A new look at the A_1, B, $Q_1(1280)$, and $Q_2(1400)$ meson system. Derivation of mass formulas and selection rules

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A new study of the A_1, B, $Q_1(1280)$, and $Q_2(1400)$ mesons is carried out in the presence of large SU(3) mixing between the Q mesons. The theoretical framework used is a purely algebraic one which expresses the dynamics of confined quarks through chiral SU(3) $\otimes$ SU(3) charge algebras, related exotic-charge commutators characterizing symmetry breaking, asymptotic SU(3) symmetry, and asymptotic level realization of SU(3) in the chiral SU(2) $\otimes$ SU(2) charge algebra $[A_{\pi^+},A_{\pi^-}]=2V_{\pi^0}$. A simple mass formula $Q_2{}^2-B^2=Q_1{}^2-A^2$ is derived which predicts the mass of A_1 around 1.1 GeV with $\Gamma(A_1\to\rho\pi)\simeq 300$ MeV. The SU(3) mixing angle θ of the Q mesons is also derived in terms of the observable masses. The sum rules obtained exhibit a close interplay among the masses, θ, and asymptotic axial-vector matrix elements. There exists an "ideal angle" $\theta=30°$ for which the couplings $Q_2(1400)\to\rho K$ and ωK become forbidden as observed by experiment. $\theta=30°$ is also found to be compatible with the mass spectrum of the A_1, B, Q_1, and Q_2 system. The $Q_2(1400)$, rather than the $Q_1(1280)$, turns out to be the SU(3) counterpart of $A_1(1100)$, suggesting that an inversion in the mass ordering took place through mixing.

I. INTRODUCTION

There has been a long history of searches for the axial-vector mesons. From the naive quark model one expects (as $L=1$, $q\bar{q}$ states) two nonets with $J^{PC}=1^{++}$ and 1^{+-}. While the $I=1$ 1^{+-} meson B is well established, the status of the $I=1$ 1^{++} meson called A_1 has been obscure for a long time. However, recent observation[1] of an enhancement of the $\rho\pi$ mass distribution in the decay of heavy lepton $\tau\to\nu_\tau\rho\pi$ provides fresh evidence for the A_1 resonance with a mass around 1100 MeV and a large width. It has recently been stressed[2] that this evidence is not inconsistent with results of the diffraction data. In a recent search for backward production of 1^{++} mesons in the K^-p interactions at 4.2 GeV (for which the Deck background is claimed to be less important), evidence for the A_1 as well as the B meson has been reported. The A_1 parameters found were[3] mass $=1040\pm13$ MeV and width $=230\pm50$ MeV. The overall impression is that although the A_1 parameters have not yet been precisely established,[4] the A_1 resonance exists as predicted.

A new development seems to have been taking place also for the $I=\frac{1}{2}$ counterparts of the A_1 and B meson, denoted as Q_1 and Q_2. The analysis of the diffractive $K\pi\pi$ system by the SLAC group found evidence for two Q mesons[5] and a comprehensive and illuminating summary was recently given by Leith.[6] The masses and widths of the $Q_1(1280)$ and $Q_2(1400)$ quoted by Leith are $Q_1=1290\pm25$ and $Q_2=1400\pm10$ MeV, and $\Gamma(Q_1)=210\pm80$ and $\Gamma(Q_2)=190\pm65$ MeV. A striking indication is that some rather unexpected selection rules are involved among the decays of these Q's. The higher state Q_2 couples strongly with the $K^*\pi$ channel $[\Gamma(Q_2\to K^*\pi)\simeq154\pm52$ MeV$]$, but is *strongly* decoupled[6] from the ρK and ωK channels $[\Gamma(Q_2\to\rho K)\simeq2\pm1$ MeV and $\Gamma(Q_2\to\omega K)\simeq0]$. Although the suppression may not be so conspicuous as in the case of the $Q_2\to\rho K$ (and ωK) modes, the $Q_1\to K^*\pi$ decays are also suppressed[6] relative to the $Q_1\to\rho K$ mode. A unitary, analytic, coupled-channel reanalysis of the same data by Basdevant and Berger[7] is also compatible with the above interpretation of the data. They found $Q_1=1.28\pm0.02$ GeV, $70<\Gamma(Q_1)<140$ MeV, and $Q_2=1.42\pm0.06$ GeV, $\Gamma(Q_2)=230\pm50$ MeV. The Q_1 is coupled mainly to ρK and the Q_2 to $K^*\pi$. The Amsterdam-CERN-Nijmegen-Oxford (ACNO) collaboration (Ref. 3) which observed the A_1 as well as the B meson also found some evidence for $Q_1(1280)$. They found[3,8] $Q_1=1275\pm10$ MeV, $\Gamma(Q_1)=75\pm15$ MeV, and $\Gamma(Q_1\to\rho K)=57\pm14$, $\Gamma(Q_1\to K^*\pi)=14\pm11$, and $\Gamma(Q_1\to\omega K)=4\pm4$ MeV. Therefore, except for fine details (branching ratios and widths), the SLAC and ACNO data on $Q_1(1280)$ seem to be in qualitative agreement.

From the naive picture of SU(3) symmetry, the $Q_2\to\rho K$ and ωK amplitudes, for example, should be comparable in magnitude with the $Q_2\to K^*\pi$ amplitude. The quark-line selection rule is of no help in forbidding, for example, the $Q_2\to\rho K$ and ωK decays relative to the $Q_2\to K^*\pi$ decay. We therefore need to find a more sophisticated explanation.

Since SU(3) symmetry is certainly broken, SU(3) particle mixing can take place between the $I=\frac{1}{2}$ SU(3) counterparts of the A_1 and B meson, called Q_A and Q_B, respectively. The observed two states Q_1 and Q_2 will become a superposition of these (hypothetical) SU(3) states Q_A and Q_B. In the framework of conventional treatment of broken SU(3)—"exact SU(3) plus mixing" for the physical couplings—it then becomes possible to derive selection rules which forbid *both* the $Q_2 \to \rho K(\omega K)$ and $Q_1 \to K^*\pi$ decays. For this the Q_A-Q_B mixing angle θ is required to be 45° and a certain condition[6,8] ($g_A/g_B=-3/\sqrt{5}$) has to be met for the ratio of the two independent D- and F-type $1^+ \to 1^- + 0^{-+}$ couplings, for example, the $A_1\rho\pi$ (g_A) and $B\omega\pi$ (g_B) couplings. By fitting the observed rates of various decay modes of the Q's with the conventional SU(3) recipe, Leith obtained[6] a SU(3) fit with a value $\theta = 41 \pm 4°$. The result seems to demonstrate beyond doubt the presence of large Q_A-Q_B mixing. However, there is, so far no further independent argument (observable mass formulas, etc.) which *substantiates* the value of the mixing angle found.

As to the value of θ which is determined solely on the basis of SU(3) fit, one probably needs to be more open-minded. First, there is *no a priori* theoretical reason which justifies the conventional recipe for the SU(3) parametrization of the physical couplings. Second, broad resonances are involved both in the parent and daughter particles of the decays under consideration. For example, for the decay $Q_1(1280) \to \omega K$ the threshold for the decay occurs within the width of the parent resonance, distorting its line shape and, therefore, affecting the determination of the average momentum $\langle q \rangle$ available in the decay. Another difficulty, which is apparently more serious,[8] is associated with the arbitrariness in determining $\langle q \rangle$ when one of the decay products is a resonance. The SLAC group used the recipe which the ACNO group calls "undistorted" phase-space factor. The ACNO group also studied the recipe called "physical" phase factor. Although these alternative definitions are identical for parent resonances far above the decay channel threshold, they can differ[8] by a factor of 2 for $Q_1 \to \rho K$ or $Q_1 \to \omega K$.

