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Gluon final states in Higgs-boson decay

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We examine the decays of $H \to 2$ gluons, $H \to q\bar{q}g$, and $H \to f\bar{f}$ ($f =$ some fermion) over an extended range of Higgs-boson and t-quark masses. In particular, we show that the branching ratio for $H \to 2$ gluons may be sizable ($\sim 10\%$) without requiring the existence of superheavy quarks. We calculate the differential spherocity and thrust distributions as well as the values of $\langle 1 - T\rangle$ and $\langle S\rangle$ for the $H \to q\bar{q}g$ decay.

I. INTRODUCTION

Since the recent model-independent determination of the neutrino-quark coupling constants based on the work of Sehgal,[1] Ecker,[2] Hung and Sukarai,[3] Abbot and Barnett,[4] and Langacker and Sidhu,[5] as well as several other authors[6] and the recent results of the SLAC-Yale experiment[7] on parity violation in electron deuteron scattering, the $SU(2)\times U(1)$ model[8] has become *the* standard model of weak and electromagnetic interactions. Even though the "correct" gauge group for this unification may be larger than $SU(2)\times U(1)$, this larger group must contain $SU(2)\times U(1)$ as a subgroup describing the interactions at present energies.

In order to show that $SU(2)\times U(1)$ is the correct group one must also find the W and Z gauge bosons with the properties expected from the model. Currently, plans are underway at CERN and at Brookhaven[9,10] to produce these particles in $\bar{p}p$ and pp collisions, respectively; plans are also being made to produce these particles in e^+e^- using LEP.[11] Since the model gives explicit predictions of their properties it can be tested in a straightforward manner.

Another important element of the standard model is the existence of a neutral scalar particle which couples to fermions and the gauge bosons and is the "debris" of the spontaneous-symmetry-breaking mechanism. Although its couplings are well known, the mass of the Higgs boson remains undetermined although reasonable bounds on the mass do exist, say, 7–500 GeV (not quite a narrow range).[12] The experimental discovery of a Higgs boson would be necessary before one could hope to conclude that the present scenario is correct; however, since the Higgs-boson mass is unknown, looking for Higgs bosons will not be an easy task.

In this paper we will examine two of the several decay mechanisms of Higgs bosons which involve gluons in the final state. In Sec. II we discuss the decay $H \to 2$ gluons and show that it may occur with a significant rate (~10%) even though superheavy quarks are assumed not to exist (the only contribution being from the six "conventional" quarks). Section III summarizes our calculations of the differential spherocity and thrust distribution for $H \to q\bar{q}g$. We also calculate the average values of S and $1-T$ to lowest nontrivial order in α_s and compare our results with those found in three-jet final states in e^+e^-. A discussion of our results and our conclusions can be found in Sec. IV.

II. THE DECAY $H \to 2g$

In order to search for the Higgs boson it is necessary to have as much information as possible on its decay modes (lifetime, branching ratios, etc.). Within the standard model,[8] the decay $H \to 2g$ proceeds through a quark triangle as shown in Figs. 1(a) and 1(b); although this decay has been discussed in the literature[13] only the limit where the mass of quark on the loop is very much larger than the Higgs-boson mass has been examined. Except, possible, for the t quark, just the opposite is true in the standard model using current-algebra quark masses.[14] Also, in this same limit, the two calculations in the literature

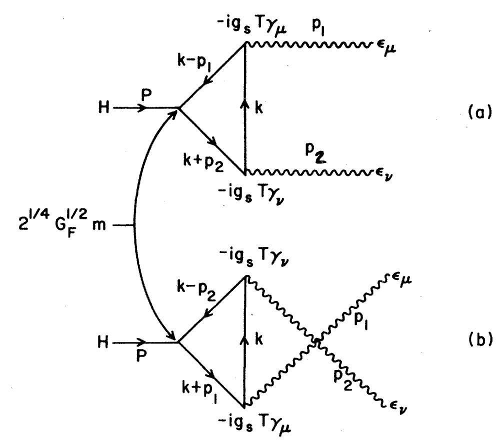

FIG. 1. Graphs contributing to $H \to 2$ gluons. m is the mass of the quark propagating around the triangle.

differ in their prediction of this decay by a factor of 4; we hope to resolve this issue in this calculation as well. We will show that the decay rate may be appreciable even though superheavy quarks are not propagating in the fermion loop.

