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# Symmetry breaking as configuration mixing

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It is shown that the best established deviations from SU(6) symmetry can be accounted for by a mixing of the ground-state nucleon with $s$ (3%) and $d$ (19%) waves with mixed symmetry (70 representation).

## I. INTRODUCTION

The possibility of reproducing the violations of SU(6) symmetry with configuration mixing and of understanding this in terms of quarks and gluons has been investigated recently. We considered first[1] semileptonic form factors ($G_A/G_V$) and magnetic moments of nucleons together with decays of $\Delta$ within the most general double excitation mixing [$(70,0^+)$, $(70,2^+)$, and $(20,1^+)$ or $s$, $d$, and $p$ waves]. We obtained similar results for the three possibilities and found that the $p$ waves in (20,1) turned out to satisfy best the requirement there that a single mixing should account for $G_A/G_V$ without conflicting with the other experimental data. The connection of those possible mixings with hyperfine interactions induced by one-gluon exchange[2] turned out to be essentially qualitative, since the predicted angles, apart from their model dependence, were too small to account for $G_A/G_V$. On the other hand, Isgur, Karl and Koniuk[3] considered the SU(6) violations in the neutron charge radius and in the selection rules $D_{15}(1670)\to p\gamma$ and $D_{05}(1830)\to \bar{K}N$, showing that they could be explained with an $s$-wave component in the nucleon of amplitude $\frac{1}{4}$, compatible with their oscillator and hyperfine analysis of the baryon spectrum. At this point one would have the unsatisfactory situation of requiring another origin for the 25% violation in $G_A/G_V$ as emphasized by Morpurgo,[4] besides having relied on nonrelativistic perturbation theory, which is questionable for the ground state.[5]

The purpose of this paper is then to investigate the possibility of a unified description of all current SU(6) violations in a mixing scheme and to discuss possible dynamical origins of the solutions. In Sec. II we consider the nucleon properties, including[6] now $\langle r_n^2\rangle$ and in Sec. III, devoted to resonances, we extend our previous analysis[1] to the neutrino production of $\Delta$ and to the decays of $D_{05}$ and $D_{15}$. The solution of the phenomenological analysis is interpreted in Sec. IV, where we discuss whether the resulting $s$- and $d$-wave components which account for all data from $G_A/G_V$ to $\langle r_n^2\rangle$ can be induced by gluons.

We shall refer sometimes to previous results, but all useful formulas are included in an appendix.

## II. NUCLEON PROPERTIES

We compute here with double excitation mixing the semileptonic form factor and the neutron/proton charge-radius ratios, which are the main SU(6) violations in nucleons, together with the magnetic-moment ratio $\mu_p/\mu_n$, where the SU(6) prediction agrees so well with experiment that it constitutes a stringent constraint for mixings.

The results for $G_A/G_V$ and $\mu_p/\mu_n$, as obtained in our previous work,[1] read

$$G_A/G_V=\tfrac{5}{3}-\tfrac{4}{3}\nu_{70,0}{}^2-\tfrac{4}{3}\nu_{20,1}{}^2-2\nu_{70,2}{}^2\,, \qquad (1)$$

$$\mu_p/\mu_n=-\tfrac{3}{2}\left(1+\tfrac{1}{3}\,\frac{\nu_{70,0}{}^2}{\nu_{56,0}{}^2}\right), \qquad (2)$$

where the $\nu$ refer to the amplitudes of the different components of the nucleon, characterized by the dimension of the SU(6) representation and the angular momentum, which are given in the Appendix.

As for the charge radius, one can immediatly see with those wave functions that it is proportional to the charge for diagonal matrix elements, so that one is left, for the neutron, with the nondiagonal $s$-wave component (because of orthogonality in angular momentum)

$$\langle n;56,0|\Sigma e_i r_i^2|n;70,0\rangle=-R^2/\sqrt{6}\,, \qquad (3)$$

where $R$ is the oscillator radius. The neutron/proton ratio is then, to first order in the mixing coefficient,

$$\frac{{}_n\left\langle\sum_i e_i r_i^2\right\rangle_n}{{}_p\left\langle\sum_i e_i r_i^2\right\rangle_p}=-\left(\tfrac{2}{3}\right)^{1/2}\frac{\nu_{70,0}}{\nu_{56,0}}\,. \qquad (4)$$

