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Inclusive semileptonic decays of heavy mesons

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We discuss the deficiency of the standard quark-parton model for estimation of lifetime of heavy mesons. The inclusive semileptonic decays of heavy mesons are investigated in detail to show difficulties which the quark-parton model embodies.

The main purpose of this paper is to question the validity of the quark-parton model for estimating lifetime of heavy mesons. We will discuss in detail the inclusive semileptonic decays which are easier to handle than the nonleptonic decays, in order to present difficulties that the quark-parton model causes. We show that the simple estimate by Gaillard, Lee, and Rosner[1] for the inclusive semileptonic decay rate should be modified, but the modification worsens the situation. The experimental observation of the lifetime difference between D^+ and D^0 (Ref. 2) also indicates that the use of the quark-parton picture is doubtful.

The weak-interaction Hamiltonian responsible for the semileptonic decays can be written as

$$H_w = \frac{G_F}{\sqrt{2}} J_\mu(x)\, L^\mu(x) + \text{H.c.}, \tag{1}$$

where $J_\mu(x)$ $[L_\mu(x)]$ denotes the hadronic [leptonic] charged current. The inclusive semileptonic decay rate for $M \to Xl\bar{\nu}$ (X denotes "anything") is given by

$$\Gamma = \frac{G_F^2}{2}\frac{1}{(2\pi)^2}\frac{1}{2s+1}\int d^4k\, \delta(k^2)\,\theta(k_0)\int d^4k'\,\delta(k'^2)\,\theta(k_0')\,\delta^4(P_X + k + k' - P)$$
$$\times\, l^{\mu\nu}\sum_X \frac{1}{2M}\langle P|J_\nu^\dagger(0)|X\rangle\langle X|J_\mu(0)|P\rangle\,, \tag{2}$$

where

$$l_{\mu\nu} = \sum_{\text{spin}} \langle 0|L_\nu^\dagger(0)|l\bar{\nu}\rangle\langle l\bar{\nu}|L_\mu(0)|0\rangle\,. \tag{3}$$

The momentum variables P_μ and Q_μ denote the momentum of the heavy meson with mass M and that of the lepton pairs, respectively, while k_μ and k'_μ are those of the charged lepton l and the corresponding neutrino ν (see Fig. 1).

We introduce the hadronic tensor $W_{\mu\nu}$ by

$$W_{\mu\nu} = \frac{1}{2s+1}\frac{1}{2M}\sum_X (2\pi)^3\,\delta^4(P_x + Q - P)\,\langle P|J_\nu^\dagger(0)|X\rangle\langle X|J_\mu(0)|P\rangle$$
$$= \frac{1}{2s+1}\frac{1}{2M}\frac{1}{2\pi}\int d^4x\, e^{iQ\cdot x}\langle P|[J_\nu^\dagger(x), J_\mu(0)]|P\rangle$$
$$= \left(-g_{\mu\nu} + \frac{Q_\mu Q_\nu}{Q^2}\right) W_1 + \left(P_\mu - \frac{P\cdot Q}{Q^2}Q_\mu\right)\left(P_\nu - \frac{P\cdot Q}{Q^2}Q_\nu\right)\frac{W_2}{M^2} + i\epsilon_{\mu\nu\alpha\beta}\,Q^\alpha P^\beta\,\frac{W_3}{2M^2}\,. \tag{4}$$

Note that the weak hadronic current $J_\mu(x)$ can be regarded as divergenceless, since the leptonic current L_μ satisfies the conservation law $\partial^\mu L_\mu = 0$, neglecting small lepton masses. Thus, we can ignore all terms proportional to Q_μ or Q_ν. This hadronic tensor is the same as the one we see in the deep-inelastic scatterings of lepton by nucleons, with one exception: In our case Q is *timelike*.

