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Multiplicity distributions in a quark model for high-energy nucleon-nucleon scattering

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A quark model for high-energy nucleon-nucleon scattering is outlined. The ratio of elastic to total cross section and the average transverse momentum of secondary hadrons are constant in this model. A constant value for the average three-momentum transfer in high-energy collisions is assumed based on the physical picture of two Lorentz-contracted disks colliding at infinite momentum in the c.m. Besides the nucleon and pion masses, only the three constants mentioned previously are needed to calculate hadronic multiplicity distributions, and these constants can be determined from experimental data. The average charged-hadron multiplicity calculated from the model grows as $E^{1/4}$ at asymptotic energies, where E is the laboratory energy of the projectile. The calculated values of the higher moments of the multiplicity distribution show the same qualitative behavior as is experimentally observed.

INTRODUCTION

This paper outlines a simple model for high-energy nucleon-nucleon scattering. The model can be used to estimate hadronic multiplicity distributions at high energies, it is being extended to calculate longitudinal-momentum distributions, and it may provide a starting point for development of more realistic models in the future.

Consider a model in which two (Lorentz-contracted) nucleons collide with infinite momentum in the c.m. frame,[1] and inelastic collisions involve only longitudinal-momentum transfer.[2] Let us make the following assumptions.

(1) Final-state hadrons are either nucleons or pions.

(2) Each of the colliding nucleons is composed of three constituent quarks, and nucleon-nucleon scattering is dominated by events in which one quark in the beam nucleon (the beam quark) scatters off one quark in the target nucleon (the target quark), while the other constituent quarks acts as spectators, retaining their initial fraction of the incident nucleon's momentum. This is the standard assumption of the additive quark model for high-energy hadronic scattering.[3]

(3) Each constituent quark consists of a valence quark plus an accompanying color-gluon cloud.[4] The probability of a constituent quark (moving at infinite momentum) containing a valence quark plus n gluons is given by a Poisson distribution[5]

$$P_Q(n) = \mu^n e^{-\mu}/n!\,, \tag{1}$$

where μ is the average number of gluons accompanying a valence quark.

(4) The force between two colliding quarks at very high energies can be modeled by an impulsive force in the c.m. system.[6] Consequently, in inelastic collisions, where some of the kinetic energy of the colliding nucleons goes to produce secondary hadrons, the total c.m. momentum p' of the system of final-state hadrons arising from the beam (or target) hadron is given by

$$p' = p - \Delta\,, \tag{2}$$

where p is the c.m. momentum of the incident nucleons and Δ is the impulse (three-momentum transfer). In elastic collisions, of course, only the direction of the incident nucleon's momentum is changed, while the magnitude is unaltered.

(5) There are two mechanisms for the production of secondary hadrons. In the first mechanism, which operates if the beam (or target) constituent quark contains one or more gluons before the collision, the collision breaks up the constituent quark, converting the gluons into quark-antiquark ($Q\overline{Q}$) pairs. The spectator quarks pick up a quark with the appropriate color charge out of the fragments of the scattered constituent quark to form the leading nucleon. The remaining $Q\overline{Q}$ pairs are the precursors of a fireball of "centrally produced" secondary pions.

In the second production mechanism, which operates only if the incident beam (or target) constituent quark consists of a valence quark without accompanying gluons just prior to the collision, the collision may cause this valence quark to emit gluons, resulting in a constituent quark containing the valence quark plus one or more gluons. If this occurs, an unstable baryon isobar is created when the scattered constituent quark is picked up by the spectators. This leading fireball then decays into a nucleon plus secondary pions. It is further assumed that the probability that a constituent quark composed of a valence quark without accompanying gluons will emit n gluons during the scattering process to become a constituent quark containing a valence quark plus n gluons equals $P_Q(n)$, which is the probability that an incident quark contains n gluons.