The amplitude for the process $H\to 2g$ can be read off directly from Fig. 1; denoting the amplitude in Fig. 1(a) by $T_{\mu\nu}(p_1,p_2)$ we see that the total amplitude can be written as

$$A_{\mu\nu}(p_1,p_1)=T_{\mu\nu}(p_1,p_2)+T_{\nu\mu}(p_2,p_1)\,. \tag{2.1}$$

Let us first consider the amplitude $T_{\mu\nu}(p_1,p_2)$; we may write this as

$$T_{\mu\nu}(p_1,p_2)=2^{1/4}G_F{}^{1/2}g_s{}^2\sum_{i=1}^{n}m_i\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_i{}^2}I^i_{\mu\nu}\frac{1}{(k-p_1)^2-m_i{}^2}\frac{1}{(k+p_2)-m_i{}^2} \tag{2.2}$$

with m_i being the mass of ith quark and

$$I^i_{\mu\nu}=\mathrm{Tr}[(\not k+m_i)\gamma_\mu(\not k-\not p_1+m_i)(\not k+\not p_2+m_i)\gamma_\nu]\,. \tag{2.3}$$

The sum extends over all quark flavors; g_s is the quark-gluon coupling constant. We have suppressed all references to color in these expressions for simplicity; below we consider a single quark on the loop of mass m and perform the sum over the various species at the end of the calculation. An explicit calculation yields

$$I_{\mu\nu}=4m[4k_\mu k_\nu+2(k_\mu p^1_\nu-k_\nu p^2_\mu)-(p^1_\mu p^2_\nu-p^2_\mu p^1_\nu)+g_{\mu\nu}(m^2-k^2-p_1\cdot p_2)]\,, \tag{2.4}$$

where we have set $p_1{}^2=p_2{}^2=0$ since the final-state gluons are on-shell. Note that $I_{\mu\nu}$ is symmetric under the interchange $(p_1,\mu)\leftrightarrow(p_2,\nu)$; this implies that $T_{\mu\nu}(p_1,p_2)=T_{\nu\mu}(p_2,p_1)$ and hence

$$A_{\mu\nu}(p_1,p_2)=2T_{\mu\nu}(p_1,p_2)\,. \tag{2.5}$$

Introducing Feynman parameters and employing dimensional regularization we have

$$T_{\mu\nu}=2(2^{1/4}G_F{}^{1/2}m)g_s^2\int_0^1 dx\int_0^{1-x}dy\int\frac{d^nk}{(2\pi)^n}\frac{I_{\mu\nu}}{(k^2+2kQ-m^2)^3}\,, \tag{2.6}$$

where $Q\equiv yp_2-xp_1$. A short calculation shows that $T_{\mu\nu}$ is *finite* in the limit $n\to 4$ and is proportional to $\Gamma(3-n/2)$. Introducing $\alpha_s=g_s^2/4\pi$ and recalling the factor of 2 from Eq. (2.5) we find

$$A_{\mu\nu}=\frac{2^{5/4}G_F{}^{1/2}\alpha_s}{\pi}[I_1(p^2_\mu p^1_\nu-p^1\cdot p^2g_{\mu\nu})-I_2(p^1_\mu p^2_\nu)] \tag{2.7}$$

with

$$I_1=m^2\int_0^1 dx\int_0^{1-x}dy(m^2-2xyp_1\cdot p_2)(1-4xy)\,, \tag{2.8a}$$

$$I_2=m^2\int_0^1 dx\int_0^{1-x}dy(m^2-2xyp_1\cdot p_2)(1-2x)(1-2y)\,. \tag{2.8b}$$

Upon squaring $A_{\mu\nu}$ and multiplying by the gluon polarization sum we see that only the integral I_1 contributes. Upon multiplying by phase space and color factors we find

$$\Gamma(H\to 2g)=\frac{\sqrt{2}G_Fm_H{}^3}{8\pi}\left(\frac{\alpha_s}{\pi}\right)^2|I|^2 \tag{2.9}$$

with $(I\equiv I_1)$

$$I=\sum_i\lambda_i\int_0^1 dx\int_0^{1-x}dy\,\frac{1-4xy}{\lambda_i-xy} \tag{2.10}$$

and $\lambda_i\equiv m_i{}^2/m_H{}^2$. (The sum is over the various quark species which go around the triangle loop.) Upon integration over y, I can be written as a single integral of the form

$$I=\sum_i\left[2\lambda_i+\lambda_i(4\lambda_i-1)\int_0^1\frac{dx}{x}\ln\left(1-\frac{x(1-x)}{\lambda_i}\right)\right]. \tag{2.11}$$

