$WZH^{\pm}$ coupling in $SU(2)\times U(1)$ gauge models

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In the framework of $SU(2)\times U(1)$ electroweak gauge models with standard fermion assignments we derive the most general expression for the *WZH* vertex which is the most relevant for the production of charged Higgs particles. We show that the vertex only exists with an appreciable strength in models with a rather complicated Higgs sector. Experimental detection of charged Higgs particles seems therefore very difficult.

I. INTRODUCTION

The spontaneous-symmetry-breaking mechanism is a cornerstone in our present understanding of unified gauge-field theories. In this context, the standard $SU(2)\times U(1)$ Weinberg-Salam (WS) model[1-2] has successfully explained a huge amount of experimental information (for a review see e.g., Ref. 3) but one cannot be certain of the correctness of present-day gauge theories until the fundamental mechanism of mass generation is elucidated. For this reason, direct observation of Higgs bosons and the consequent verification of its role is an important goal in high-energy physics research. At present, the Higgs-boson sector still contains a high degree of arbitrariness and one is compelled to contemplate and explore a rich variety of still open options. In particular, one may enlarge the minimal WS choice (only one Higgs doublet) to incorporate charged Higgs bosons in the model by introducing extra Higgs multiplets. The potential phenomenology of this enlargement has already been discussed in the literature.[4-7] Since the main reason for the present experimental elusiveness of Higgs particles is their extremely weak coupling to fermions, the most interesting processes from the phenomenological point of view involve the triple-boson ($WZH^{\pm}$) vertex, exploiting the fact that Higgs bosons couple proportionally to the mass of the particles they couple to. As a consequence, these processes will become relevant in future accelerator facilities.

In the present paper we focus on the $WZH^{\pm}$ vertex (see Fig. 1) and derive its value in the most general case giving an upper bound to its intensity (Sec. II). We then discuss what possible realistic models do embody it and determine its strength in a number of phenomenologically relevant cases (Sec. III). Section IV is devoted to the conclusions.

II. GENERAL FORM OF THE $WZH^{\pm}$ COUPLING

To set up our framework, we consider an $SU(2)\times U(1)$ model with the standard WS assignments for leptons and quarks but leave open the Higgs-boson sector which will consist of several multiplets ϕ_j belonging to various representations of $SU(2)\times U(1)$.

The piece of the Lagrangian which is relevant for our purposes is

$$\mathcal{L}_{H(\mathrm{kin})}=\sum_j (D_\mu\phi_j)^+(D^\mu\phi_j)\,, \qquad (1)$$

where in an obvious notation (see e.g., Ref. 8)

$$\begin{aligned}D_\mu &= \partial_\mu - i(g\vec{\mathrm{T}}\cdot\vec{\mathrm{W}}_\mu + g'YB_\mu)\\ &= \partial_\mu - i\{2^{-1/2}g(T_-W_\mu^- + T_+W_\mu^+) + eQA_\mu\\ &\qquad + g\sec\theta_W[T_3-(\sin^2\theta_W)Q]Z_\mu\}\,,\end{aligned}$$

where $Q=T_3+Y$ is the electric charge operator and $\tan\theta_W=g'/g$.

After the spontaneous symmetry breaking, the Higgs fields ϕ_j acquire nonzero vacuum expectation values (VEV's) $\langle\phi_j\rangle_0$. Defining the displaced fields $\phi_j'\equiv\phi_j-\langle\phi_j\rangle_0$ and introducing them in Eq. (1) we can extract the *WZH* couplings which are obtained by replacing one of the ϕ_j by its VEV. This suggests immediately the following trick in order to eliminate redundant degrees of freedom.

If there is a subset of identical multiplets we can always define new rotated fields $\psi_i=\sum_j' R_{ij}\phi_j$, where R_{ij} is a unitary matrix and the indices i,j run over the subset of repeated multiplets. Obviously, this rotation leaves Eq. (1) unchanged and we can always choose R_{ij} in such a way that the VEV of the rotated fields are $\langle\psi_i\rangle_0\propto\delta_{1i}$, i.e., only ψ_1 develops a nonvanishing VEV. The rotated fields ψ_i, $i\geq 2$, thus decouple from the vertex we are interested in. Of course, this can be done for each subset of repeated multiplets and there-

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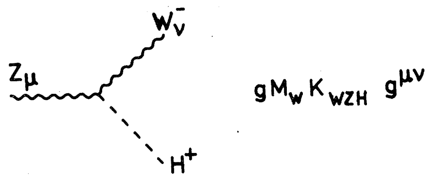

FIG. 1. The $WZH^\pm$ vertex.

fore, in order to obtain the WZH vertex, we can take without loss of generality all Higgs multiplets belonging to *different* representations of SU(2) $\times$ U(1).