(6) The fireballs produced by either of the two mechanisms discussed above emit pions isotropically in the fireball c.m. frame, in a manner similar to the fireballs produced in a previously published model for e^+e^- annihilation into hadrons.[7] Each fireball must create at least one pion, and the number n of additional pions produced follows a truncated Poisson distribution[8]

$$P_F(n) = \alpha \xi^n e^{-\xi}/n!\,, \quad n \le m_A/m_\pi - 1$$

and (3)

$$P_F(n) = 0\,, \quad n > m_A/m_\pi - 1\,.$$

The average number ξ of additional pions produced by a fireball is given by

$$\xi = [m_A/(p_H^2 + m_\pi^2)^{1/2}] - 1\,, \tag{4}$$

where m_A is the available energy for pion production and $(p_H^2 + m_\pi^2)^{1/2}$ is the average energy needed to produce a pion in the fireball c.m. frame. The available energy for pion production from a central production fireball is $m_A = m_C^*$ where m_C^* is the invariant mass of the central production fireball. The available energy for pion production from a leading fireball is $m_A = m_L^* - m_N$, where m_L^* is the invariant mass of the leading fireball and m_N is the nucleon mass, since the nucleon remains after the leading fireball radiates pions. The average energy required to produce a pion, $(p_H^2 + m_\pi^2)^{1/2}$, is determined by the pion mass m_π and $|\vec{p}_H|$, which is the average magnitude of the momentum in the fireball c.m. frame of the pions produced from a fireball.[9] The normalization constant α is given by

$$\alpha^{-1} = \sum_{n=0}^{n_{max}} \xi^n e^{-\xi}/n!\,,$$

where n_{max} is the largest integer less than or equal to $(m_A/m_\pi) - 1$.

In the subsequent development of this model, the motion of constituent quarks within the nucleon and the motion of valence quarks and gluons within the constituent quarks will be incorporated through the use of probability functions, which specify the probability that a constituent quark carries a fraction x of the nucleon's momentum and the probability that a valence quark carries a fraction y of the constituent quark's momentum. However, in this paper, multiplicity distributions will be calculated in the first approximation, relying on

(a) the fact that, on the average, each constituent quark must carry $\frac{1}{3}$ of the nucleon's momentum and

(b) the assumption that a gluon carries, on the average, twice as much momentum as a valence quark, as suggested by the equilibrium gluon $\to Q\bar{Q}$ pair.

Then, when an incident beam or target quark composed of a valence quark plus n gluon breaks up after being decelerated in the scattering event, the valence quark carries, on the average, $1/(2n+1)$ of the momentum of the decelerated constituent quark, while the quarks and antiquarks created from the gluons carry $2n/(2n+1)$ of the decelerated constituent quark's momentum, on the average.

The model developed in this paper is related to earlier work by Satz[10] and by Altarelli, Cabibbo, Maiani, and Petronzio.[4] The model is not the same as the usual quark-parton model, and contact with the deep-inelastic lepton scattering data will have to be made via the methods of Altarelli *et al.*[4]

DETERMINING THE PARAMETERS OF THE MODEL

In this model, the probability of elastic scattering is the probability $P_Q^2(0)$ that both the beam and target constituent quarks contain no gluons just before the collision, times the probability $P_Q^2(0)$ that neither of the colliding valence quarks emit gluons during the collision. At very high energies, the ratio σ_{el}/σ_{tot} is approximately 0.175, and therefore

$$P_Q^4(0) = e^{-4\mu} = 0.175\,. \tag{5}$$

Consequently, $\mu = 0.436$ and this establishes the average number of gluons in a constituent quark moving at infinite momentum.

The average transverse momentum of secondary pions in this model is

$$\langle |p_T| \rangle = (\tfrac{2}{3})^{1/2} |\vec{p}_H|\,. \tag{6}$$

If $|\vec{p}_H|$ is fixed at 440 MeV/c, the average transverse momentum of secondary pions is 360 MeV/c, in rough agreement with experiment.

At high energies in the c.m. frame, the physical picture of two Lorentz-contracted colliding disks (the hadrons or their constituent quarks) approaching each other at infinite momentum is always the same. Thus, it is appropriate to assume that the average value of the three-momentum transfer Δ does not depend on the c.m. energy. Therefore, the final-state momentum p' of the beam (or target) system of hadrons produced in an inelastic collision in this model is related to p, the incident hadron momentum, by $p' = p - \Delta$. A value of $\Delta = 0.75$ GeV/c was arbitrarily chosen to get the average charged multiplicities approximately correct.[11]