I is, in general, complex but takes on real values for $\lambda\geq\frac{1}{4}$; for $\lambda\gg 1$, I can be expanded in powers of λ^{-1}. We find

$$I=\frac{1}{3}\left(1+\frac{7}{120}\frac{1}{\lambda}+O(1/\lambda^2)\right), \tag{2.12}$$

so that in the limit $m^2\gg m_H{}^2$ we find (for one superheavy quark)

$$\Gamma=\frac{G_Fm_H{}^3}{\sqrt{2}\times 36\pi}\left(\frac{\alpha_s}{\pi}\right)^2\left(1+\frac{7}{60}\frac{1}{\lambda}+O(1/\lambda^2)\right). \tag{2.13}$$

This confirms the result of Ellis *et al.*[13] and disagrees with that of Wilczek.[13]

We are primarily interested in calculating I for the known quarks (and a presumed t quark); we will take

$$\begin{aligned}&m_u=4.0\text{ MeV},\quad m_d=7.5\text{ MeV},\\&m_s=150\text{ MeV},\quad m_c=1.5\text{ GeV},\\&m_b=5\text{ GeV}\,.\end{aligned} \tag{2.14}$$

We will also assume that

$$15 \leqslant m_t \leqslant 50 \text{ GeV}, \qquad 10 \leqslant m_H \leqslant 100 \text{ GeV} \tag{2.15}$$

in the remainder of our calculation. For $\lambda < \frac{1}{4}$ (which applies for the u, d, s, c, and probably b quarks) the imaginary part of I can be calculated in a straightforward manner:

$$\operatorname{Im} I = \sum_i -i\pi\lambda_i(1-4\lambda_i)\ln(r_+^i/r_-^i), \tag{2.16}$$

where

$$r_\pm^i = \tfrac{1}{2}[1 \pm (1-4\lambda_i)^{1/2}]. \tag{2.17}$$

The real part of I must, however, be computed numerically.

In Fig. 2 we have plotted $|I|^2$ for various Higgs-boson masses as a function of m_t; as can be easily seen, for a wide range of m_H and m_t the values of $|I|^2$ lie in the range ~0.2–0.5. We thus find

$$\Gamma(H \to 2g) \simeq \Gamma_0\left(\frac{\alpha_s}{\pi}\right)^2(0.3-0.5), \tag{2.18}$$

where

$$\Gamma_0 \equiv \frac{G_F m_H{}^3}{4\sqrt{2}\pi} = (6.51\times10^{-1}\text{ MeV})\left(\frac{m_H}{10\text{ GeV}}\right)^3. \tag{2.19}$$

To estimate a branching ratio we must examine the usual decay: $H \to f\bar{f}$ (where f is a quark or lepton); this decay rate is given by

$$\Gamma(H \to f\bar{f}) = (3)\Gamma_0\left(\frac{m_f}{m_H}\right)^2(1-4m_f{}^2/m_H{}^2)^{3/2} \leqslant (3)\Gamma_0(0.0465). \tag{2.20}$$

Figure 3 shows $\Gamma(H \to \sum_i \bar{f}_i f_i)/\Gamma_0$ for the same range of m_t and m_H as in Fig. 2; note that this

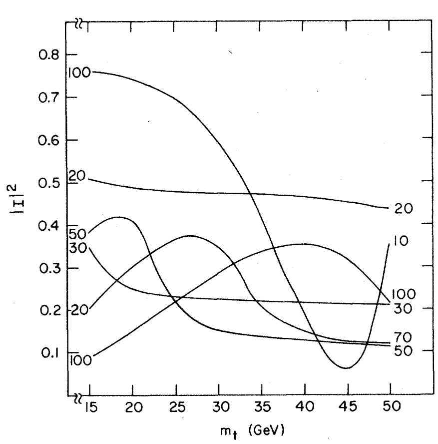

FIG. 2. $|I|^2$ as a function of m_t for various values of the Higgs-boson mass.