An independent determination of the angle θ by SU(3) fit was recently attempted[8] by the ACNO group based on the branching ratios of the $Q_1(1280)$ decays as well as the rates of the $A_1\rho\pi$ and $B\omega\pi$ decays. Using the "undistorted" phase-space factor, they found that the parameters obtained by their analysis can also fit the feature of SLAC Q_2 well, especially the very small rates of the $Q_2 \to \rho K$ and ωK decays. This indicates[8] a satisfactory common interpretation of both the ACNO and SLAC data in terms of the SU(3) mixing scheme. However, the mixing angle in the fit to the ACNO data is $27 \pm 8°$, which is *less than the complete decoupling angle* 45°.

A simple dynamical model of Q_A-Q_B mixing which produces, in its simplest treatment, the 45° mixing and the *complete* decoupling of the two Q mesons from either the $K^*\pi$ or $K\rho$ channels was, in fact, proposed earlier by Lipkin.[9] However, a more detailed treatment revealed that the result is sensitive to the relative amplitude of the phase of the S to D waves of the $Q_1, Q_2 \to K^*\pi$ and ρK decays. In particular, for the ratios of S- to D-wave amplitudes predicted by the $SU(6)_W$ quark model, the Q_A-Q_B mixing angle was actually found to become zero. Therefore, a more sophisticated treatment becomes necessary.[9]

In this paper we study the same problem from a purely algebraic theoretical framework which expresses the dynamics of confined quarks through chiral SU(3)⊗SU(3) charge algebras, related exotic commutators which characterize symmetry breaking, the hypothesis[10] of asymptotic SU(3), and asymptotic level realization of SU(3) in the chiral SU(2)⊗SU(2) charge algebra $[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0}$.

We find two possible solutions for the set of constraints obtained in this theoretical framework. One of them requires no mixing ($\theta=0$). This case was already studied[11] by Laankan and Oneda in 1973. The other solution, which is the central topic of this paper, produces a large mixing and lends to a simple but rather surprising result, suggesting that an inversion takes place in the mass spectrum of Q mesons through mixing. It also demonstrates that dynamical selection rules exist for the Q-meson decays, $Q_2(1400) \to \rho K$ and ωK.

II. DERIVATION OF SUM RULES FROM REALIZATION OF CHIRAL SU(3)⊗SU(3) CHARGE ALGEBRAS

To cope with broken SU(3) symmetry, we use the concept of asymptotic SU(3) symmetry.[10] The Q_A-Q_B mixing angle θ is introduced (in the asymptotic limit $\vec{k} \to \infty$) among the creation and annihilation operators of the two *physical* Q mesons Q and Q' [we do *not* yet specify whether $Q \equiv Q_1(1280)$ and $Q' \equiv Q_2(1400)$, or $Q \equiv Q_2(1400)$ and $Q' \equiv Q_1(1280)$] and those of the hypothetical SU(3) states Q_A and A_B as follows ($\vec{k} \to \infty$):

$$\begin{aligned} a_Q(\vec{k},\lambda) &= \cos\theta\, a_{Q_A}(\vec{k},\lambda) + \sin\theta\, a_{Q_B}(\vec{k},\lambda)\,, \\ a_{Q'}(\vec{k},\lambda) &= -\sin\theta\, a_{Q_A}(\vec{k},\lambda) + \cos\theta\, a_{Q_B}(\vec{k},\lambda)\,. \end{aligned} \tag{1}$$

λ denotes the helicity and $\vec{k}$ the momentum. With the use of asymptotic SU(3) and the chiral charge algebras $[V,V]=V$ and $[V,A]=A$, which are valid in broken SU(3) symmetry, parametrizations of the *asymptotic* matrix elements of the vector (V_α) and axial-vector (A_α) charges (but *not* the coupling constants) in terms of the conventional recipe—exact SU(3)-plus mixing—are justified in the present theory.

The SU(3) breakings are (algebraically) characterized[10] by the presence of exotic commutators involving $\dot{V}=dV/dt$,

$$[\dot{V}_\alpha, V_B]=0 \tag{2}$$

and

$$[\dot{V}_\alpha, A_\beta]=0, \tag{3}$$

where (α,β) is the *exotic* combination of the physical SU(3) indices (K^0,K^0), (K^0,π^-), ..., etc. Equation (2) expresses the usual assumption of SU(3) breaking and produces, when combined with asymptotic SU(3), the quadratic Gell-Mann–Okubo (GMO) mass formula including SU(3) particle mixing, as an *exact* constraint in the theory. Equation (3), which is actually a weaker assumption than the pure $(3,3^*)\oplus(3^*,3)$ chiral symmetry breaking, yields, with asymptotic SU(3), many powerful intramultiplet and intermultiplet constraints involving masses, mixing angles, and asymptotic axial-vector matrix elements. It will be fully utilized as explained below.

A. $[\dot{V},A]=0$ asymptotic SU(3) sum rules

Let us consider, for example, an exotic commutator $[\dot{V}_{K^0},A_{\pi^-}]=0$. Insert the commutators $[V_{K^0}(\dot{V}_{K^0}),A_{\pi^-}]=0$ between the states $\langle K^{*0}(\vec{p},\lambda)|$ and $|A_1^+(\vec{p},\lambda)\rangle$, and also between $\langle K^{*0}(\vec{p},\lambda)|$ and $|B^+(\vec{p},\lambda)\rangle$. Asymptotic SU(3) implies with $\vec{p}\to\infty$ that

$$\langle K^{*0}|V_{K^0}|\rho^0\rangle\langle\rho^0|A_{\pi^-}|A_1^+\rangle - \sum_{n=Q,Q'}\langle K^{*0}|A_{\pi^-}|n\rangle\langle n|V_{K^0}|A_1^+\rangle=0, \tag{4}$$

$$\langle K^{*0}|\dot{V}_{K^0}|\rho^0\rangle\langle\rho^0|A_{\pi^-}|A_1^+\rangle - \sum_{n=Q,Q'}\langle K^{*0}|A_{\pi^-}|n\rangle\langle n|\dot{V}_{K^0}|A_1^+\rangle=0, \tag{5}$$

$$\sum_{n'=\omega,\phi}\langle K^{*0}|V_{K^0}|n'\rangle\langle n'|A_{\pi^-}|B^+\rangle - \sum_{n=Q,Q'}\langle K^{*0}|A_{\pi^-}|n\rangle\langle n|V_{K^0}|B^+\rangle=0, \tag{6}$$

$$\sum_{n'=\omega,\phi}\langle K^{*0}|\dot{V}_{K^0}|n'\rangle\langle n'|A_{\pi^-}|B^+\rangle - \sum_{n=Q,Q'}\langle K^{*0}|A_{\pi^-}|n\rangle\langle n|\dot{V}_{K^0}|B^+\rangle=0. \tag{7}$$

Analogous to Eq. (1), we also introduce[10] the ω-ϕ mixing angle χ,

$$a_\phi=\cos\chi\, a_8+\sin\chi\, a_0 \text{ and } a_\omega=-\sin\chi\, a_8+\cos\chi\, a_0 .$$

