrightarrow}\frac{\pi}{4}\,. \tag{40}$$

The large-k limit is then

$$\frac{dK}{dk\,d\cos\theta_k}\underset{k\to\infty}{\longrightarrow}\frac{1}{128\pi}\left(\frac{2B}{F_\pi^{\,2}}\right)^2\frac{\rho^2\tau}{k^2}\,. \tag{41}$$

The high-k limit is isotropic, while at low k the pions are emitted preferentially perpendicular to the jet axis.

Interpolating these limits and then integrating over θ_k yields

$$\frac{dK}{dk}=\frac{1}{32\pi}\left(\frac{2B}{F_\pi^{\,2}}\right)^2\rho^4\tau\ln\left(1+\frac{1}{1+2(k\rho)^2}\right)\,, \tag{42}$$

and

$$K=\frac{1-1/\sqrt{2}}{32}\left(\frac{2B}{F_\pi^{\,2}}\right)^2\rho^3\tau\,. \tag{43}$$

In the large-momentum limit, the treatment of the pion as a point particle is probably not correct. Pionic form factors would presumably suppress the production of a pair with large relative momentum. For example, a dipole form factor

$$F(k,k')=\frac{1}{[1+(k+k')^2/m_\rho{}^2]^2} \quad (44)$$

would lead to a high-momentum limit falling like k^6. However, since most of the particles are produced with energies less than m_ρ, the form factor leads to only a small suppression of the total number.

The parameter ρ (the radius of the tube) and τ (the half length) must now be estimated. The radius is simply given by Gauss's Law. The electric field of color a, assumed to be roughly constant across a tube of cross-sectional area A, is

$$E^a A = g T^a\,, \quad (45)$$

where T^a is the color representation matrix for the source (i.e., for quarks $T^a=\lambda^a/2$). The condition for the vacuum to be in the perturbative phase is

$$\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}=\tfrac{1}{2}\sum_a \vec{E}^a\cdot\vec{E}^a \geq B\,. \quad (46)$$

Assuming a circular tube, the radius is then required to be

$$\rho^4=\frac{2\alpha_s}{\pi}\,\frac{1}{B}C\,, \quad (47)$$

where C is the Casimir operator for the sources

$$\begin{aligned} C &= \sum_a (T^a)^2 \\ &= \tfrac{4}{3}\ \text{(quarks)} \\ &= 3\ \text{(gluons)}. \end{aligned} \quad (48)$$

To obtain the effective half length τ, let us estimate the total volume of the perturbative vacuum produced in the jet, and then equate it to the volume of the tube $V_T=\pi\rho^2(2\tau)$. Since each of the hadrons produced in the standard fragmentation process is itself a bubble of volume V_0 of the higher-energy vacuum, the minimum volume produced is

$$V_T = n_F V_0\,, \quad (49)$$

where n_F is the number of fragments. While this is probably a considerable underestimate, I will use it as a rough conservative figure. Note, however, that the proportionality between the volume and the fragment multiplicity is probably more general than the above estimate. One would expect a given amount of string length to lead to a fixed number of fragments.

The above estimates lead to

$$K=\frac{(1-1/\sqrt{2})}{64\pi}\left(\frac{2B}{F_\pi{}^2}\right)^2\rho V_0 n_F\,. \quad (50)$$

Using V_0 given by the pion's volume in the bag model[4] ($R_\pi=0.7$ fm),

$$K=0.014 n_F \quad (51)$$

and the number of particles produced is

$$N_{\text{out}}=0.16 n_F\,. \quad (52)$$

In e^+e^- reactions, with beam energy E (in GeV)[14] I will use the observed multiplicity as a first estimate of the fragment multiplicity

$$n_F \approx \tfrac{3}{2}\langle n_{\text{ch}}\rangle = 3 + 2\ln E\,, \quad (53)$$

so that

$$N_{\text{tot}} \approx (0.5+0.32\ln E)\,. \quad (54)$$

If the fragment multiplicity contained a term proportional to $\ln^2 E$, as has been suggested,[14] N_{out} would also reflect this. Including the form factor, Eq. (44), would lead to

$$N_{\text{tot}} = (0.36+0.23\ln E)\,. \quad (55)$$

For gluons a similar form would be expected to hold; however, the fragment multiplicity of a gluon jet is most likely different from that of a quark jet.

The long-tube model thus leads to the emission of pions which are dominantly back to back, and exclusively low energy. The number of such particles is estimated to be about $\frac{1}{6}$ of the average multiplicity at moderate energies. It is difficult to fully assess the uncertainties in this estimate. Clearly, numerical uncertainties in the coupling $\sigma^2(x)$ can effect the final result. However, in my opinion, the most serious unknown is the space-time development of the hadron jet. The long-tube example was made as an exploratory calculation, and the above estimate makes clear that quark-vacuum scattering is capable of producing a sizable number of particles. Beyond this, however, the long-tube model has some defects. The screening of the flux tube by quark pairs divides the tube into several segments, which propagate and further divide. To provide distributions of particles which are to be compared with experiment, one must be more specific about the fragmentation process, and construct a full theory of the jet development. This will be attempted in the future.