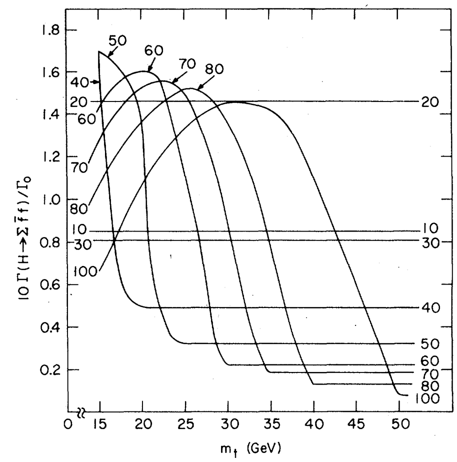

FIG. 3. $\Gamma(H \to \Sigma f\bar{f})/\Gamma_0$ as a function of m_t for various values of the Higgs-boson mass. Note that all the curves become flat for $m_t \geqslant m_H/2$.

ratio is bounded by $\simeq 0.17$. Hence we expect

$$B(H \to 2g) \gtrsim (2-4\%)(\alpha_s/\pi)^2 \simeq 2-4\%. \tag{2.21}$$

The detailed results are shown in Fig. 4 where we have plotted $(\alpha_s/\pi)^{-2}\Gamma(H \to 2g)/\Gamma(H \to \sum \bar{f}f)$ as a function of m_t for various values of m_H. Figure 4 shows that the curves for a particular m_H exhibit a peaking as a function of m_t for $m_t = \frac{1}{2}m_H$; the peaking increases substantially as m_H is increased.

Figure 4 shows that the rate for the two-gluon process may be larger than 10%; a not unlikely possibility, for example, is $m_H = 10$ GeV (Ref. 13) and $m_t \sim 20$ GeV. This would give a branching ratio close to 9%. If, however, $m_H = 100$ GeV and $m_t = 50$ GeV the two-gluon branching ratio could be larger than 20%.

Our calculation clearly demonstrates that the two-gluon final state may, indeed, be more important in Higgs-boson decay than might be naively

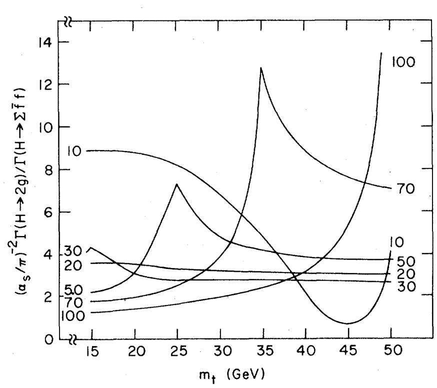

FIG. 4. The ratio $\Gamma(H \to 2g)/\Gamma(H \to \Sigma f\bar{f})$ scaled by $(\alpha_s/\pi)^{-2}$ as a function of m_t for various values of the Higgs-boson mass. Note the peaking that occurs for $m_t = m_H/2$.

expected in the absence of superheavy fermions ($m_f \gg m_H$). The gluon jets which materialize from this decay should be wider and have a higher multiplicity than quark jets due to the fact that gluon-gluon coupling is stronger than quark-quark coupling by a Clebsch-Gordon coefficient (9/4). The final state into which the Higgs boson decays may have substantially more light particles than expected from the usual coupling proportional to the mass of the coupled particle. Any excess observed may be attributed to gluon-jet final states which would tend to fragment into light (π's and k's) particles.

Thus, a signal for Higgs-boson decay into two gluons may be a large fraction of final states having no heavy particles such as t's, b's, c's, or τ's.

III. JETS FROM HIGGS-BOSON DECAY

As is well known, the usual two-body decay of the Higgs boson leads to $q\bar{q}$ jets in the final state; in this section we consider the decay $H \to q\bar{q} + \text{gluon}$ and calculate the differential spherocity and thrust distribution as well as the average values of S and $1-T$. For two-body decays, apart from nonperturbative effects, $S=1$, $T=0$.

The detailed study of two- and three-jet final states in e^+e^- reactions exists in the literature.[15] The two diagrams leading to three-jet final states in Higgs-boson decay are shown in Fig. 5. In our calculation we will neglect the mass of the quarks on the external lines to simplify our calculation.