An important situation occurs when *all* multiplets are identical in which case, for our purposes, the model is as if it had only one multiplet. In particular, if they are all doublets the WZH vertex *does not* exist. This is because the model would be like the standard WS model, the only difference being the additional doublets ψ_i, $i \geq 2$, which would not contribute to the $WZH^\pm$ vertex.

At this point it is convenient to distinguish between multiplets and antimultiplets. We shall arbitrarily call *multiplets* (*antimultiplets*) those with $Y \geq 0$ ($Y \leq 0$) or, equivalently, with predominantly positively (negatively) charged fields. Of course, the case $Y = 0$ corresponds to self-conjugate multiplets. We shall also classify the *multiplets* into two types.

(*a*) *Type* I: characterized by either $Y = 0$ (self-conjugated) or $Y = T$. In either case they involve only one field with $Q = +1$.

(*b*) *Type* II: characterized by $0 < Y < T$. They involve two distinct fields with $Q = +1$, one contained in the multiplet and the other one in its antimultiplet.

The above statement about the nonexistence of the WZH vertex when all Higgs multiplets are doublets is easily generalized to the case wherein all the multiplets are identical and of type I. Indeed, in this case the sole singly charged field of the first multiplet (the only one with nonzero VEV) becomes a Goldstone boson which is absorbed to give mass to the W boson. If, instead, the multiplet is of type II, only one combination of the two singly charged fields is the Goldstone boson. The orthogonal combination remains physical and, in general, coupled to the W and Z bosons.

For definiteness we shall consider the W^-ZH^+ vertex. Let us denote by ϕ_α^+, $\alpha = 1, 2, \ldots$, *all* singly positively charged fields of the Higgs sector and by ϕ_α the multiplet or antimultiplet which contains ϕ_α^+. The VEV of these multiplets are $\langle\phi_\alpha\rangle_0 = v_\alpha \chi_\alpha$, where v_α is, in general, a complex number and χ_α is a "spinor" with zeros everywhere except for a 1 in the position of the neutral field (this guarantees the photon remaining massless). Notice that, since type II multiplets provide two charged fields, they both (as well as the multiplet *and* antimultiplet they respectively belong to) are labeled with different values of the subscript α.

It is now straightforward to obtain from Eq. (1) the explicit form of the $W^-Z\phi_\alpha^+$ vertex. We find

$$\mathcal{L}_{WZ\Phi_\alpha} = 2^{-1/2} g^2 C_\alpha v_\alpha^* (2T_{3\alpha}/\cos\theta_W + \cos\theta_W) \times W_\mu^- Z^\mu \phi_\alpha^+ \tag{2}$$

with

$$C_\alpha \equiv [(T_\alpha - T_{3\alpha})(T_\alpha + T_{3\alpha} + 1)]^{1/2},$$

where T_α and $T_{3\alpha}$ are, respectively, the eigenvalues of T and T_3 corresponding to the neutral component of ϕ_α.

The ϕ_α^+, however, are not physical fields and we must diagonalize the Higgs-boson mass matrix with a unitary transformation to get the physical mass eigenstates H_α^+, namely,

$$H_\alpha^+ = \sum_\beta U_{\alpha\beta} \phi_\beta^+ , \tag{3}$$

where we have ordered our set of mass eigenstates H_α^+ in such a way that the first one H_1^+ corresponds to the Goldstone boson G^+ responsible for the generation of the mass of the W^+ boson. The Goldstone theorem fixes the first row of the matrix U to be

$$U_{1\beta} = C_\beta v_\beta^* / V , \tag{4}$$

where $V \equiv (\sum_\alpha C_\alpha{}^2 |v_\alpha|^2)^{1/2}$, which is related to the W-boson mass through

$$V = \sqrt{2}\, M_W / g . \tag{5}$$

From Eqs. (2)–(5) we obtain the following expression for the coupling of the *physical* charged Higgs bosons H_α^+, $\alpha \geq 2$, to W^- and Z:

$$\mathcal{L}_{WZH_\alpha} = K_{WZH_\alpha} g M_W W_\mu^- Z^\mu H_\alpha^+ \quad (\alpha \geq 2) , \tag{6}$$

with[9]

$$K_{WZH_\alpha} = 2 \sec\theta_W \sum_\beta U_{1\beta} U_{\alpha\beta}^* T_{3\beta} . \tag{7}$$