MODEL DEVELOPMENT

Consider one of the colliding constituent quarks in one of the incident hadrons. The probability, in the infinite-momentum frame, of this constituent quark consisting of a valence quark plus n gluons is given by the Poisson distribution (1), which is normalized according to

$$1=\sum_{n=0}^{\infty} P_Q(n). \tag{7}$$

If the colliding constituent quark consists of a valence quark plus one or more gluons just prior to the collision, the collision process breaks up the constituent quark and converts the gluons into $Q\bar{Q}$ pairs. A "bare" quark of the appropriate color is picked up by the spectator constituent quarks to form the leading nucleon, and the remaining fragments of the colliding constituent quark are the progenitors of the central production fireball. There is no further production of secondary hadrons by the leading nucleon in this case. If Eq. (7) is rewritten as

$$1=P_Q(0)+\sum_{n=1}^{\infty} P_Q(n), \tag{8}$$

the first term $P_Q(0)=e^{-\mu}$ gives the probability of events in which no central production results from the colliding quark under consideration, while the second term, $\sum_{n=1}^{\infty} P_Q(n)=1-P_Q(0)$, gives the probability of central production from the colliding quark.

If the colliding constituent quark consists of a valence quark unaccompanied by gluons just prior to the collision, it may emit gluons during the collision. When this "excited" constituent quark is picked up by the spectator quarks to form a leading baryonic system, the resulting excited baryonic system subsequently decays by radiating secondary pions. Since the model under development does not incorporate a detailed description of the evolution of the leading baryonic system into final-state hadrons, events in which different numbers of gluons are radiated by an unaccompanied valence quark during a collision are treated on the same footing. The probability of an unaccompanied valence quark emitting n gluons during a collision is assumed to equal $P_Q(n)$, so Eq. (8) can be rewritten as

$$1=P_Q(0)\{P_Q(0)+[1-P_Q(0)]\}+\sum_{n=1}^{\infty} P_Q(n), \tag{9}$$

where $[1-P_Q(0)]=\sum_{n=1}^{\infty} P_Q(n)$ is the probability of an unaccompanied valence quark emitting one or more gluons during a collision. If Eq. (9) is rewritten as

$$1=P_Q^{\,2}(0)+P_Q(0)[1-P_Q(0)]+\sum_{n=1}^{\infty} P_Q(n), \tag{10}$$

the first term gives the probability that no secondary hadronic fireball is created by the colliding constituent quark, the second term gives the probability that the initially unaccompanied valence quark in the colliding constituent quark emits gluons during the collision resulting in a leading excited baryonic system or fireball, and the third term gives the probability of the colliding constituent quark fragmenting to create a central production fireball.

Since one constituent quark in each colliding nucleon participates in a high-energy nucleon-nucleon scattering event, the product of two expressions like Eq. (10)

$$1=\left\{P_Q^{\,2}(0)+P_Q(0)[1-P_Q(0)]+\sum_{n=1}^{\infty} P_Q(n)\right\} \times\left\{P_Q^{\,2}(0)+P_Q(0)[1-P_Q(0)]+\sum_{n=1}^{\infty} P_Q(n)\right\} \tag{11}$$

can be used to determine the probability of different types of events in nucleon-nucleon scattering.

The invariant masses of the fireballs created in inelastic scattering events can be calculated from kinematics. In those cases where both the beam and target quarks generate fireballs through one or the other of the two postulated mechanisms, the invariant mass m_2^* of the beam (or target) hadronic system can be calculated from

$$(p^2+m_N^{\,2})^{1/2}=[(p-\Delta)^2+m_2^{*2}]^{1/2}, \tag{12}$$

assuming that momentum, but not energy, is transferred between the systems. In those cases where only one of the colliding quarks generates a fireball, both momentum and energy must be transferred, and the invariant mass m_1^* of the (beam or target) hadronic system containing the fireball is found from

$$2(p^2+m_N^{\,2})^{1/2}=[(p-\Delta)^2+m_1^{*2}]^{1/2}+[(p-\Delta)^2+m_N^{\,2}]^{1/2}. \tag{13}$$

Once the invariant masses of the beam and target hadronic systems subsequent to the scattering are known, the fireball masses are readily determined. In this model, the invariant mass m_L^* of a leading fireball (the excited baryonic system generated when a colliding constituent quark, which initially contains no gluons, emits one or more gluons during a collision) is identical to the invariant mass of the corresponding beam or target hadronic system. On the other hand, when a colliding constituent quark which contains a valence quark and n gluons immediately prior to the collision fragments to create a central production fireball, the average total momentum of the resulting n $Q\bar{Q}$ pairs which are the progenitors of the central production fireball is

$$p_C=\frac{2n}{2n+1}\left(\tfrac{1}{3}p-\Delta\right), \tag{14}$$

while the average momentum of the leading nucleon created when the spectator quarks pick up one of the fragment quarks is

$$p_L = \tfrac{2}{3}p + \frac{1}{2n+1}\left(\tfrac{1}{3}p - \Delta\right). \qquad (15)$$