The square of the matrix element is proportional to

$$\frac{\alpha_s}{\pi}\,\frac{(p_1\cdot k)^2+(p_2\cdot k)^2+2p_1\cdot(p_2+k)p_2\cdot(p_1+k)}{(p_1\cdot k)(p_2\cdot k)}\,. \tag{3.1}$$

In the usual notation[15] we may write

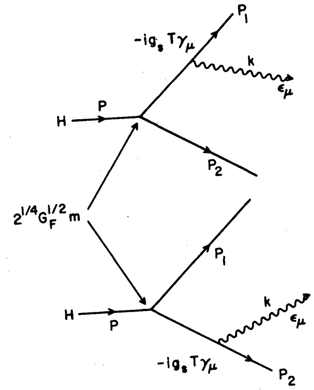

FIG. 5. Higgs-boson decay into $\bar{q}q$ and a gluon in lowest order. m is the mass of the quark or antiquark.

$$\begin{aligned} p_1\cdot p_2 &= \tfrac{1}{2}s(1-x_3)\,,\\ p_1\cdot k &= \tfrac{1}{2}s(1-x_2)\,,\\ p_2\cdot k &= \tfrac{1}{2}s(1-x_1)\,, \end{aligned} \tag{3.2}$$

with $s \equiv m_H^{\,2}$. The x_i satisfy $\sum_i x_i = 2$ with the definition

$$x_i = \{2E_1/\sqrt{s},\ 2E_2/\sqrt{s},\ 2k/\sqrt{s}\}\,. \tag{3.3}$$

Putting in numerical constants and phase-space factors we find

$$\frac{d\Gamma_3(H\to q\bar{q}g)}{dx_1dx_2} \simeq \frac{G_F m_H^{\,3}}{2\sqrt{2}\pi}\,\frac{\alpha_s}{\pi}\left(\frac{m_q}{m_H}\right)^2 \times \frac{(1-x_1)^2+(1-x_2)^2+2x_1x_2}{(1-x_1)(1-x_2)} \tag{3.4}$$

or simply

$$\frac{1}{\Gamma_2}\,\frac{d\Gamma_3}{dx_1dx_2} \simeq \frac{2\alpha_s}{3\pi}\,\frac{(1-x)^2+(1-x_2)^2+2x_1x_2}{(1-x_1)(1-x_2)}\,, \tag{3.5}$$

where Γ_2 is the two-body decay rate for $H\to q\bar{q}$ to lowest order in $(m_q/m_H)^2$ which can be obtained from Eq. (2.20). Following De Rújula *et al.*[15] we may rewrite this in terms of the variables' sphericity (S) and thrust (T):

$$\begin{aligned} \frac{1}{\Gamma_2}\,\frac{d\Gamma_3}{dS\,dT} \simeq \frac{2\alpha_s}{3\pi}\,\frac{\pi^2T}{64(1-T)[1-\pi^2S/16(1-T)]^{1/2}}\Bigg[&2\left(\frac{(1-T)^2+(1-x_{2^+})^2+2x_{2^+}T}{(1-T)(1-x_{2^+})}+\frac{(1-T)^2+(1-x_{2^-})^2+2x_{2^-}-T}{(1-T)(1-x_{2^-})}\right)\\ &+\frac{(1-x_{2^+})+(T+x_{2^+}+1)^2+2x_{2^+}(2-T-x_{2^+})}{(1-x_{2^+})(T+x_{2^+}-1)}\\ &+\frac{(1-x_{2^-})^2+(T+x_{2^-}+1)^2+2x_{2^-}(2-T-x_{2^-})}{(1-x_{2^-})(T+x_{2^-}-1)}\Bigg]\,, \end{aligned} \tag{3.6}$$

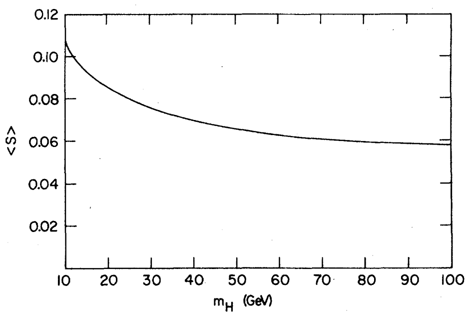

FIG. 6. Plot of $\langle S\rangle$ as a function of m_H; we have taken $\Lambda^2 = 0.5$ GeV^2 and $N=5$ for purposes of demonstration.