The decay width can be expressed in terms of the structure functions W_1 and W_2,

$$\Gamma = \frac{G_F^2}{3\times 2^3\pi^3}\int dQ^2\, d\nu\, \theta(Q^2)\,\theta(\nu)\,(\nu^2 - Q^2)^{1/2}$$
$$\times [3Q^2 W_1 + (\nu^2 - Q^2)W_2], \tag{5}$$

where $\nu = P\cdot Q/M$ and we have neglected lepton masses. The reason we do not have W_3 in the

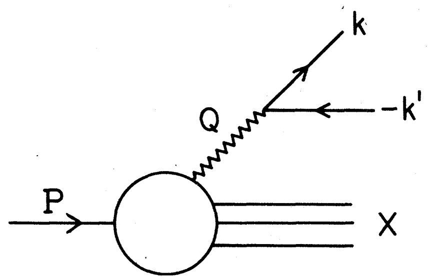

FIG. 1. Inclusive semileptonic decay.

total decay width, in contrast to the case of deep-inelastic scatterings, is that the lepton-part integral yields

$$\theta(Q^2)\ \theta(\nu)\ (Q_\mu Q_\nu - g_{\mu\nu}Q^2)\ , \tag{6}$$

which leads to the vanishing contribution from W_3. Thus, in principle, we can[3] measure the structure functions of hadrons by careful observations of the ν^2 dependence (which gives W_2) and the Q^2 dependence (which gives $3W_1 - W_2$) for the inclusive semileptonic decays, provided that Q^2 available to the decay is large enough. We can also measure[3] W_3 from angular correlations between k and Q.

We now present the following argument: Because of the high mass of heavy mesons, the momentum transfer Q_μ in Eq. (4) will be large, although Q^2 is limited by M^2. Therefore, the integral in the second line of Eq. (4) will be dominated by short distances. Using the Wilson expansion for the commutator $[J_\nu^\dagger(x),\ J_\mu(0)]$ we will have the standard quark-parton picture, neglecting quantum-chromodynamic corrections due to gluons. This observation would justify the use of the quark-parton model.

We introduce the Feynman scaling variable x by (see Fig. 2)

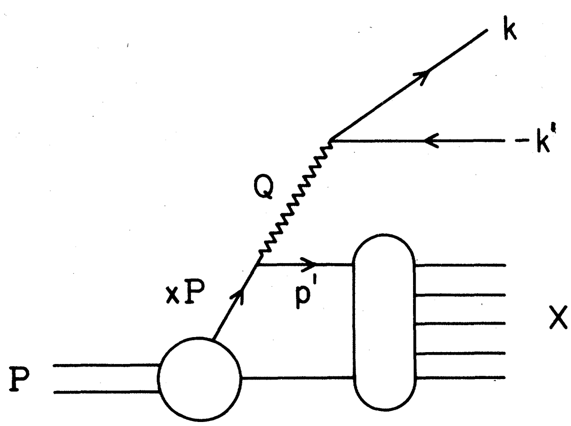

FIG. 2. The quark-parton picture of inclusive semileptonic decay.

$$Q_\mu = xP_\mu - p'_\mu\ ,\qquad (p')^2 = m'^2\ . \tag{7}$$

The rest of the quarks in the decay process are assumed to be massless. Moreover, we will have

$$MW_1 = C(x)\ , \tag{8}$$

$$W_2 = \frac{2M\ x}{P\cdot p'}\ C(x)\ , \tag{9}$$

where $C(x)$ denotes the probability function of finding a heavy quark with four-momentum xP_μ inside the heavy meson. Then, the decay rate becomes

$$\Gamma = \Gamma_0 \int_\alpha^1 dx\ x^5\ C(x) \left(1 - 8\frac{\alpha^2}{x^2} + 8\frac{\alpha^6}{x^6} - \frac{\alpha^8}{x^8} - 24\frac{\alpha^4}{x^4}\ \ln\frac{\alpha}{x}\right)\ , \tag{10}$$

where

$$\Gamma_0 = \frac{G_F^2 M^5}{192\ \pi^3} \tag{11}$$

and

$$\alpha = m'/M\ . \tag{12}$$

The choice where $C(x) = \delta(1-x)$ and M = (heavy-quark mass) yields the standard result of the simplest quark-parton picture by Gaillard, Lee, and Rosner.[1] However, this structure function is rather unrealistic, since it tells us that all the momentum is carried by a heavy quark. Moreover, the experimentally observed pion structure function $F_2^\pi(x)$ is flat.[4]