The invariant mass of the central production fireball $m_C^*(n)$ is then found from

$$[(p-\Delta)^2 + m_X^{*2}]^{1/2} = (p_L{}^2 + m_N{}^2)^{1/2} + [p_C{}^2 + m_C^{*2}(n)]^{1/2}, \qquad (16)$$

where m_X^* takes on the values m_2^* and m_1^* determined in Eq. (12) or (13), respectively, depending on whether or not hadronic fireballs are generated in association with both or only one of the beam and target hadronic systems. Note that the average mass of the central production fireball $m_C^*(n)$ depends upon the number n of gluons dressing the colliding constituent quark which created the fireball.

The probability of creating n pions, in addition to the two colliding nucleons, in a nucleon-nucleon scattering event is determined by finding the probability of creating m pions from one of the colliding nucleons and convoluting it with the (identical) probability distribution for the other colliding nucleon. If $P_L(k)$ denotes the probability [given by Eq. (3)] of producing $k+1$ pions from a leading fireball with invariant mass m_L^* and $P_C(k;n)$ denotes the probability [also given by a function like that shown in Eq. (3)] of producing $k+1$ pions from a central production fireball of invariant mass $m_C^*(n)$ created from a colliding constituent quark containing n gluons, Eq. (10) can be rewritten as

$$1 = P_Q{}^2(0) + P_Q(0)[1-P_Q(0)] \sum_{k=0}^{m_L^*/m_\pi - 1} P_L(k) + \sum_{n=1}^{\infty} P_Q(n) \sum_{k=0}^{m_C^*(n)/m_\pi - 1} P_C(k;n). \qquad (17)$$

Consequently, the probability of one of the colliding nucleons producing m pions is

$$P_1(m) = \delta_{m0} P_Q{}^2(0) + P_Q(0)[1-P_Q(0)] P_L(m-1) + \sum_{n=1}^{\infty} P_Q(n) P_C(m-1;n). \qquad (18)$$

The probability of producing n secondary pions from both colliding nucleons is given by the convolution

$$P_\pi(n) = \sum_{k=0}^{n} P_1(n-k) P_1(k). \qquad (19)$$

Experiments generally measure charged-particle production, and since charge conservation requires that charged particles are produced in pairs, it is convenient to develop the model in terms of the number of hadron *pairs* produced in addition to the two initial colliding hadrons. Using Eq. (18), the probability of one of the colliding nucleons producing i pion pairs is

$$P_p(i) = P_1(2i) + P_1(2i+1). \qquad (20)$$

Since the probability of a pion pair being charged is $\frac{2}{3}$, the probability of producing j charged-pion pairs in addition to the hadron carrying the charge on the incident proton is

$$P'_{\text{ch}}(j) = \sum_{i=j}^{\infty} \left(\frac{2}{3}\right)^{j} \left(\frac{1}{3}\right)^{i-j} \left[\frac{i!}{j!(i-j)!}\right] P_p(i). \qquad (21)$$

The probability of producing n charged-pion pairs in addition to the hadrons carrying the charge of the incident protons is then given by the convolution

$$P''_{\text{ch}}(n) = \sum_{k=0}^{n} P'_{\text{ch}}(n-k) P'_{\text{ch}}(k). \qquad (22)$$

Since charge is conserved and the colliding hadrons in a pp collision are both charged, the probability $P_{\text{ch}}(m)$ of finding m charged hadrons in the final state of a pp scattering event is

$$P_{\text{ch}}(m) = P_{\text{ch}}(2n+2) = P''_{\text{ch}}(n) \qquad (23)$$

and it is this distribution which is used to make comparisons with experiment. Most experiments involve the properties of the multiplicity distribution of the charged hadrons produced in inelastic collisions. The average number of charged hadrons produced in the inelastic scattering of two positively (or negatively) charged hadrons is given by

$$\bar{n} = \left[\sum_{n=2}^{\infty} n P_{\text{ch}}(n) - 2P_{\text{el}}\right] \Big/ (1 - P_{\text{el}}) \qquad (24)$$

and the higher moments of the distribution are determined by

$$\overline{n^k} = \left[\sum_{n=2}^{\infty} n^k P_{\text{ch}}(n) - 2^k P_{\text{el}}\right] \Big/ (1 - P_{\text{el}}), \qquad (25)$$

where P_{el} is the probability of elastic scattering.