In the ideal configuration (where ϕ becomes a pure $s\bar{s}$ state), $\chi=\chi_0$ and $\sin\chi_0=-\sqrt{1/3}$ in our convention. We now define the relevant asymptotic axial-vector matrix elements as follows ($\vec{p}\to\infty$):

$$\sqrt{1/2}\langle\rho^0(\vec{p},\lambda)|A_{\pi^-}|A_1^+\rangle\equiv h_A(\lambda),$$
$$\sqrt{1/2}\langle\omega(\vec{p},\lambda)|A_{\pi^-}|B^+\rangle\equiv -h_B(\lambda),$$
$$\sqrt{1/2}\langle\phi(\vec{p},\lambda)|A_{\pi^-}|B^+\rangle\equiv -h_B'(\lambda),$$
$$\langle K^{*0}(\vec{p},\lambda)|A_{\pi^-}|Q^+\rangle\equiv -h_Q(\lambda),$$

and

$$\langle K^{*0}(\vec{p},\lambda)|A_{\pi^-}|Q'^+\rangle\equiv -h_{Q'}(\lambda).$$

From now on we often suppress $\vec{p}$ (and also λ), since all the computations are $\vec{p}\to\infty$ limit and the sum rules we obtain hold for any helicity λ. Equations (4)–(7) then read ($\rho^2\equiv m_\rho{}^2$ etc.)

$$h_A=-\sin\theta\, h_{Q'}+\cos\theta\, h_Q, \tag{8}$$

$$(K^{*2}-\rho^2)h_A=-(Q'^2-A_1{}^2)\sin\theta\, h_{Q'}+(Q^2-A_1{}^2)\cos\theta\, h_Q, \tag{9}$$

$$-\sqrt{3}\sin\chi\, h_B+\sqrt{3}\cos\chi\, h_{B'}=\cos\theta\, h_{Q'}+\sin\theta\, h_Q, \tag{10}$$

$$-\sqrt{3}\sin\chi(K^{*2}-\omega^2)h_B+\sqrt{3}\cos\chi(K^{*2}-\phi^2)h_B' =\cos\theta(Q'^2-B^2)h_{Q'}+\sin\theta(Q^2-B^2)h_Q. \tag{11}$$

Equations (8)–(11) alone already enable us to compute[12] the Q_A-Q_B mixing angle θ and the ratio of the couplings of $Q, Q'\to K^*\pi$ [via partially conserved axial-vector current (PCAC) hypothesis] in terms of the masses A_1, B, Q, Q', ρ, and K^*, if we assume that the 1^{--} nonet is ideal (i.e., $\sin\chi=\sin\chi_0=-\sqrt{1/3}$, $\omega^2=\rho^2$, and $h_B'=0$). However, one can make the predictions much more powerful by adding further constraints which will be discussed in Sec. II B.

Before doing this we summarize, for later comparison, the implication of the sum rules, Eqs. (8)–(11), in the *absence* of Q_A-Q_B mixing.[11] First, Eqs. (8) and (9) immediately lead to an intermultiplet mass relation $K^{*2}-\rho^2=Q^2-A_1{}^2$. Next, even though actually *unnecessary*,[10] let us assume for the purpose of *simple* demonstration that the ϕ is a pure $s\bar{s}$ state, i.e., $\sin\chi=\sin\chi_0=-\sqrt{1/3}$. Then, the well-known results $\omega^2=\rho^2$ and $h_B'\equiv\langle\phi|A_{\pi^-}|B^+\rangle=0$ also follow[10] from the same exotic commutators $[V_{K^0}(\dot{V}_{K^0}),A_{\pi^-}]=0$, which are inserted between the states $\langle K^{*0}(\vec{p},\lambda)|$ and $|\rho^+(\vec{p},\lambda)\rangle$ with $\lambda=\pm1$ and $\vec{p}\to\infty$. With $h_B'=0$ and $\omega^2=\rho^2$, Eqs. (10) and (11) immediately imply $K^{*2}-\rho^2=Q'^2-B^2$. As a matter of fact, the above result

$$K^{*2}-\rho^2=Q^2-A_1{}^2=Q'^2-B^2 \tag{12}$$

is a particular case of the *general* intermultiplet mass relations[13] for the meson nonets B_α ($\alpha \equiv J^{PC}$),

$$K_\alpha{}^2 - \pi_\alpha{}^2 = \text{const, i.e., } K^2 - \pi^2 = K^{*2} - \rho^2 = K^{**2} - A_2{}^2 = \cdots . \quad (13)$$

Equation (13) is *always* valid in the present theoretical framework, as long as we consider *only* the $I = Y = 0$ singlet-octet mixing within each nonet. $K^2 - \pi^2 = K^{*2} - \rho^2 = K^{**2} - A_2{}^2$ is in reasonable agreement with experiment. Small discrepancies may be cured if we include the effect of small SU(3) mixings between these particles and their *higher-lying* excited states. The straightforward extension of Eq. (13) into SU(4) predicts[14]

$$D^2 - \pi^2 = D^{*2} - \rho^2 = \cdots , \quad (14)$$

which is also in surprisingly good agreement with experiment (i.e., $3.46 = 3.43\ \text{GeV}^2$). However, we notice that neither the assignment $Q = Q_1(1280)$ and $Q' = Q_2(1400)$, nor $Q = Q_2(1400)$ and $Q' = Q_1(1280)$ in Eq. (12) are in good agreement with experiment. However, in this paper we show that the situation changes drastically if we consider the Q_A-Q_B mixing.

B. Level-realization sum rules from $[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0}$

We now insert the well-known chiral SU(2)⊗SU(2) charge algebra $[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0}$ between the same SU(3) multiplet B_α ($\alpha = \pi$ and K), i.e., between $\langle B_\alpha(\vec{p}, \lambda)|$ and $|B_\alpha(\vec{p}\,', \lambda)\rangle$, with $\vec{p} \to \infty$. The right-hand side of this equation is denoted as $g_\alpha(\lambda)$. g's are pure numbers, i.e., $g_{\pi^+} = 2$ and $g_{K^+} = 1$, except for the factor $(2\pi)^3 \delta^3(p - p')$:

$$\sum_{n_L} \big(\langle B_\alpha | A_{\pi^+} | n_L\rangle\langle n_L | A_{\pi^-} | B_\alpha\rangle - \langle B_\alpha | A_{\pi^-} | n_L\rangle\langle n_L | A_{\pi^+} | B_\alpha\rangle\big) = g_\alpha(\lambda) . \quad (15)$$

Among the sum over the single-particle hadron intermediate states appearing in Eq. (15), we distinguish for given α the fractional contribution $f_\alpha^L(\lambda)$ to $g_\alpha(\lambda)$, $g_\alpha(f_\alpha^0 + f_\alpha^1 + f_\alpha^2 + \cdots) = g_\alpha$, coming from *all* the states n_L belonging to a level L. (For example, $L = 0, 1, 2, \ldots$. We may also add radial quantum number.) The hypothesis introduced[15] is that the fraction $f_\alpha^L(\lambda)$ will depend on λ but not on the SU(3) index α. Namely, the SU(3) contents of the algebra [which we can study by varying the SU(3) index α] are realized by each level L *separately*. Therefore, the fractional contribution from each level L to the algebra under consideration is assumed invariant under SU(3) rotation. If the concept of levels were really applicable to the hadrons which are the manifestations of the bound states of confined quarks, the hypothesis may not be as drastic and unrealistic as it may sound. Actually, in Eq. (15) the fractional contributions which come from each of the intermediate states n belonging to the SU(3) multiplets satisfying $C(n) = -C(B_\alpha)$ (i.e., $\langle B_\alpha | A_\pi | n\rangle$ is of F type as is the case with the vector matrix elements) are independent of α by themselves fulfilling the requirement. This is, however, not the case with those from the states with $C(n) = C(B_\alpha)$ ($\langle B_\alpha | A_\pi | n\rangle$ is now D type). However, here the notion of levels may play a role. We assume that if we sum over the contributions coming from *all* the states belonging to a level L, SU(3) under consideration is restored at each level L. Therefore, the hypothesis imposes constraints on the D-type asymptotic axial-vector matrix elements. In the case of bosons a stronger assumption is to assume that each SU(3) nonet does the job.[15] This is a kind of algebraic alternative to the Okubo's nonet ansatz,[16] but it is a weaker assumption. We stress that the hypothesis provides us [without introducing a perturbative point of view towards broken SU(3) symmetry] a means to recognize unambiguously SU(3) multiplets in the broken SU(3) world. For example, let us consider the ρ meson ($L = 0$). We have its SU(3) counterpart K^* ($L = 0$), $K^{*\prime}$ ($L = 2$) meson, radially excited counterparts, etc. However, a realization hypothesis will be satisfied only for the pair (ρ, K^*) but not for other pairs $(\rho, K^{*\prime}), \ldots$. Therefore, if the hypothesis were correct, we would not have any difficulty, in principle, in identifying the members of the SU(3) multiplet in *broken* SU(3) symmetry.