The more reasonable choices, for example, give

$$\Gamma = 0.02\ \Gamma_0 \quad \text{for } C(x) = \frac{3}{4}\frac{1-x}{\sqrt{x}}\ , \tag{13}$$

$$\Gamma = 0.17\ \Gamma_0 \quad \text{for } C(x) = 1\ , \tag{14}$$

where we have assumed $m' = 0$. For the case where $m' \neq 0$, we will obtain smaller values than those in Eqs. (13) or (14). A similar result has been obtained by Pham and Nabavi,[3] although we disagree with them about the proper definition of x. Usually it is hard to distinguish the various forms of structure function $C(x)$, but in our case, the x^5 factor is very good at discriminating the power of $(1-x)$: For example, $C(x) = \frac{15}{16}[(1-x)^2/\sqrt{x}]$ gives only $\frac{1}{3}$ of Eq. (13). The smaller the decay width is, the more the power to the $(1-x)$ is.

We argue that such small values for Γ as in Eqs. (13) or (14) are incompatible with reality. The reason is as follows: The exclusive mode $0^-(M) \to 0^-(m)l\nu$ can be estimated rather reliably:

$$\Gamma(0^- \to 0^- l\nu) = \Gamma_0\ \tfrac{1}{4}\ |f_+|^2[1 - 8\alpha^2 + 8\alpha^6 - \alpha^8 - 24\alpha^4 \ln\alpha]\ , \tag{15}$$

where $\alpha = m/M$. For the case where $\alpha = 0.3$ (which corresponds to the mass ratio of kaon to D meson or D meson to B meson), we have

$$\Gamma(0^- \to 0^- l\nu) = 0.13\ |f_+|^2\ \Gamma_0\ , \tag{16}$$

which is in conflict with Eq. (13), since the inclusive width has to be larger than the exclusive width and we expect the form factor f_+ to be near one. Unless the form factor is much smaller than one, the naive counting structure function is *unacceptable* for heavy mesons.

One may think that the use of the Q^2-dependent form factor will decrease the estimate, but this is wrong. For example, the form factor for D meson will take the F^*-pole-dominant form

$$f_+(Q^2) \sim \frac{m_{F^*}{}^2}{m_{F^*}{}^2 - Q^2} \tag{17}$$

for any Cabibbo-favored decay, such as $D^+ \to K^0 \bar{l}\nu$. The problem is that our Q^2 lies in the timelike range $0 < Q^2 < m_D{}^2$. Thus the introduction of the Q^2-dependent form factor has a tendency to increase the naive estimate, instead of decreasing it.

Note that the main contribution for the integral comes from the larger x region, because of the x^5 factor. Thus the larger-Q^2 region contributes to the integral. Even though our integration extends to $Q^2 = 0$, our estimate is good as long as the parent hadron is heavy enough to provide large Q^2, which is expected to be more than (1 GeV)^2. Consequently, one of the following is true: Either the structure functions of heavy mesons are flatter than the naive counting prediction, or the quark-parton picture is not valid at all.

If we use the experimental fact[5] that the exclusive mode, $D \to Ke\nu$ is around 40–50% of the total semileptonic decays of $D \to e\nu X$, Eq. (16) indicates the inclusive width as $\Gamma \sim 0.3\ \Gamma_0$. This value is again[6] in conflict with Eq. (13).