Since it is the only contribution, the experimental[7] value $-0.15\pm0.1$ fixes $\nu_{70,0}/\nu_{56,0}=0.18$. This value then modifies $\mu_p/\mu_n$ just by 1%, far below the expected accuracy of the prediction, in spite of its wrong direction. It is worth remarking that the induced modifications for the other baryons are of the same order, so that the recently measured 15% deviations in the magnetic moments of $\Sigma^+$ and $\Xi^0$ must have another origin, such as the SU(3) mixing advocated by Lipkin,[8] which should be complementary to the mixing considered here. The $s$-wave component thus obtained reduces in turn $G_A/G_V$ only by 2%, so that the experimental deviation of 25% from $\frac{5}{3}$ requires an additional mixing with $p$ and/or $d$ waves, as one can see in Eq. (1). To decide this question and check the consistency we investigate next the resonances.

## III. RESONANCES

An appropriate place to look for the effects of configuration mixing, as proposed by Isgur and Karl,[3] are the $\bar{K}N$ decays of $D_{05}$, whose analysis for the 70, 0 component is extended here to the general case. The decays are described by the quark diagram in Fig. 1, where one can see at once that it is forbidden if the nucleon is unmixed. In fact, the $u,d$ spectator quarks are necessarily in a spin-1 state (to yield the maximal selected total spin), so that if the nucleon is unmixed in the 56 groundstate, the isospin in the initial state must be also 1 and the selection rule follows that only $\Sigma(D_{15})$ can decay. From this simple argument it is obvious that any admixture which breaks the isospin-spin symmetrical correlation of the 56, 0 allows for the decay, which is computed next for $s,p$, and $d$ waves.

Carrying out the usual analysis of baryon decays in the [SU(6)] quark shell model[9] for the various nucleon wave functions specified in the Appendix, one obtains immediately, again to first order in the mixing angle, the ratio

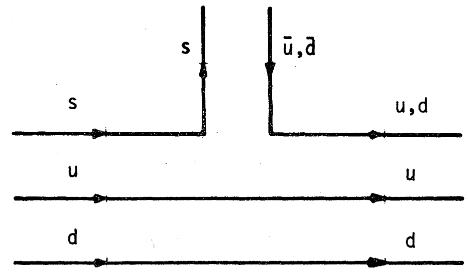


FIG. 1. Quark diagram for the decays of $D_{05}$ and $D_{15}$ into $\bar{K}N$.

$$\frac{A(D_{05}\to\bar{K}N)}{A(D_{15}\to\bar{K}N)}=\frac{1}{\sqrt{2}}\frac{\nu_{70,0}}{\nu_{56,0}}-\frac{1}{\sqrt{2}}\frac{\nu_{20,1}}{\nu_{56,0}}-\frac{1}{2\sqrt{5}}\frac{\nu_{70,2}}{\nu_{56,0}}, \tag{5}$$

where the numerical coefficients follow simply from the spin matrix elements and a ratio of the space integrals, which reduces to a ratio of Clebsh-Gordon coefficients.

Another good case for SU(6) violations is the Moorhouse selection rule[10] for the radiative decay $D_{15}\to p\gamma$, which is also forbidden for a pure 56, 0 proton for similar reasons as above. There are two amplitudes, $A_{3/2}$ and $A_{1/2}$, corresponding to the decay of the $J_z=\frac{3}{2}$ and $\frac{1}{2}$ components of the resonance into a photon, say, moving along the $z$ axis with positive helicity. From the nonrelativistic reduction of the point quark-photon interaction, it is straightforward to obtain

$$A^p_{1/2}/A^p_{3/2}=\frac{1}{\sqrt{2}}, \tag{6}$$

$$A^p_{3/2}/A^n_{3/2}=-(\tfrac{2}{3})^{1/2}\frac{\nu_{70,0}}{\nu_{56,0}}+(\tfrac{2}{3})^{1/2}\frac{\nu_{20,1}}{\nu_{56,0}}+\frac{1}{\sqrt{15}}\frac{\nu_{70,2}}{\nu_{56,0}}. \tag{7}$$