It has been shown[15,16] that this hypothesis works well, actually including the use of the full set of chiral SU(3)⊗SU(3) algebra $[A_\alpha, A_\beta] = if_{\alpha\beta\gamma}V_\gamma$, for meson nonets and the ground-state baryons producing SU(6)-like good constraints. A bad result of SU(6) (such as $g_A = \frac{5}{3}$) is replaced by a good one.[15] The approach may provide a viable alternative to the usual SU(6)⊗O(3) approach without imposing $\text{SU}(6)_W$ from outside. The concept of levels of hadrons could be more fundamental than that of SU(6) symmetry. The sum rules thus obtained are also found to be compatible with the quark-line selection rule, as we encounter with an example in this paper in Sec. III A. Recently, the approach has been extended[17] to include the charge-current algebra such as $[[j_3^\mu(0), A_{\pi^+}], A_{\pi^-}] = 2j_3^\mu(0)$, opening up a way to treat the processes involving helicity change. For example, a good nucleon anomalous-moment relation $k_p = -k_n$ has been obtained[18] among others. We therefore feel that there is sufficient support for pursuing this hypothesis.

In exact SU(3) symmetry, the 1^{++} and 1^{+-} octet identifications are $(A_1, Q_A, \ldots)$ and $(B, Q_B, \ldots)$. We now proceed assuming, for later convenience, that $(A_1, Q', \ldots)$ and $(B, Q, \ldots)$ are the physical octet assignments in broken SU(3) which enable us to apply the hypothesis of level realization in the algebra of Eq. (15). This, however, does not imply any special assignment, since Q and Q' are merely defined by Eq. (1). Only after the application of the level-realization hypothesis do we know the correct identification of Q and Q' with the observed $Q_1(1280)$ and $Q_2(1400)$. The values of g_α's in Eq. (15) are independent of the mixing angle, as long as we keep exact SU(2) symmetry, i.e., $\langle Q^+(\vec{p})|V_{\pi^0}|Q^+(\vec{p}')\rangle = \langle Q'^+(\vec{p})|V_{\pi^0}|Q'^+(\vec{p})\rangle = (\frac{1}{2})(2\pi)^3\delta^3 \times(\vec{p}-\vec{p}')$. Therefore, we have no problem (as in the case of finding the $I=\frac{1}{2}$ counterpart of the ρ meson discussed above) in looking for the $I=\frac{1}{2}$ SU(3) counterparts of the A_1 and B mesons as long as we use[19] the chiral SU(2)⊗SU(2) charge algebra, Eq. (15).

We now take, in Eq. (15), $B_\alpha = A_1^+(\vec{p},\lambda)$ and also $B_\alpha = Q'^+(\vec{p},\lambda)$, with $\vec{p}\to\infty$. We study the fractional contribution to the algebra coming from the ground states $L=0$ (1^{--} and 0^{-+} mesons). However, the 0^{-+} mesons do not make a contribution because of parity conservation:

$$\langle A_1^+|A_{\pi^+}|\rho^0\rangle\langle\rho^0|A_{\pi^-}|A_1^+\rangle+\cdots=2 \tag{16}$$

and

$$\langle Q'^+|A_{\pi^+}|K^{*0}\rangle\langle K^{*0}|A_{\pi^-}|Q'^+\rangle+\cdots=1. \tag{17}$$

In Eqs. (16) and (17) only the ground-state contribution is explicitly written. The condition $f^0_{A_1^+}=f^0_{Q'^+}(\lambda)$ implies $|\langle A_1^+|A_{\pi^+}|\rho^0\rangle|^2=2|\langle Q'^+|A_{\pi^+}|K^{*0}\rangle|^2$ which, suppressing the helicity indices, leads to

$$h_A{}^2=h_{Q'}{}^2. \tag{18}$$

Similarly, for the pair (B^+,Q^+), we obtain from Eq. (15)

$$|\langle B^+|A_{\pi^+}|\omega\rangle|^2+|\langle B^+|A_{\pi^+}|\phi\rangle|^2+\cdots=2 \tag{19}$$

and

$$|\langle Q^+|A_{\pi^+}|K^{*0}\rangle|^2+\cdots=1, \tag{20}$$

which implies

$$h_B{}^2+h_B'^2=h_Q{}^2. \tag{21}$$

Our new task is to solve the set of sum rules Eqs. (8)–(11), (18), and (21) in the presence of Q_A-Q_B mixing.

III. SOLUTION OF THE CONSTRAINTS; MASS FORMULAS, MIXING ANGLE AND SU(3) COUPLINGS

A. Selection rule for the $B\to\phi\pi$ decay

By squaring Eq. (8) and using the realization constraint Eq. (18), we obtain

$$h_Q h_{Q'}\sin 2\theta=-\cos^2\theta(h_{Q'}{}^2-h_Q{}^2). \tag{22}$$

Next, by squaring Eq. (10) and eliminating h_Q and h_Q' [by using Eq. (22) and the realization constraint Eq. (21)] from this equation, we obtain

$$(3\cos^2\chi-1)R^2-6\sin\chi\cos\chi R+(3\sin^2\chi-1)=0, \tag{23}$$

where $R\equiv h_B'/h_B$. Solving Eq. (23) for R and discarding an unphysical solution[20] we obtain

$$R\equiv\frac{h_B'}{h_B}\equiv\frac{\langle\phi|A_{\pi^-}|B^+\rangle}{\langle\omega|A_{\pi^-}|B^+\rangle}=\frac{3\sin\chi\cos\chi+\sqrt{2}}{3\cos^2\chi-1}=\tan(\chi-\chi_0), \tag{24}$$

where χ_0 is the ideal angle, $\sin\chi_0=-\sqrt{1/3}$ ($\chi_0\simeq-35°$). The value of the ω-ϕ mixing angle χ, which can be evaluated in the present theory from the quadratic GMO mass formula of the 1^{--} nonet obtained by using the exotic commutator $[\dot{V}_{K^0},V_{K^0}]=0$ and asymptotic SU(3), is around $-40°$. Hence, $R\simeq-0.083$. In the ideal limit of the 1^{--} nonet, i.e., $\chi=\chi_0$, Eq. (24) leads to a strict selection rule $\langle\phi|A_{\pi^-}|B^+\rangle=0$, which implies via PCAC that the $B\to\phi\pi$ decay is forbidden. The degree of actual violation of this selection rule is predicted by Eq. (24) and PCAC:

$$\frac{\Gamma(B\to\phi\pi)}{\Gamma(B\to\omega\pi)}=\left(\frac{B^2-\phi^2}{B^2-\omega^2}\right)^2\left(\frac{q_\phi}{q_\omega}\right)R^2\simeq7.3\times10^{-4}.$$