The symmetry argument can be used in the exclusive modes and provides interesting properties. The SU(2) invariance alone guarantees

$$\Gamma(D^0 \to K^- \bar{l}\nu) = \Gamma(D^+ \to \bar{K}^0\ \bar{l}\nu). \tag{18}$$

Similarly, SU(3) invariance together with the quark-line rule[7] demands the following for Cabibbo-favored decays:

$$\Gamma(F^+ \to \eta\ \bar{l}\nu) = \tfrac{1}{3}(\sqrt{2}\ \cos\theta_P + \sin\theta_P)^2\ \Gamma(D^0 \to K^-\ \bar{l}\nu), \tag{19a}$$

$$\Gamma(F^+ \to \eta' \bar{l}\nu) = \tfrac{1}{3}(\cos\theta_P - \sqrt{2}\ \sin\theta_P)^2\ \Gamma(D^0 \to K^-\ \bar{l}\nu)\ , \tag{19b}$$

$$\Gamma(F^+ \to \phi\ \bar{l}\nu) = \Gamma(D^0 \to K^{*-}\ \bar{l}\nu) = \Gamma(D^+ \to \bar{K}^{*0}\ \bar{l}\nu)\ , \tag{19c}$$

$$\Gamma(F^+ \to \omega\ \bar{l}\nu) = \Gamma(F^+ \to \rho^0\ \bar{l}\nu) = 0\ , \tag{19d}$$

where θ_P is the mixing angle between η-η' complex with the value of $\theta_P \sim -20^\circ$ (-10°) for the linear (quadratic) mass mixing and we have assumed ideal mixing for ω-ϕ complex. Furthermore, for the inclusive semileptonic decays, SU(2) symmetry alone requires (independent of the quark-parton model)

$$\Gamma(D^0 \to \bar{l}\nu + \text{anything}) = \Gamma(D^+ \to \bar{l}\nu + \text{anything})\ , \tag{20}$$

which has been noted by many authors. We have neglected the Cabibbo-suppressed contributions. However, the equality

$$\Gamma(F^+ \to \bar{l}\nu + \text{anything}) = \Gamma(D^+ \to \bar{l}\nu + \text{anything}) \tag{21}$$

is not guaranteed by SU(3) symmetry alone, unless we use any one of the following three: a generalization of the quark-line rule,[7] the quark-parton model, or the Pomeron-exchange model (which will be explained below).

The reason why the quark-parton model may not work is not difficult to find: The short-distance expansion for the commutator $[J_\nu^\dagger(x),\ J_\mu(0)]$ is valid only in the limit where $Q_\mu \to \infty$. However, in our case, Q^2 is limited by the parent mass M^2. Thus, we have a situation where nonleading higher-twist terms may not be negligible. Or, the variables Q^2 and ν are not large enough to use the Wilson expansion, even though Q^2 reaches up to M^2. It may happen that main contributions come from the kinematical region where ν is large, but Q^2 is moderate, say order of a few hundred MeV^2, instead of GeV^2. This region corresponds to the Reggeon-exchange domain. Thus we are led to consider the contributions from the Regge region.

We dominate the current $J_\mu(x)$ by F^*. Assuming the equality of the F^*-Pomeron and the photon-Pomeron coupling, we roughly estimate

$$\Gamma = \frac{G_F{}^2}{3\times 2^3\pi^3}\int dQ^2 \int^M d\nu\ \nu 2Q^2\ \frac{2\nu}{4\pi^2\alpha}\ \sigma^{\gamma M}_{\text{total}}$$

$$= \Gamma_0\ \frac{4}{3\pi^2\alpha}\ \frac{\langle Q^2\rangle^2}{M^2}\ \sigma^{\gamma M}_{\text{total}}\ , \tag{22}$$

where we have assumed that VV contributions are equal to AA contributions, and $\langle Q^2\rangle$ denotes the average value of Q^2. The factorization of Regge residues and the real-photoproduction data yield an estimate of

$$\sigma^{\gamma M}_{\text{total}} \sim 0.067\ \text{mb}. \tag{23}$$

Thus,

$$\Gamma \sim 0.27\Gamma_0 \quad \text{for } \langle Q^2\rangle = 0.5\ \text{GeV}^2 \text{ and } M = 1.86\ \text{GeV}, \tag{24}$$

$$\Gamma \sim 0.13\Gamma_0 \quad \text{for } \langle Q^2\rangle = 1\ \text{GeV}^2 \text{ and } M = 5\ \text{GeV}. \tag{25}$$