where

$$x_{2\pm} = 1 - \tfrac{1}{2}T\{1 \pm [1 - \pi^2 S/16(1-T)]^{1/2}\}. \tag{3.7}$$

Using this distribution we can calculate $\langle S\rangle$ and $\langle 1-T\rangle$ numerically; we find

$$\langle S\rangle \simeq (1.78)\,\frac{2\alpha_s}{3\pi}, \qquad \langle 1-T\rangle \simeq (1.68)\left(\frac{2\alpha_s}{3\pi}\right). \tag{3.8}$$

These are comparable to the values found in e^+e^- [15]:

$$\langle S\rangle \simeq (1.64)\,\frac{2\alpha_s}{3\pi}, \qquad \langle 1-T\rangle \simeq (1.57)\,\frac{2\alpha_s}{3\pi}. \tag{3.9}$$

These expressions (3.8) depend on the value of m_H through the Q^2 dependence of α_s:

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2N)\ln(Q^2/\Lambda^2)}. \tag{3.10}$$

Figure 6 shows a plot of $\langle S\rangle$ as a function of the Higgs-boson mass for $\Lambda^2 = 0.5$ GeV^2 and $N=5$. We would expect nonperturbative effects to be small for $m_H \geqslant 10$–20 GeV. (A plot of $\langle 1-T\rangle$ would be simply obtained by scaling Fig. 6 by ~0.96.)

Equation (3.6) can be separately integrated over S or T to give dN/dT or dN/dS; we will not do so here.

We expect that the branching ratio for $H \to q\bar{q}g$ to be roughly $\sim\alpha_s/\pi$ although a careful analysis, paralleling that of Sterman and Weinberg,[15] is necessary to determine how this rate depends on such things as jet width, etc.

IV. DISCUSSION AND CONCLUSION

In this paper we have discussed mechanisms by which gluons can appear in the final state of Higgs-boson decay in the standard model. Along with the "conventional" decay, $H \to q\bar{q}$ + gluon, which results from bremsstrahlung, we have also considered the decay $H \to 2$ gluons through a quark loop.

One might worry that if we are considering the two final-state gluons as jets we must be careful in analyzing the infrared finiteness and possible dangerous mass singularities associated with the graphs of Fig. 1. These same problems arose in our previous discussion of the $Z \to 2g$ decay[16]; we will summarize those arguments here.

First, as was shown explicitly in Sec. II, the graphs of Fig. 1 are ultraviolet finite and vanish in the limit $m \to 0$; the finite terms which remain in this limit will then behave no worse than $\sim m^2 \ln(Q^2/m^2)$ as $m \to 0$. When considering the infrared behavior of $H \to 2g$ *jets* we must also consider $H \to q\bar{q} + 2$ gluons to the same order in α_s with the quark pair soft and the gluons hard. Since the graphs of Fig. 1 are finite as $m \to 0$ and all the ultraviolet divergences must cancel between the two sets of graphs we conclude that the sum of all graphs for $H \to q\bar{q} + 2$ gluons is infrared finite and free of dangerous mass singularities. Now, although the graphs for $H \to q\bar{q}$(soft) + 2 hard gluons can contribute to the rate for $H \to 2g$ (jets), they are substantially suppressed by phase-space factors; in general, the four particles in the final state $H \to q\bar{q} + 2g$ would prefer to have roughly equal energies. We are thus able to conclude that the absence of dangerous mass singularities which do not factorize allows us to consider only the contribution of Fig. 1 in our calculation.

Our results for the calculation of $H \to 2g$ can be found essentially in Fig. 4; with $\alpha_s/\pi \sim 0.1$ we see that this branching ratio may be larger than 10% depending on the values of m_t and m_H. Thus, without relying on superheavy fermions, we are still able to obtain a large branching fraction for this decay mode.

For the decay $H \to q\bar{q}$ + gluon we have obtained the differential distribution in spherocity and thrust; the average values of S and T that we obtain are comparable to those found for three-jet final states in e^+e^- reactions. We expect three-jet events from Higgs-boson decay ~10% of the time.

ACKNOWLEDGMENT

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