We note that in deriving Eq. (23), only the realization constraints Eqs. (18) and (21) are used, but not the $[\dot{V},A]=0$ constraints Eqs. (9) and (11). Equation (23), therefore, illustrates that the asymptotic level-realization hypothesis being utilized is compatible with the quark-line rule[21] (even in the presence of Q_A-Q_B mixing). The $[\dot{V},A]=0$ constraints are also consistent with the quark-line rule.[10] Since R is reasonably small, we make for the bulk of this paper an approximation[22] that the 1^{--} nonet is *ideal*, i.e., $\chi=\chi_0$. Then as mentioned before, $\omega^2=\rho^2$ and $h_B'\equiv\langle\phi|A_{\pi^-}|B\rangle=0$. Equations (10), (11), and (21) then become

$$h_B=\cos\theta\, h_{Q'}+\sin\theta\, h_Q, \tag{10'}$$

$$(K^{*2}-\rho^2)h_B=(Q'^2-B^2)\cos\theta\, h_{Q'}+(Q^2-B^2)\sin\theta\, h_Q, \tag{11'}$$

and

$$h_B{}^2=h_Q{}^2. \tag{21'}$$

B. Mass formulas and mixing angle

Eliminating h_Q from Eqs. (8) and (9) and using Eq. (18) we immediately obtain

$$\sin^2\theta=\frac{[(Q^2-A_1{}^2)-(K^{*2}-\rho^2)]^2}{(Q'^2-Q^2)^2}. \tag{25}$$

Similarly, by eliminating $h_{Q'}$ from Eqs. (10′) and (11′), and combining with Eq. (21′) (in the ideal limit of the 1^{--} nonet), we also get

$$\sin^2\theta=\frac{[(Q'^2-B^2)-(K^{*2}-\rho^2)]^2}{(Q'^2-Q^2)^2}. \tag{26}$$

From Eqs. (25) and (26) we obtain two possible solutions:

$$(Q^2-A_1^{\,2})-(K^{*2}-\rho^2)=\mp[(Q'^2-B^2)-(K^{*2}-\rho^2)]\,,$$

i.e., either

(i) $(Q^2-A_1^{\,2})+(Q'^2-B^2)=2(K^{*2}-\rho^2)$,

or

(ii) $Q'^2+A_1^{\,2}=Q^2+B^2$.

Eliminating h_A from Eqs. (8) and (9), we have

$$[(K^{*2}-\rho^2)-(Q^2-A_1^{\,2})]\cos\theta\, h_Q =[(K^{*2}-\rho^2)-(Q'^2-A_1^{\,2})]\sin\theta\, h_{Q'}\,. \quad (27)$$

Similarly, by eliminating h_B from Eqs. (10′) and (11′),

$$[(K^{*2}-\rho^2)-(Q'^2-B^2)]\cos\theta\, h_{Q'} =-[(K^{*2}-\rho^2)-(Q^2-B^2)]h_Q\sin\theta\,. \quad (28)$$

Eliminating h_Q and $h_{Q'}$ from Eqs. (27) and (28) (assuming[23] $h_Q h_{Q'}\neq 0$), we obtain

$$\cos^2\theta[(K^{*2}-\rho^2)-(Q^2-A_1^{\,2})][(K^{*2}-\rho^2)-(Q'^2-B^2)]=-\sin^2\theta[(K^{*2}-\rho^2)-(Q'^2-A_1^{\,2})]\,[(K^{*2}-\rho^2)-(Q^2-B^2)]\,. \quad (29)$$

If we adopt the mass formula, case (i), Eq. (29) becomes

$$-\cos^2\theta[(K^{*2}-\rho^2)-(Q^2-A_1^{\,2})]^2=\sin^2\theta[(K^{*2}-\rho^2)-(Q^2-B^2)]^2\,.$$

With Eqs. (25) and (26), this equation can only be satisfied with $\theta=0$ (or 180°) and $K^{*2}-\rho^2=Q^2-A_1^{\,2}=Q'^2-B^2$ which is Eq. (12). Therefore, case (i) corresponds to the case of *no* Q_A-Q_B mixing discussed in Sec. II A. On the other hand, case (ii) provides us with a genuine mass formula in the *presence* of Q_A-Q_B mixing. In this case, Eq. (29) becomes

$$\cos^2\theta[(K^{*2}-\rho^2)-(Q^2-A_1^{\,2})]^2=-\sin^2\theta[(K^{*2}-\rho^2)-(Q'^2-A_1^{\,2})][(K^{*2}-\rho^2)-(Q^2-B^2)]$$

which implies (assuming[24] $\cos\theta\neq 0$) [smaller of $(Q'^2-A_1^{\,2})$ and (Q^2-B^2)] $<K^{*2}-\rho^2<$ [larger of $(Q'^2-A_1^{\,2})$ and (Q^2-B^2)]. Combining with the relation $Q'^2+A^2=Q^2+B^2$ we then obtain, if we *take* the experimentally indicated mass ordering $B^2>A_1^{\,2}$,

$$Q'^2-B^2=Q^2-A_1^{\,2}\quad (B^2>A_1^{\,2}) \quad (30)$$

and

$$Q'^2-A_1^{\,2}>K^{*2}-\rho^2>Q^2-B^2\,. \quad (31)$$

We already find here a rather surprising result, i.e., Q' mass should be larger than the Q mass. The choice $Q'=Q_2(1400)$ and $Q=Q_1(1280)$ is consistent with Eqs. (30) and (31). Although there are *a priori* two possible solutions for $\sin\theta$, we choose one of them which leads to the SU(3) parametrization adopted by the ACNO group[8] (the other solution merely corresponds to the parametrization with an angle whose sign is opposite to that of ACNO's). Namely, we write from Eq. (25)

$$\sin\theta=\frac{-(Q^2-A_1^{\,2})+(K^{*2}-\rho^2)}{Q'^2-Q^2}\quad (-90°<\theta<90°)\,. \quad (32)$$

Equations (30)–(32) are the constraints on the masses and mixing angle in the present theoretical framework. Our new mass relations, Eqs. (30) and (31), which are valid in the presence of Q_A-Q_B mixing, should be compared with Eq. (12) which was derived on the assumption of *no* Q_A-Q_B mixing. Mixing gives a substantial improvement on the agreement with experiment. Taking $Q=1280$, $Q'=1400$, and $B=1231$ MeV, Eq. (30) predicts

$$A_1^{\,2}=Q^2-Q'^2+B^2\,,\quad A_1\simeq 1.093\ \mathrm{GeV}. \quad (33)$$

The value of the mass of A_1 obtained seems reasonable.