The numerical results are summarized in Table I where the first column (second) corresponds to an amplitude of $p(d)$ wave fixed to reproduce $G_A/G_V$

TABLE I. Predictions of the various mixings in the nucleon (columns). Underlined values are input.

| | $p$ | $d$ | $s$ | $p+s$ | $d+s$ | Experiment |
|---|---|---|---|---|---|---|
| $G_A/G_V$ | $\underline{1.25}$ | $\underline{1.25}$ | 1.62 | $\underline{1.25}$ | $\underline{1.25}$ | 1.25 |
| $\langle r_n^2\rangle/\langle r_p^2\rangle$ | 0 | 0 | $\underline{-0.15}$ | $\underline{-0.15}$ | $\underline{-0.15}$ | −0.15 |
| $A^p_{1/2}/A^p_{3/2}$ | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.95 ± 0.93 |
| $A^p_{3/2}/A^n_{3/2}$ | −0.55 | −0.13 | −0.15 | −0.69 | −0.28 | −0.33 ± 0.28 |
| $\frac{A(D_{05}\to\bar{K}N)}{A(D_{15}\to\bar{K}N)}$ | 0.48 | 0.11 | 0.13 | 0.60 | 0.24 | 0.22 ± 0.09 |

and the third column is for the $s$ wave required by the neutron charge radius. The fourth column shows the implications of the mixing with $s$ and $p$ waves (again with $G_A/G_V$ and $r^2$ as inputs) and they are in clear contradiction with the data, which lead us finally to the following mixture of $s$ and $d$ waves

$$|N\rangle = 0.88|N;56,0\rangle - 0.44|N;70,2\rangle + 0.16|N;70.0\rangle \tag{8}$$

for which there is very good agreement in sign and magnitude (fifth column) with the experimental[11] values in the last column.

In the rest of the section we check this solution to the SU(6) violations considering decays and production of $\Delta$, where the main fact to worry about for $d$ waves is the absence of $E2$ transitions[12] in $\Delta^+ \to p\gamma$ to the level of 4% (relative to $M1$). The evaluation, which has the problems of involving a new mixing coefficient for $\Delta$ (labeled $\delta_{70,2}$) and of model dependence, yields for the harmonic oscillator

$$\frac{E2}{M1} = \frac{1}{30\pi}|\vec{k}|^2 m^2 R^4 \frac{(\nu_{56,0}\delta_{70,2} - \nu_{70,2}\delta_{56,0})^2}{\left(\frac{2\sqrt{2}}{3}\nu_{56,0}\delta_{56,0} - \frac{\sqrt{2}}{3}\nu_{70,2}\delta_{70,2}\right)^2}, \tag{9}$$

where $k$ is the momentum transfer, $m$ the quark mass and $R$ the radius. For the case $\delta_{70,2} = -\nu_{70,2}$, as predicted, e.g., by hyperfine interactions, the experimental 4% would correspond to $R \sim 1$ fm, and for the reasonable value of 0.68 fm, given by the spectrum,[13] the ratio is safely less than 1% and the Morpurgo selection rule is no problem for our 19% admixture of $d$ waves.

One can also consider the production of $\Delta$, $\nu p \to \Delta^{++}\mu^-$, and compute the axial-vector and vector form factors,[14] for which we take again the ratio trying to avoid the problems of extending SU(6) from the static limit and of the model dependence of the parameters. The result for this ratio, given by experiment as 0.58, is

$$F_A/F_V = \tfrac{2}{3}\left(1 - \frac{1}{2\sqrt{2}}\frac{\nu_{70,2}\delta_{56,2}}{\nu_{56,0}\delta_{56,0}}\right). \tag{10}$$

The conclusions from this equation are obscured by the presence of the new mixing amplitudes $\delta_{56,2}$ (now possible for $\Delta$) and $\delta_{70,2}$. If one takes again into account the relation[1]

$$\frac{\nu_{70,2}}{\nu_{56,0}} = -\frac{\delta_{70,2}}{\delta_{56,0}} = \frac{1}{\sqrt{2}}\frac{\delta_{56,2}}{\delta_{56,0}} \tag{11}$$

given by the hyperfine tensor, the prediction of the mixing in Eq. (8) agrees exactly with the experimental result.

