Baryons with strangeness and charm in a quark model with chromodynamics

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The low-lying spectrum of baryons containing a charmed quark and one or two strange quarks is calculated in a a quark model with chromodynamics.

Recent work on low-energy hadronic structure has explored the possibility that quantum chromodynamics (QCD) may provide the framework for a new understanding of hadronic properties. More specifically, a model for baryons[1] which incorporates into the nonrelativistic quark model several ingredients suggested by chromodynamics has had some success in understanding both the spectroscopy[2] and decays[3] of the uncharmed baryonic states. This model has already been extended to include the nonstrange singly charmed baryons[4]; here we complete the discussion of the singly charmed sector by considering *usc*, *dsc*, and *ssc* states.[5] The parameters required for these calculations are completely determined by previously studied sectors so the results are entirely predictive.

As the model employed here has been described extensively elsewhere,[1-4] we provide only a brief summary. The baryon Hamiltonian is assumed to be

$$H=H_0+\sum_{i<j}H^{ij}_{\rm hyp}\,, \tag{1}$$

where

$$H_0=\sum_i\left(m_i+\frac{p_i^2}{2m_i}\right)+\sum_{i<j}V^{ij}_{\rm conf}(r_{ij}) \tag{2}$$

and

$$H^{ij}_{\rm hyp}=\frac{2\alpha_s}{3m_im_j}\left[\frac{8\pi}{3}\,\vec{S}_i\cdot\vec{S}_j\,\delta^3(\vec{r}_{ij})+\frac{1}{r_{ij}^3}\left(\frac{3\vec{S}_i\cdot\vec{r}_{ij}\,\vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^2}-\vec{S}_i\cdot\vec{S}_j\right)\right], \tag{3}$$

where $V^{ij}_{\rm conf}$ is a flavor-independent "confinement" potential and $H^{ij}_{\rm hyp}$ is the magnetic-dipole–magnetic-dipole color hyperfine interaction. If we choose

$$V^{ij}_{\rm conf}=\tfrac{1}{2}Kr_{ij}^2+U(r_{ij})\,,$$

then for $U=0$ the Hamiltonian H_0 is separable; for this reason we elect to do perturbation theory about this limit. In the case of equal mass quarks it has been shown[1,2] that an arbitrary potential $U(r_{ij})$ gives (in first-order perturbation theory) the same three-parameter spectrum for states carrying up to one radial or two orbital excitations. (These states encompass what in SU(6) language would be called the $[56,0^+]$, $[70,1^-]$, $[56',0^+]$, $[70,0^+]$, $[56,2^+]$, $[70,2^+]$, and $[20,1^+]$ supermultiplets. The three potential parameters are then determined from the study of the spectroscopy of the nonstrange sector of these multiplets.) When the quarks have different masses, the spectrum of H_0 is shifted in three ways: (1) The $\sum_i m_i$ term is changed. This effect is calculated by simple addition. (2) The zero-point energy of the ground state is changed. This shift is obtained by treating the change in $\sum_i(p_i^2/2m_i)$ as a perturbation and calculating its expectation value in the equal-mass ground state.[6] (3) Finally, the excitation energies are changed because the effective masses of the normal modes have changed: In the harmonic limit the excitation energies would be multiplied by $(m/m')^{1/2}$ where m (m') is the old (new) effective mass of the mode, and we assume that this scaling law remains approximately valid even when $U\neq 0$.

When all three quark masses are different the problem of reducing H_0 to separable form in the harmonic limit, although trivial, is considerably more tedious than in the case where two masses are equal (the most general case treated previously in this model). However, under a change of variables

$$\vec{y}_i=m_i^{1/2}\vec{r}_i$$

the kinetic-energy term becomes

$$-\sum_{i=1}^{3}\frac{1}{2m_i}\nabla_{r_i}{}^2=-\sum_{i=1}^{3}\tfrac{1}{2}\nabla_{y_i}{}^2\,,$$

which is invariant under orthogonal transformations of the $\vec{y}_i$. Writing $\frac{1}{2}\sum_{i<j}Kr_{ij}^2$ in terms of the $\vec{y}_i$ one obtains a quadratic form which is straightforward to diagonalize and leads to the following form for H_0:

$$H_0=-\frac{1}{2M}\nabla_{\rm c.m.}{}^2+\left(-\frac{1}{2m_+}\nabla_{w_+}{}^2+\tfrac{3}{2}Kw_+^2\right)+\left(-\frac{1}{2m_-}\nabla_{w_-}{}^2+\tfrac{3}{2}Kw_-^2\right),$$

where

$$\vec{r}_{\rm c.m.}=\frac{1}{M}\sum_i m_i\vec{r}_i\,,$$

$$M=\sum_i m_i\ ,$$

$$\vec{w}_\pm = m_\pm^{-1/2}(1+f_\pm^2+g_\pm^2)^{-1/2}\times(m_1^{1/2}\vec{r}_1+m_2^{1/2}f_\pm\vec{r}_2+m_3^{1/2}g_\pm\vec{r}_3)\ ,$$

$$m_\pm=\left(\frac{\sum_i m_i^{-1}\pm A}{3}\right)^{-1},$$

in which

$$A=\left(\sum_i\frac{1}{m_i^2}-\sum_{i<j}\frac{1}{m_im_j}\right)^{1/2},$$

$$f_\pm=\frac{\dfrac{1}{m_1}-\dfrac{1}{2m_2}-\dfrac{1}{2m_3}\mp\dfrac{A}{2}}{\left(\dfrac{1}{m_1m_2}\right)^{1/2}-\dfrac{1}{2}\left(\dfrac{m_2}{m_1^3}\right)^{1/2}-\dfrac{1}{2}\left(\dfrac{m_2}{m_1m_3^2}\right)^{1/2}\mp\dfrac{1}{2}\left(\dfrac{m_2}{m_1}\right)^{1/2}A},$$

$$g_\pm=(m_1m_3)^{1/2}\left(\frac{1}{m_1}-\frac{1}{m_2}-\frac{1}{m_3}\mp A\right)-\left(\frac{m_3}{m_2}\right)^{1/2}f_\mp\ .$$

In terms of the normal coordinates $\vec{w}_\pm$ one may immediately write down the eigenfunctions of H_0 in the harmonic limit, subsequently approximating their energies in the presence of a nonzero U by the methods outlined above.

The three relevant flavor wave functions for this system are $|usc\rangle$, $|dsc\rangle$, and $|ssc\rangle$. In the first two cases there is no symmetry between the three particles so that even the "*uds* basis"[1,2] becomes inappropriate and the overall wave functions are without any special permutational symmetry, while in the third case the overall wave function must be, as in the *uds* basis, antisymmetric in the first two quarks. One may in any case choose the standard spin wave functions

$$\chi_+^\rho=\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)\ ,$$

$$\chi_+^\lambda=-\frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow)\ ,$$

etc., for spin $\frac{1}{2}$ and

$$\chi_{3/2}^s=\uparrow\uparrow\uparrow\ ,$$

etc., for spin $\frac{3}{2}$, but only in the $|ssc\rangle$ sector where the spatial wave functions in $\vec{w}_\pm$ collapse into wave functions of the ρ and λ type[1,2] will these spin wave functions diagonalize the Hamiltonian. Thus in the absence of hyperfine interactions the ground states of H_0 may be taken to be

$$|usc\rangle_\rho^{00}\equiv|usc\rangle\chi^\rho\psi_{00}\ ,$$

$$|usc\rangle_\lambda^{00}\equiv|usc\rangle\chi^\lambda\psi_{00}\ ,$$

$$|usc\rangle_s^{00}\equiv|usc\rangle\chi^s\psi_{00}\ ,$$

with similar states for $|dsc\rangle$, and

$$|ssc\rangle_\lambda^{00}\equiv|ssc\rangle\chi^\lambda\psi_{00}\ ,$$

$$|ssc\rangle_s^{00}\equiv|ssc\rangle\chi^s\psi_{00}\ .$$

TABLE I. The masses of some low-lying *usc*, *dsc*, and *ssc* baryons. These masses are based on a value for the charmed-quark mass which is 15 MeV heavier than that used in Refs. 4 and 8; if the Λ_c mass is really 2260 MeV rather than 2275 MeV then all these states will be 15 MeV lighter.