We mention here the modifications which take place when we do *not* make the simplifying ideal 1^{--} nonet approximation. Equation (25) remains unchanged. However, Eqs. (26) and (33) now read

$$\sin^2\theta=\frac{3[(B^2-Q'^2+K^{*2}-\omega^2)\sin\chi\cos(\chi-\chi_0)+(Q'^2-B^2)\cos\chi\sin(\chi-\chi_0)]^2}{(Q'^2-Q^2)^2} \quad (26')$$

and

$$(Q'^2+A_1^{\,2})-(Q^2+B^2)=-\sqrt{3}[(K^{*2}-\omega^2)\sin\chi\cos(\chi-\chi_0)-(K^{*2}-\phi^2)\cos\chi\sin(\chi-\chi_0)]-(K^{*2}-\rho^2)\,. \quad (33')$$

Equation (33′) predicts the mass of A_1, $A_1 \simeq 1.105$ GeV, i.e., the error[25] produced by the ideal 1^{--} nonet approximation is only of the order 1.1%. A comment is in order for the determination of the masses of A_1 and B from Eqs. (25) and (26′) [or (26)] when θ is given. One may suspect that there are two possible mass values for the A_1 and B. However, this is not the case. Equations (33) or (33′) dictate us to choose *only* one of the possible values.

We now discuss the mixing angle θ. It turned out that, once the values $Q' = 1400$ GeV and $Q = 1.280$ GeV are fixed, the mass values of A_1 and B, which are predicted by Eqs. (25) and (26′) [or (26)] and which should be consistent with our mass formula Eq. (33′) [or (33)], do *not* vary significant over a rather large range of values of θ, i.e., $30° \lesssim \theta \lesssim 50°$. For example, for the mass values $A_1 \simeq 1.105$ GeV and $B \simeq 1.231$ GeV, we obtain $\theta \simeq 41°$ from Eq. (25) and also from Eq. (26′). If we take the ideal 1^{--} nonet approximation [Eqs. (25) and (26)], $\theta \simeq 49°$. For the value of θ, $\theta \simeq 30°$, we obtain $A_1 \simeq 1.129$ GeV from Eq. (25) and $B \simeq 1.251$ GeV from Eq. (26′). Therefore, in view of the uncertainties associated with the central mass values of broad resonances ρ, K^*, A_1, B, Q_1, and Q_2, the values of θ in the range $30° \lesssim \theta \lesssim 50°$ are consistent with our sum rules. Therefore, it is difficult to single out the value of θ from the consideration of masses alone.

However, as will be discussed below, we strongly suspect that the mixing angle θ lies in the vicinity of 30°, since in the present formulation the decays $Q' = Q_2(1400) \to \rho K$ (and ωK) are strictly forbidden as suggested by recent experiment[6] for $\theta = 30°$. In this connection, it is very interesting to note that the ACNO group obtained[8] a value of mixing angle $(27° \pm 8°)$ *solely* from their SU(3) fit to the A_1, B, and $Q(1280)$ decays and they state that their parameters can also forbid the decays $Q_2(1400) \to \rho K$ and ωK, as is the case with the present theory with $\theta = 30°$.

C. SU(3) couplings

In the conventional SU(3) parametrization[6,8] three input parameters are needed—two independent D and F couplings (for example, the $A_1\rho\pi$ and $B\omega\pi$ couplings) and the mixing angle θ. However, in the present theoretical framework we need only one input, for example, h_B, which can be related to the physical $B\omega\pi$ coupling via PCAC.

We first note that either from Eqs. (27) or (28) we obtain, using Eqs. (30) and (32),

$$\frac{\langle K^{*0}|A_{\pi^-}|Q'^+\rangle}{\langle K^{*0}|A_{\pi^-}|Q^+\rangle} \equiv \frac{h_{Q'}}{h_Q} = -\left(\frac{1+\sin\theta}{\cos\theta}\right), \tag{34}$$

which also implies through Eqs. (8) and (10′) that

$$\frac{\langle \rho^0|A_{\pi^-}|A_1^+\rangle}{-\langle \omega|A_{\pi^-}|B^+\rangle} \equiv \frac{h_A}{h_B} = \frac{h_{Q'}}{h_Q} = -\left(\frac{1+\sin\theta}{\cos\theta}\right). \tag{35}$$

Furthermore, a close look at the set of constraints Eqs. (8), (10′), (34), and (35), reveals that

$$h_A = -h_{Q'} \text{ and } h_B = -h_Q. \tag{36}$$

Therefore, h_A and h_B are no longer independent. Equations (30)–(32) and (34)–(36) are the *whole* set of independent constraints in the theory which are imposed upon the masses, mixing angle and the asymptotic axial-vector matrix elements involving the A_1, B, Q, and Q' system. Other relevant asymptotic axial-vector matrix elements in terms of h_A and h_B [actually *solely* in terms of h_B owing to Eq. (35)] are as follows:

$$\langle\rho^0|A_{K^-}|Q^+\rangle = \frac{1}{\sqrt{2}}(h_A\cos\theta - h_B\sin\theta) = -\frac{1}{\sqrt{2}}h_B(1+2\sin\theta), \tag{37}$$

$$\langle\rho^0|A_{K^-}|Q'^+\rangle = \frac{-1}{\sqrt{2}}(h_A\sin\theta + \cos\theta h_B) = \frac{1}{\sqrt{2}}h_A(1-2\sin\theta), \tag{38}$$

$$\langle\omega|A_{K^-}|Q^+\rangle = \frac{-1}{\sqrt{2}}(h_A\cos\theta - h_B\sin\theta) = -\frac{1}{\sqrt{2}}h_B(1+2\sin\theta), \tag{39}$$

$$\langle\omega|A_{K^-}|Q'^+\rangle = \frac{-1}{\sqrt{2}}(h_A\sin\theta + h_B\cos\theta) = \frac{1}{\sqrt{2}}h_A(1-2\sin\theta). \tag{40}$$

We immediately notice that the angle $\theta = 30°$ plays a particular role. Namely, for $\theta = 30°$, $\langle\rho^0|A_{K^-}|Q'^+\rangle = \langle\omega|A_{K^-}|Q'^+\rangle = 0$. We now relate the asymptotic axial-vector matrix elements $\langle 1^-|A_\gamma|1^+\rangle$ to the physical $1^+ \to 1^- + P$ couplings via PCAC, $\partial^\mu A_\gamma^\mu = F_\gamma m_\gamma^2 \phi_\gamma$, where ϕ_γ ($\gamma = \pi$ or K) is the pseudoscalar meson with mass m_γ. F_γ is the decay constant of the $\pi \to \mu\nu$ or $K \to \mu\nu$ decays. We obtain in the limit $\vec{p} \to \infty$,

$$\langle B_\beta(1^-), \lambda, \vec{p}|A_\gamma|B_\alpha(1^+), \lambda, \vec{p}\rangle = F_\gamma t_{\alpha\beta\gamma}(\lambda) g_{\alpha\beta\gamma}(m_\gamma^2 = 0), \tag{41}$$

where $g_{\alpha\beta\gamma}$ is the physical coupling for the process $B_\alpha(1^+) \to B_\beta(1^-) + P_\gamma$, except that it involves the mass-shell extrapolation $m_\gamma^2 \to 0$. $t_{\alpha\beta\gamma}$ is the kinematical factor which arises via PCAC in the $\vec{p} \to \infty$ limit. For the case $\lambda = \pm 1$, $t_{\alpha\beta\gamma}$ is given by[12,26]

$$t_{\alpha\beta\gamma} = \frac{1}{m_\alpha^2 - m_\beta^2}. \tag{42}$$

With Eqs. (41) and (42), we obtain for the SU(3) comparison of the *S-wave* decays of $1^+ \to 1^- + P$

$$\frac{\Gamma(B_\alpha(1^+) \to B_\beta(1^-) + P_\gamma)}{\Gamma(B_{\alpha'}(1^+) \to B_{\beta'}(1^-) + P_{\gamma'})} = \left(\frac{F_{\gamma'}}{F_\gamma}\right)^2 \left(\frac{m_\alpha^2 - m_\beta^2}{m_{\alpha'}^2 - m_{\beta'}^2}\right)^2 \left(\frac{m_{\alpha'}}{m_\alpha}\right)^2 \times \left(\frac{q_\beta}{q_{\beta'}}\right) \left|\frac{\langle B_\beta|A_\gamma|B_\alpha\rangle}{\langle B_{\beta'}|A_{\gamma'}|B_{\alpha'}\rangle}\right|^2, \tag{43}$$

where q_β and $q_{\beta'}$ are the momenta of the secondary 1^- mesons in the rest frame of the parent particles.