There are in principle other tests, like the $\mu(\Delta^{++})$ and the radiative decays of $F_{17}(1990)$, which are easily computed, but for which present data have so big errors that they cannot be used as checkings of the scheme, to be regarded rather as a way of predicting those numbers.

The implications of the phenomenological mixing for the forbidden decay[15] $F_{17} \to p\gamma$ are similar to those for $D_{15} \to p\gamma$. Assuming that the $F_{17}(1990)$ belong to the 70, 2 representation, one obtains from Eq. (8)

$$A^p_{1/2}/A^p_{3/2} = \left(\frac{3}{5}\right)^{1/2} \tag{12}$$

$$A^p_{3/2}/A^n_{3/2} = 2\frac{\nu_{70,0}}{\nu_{56,0}} - \left(\frac{2}{3}\right)^{1/2}\frac{\nu_{70,2}}{\nu_{56,0}}. \tag{13}$$

The magnetic moment[1,16] of $\Delta^{++}$ is predicted to be

$$\mu(\Delta^{++}) = 2(1 - \tfrac{2}{5}\delta_{56,2}{}^2 - \tfrac{3}{5}\delta_{70,2}{}^2) = 1.78 \tag{14}$$

for the coefficients given in Eq. (8) and the relation in Eq. (11). One can also compute $F/D$ from strange semileptonic decays

$$F/D = \tfrac{2}{3}\left(1 + \tfrac{1}{2}\frac{\nu_{70,0}{}^2}{\nu_{56,0}{}^2} - \tfrac{1}{2}\frac{\nu_{70,2}{}^2}{\nu_{56,0}{}^1} - \tfrac{1}{3}\frac{\nu_{20,1}{}^2}{\nu_{56,0}{}^2}\right). \tag{15}$$

For the $s$ and $d$ wave mixing in Eq. (8) the SU(6) prediction $F/D = \frac{2}{3}$ is reduced to the experimental value of 0.58, in contrast with the case of pure (70, 0) mixing, which increases it.[17]

## IV. CONCLUSIONS

The idea of factorization of flavor and spin parts of hadronic wave functions [quark shell model or SU(6)] has been very useful both for phenomenology and to gain insight into the quark dynamics (color). Lacking still a theory for the spectrum (confinement), we have investigated the possibility of accounting for the deviations of the simple scheme in its own framework. Combining all symmetry deviations which had been considered partially before,[1,3] we conclude that the mixing of the ground state with mixed symmetry in $s$ and $d$ waves [(70, 0) and (70, 2) in SU(6) language], as given in Eq. (8), accounts in fact for the experimental data. Besides the phenomenological interest because of its simplicity,[18] the mixing brings us to the question of understanding its origin dynamically, in terms of quarks and gluons.

It is well known that spin-contact and tensor terms, like the ones present in the Pauli reduction of one-gluon exchange, induce, respectively, a mixing of the ground state with the $s$ and $d$ waves in the 70 representation. But in order to obtain their specific amplitudes one has to specify the potentials and what is usually done is to combine a Coulomb term with an oscillator (or linear) term in nonrelativistic perturbation theory.[1,2,3] For a reasonable choice of the radius and mass matrix one can reproduce the $s$-wave coefficient of 3% within a factor of 2 (above), but one would obtain this way $\nu_{70,2}/\nu_{70,0}\sim\frac{1}{4}$, in apparent conflict with the phenomenological solution in Eq. (8), which requires $d/s\sim 2.5$. The consistency of the ingredients entering those numerical "predictions" has been considered recently by Bohm,[5] and there are two facts in his work which can be relevant here, namely that the matrix elements of the contact term ($s$-wave mixing) are understood to be overestimated for the ground state and that the tensor term has little effect on the excited states. The nonrelativistic estimate of $\nu_{70,0}$ should therefore decrease while $\nu_{70,2}$ could be increased without contradicting spectral data, which is precisely in the direction of the phenomenological result. The situation could be summarized by stating that the presence of 70, 0 and 70, 2 mixing follows from one-gluon exchange (due essentially to its vector nature) but their required weights do not agree with nonrelativistic estimates, even though there is some hope to understand the disagreement. We remind the reader also that the spin-orbit terms present in the Breit-Fermi form are not yet established.