For smaller values of $\langle Q^2\rangle$, the decay width will be even smaller. Although ambiguities exist, it is clear that the width tends to be *smaller* than the most naive estimate Γ_0, just like what we have found in the quark-parton model. Therefore, we doubt the validity of the most naive estimate $\Gamma\sim\Gamma_0$.

We also note that the popular objection to the use of the quark-parton model is as follows: The semileptonic decay modes are mainly limited to pole contributions, since PCAC (partial conservation of axial-vector current) tells us that soft-pion emission is reduced to the one with commutator of axial charge with hadronic current and for heavy mesons their hadronic current is singlet (the main part determined by the Kobayashi-Maskawa angles), and therefore, the soft-pion emission matrix elements vanish. This argument might be verified by data on D mesons: Experimentally, semileptonic decays for D mesons are almost saturated by $Kl\nu$ and $K^*l\nu$ (or $K\pi l\nu$).[5] They claim that contributions from many-body modes are very small. Pole dominance completely devastates the situation, since we cannot use either the quark-parton model or the Pomeron-exchange model. Pole dominance in the s channel indicates exchanges of f and f' Regge trajectories (not the Pomeron) in the t channel, because of duality.[8] The non-Pomeron exchange will yield much smaller values, compared with Eqs. (24) or (25), since the f (or f') will not couple with F^* (or D^*) mesons.

In summary, we feel that both the quark-parton approach and the Regge-exchange model are not suitable for the inclusive semileptonic decays of heavy mesons, although both give similar values. The quark-parton model will not work *much less* for nonleptonic decays. The reason is as follows: The inclusive nonleptonic decay width is given by

$$\Gamma=\frac{(2\pi)^4}{2M}\sum_X \delta(P_X-P)\langle P|H_w(0)|X\rangle\langle X|H_w(0)|P\rangle$$
$$=\int\frac{d^4x}{2M}\langle P|[H_w(x),H_w(0)]|P\rangle\,, \qquad (26)$$

where $H_w(x)$ is the weak Hamiltonian responsible for nonleptonic decays. In contrast to the case of semileptonic decays, Eq. (4), there is no e^{iQx} factor in the integrand. Therefore, we are no longer able to justify even the use of the short-distance expansion for the commutator $[H_w(x), H_w(0)]$. This implies that the simplest decay-width estimate by Gaillard, Lee, and Rosner[1] is doubtful, in spite of its attractive simplicity. Especially, there is no compelling reason why the lifetimes of D^0 and D^+ mesons should be the same. Actually, many people[9-11] have proposed modifying the simple quark-parton picture in order to accommodate the lifetime difference. However, it is interesting to note that the SU(3) invariance alone implies[12]

$$\Gamma(D^0\to\text{hadrons})=\Gamma(F^+\to\text{hadrons}) \qquad (27)$$

if the nonleptonic Hamiltonian H_w has the customary SU(4) 20-plet dominant structure.

Finally, the F^* dominance of the hadronic current $J_\mu(x)$ for Cabibbo-favored modes leads to the form factor

$$\left(\frac{m_{F^*}{}^2}{m_{F^*}{}^2-Q^2}\right)^2,$$

while D^* dominance for Cabibbo-suppressed modes yields a larger enhancement factor

$$\left(\frac{m_{D^*}{}^2}{m_{D^*}{}^2-Q^2}\right)^2$$

in the case of charmed-meson semileptonic decays. It is interesting to see such effect in data, although it will be difficult.

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