In Eq. (43), $\langle B_\beta|A_\gamma|B_\alpha\rangle$ and $\langle B_{\beta'}|A_{\gamma'}|B_{\alpha'}\rangle$ can be parametrized by the prescription of *exact SU(3) plus mixing*. In place of Eq. (43) the conventional SU(3) analysis (which was employed by the SLAC[6] and ACNO groups[8]) uses

$$\frac{\Gamma(B_\alpha(1^+)\to B_\beta(1^-)+P_\gamma)}{\Gamma(B_{\alpha'}(1^+)\to B_{\beta'}(1^-)+P_{\gamma'})}=\left(\frac{m_{\alpha'}}{m_\alpha}\right)^2\left(\frac{q_\beta}{q_{\beta'}}\right)\left|\frac{g_{\alpha\beta\gamma}}{g_{\alpha'\beta'\gamma'}}\right|^2, \tag{44}$$

where $g_{\alpha\beta\gamma}$'s are parametrized by the prescription of *exact SU(3) plus mixing*. Equation (43) contains an extra factor $(F_{\gamma'}/F_\gamma)^2(m_\alpha^2-m_\beta^2)^2/(m_{\alpha'}^2-m_{\beta'}^2)^2$ [compared with Eq. (44)] which may deviate from unity depending on the masses involved. Of course, in Eq. (43) some allowance has to be made for the mass-shell extrapolation involved in the coupling constants. Takasugi and Oneda have also shown[27] that, in the strict soft-pseudoscalar-meson limit (i.e., zero mass limit in the phase space as well as in the couplings), the effect of the PCAC kinematical factor $t_{\alpha\beta\gamma}$ can be expressed as an effective q^3 angular momentum barrier in the decay rate. Namely, Eq. (43) can be replaced by

$$\frac{\Gamma(B_\alpha(1^+)\to B_\beta(1^-)+P_\gamma)}{\Gamma(B_{\alpha'}(1^+)\to B_{\beta'}(1^-)+P_{\gamma'})}=\left(\frac{F_{\gamma'}}{F_\gamma}\right)^2\left(\frac{q_\beta}{q_{\beta'}}\right)^3\left|\frac{\langle B_\beta|A_\gamma|B_\alpha\rangle}{\langle B_\beta|A_\gamma|B_{\alpha'}\rangle}\right|^2. \tag{45}$$

This formula is very general, including all the partial waves involved. If we assume[28] $F_\pi=F_K$, Eq. (45) gives a very simple prescription. However, strickly speaking Eq. (45) is exact only in the soft-meson ($m_\gamma=m_{\gamma'}=0$) limit. In general, Eq. (45) [and, of course, Eq. (43)] will provide a reasonably good prescription if the mass differences between the 1^+ and 1^- mesons are much larger than the mass of the pseudoscalar meson involved. It would be desirable to establish a good recipe[29] for computing $\langle(q_\beta/q_{\beta'})^3\rangle$ in Eq. (45). In any case, the comparison of Eq. (43) with (44) suggests that the conventional SU(3) prescription, Eq. (44), does *not* necessarily have an automatic theoretical justification and one should *not* draw too strong a conclusion from the use of Eq. (44).

As it stands, present theory does not fix the value of θ to a certain value algebraically. However, we note that according to Eqs. (38) and (40), the physical couplings Q' [$\equiv Q_2(1400)$] $\to \rho K$ and ωK strickly vanish if $\theta=30°$ (tolerating, of course, the kaon PCAC). Since $\theta=30°$ also gives a reasonable prediction [consistent with Eq. (33)] on the mass of the A_1 meson as discussed in III B, we may be tempted to assume that θ is close to 30° *if* the rates of the $Q_2(1400)\to\rho K$ and ωK were indeed *very small* as observed by the SLAC group.[5,6] For the value $\theta=30°$, the rates of $Q_1(1280)\to\rho K$ and ωK will be enhanced by a factor of 4, compared with the case of no Q_A-Q_B mixing. [See Eqs. (37) and (39).] However, Eq. (36) shows that in the present theory one *cannot* forbid the $Q_1(1280)\to K^*\pi$ decay, although the ratio $\Gamma(Q_1\to K^*\pi)/\Gamma(Q_1\to\rho K)$ certainly depends on the value of θ.

In the following crude analysis we now assume the *ideal* value of θ, $\theta=30°$. Then Eq. (35) becomes

$$\frac{h_A}{h_B}=-\frac{\langle\rho^0|A_{\pi^-}|A_1^+\rangle}{\langle\omega|A_{\pi^-}|B^+\rangle}=\sqrt{3}\,, \tag{46}$$

which predicts for the ratio of the S-wave $A_1\to\rho\pi$ and $B\to\omega\pi$ couplings via Eqs. (41) and (42) that

$$\gamma\equiv\frac{\sqrt{2}g_{A^+\rho\pi^-}}{g_{B^+\omega\pi^-}}=-\sqrt{6}\left(\frac{A^2-\rho^2}{B^2-\omega^2}\right). \tag{47}$$

If we use, for comparison, ACNO's mass value of A_1, 1.040 GeV, Eq. (47) predicts $\gamma\simeq-1.31$, while the ACNO group deduced[29] $\gamma\simeq-1.47$ from the rates of $A_1\to\rho\pi$ and $B\to\omega\pi$ decays. Therefore, our prediction [Eq. (47)] agrees rather well with the result of the ACNO group and yields $\Gamma(A_1\to\rho\pi)\simeq 200$ MeV for the mass of A_1 chosen. If the A_1 mass is heavier, say 1.1 GeV, the width will be of the order of 300 MeV.[30] Actually, the result is also in good agreement with the old (but quite independent of the present calculation) result by Matsuda and Oneda[31] based on the chiral SU(2)⊗SU(2) charge-charge-density algebra and PCAC. There they obtained for pure S-wave decay $\Gamma(A_1\to\rho\pi)\simeq 275$ MeV for the mass of A_1, $A_1\simeq\sqrt{2}m_\rho$. Our PCAC approach to the SU(3) test will probably be most accurate for the ratio $\Gamma(Q'\to K^*\pi)/\Gamma(Q\to K^*\pi)$, since the Q values of these decays are relatively large compared with the pion mass, although we may still need to include the broad width correction to the average $\langle q\rangle$'s. Indeed, Eq. (45) gives

$$\frac{\Gamma(Q'^+\to K^*\pi)}{\Gamma(Q^+\to K^*\pi)}=\left(\frac{q_{Q'}}{q_Q}\right)^3\left(\frac{h_A}{h_B}\right)^2=3\left(\frac{q_{Q'}}{q_Q}\right)^3\simeq 6.6\,, \tag{48}$$

whereas Eq. (43) yields $\simeq 6.2$ for this ratio. For the value of $(q_{Q'}/q_Q)$, the zero-width approximation is used for the resonances. The $Q(1280)\to K^*\pi$ is relatively suppressed compared with the $Q'(1400)\to K^*\pi$. The quantitative treatment of $Q(1280)\to\rho K$ and ωK is more difficult since kaon PCAC has to be involved. With $F_\pi=F_K$ (Ref. 27) and using Eq. (43) (assuming S-wave decays) we obtain