In general, we cannot say much about the matrix U of Eqs. (3) and (7) since it depends on the Higgs potential. Nevertheless, a useful relation which only involves the first row [given by Eq. (4)] of the matrix U can be obtained by squaring Eq. (7) and summing over $\alpha \geq 2$, namely

$$\sum_{\alpha \geq 2} |K_{WZH_\alpha}|^2 = 4 \sec^2\theta_W \left[\sum_\beta |U_{1\beta}|^2 T_{3\beta}{}^2 - \left(\sum_\beta |U_{1\beta}|^2 T_{3\beta} \right)^2 \right] \tag{8a}$$

$$= 4 \sec^2\theta_W \sum_{\beta < \beta'} |U_{1\beta} U_{1\beta'} (T_{3\beta} - T_{3\beta'})|^2 , \tag{8b}$$

where the last equality (8b) follows, after some algebra, from Eq. (8a) by multiplying the first term inside the brackets by $\sum_\beta |U_{1\beta}|^2 = 1$. Equation (8) gives an upper bound to each one of the constants $|K_{WZH_\alpha}|^2$, $\alpha \geq 2$. It shows explicitly the already mentioned fact that when all the multiplets are identical and of type I all the constants K_{WZH_α} vanish. In models with one single physical H^+, Eqs. (7) or (8) determine completely $|K_{WZH}|$. We shall come back to this circumstance later.

III. PHENOMENOLOGICAL CONSTRAINTS ON MODELS

We first turn our attention to a very important phenomenological constraint to the models considered in this paper. The ratio

$$\rho^2 \equiv M_W{}^2/M_Z{}^2 \cos^2\theta_W$$

is predicted to be 1 in the standard WS model.[10] For an SU(2) × U(1) model with an arbitrary Higgs sector the value of ρ^2 is given by

$$\rho^2 = \sum_j [T_j(T_j+1) - T_{3j}{}^2]|v_j|^2 \Big/ 2\Big(\sum_j T_{3j}{}^2 |v_j|^2\Big), \tag{9}$$

where, as in Eq. (1), the subscript j runs over *multiplets* only. T_j and T_{3j} are the eigenvalues of T and T_3 corresponding to the neutral component of the multiplet ϕ_j whose VEV is v_j.

The present experimental value[3] is $\rho^2 = 1 \pm 3\%$. If, in general, we require

$$1 - \delta \leq \rho^2 \leq 1 + \delta, \tag{10}$$

we shall say that the constraint (10) is *naturally* satisfied if it is true for any choice of the v_j of Eq. (9). The following statement is true: Eq. (10) is *naturally* satisfied if and only if

$$1 - \delta \leq \lambda_j \leq 1 + \delta \tag{11}$$

for all j, where

$$\lambda_j \equiv [T_j(T_j+1) - T_{3j}{}^2]/2T_{3j}{}^2. \tag{12}$$

It is easy to see that naturality implies Eq. (11). Just choose in Eq. (9) all VEV but one equal to zero.

The converse is also straightforward. From Eqs. (9) and (12) we have

$$\rho^2 = \sum_j \lambda_j T_{3j}{}^2 |v_j|^2 \Big/ \sum_j T_{3j}{}^2 |v_j|^2. \tag{13}$$

It is then easy to convince oneself that Eqs. (11) and (13) imply

$$1 - \delta \leq (\lambda_j)_{\min} \leq \rho^2 \leq (\lambda_j)_{\max} \leq 1 + \delta.$$

The condition (11) provides us with a powerful relation to select phenomenologically allowed models. The smallest values of T_j (T_{3j}) allowed by Eq. (11) are

$\delta = 0$: $\frac{1}{2}$ ($\pm\frac{1}{2}$), 3 (± 2), $\frac{25}{2}$ ($\pm\frac{15}{2}$), . . . ,

$\delta = 0.03$: $\frac{1}{2}$ ($\pm\frac{1}{2}$), 3 (± 2), 10 (± 6), $\frac{25}{2}$ ($\pm\frac{15}{2}$), ...