IV. DISCUSSION

The preceding sections have outlined a novel mechanism for particle production in processes with separating quark or gluon charges, valid if the idea of a two-phase vacuum is correct. The phase change in the presence of quark or gluon fields excites the quanta of the true vacuum. An analysis of a simple model has been given in Sec.

III. I argued that the multiplicity from this process should be related to the multiplicity of more standard fragmentation processes (n_F) by

$$N_{\text{out}} = \beta n_F ,$$

where β in our example was 0.16. The precise value of β has very many uncertainties, but the estimate suggests that quark-vacuum scattering may be a sizable contribution to observed processes. In a measurement of the quark's fragmentation function this mechanism would populate the region near $z=0$. It is interesting to contrast various dynamical mechanisms for particle production in quark jets. At high transverse momentum we have the perturbative bremsstrahlung of gluons off of a quark. Brodsky and Gunion have suggested this mechanism also for low transverse momentum.[15] A contrasting model is that of Casher *et al.*[10] involving the breakup of a flux tube. The present work is related to the latter picture; however, it is distinct. This can be seen from the lack of leading-particle effect in quark-vacuum scattering. Pions are always produced in pairs and the flavor of the quark which triggered the transition is irrelevant. In the flux-tube model, each pair-creation event leads to one extra meson which contains the quark that generated the original flux. If the ideas of the QCD vacuum assumed by this paper are in fact correct, it is clear that each of these three mechanisms (gluon bremsstrahlung, flux breakup, and vacuum transitions) do play a role in producing particles. It would be attractive, though nontrivial, to combine them and attempt a complete description of jet phenomena.

The production of particles by changes in the vacuum structure is a necessary consequence of the two-phase model. Despite the uncertainties in the magnitude of the signal, it appears that this process can produce a sizable component in the hadronic development of quark jets.

ACKNOWLEDGMENTS

I would like to thank D. Toussaint and J. Gunion for crucial suggestions. This work is supported in part through funds provided by the U. S. Department of Energy under Contract No. EY-76-C-02-3069.

---

[1] G. K. Savvidy, Phys. Lett. 71B, 133 (1977); S. G. Malinyan and G. K. Savvidy, Nucl. Phys. B134, 539 (1978); H. Pagels and E. Tomboulis, *ibid.* B143, 485 (1978); N. K. Nielsen and P. Olesen, *ibid.* B144, 376 (1978); Ambjorn, N. K. Nielsen, and P. Olesen, *ibid.* B152, 75 (1977); T. Saito and K. Shigemoto, Prog. Theor. Phys. 63, 256 (1980).

[2] J. Goldstone, Nuovo Cimento 19, 154 (1961); Y. Nambu and Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[3] C. Callan, R. Dashen, and D. Gross, Phys. Rev. D 19, 1826 (1979); C. Aragao de Carvalho, *ibid.* 21, 1100 (1980).

[4] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974); T. De-Grand, R. L. Jaffe, K. Johnson, and J. Kiskis, *ibid.* 12, 2060 (1975); J. F. Donoghue, E. Golowich, and B. R. Holstein, *ibid.* 12, 2875 (1975); J. F. Donoghue and K. Johnson, *ibid.* 21, 1975 (1980).

[5] B. S. DeWitt, Phys. Rep. 196, 295 (1975).

[6] G. T. Moore, J. Math. Phys. 11, 2679 (1970); S. A. Fulling and P. C. W. Davies, Proc. R. Soc. London A348, 199 (1975).

[7] L. Parker, Phys. Rev. 183, 1057 (1969); Phys. Rev. D 3, 346 (1971).

[8] S. W. Hawking, Nature (London) 248, 30 (1974); Commun. Math. Phys. 43, 199 (1975).

[9] A standard reference which contains further sources is R. D. Field and R. P. Feynman, Nucl. Phys. B136, 1 (1978).

[10] A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D 20, 179 (1979). See also A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Lett. 31, 793 (1973); A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D 21, 1966 (1980); H. Neuberger, *ibid.* 20, 2936 (1979).

[11] M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960). For reviews, see S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969); H. Pagels, Phys. Rep. 16C, 219 (1975).

[12] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) 13, 379 (1961).

[13] K. Johnson and C. Thorn, Phys. Rev. D 13, 1934 (1977).

[14] R. F. Schwitters, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 5.

[15] S. J. Brodsky and J. F. Gunion, Phys. Rev. Lett. 37, 402 (1976).