State(s) [a]	Mass or mass difference (MeV)
usc ground states	
$(usc)_a\ \frac{1}{2}^+$	2495
$(usc)_a\ \frac{1}{2}^+-(dsc)_a\ \frac{1}{2}^+$	−3.4
$(usc)_b\ \frac{1}{2}^+$	2590
$(usc)_b\ \frac{1}{2}^+-(dsc)_b\ \frac{1}{2}^+$	−1.1
$(usc)\ \frac{3}{2}^+$	2660
$(usc)\ \frac{3}{2}^+-(dsc)\ \frac{3}{2}^+$	−2.4
$(usc)\ \frac{1}{2}^-$	2760, 2845, 2880,. . .
$(usc)\ \frac{1}{2}^+$	2815, 2885,. . .
$(usc)\ \frac{3}{2}^-$	2760, 2880,. . .
$(usc)\ \frac{3}{2}^+$	2970,. . .
$(usc)\ \frac{5}{2}^-$	2915,. . .
ssc ground states	
$(ssc)\ \frac{1}{2}^+$	2745
$(ssc)\ \frac{3}{2}^+$	2805
$(ssc)\ \frac{1}{2}^-$	3015, 3040,. . .
$(ssc)\ \frac{1}{2}^+$	3020,. . .
$(ssc)\ \frac{3}{2}^-$	3030, 3065,. . .
$(ssc)\ \frac{3}{2}^+$	3090,. . .
$(ssc)\ \frac{5}{2}^-$	3050,. . .

[a] $(usc)_{a(b)}$ is dominantly the state $|usc\rangle_{\rho(\lambda)}^{00}$.

When the hyperfine interactions are turned on they shift these states and also cause some mixing between $|usc\rangle_\rho$ and $|usc\rangle_\lambda$. Similar effects occur in excited states. After including all hyperfine shifts and "nearest-neighbor" interband as well as intraband mixings, we find the results of Table I where we have listed only the ground states plus a few of the most accessible excited states in an effort to be realistic. Note that, following Ref. 7, we have calculated the expected isomultiplet mass differences in these states.

It can be seen from Table I that both $(usc)\,\frac{1}{2}^+$ states should be stable against strong decays, while the $(usc)\,\frac{3}{2}^+$ at 2660 MeV should just barely be able to decay by pion emission (with a width of approximately 3 MeV) to the *usc* ground state. On the other hand, we predict an unusual situation in the *ssc* states: Both the analog of the N and the Δ should be stable against strong decays. This comes about because the 300-MeV Δ-N splitting has been almost entirely eroded by the weakened chromomagnetic moments of the c and s quarks. In addition to these states we show in the table a few of the lowest-lying excited states. Those in the *usc* sector may be difficult to detect since their allowed strong decays have considerable phase space. In the *ssc* sector, however, the single-pion decay is forbidden and the two-pion and $\bar{K}$-$(usc)_a\frac{1}{2}^+$ channels are just barely open for the lowest-lying states which should, therefore, be narrow and thereby possibly more easily detectable.

In summary, we have calculated in terms of a model that was completely determined by other baryonic sectors the spectrum of low-lying states of the *usc*, *dsc*, and *ssc* systems. In addition to the intrinsic interest of observing such states, our calculation shows that they will provide tests of several of the features of QCD-based quark models, especially the flavor independence of confinement and the assumed characteristics of the color hyperfine interaction.

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[1]This model has now been reviewed in several places. See Nathan Isgur, in *New Aspects of Subnuclear Physics*, proceedings of the XVI International School of Subnuclear Physics, Erice, Italy, 1978, edited by A. Zichichi (Plenum, New York 1980); Gabriel Karl, in *Proceedings of the XIX International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), p. 135; O. W. Greenberg, Ann. Rev. Nucl. Part. Phys. 28, 327 (1978); A. J. G. Hey, presented at the 1979 EPS International Conference on High Energy Physics; and Southampton report, 1979 (unpublished).

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[3]Roman Koniuk and Nathan Isgur, Phys. Rev. Lett. 44, 845 (1980); Phys. Rev. D 21, 1868 (1980).

[4]L. A. Copley, Nathan Isgur, and Gabriel Karl, Phys. Rev. D 20, 768 (1979).

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[6]This "perturbation" may be rather large when the states in question contain charmed quarks. However, errors incurred in this treatment tend to be hidden by the choice of the charmed quark mass. We estimate that the net error in this procedure is less than about 10 MeV.

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