Our last comment is on the recent suggestion of constituent gluons in hadrons, motivated, e.g., by charmed-meson decays.[19] This is an alternative which should be investigated as a source of symmetry breaking in baryon matrix elements. The analysis would be very involved, but one can have an idea of the implications of the gluon for the valence quarks by discussing it in terms of the flavor, spin, and space of the present approach.

The requirement of color-singlet hadrons and the space symmetry of the ground state selected the spin-flavor symmetrical state of the quarks, but they can can be also antisymmetrical or mixed in spin-flavor when they are with one gluon. Still, the requirement of color-singlet hadrons imposes mixed symmetry in color for the quarks in such a state and this fact organizes the possible spin-flavor states in one antisymmetrical [20 in SU(6) language], one symmetrical (56), and six of mixed symmetry (70), after combination with the various space symmetries. One could naively expect the 70 to be more likely mixed, because of its higher weight, but any quantitative statement on mixing amplitudes and symmetry breaking has to follow here from definite (involved) assumptions on the quark-gluon confining interaction and the behavior of the gluon in matrix elements.

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## APPENDIX

We present here the wave functions of a three-quark system in a colour singlet with one harmonic-oscillator potential for their space dependence. The requirement of total symmetry makes very useful the classification of the spin, flavor, and space factors according to their permutation symmetry. Total symmetry (antisymmetry) will be labeled by $S$ ($A$) and the mixed symmetry by $\alpha$ ($\beta$) for symmetry (antisymmetry) in the first two variables. The latter is obviously also the permutation symmetry of the standard relative coordinates

$$\rho=\frac{1}{\sqrt{2}}(r_1-r_2)\lambda=\frac{1}{\sqrt{6}}(r_1+r_2-2r_3) \tag{A1}$$

in terms of which the wave functions of the first three levels read

$$\begin{aligned}
\psi_s(0,0;0)&=(\pi R^2)^{-3/2}\exp[-(\rho^2+\lambda^2)/2R^2]\,,\\
\psi_\alpha(0,1;1)&=\left(\frac{8\pi}{3}\right)^{1/2}\frac{\lambda}{R}Y^1_m(\Omega_\lambda)\psi_s(0,0;0)\,,\\
\psi_\beta(1,0;1)&=\left(\frac{8\pi}{3}\right)^{1/2}\frac{\rho}{R}Y^1_m(\Omega_\rho)\psi_s(0,0;0)\,,\\
\psi_s(2(0),0(2);2)&=\left(\frac{8\pi}{15}\right)^{1/2}\frac{1}{R^2}[\rho^2Y^2_m(\Omega_\rho)+\lambda^2Y^2_m(\Omega_\lambda)]\\
&\quad\times\psi_s(0,0;0)\,,\\
\psi_\alpha(2(0),0(2);2)&=\left(\frac{8\pi}{15}\right)^{1/2}\frac{1}{R^2}[\rho^2Y^2_m(\Omega_\rho)-\lambda^2Y^2_m(\Omega_\lambda)]\\
&\quad\times\psi_s(0,0;0)\,,\\
\psi_\beta(1,1;2)&=\frac{8\pi}{3}\frac{\rho\lambda}{R^2}\sum_{\mu,\mu'}(1\mu1\mu'|2m)Y^1_\mu(\Omega_\rho)Y^1_{\mu'}(\Omega_\lambda)\\
&\quad\times\psi_s(0,0;0)\,,\\
\psi_\alpha(0,0;0)&=-\frac{(\rho^2-\lambda^2)}{\sqrt{3}R^2}\psi_s(0,0;0)\,,\\
\psi_\beta(1,1;0)&=\frac{8\pi}{3}\frac{1}{R^2}\sum_{\mu,\mu'}\rho\lambda(1\mu1\mu'|00)Y^1_\mu(\Omega_\rho)Y^1_{\mu'}(\Omega_\lambda)\\
&\quad\times\psi_s(0,0;0)\,,\\
\psi_A(1,1;1)&=\frac{8\pi}{3}\frac{\rho\lambda}{R^2}\sum_{\mu,\mu'}(1\mu1\mu'|1m)Y^1_\mu(\Omega_\rho)Y^1_{\mu'}(\Omega_\lambda)\\
&\quad\times\psi_s(0,0;0)\,,
\end{aligned} \tag{A2}$$

with the notation $(l_\rho, l_\lambda, L)$, $L = l_\rho + l_\lambda$.