A look at the above list shows that the most economical (natural) models are those with a Higgs sector made of doublets only. We know however that in this case $K_{WZH} = 0$. Therefore, in order to have $K_{WZH} \neq 0$, the most economical (natural) model should include at least one 7-plet ($T = 3$, $T_3 = \pm 2$) which would contain quintuply charged fields. We conclude, therefore, that a *natural* SU(2) × U(1) model with a WZH^+ vertex would be very unrealistic.

If we abandon the naturality condition, i.e., we allow for multiplets which satisfy Eq. (9) only for a specific choice of VEV's, and insist in keeping one doublet to give masses to the fermions through a Yukawa coupling, we are led to the following discussion.

If $\delta = 0$ it can easily be seen from Eqs. (9) and (10) that at least two different additional multiplets (apart from the doublet) are needed. This implies the existence of two or more physical H^+'s with K_{WZH} given by Eq. (7) for each specific model.

If $\delta \neq 0$ we can have a model with a doublet plus an additional multiplet. If this additional multiplet is of type I there will be only one charged Higgs particle and, in this case, we can compute the coupling exactly. Equation (7) [or Eq. (8)] becomes

$$K_{WZH} = 2\sec\theta_W |T_3 - \tfrac{1}{2}| C v_1^* v_2^* / (|v_1|^2 + C^2 |v_2|^2),$$

where v_1 (v_2) is the VEV of the doublet (additional multiplet), T_3 is the third component of weak isospin of the neutral component of the additional multiplet, and the constant C is equal to $[T(T+1)]^{1/2}$ or $(2T)^{1/2}$ depending on whether the (type I) additional multiplet has $Y = 0$ (self-conjugated) or $Y = T$. The constraint (10) restricts the range of values of the ratio $|v_2/v_1|$ and imposes, after some algebra, the following upper bound to K_{WZH}:

$$K_{WZH} < (\delta/2)^{1/2} \sec\theta_W \quad (Y = 0), \tag{14a}$$

$$K_{WZH} < [(2T-1)\delta]^{1/2} \sec\theta_W \quad (Y = T), \tag{14b}$$

where, as before, T and Y correspond to the additional multiplet. As expected, these couplings are zero when $\delta = 0$. The simplest example of this situation corresponds to a model with a doublet and a triplet. The upper bound (14) for $\delta = 0.03$ is, in this case, $K_{WZH} < 0.15$ (< 0.20) corresponding to a $Y = 0$ ($= 1$) triplet. As shown by Eq. (14a), the upper bound is independent of T for a self-conjugated multiplet. This is not the case when $Y=T$, although a bound of order ~1 would require $T \gtrsim 12$.

If the additional multiplet is of type II, we have again two or more positively charged Higgs particles and K_{WZH} cannot be calculated without specifying the model. The simplest situation of this type occurs with a doublet plus a quartet with $Y=\frac{1}{2}$. In this case the bound (8) and the constraint (10) imply $|K_{WZH_\alpha}|^2 \leqslant 5.6\delta$.

IV. CONCLUSIONS

We have derived the most general expression, given by Eqs. (6) and (7), for the $WZH^{\pm}$ vertex as well as an upper bound [Eq. (8)] to its strength which depends only on the VEV's of the scalar fields and not on the specific form of the Higgs potential. A few points follow from these results.

First of all, in an $SU(2)\times U(1)$ model with a *natural* Higgs sector (i.e., with the ratio M_W/M_Z independent of the VEV's) the WZH vertex does not exist unless we allow for very exotic Higgs-boson representations including fields with electric charge $Q \geqslant 5$. In this respect, we stress the fact that in a standard model with several Higgs doublets we have $K_{WZH^{\pm}}=0$.

If we give up the "naturality" condition (this would be rather unpleasant from the aesthetical point of view) we can construct phenomenologically acceptable models with the WZH vertex. However, the most economical ones (a doublet plus an additional multiplet) give only approximately the standard WS value of $\rho^2 \equiv M_W{}^2/M_Z{}^2 \cos\theta_W$ and the strength of the WZH coupling turns out to be proportional to $\delta^{1/2}$. Any improvement on the experimental error δ of the ratio ρ^2 is, therefore, very desirable.

In any case, we conclude that a value of $K_{WZH^{\pm}} \sim 0$ (1), as it is assumed in the literature,[5] seems too optimistic. Of course, it is possible to construct simple models containing charged Higgs bosons but the above considerations show that, unless we resort to a rather complicated Higgs sector, they do not couple directly to the W and Z bosons. This makes the experimental detection of charged Higgs particles an extremely difficult task.

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