$$\frac{\Gamma(Q^+\to\rho K)}{\Gamma(B^+\to\omega\pi)}=3\left(\frac{Q^2-\rho^2}{B^2-\omega^2}\right)^2\left(\frac{B^2}{Q^2}\right)\left(\frac{q_\rho}{q_\omega}\right)\simeq 0.68\,, \tag{49}$$

$$\frac{\Gamma(Q^+\to K^*\pi)}{\Gamma(B^+\to\omega\pi)}=\tfrac{3}{4}\left(\frac{Q^2-K^{*2}}{B^2-\omega^2}\right)^2\left(\frac{B^2}{Q^2}\right)\left(\frac{q_{K^*}}{q_\omega}\right)\simeq 0.50\,, \tag{50}$$

$$\frac{\Gamma(Q^+\to\omega K)}{\Gamma(B^+\to\omega\pi)}=\left(\frac{Q^2-\omega^2}{B^2-\omega^2}\right)^2\left(\frac{B^2}{Q^2}\right)\left(\frac{q_{\omega K}}{q_\omega}\right)\simeq 0.14\,. \tag{51}$$

Here again we have used for q_ρ, q_{K^*}, and $q_{\omega K}$ the values obtained by assuming that all the broad resonances involved have zero widths. For q_ω we have used[8] $\langle q_\omega \rangle = 374$ MeV. Although we believe that Eqs. (49)–(51) describe the gross feature of $Q(1280)$ decays, a rather large error may be involved, especially for the estimate of $\Gamma(Q^+ \to \rho K)$ and $\Gamma(Q^+ \to \omega K)$. In addition to the zero-width approximation for broad resonances and the assumption of pure S wave,[30] there is also an effect of mass-shell extrapolation $m_K{}^2 \to 0$ in the couplings of these decays. Finally, as a consistency check we also studied the realization of the same algebras for the following choice of the states. The sum rules involve the same axial-vector matrix elements as discussed in this paper:

$$\langle Q^0 | [V_{K^0}(\dot{V}_{K^0}), A_{\pi^-}] | \rho^+ \rangle = 0 ,$$

$$\langle Q'^0 | [V_{K^0}(\dot{V}_{K^0}), A_{\pi^-}] | \rho^+ \rangle = 0$$

and the realization of SU(3) in $[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0}$ between the pair (ρ^+ and K^{*+}) for which the contribution of $L = 1$ intermediate states is considered. We found that the constraints obtained in this paper also provide the only solution for the above case.

IV. CONCLUSION

We may conclude that the overall picture which emerged is as follows.

(1) In the presence of $Q_A - Q_B$ mixing we still maintain a simple mass relation $Q'^2 - B^2 = Q^2 - A_1{}^2$. However, this mass difference is no longer required to be equal to the usual scale $K^{*2} - \rho^2$ (or $K^2 - \pi^2$), although the masses of Q_1 and Q_2 are constrained by the relation $Q'^2 - A_1{}^2 > K^{*2} - \rho^2 > Q^2 - B^2$. With the present values of $Q = Q_1(1280)$ and $Q' = Q_2(1400)$, the A_1 mass is predicted around 1100 MeV.

(2) If the severe suppression of $Q' = Q_2(1400) \to \rho K$ and ωK were indeed the case, the mixing angle θ is required to be close to 30°. $\theta = 30°$ is also in reasonably good agreement with our mass–mixing-angle constraints Eq. (25) if the mass of A_1 is around 1100 MeV.

(3) Allowing for the theoretical and experimental uncertainties mentioned before, the predictions on the decay rates seem to be in reasonable agreement with the gross features of presently available experiments. For the $Q_1(1280)$, the ρK mode is certainly enhanced by the $Q_A - Q_B$ mixing and is the most important decay mode. However, we do not obtain a *severe* suppression of the $Q_1(1280) \to K^*\pi$ mode as in the case of $Q_2(1400) \to \rho K$ and ωK. The rate of $\Gamma(Q_1(1280) \to \rho K, K^*\pi, \omega K)$ is probably around 180 MeV and $\Gamma(Q_2(1400) \to K^*\pi) \simeq 300$ MeV.

(4) The S-wave $A_1 \to \rho\pi$ width is predicted to be rather large, around 300 MeV for the A_1 mass around 1100 MeV, which is consistent with the previous estimate based on the chiral SU(2)⊗SU(2) charge–charge-density algebra and PCAC.[31] It is also gratifying to notice that the predicted mass value of A_1 is in reasonable agreement with the old predictions (including Ref. 31) made by various authors[32] using current algebra.

(5) We have found the nonet assignments $[A_1, Q_2(1400), \ldots]$ and $[B, Q_1(1280), \ldots]$. Therefore, at the level of Q mesons, inversion in the order of masses has apparently taken place due to the $Q_A - Q_B$ mixing. This may not be so surprising, since in the conventional treatment of $Q_A - Q_B$ mixing the masses of the hypothetical SU(3) states Q_A and Q_B mixing the masses of the hypothetical SU(3) states Q_A and Q_B are found[6] to be almost degenerate around 1.34 GeV for $\theta \simeq 45°$. A somewhat similar situation also exists in the present formalism.[33]

(6) We may expect that similar inversion takes place for the two K mesons with $J^P = 2^-, 3^+, 4^-, \ldots$.

(7) It has been shown[10] that the present theoretical framework can explain the existence and violation of the quark-line selection rule in the asymptotic axial-vector matrix elements. In this paper we have shown that the same theoretical framework may also explain the recent remarkable findings on the axial-vector mesons. Therefore, the present algebraic approach to confined quarks through the quark current algebra seems to provide a promising alternative to the naive quark model.

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[23]From Eq. (18) and Eq. (21′), $h_Q = 0$ and $h_{Q'} = 0$ are excluded.

[24]If $\cos\theta = 0$, Eq. (29) implies $(Q'^2 - A_1^2) = (Q^2 - B^2) = (K^{*2} - \rho^2)$. This is again the case of no Q_A-Q_B mixing.

[25]Of course, in addition there could exist some effect of SU(3) mixing with the radially excited 1^{++} and 1^{+-} states.

[26]General formulas to compute $t_{\alpha\beta\gamma}$ for general J^{PC} assignments for B_α and B_β and their implication for the SU(3) test are given by C. Y. Huang, Ph.D. thesis, University of Maryland, 1973 (unpublished).

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[29]See Ref. 8. If we use Eq. (45) we need probably some sensible average of $\langle q_\rho / q_\omega \rangle^3$ over the distribution of broad resonances.

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[33]If we further impose asymptotic realization $\langle Q^+(\vec{p})|[\dot{V}_{K^+}, V_{K^+}]|Q'^-\rangle = 0$, $\vec{p} \to \infty$, we obtain a sum rule which implies that the effects of Q_A-Q_B mixing upon the GMO nonet mass-mixing-angle formulas for the 1^{++} and 1^{+-} nonets vanish. This is in accord with our assertion that the nonets keep their identities even in the presence of Q_A-Q_B mixing. However, the constraint also leads to $Q = Q'$. This implies that we have to include further SU(3) mixings when we consider the above realization. This realization is, in fact, very sensitive to mixing. Unlike other constraints considered in this paper, the above realization produces no constraint (i.e., $0 = 0$) if we do not introduce SU(3) mixings. This suggests that the masses of the SU(3) states, Q_A and Q_B, are close. We thank Dr. H. L. Hallock for useful discussions.