The explicit quark content of the flavor part of the wave function for the octect (mixed symmetry) is

$$p_\alpha = \frac{1}{\sqrt{6}}(2uud - udu - duu), \quad p_\beta = \frac{1}{\sqrt{2}}(udu - duu),$$

$$n_\alpha = \frac{1}{\sqrt{6}}(dud + udd - 2ddu), \quad n_\beta = \frac{1}{\sqrt{2}}(udd - dud),$$

$$\Sigma^+_\alpha = \frac{1}{\sqrt{6}}(usu + suu - 2uus), \quad \Sigma^+_\beta = \frac{1}{\sqrt{2}}(suu - usu),$$

$$\Sigma^-_\alpha = \frac{1}{\sqrt{6}}(dsd + sdd - 2dds), \quad \Sigma^-_\beta = \frac{1}{\sqrt{2}}(sdd - dsd),$$

$$\Xi^0_\alpha = \frac{1}{\sqrt{6}}(2ssu - uss - sus), \quad \Xi^0_\beta = \frac{1}{\sqrt{2}}(sus - uss),$$

(A3)

$$\Xi^-_\alpha = \frac{1}{\sqrt{6}}(2ssd - dss - sds), \quad \Xi^-_\beta = \frac{1}{\sqrt{2}}(sds - dss),$$

$$\Sigma^0_\alpha = \frac{1}{2\sqrt{3}}(dsu + sdu + usd + sud - 2uds - 2dus),$$

$$\Sigma^0_\beta = \tfrac{1}{2}(sdu - dsu + sud - usd),$$

$$\Lambda_\alpha = \tfrac{1}{2}(usd + sud - dsu - sdu),$$

$$\Lambda_\beta = \frac{1}{2\sqrt{3}}(2uds - 2dus + usd - sud + sdu - dsu).$$

The spin-$\frac{1}{2}$ factors can be read from the expressions for $p$ and $n$ in (A3) replacing $u$ by $\uparrow$ and $d$ by $\downarrow$. Combining the previous functions in all possible ways to obtain a completely symmetric object one obtains the total [SU(6)] wave functions

$$|B;56,0\rangle = \frac{1}{\sqrt{2}}(B_\alpha \tfrac{1}{2}_\alpha + B_\beta \tfrac{1}{2}_\beta)\psi_s(0,0;0)$$

$$|B;70,2\rangle = \frac{1}{\sqrt{2}} 3/2s[B_\alpha \psi_\alpha(2(0),0(2);2) + B_\beta \psi_\beta(1,1;2)]$$

$$|B;70,0\rangle = \tfrac{1}{2}[B_\alpha \tfrac{1}{2}_\beta \psi_\beta(1,1;0) - B_\alpha \tfrac{1}{2}_\alpha \psi_\alpha(0,0;0) + B_\beta \tfrac{1}{2}_\alpha \psi_\beta(1,1;0) + B_\beta \tfrac{1}{2}_\beta \psi_\alpha(0,0;0)] \qquad \text{(A4)}$$

$$|B;20,1\rangle = \frac{1}{\sqrt{2}}(B_\alpha \tfrac{1}{2}_\beta - B_\beta \tfrac{1}{2}_\alpha)\psi_A(1,1;1)$$

for the $\frac{1}{2}^+$ states.

The problem is reduced to evaluating matrix elements of one-body operators corresponding to the one-quark interaction and due to the overall symmetry one can always choose the first two quarks as spectators, so that matrix elements between states of different symmetry in those quarks vanish. The result is then three times a product of diagonal spin and flavor one-quark matrix elements and one overlap integral in the variable λ. This last factor, which is explicitly model dependent, is not present but for Clebsh-Gordan coefficients, if one takes ratios as we have tried to do here.

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