RESULTS

Three results have been built into the model:

(i) The average transverse momentum of secondary hadrons is constant. Note that the average transverse momentum is different for fireballs of different multiplicity, just as in the earlier model for e^+e^- annihilation.[7] It is only the average transverse momentum averaged over all fireball multiplicities which is constant. The value chosen for the average transverse momentum of secondary hadrons is 360 MeV/c.

(ii) The longitudinal momentum transfer in hadron scattering events is negligible. In this model, the force between two colliding hadrons is an impulsive force in the c.m. system, so the c.m. momentum p' of the beam and target hadronic systems after an inelastic collision is related to the initial c.m. momentum p by $p'=p-\Delta$. Since Δ was chosen as 0.75 GeV/c, $p'\approx p$ at high energies, and the model approximately satisfies the minimal rule of Benecke, Chou, Yang, and Yen[12] which states that only infinitesimal longitudinal momentum transfer is allowed at asymptotic energies.

(iii) The ratio of elastic to total cross section σ_{el}/σ_{tot} is constant in the model. In proton-proton scattering, the ratio σ_{el}/σ_{tot} is approximately constant at 0.175 for incident laboratory momenta above about 100 GeV/c. Since σ_{el}/σ_{tot} rises at incident proton momenta below 100 GeV/c, this indicates that finite momentum effects should become significant in the model below a c.m. energy of about 15 GeV in nucleon-nucleon scattering.

The following results indicate the kinds of information which can be obtained from the model in its present form:

(a) If the average value of the three-momentum transfer Δ is assumed to be constant at high energies, the average charged multiplicity grows as $E^{1/4}$ at asymptotic energies, where E is the incident laboratory nucleon energy. Assuming a constant value for the three-momentum transfer in the c.m. frame in high-energy hadronic scattering implies [through Eqs. (12), (13), and (16)] that the invariant mass of the fireball of secondary hadrons produced in an inelastic collision rises as $E^{1/4}$. Then the assumption implicit in Eq. (4), that the average number of secondary hadrons is a linear function of the invariant mass of the fireball, leads to an average charged-hadron multiplicity growing as $E^{1/4}$ at asymptotic energies. In fact, there are indications[13] from cosmic-ray experiments that the average charged-hadron multiplicity grows faster above about 5 TeV in the laboratory (approximately 100 GeV in the c.m.) than would be indicated by the $a+b\ \ln(s)$ form commonly used[14] at lower energies.

(b) The fraction of scattering events of different types can be determined from Eqs. (8), (10), and (11). Since μ is chosen as 0.436 to obtain $\sigma_{el}/\sigma_{tot}=0.175$, Eq. (8) states that in $P_Q(0)=64.7\%$ of events, a colliding quark does not generate central production hadrons, while in the remaining $1-P_Q(0)=35.3\%$ of events, central production hadrons are created. Similarly, Eq. (10) says that the probability of a colliding constituent quark creating no secondary hadronic fireball is $P_Q^{\,2}(0)=41.8\%$, the probability that the quark creates a leading fireball is $P_Q(0)\ [1-P_Q(0)]=22.9\%$, and the probability that the quark creates a central production fireball is $\sum_{n=1}^{\infty} P_Q(n)=1-P_Q(0)=35.3\%$.

Finally, when the colliding quarks in each incident hadron are considered as in Eq. (11), the percentage of nucleon-nucleon scattering events of different types are shown in Table I.

(c) Computer calculations were performed using the values of the input parameters discussed previously. No attempt was made to search for the best values of the input parameters. At each value of the c.m. energy the probability of producing m pions from each of the two colliding nucleons is calculated from Eq. (18) in each of two ways. In calculation A, incident constituent quarks containing a valence quark plus one, two, or three gluons were considered. To preserve the proper normalization (conserve probability), $P_Q(3)$ was taken as $P_Q(3)=1-\sum_{n=0}^{2} P_Q(n)$. The calculation was truncated at $n=3$, because the probability that a quark is dressed by fewer than four gluons is $\sum_{n=0}^{3} P_Q(n)=0.9989$. Furthermore, the probability that a constituent quark containing gluons contains only one, two, or three gluons is $\sum_{n=1}^{3} P_Q(n)/[1-P_Q(0)]=0.9970$.

Calculation B employed the simplest possible version of the model, in which a valence quark can be dressed by either zero or one gluon and the proper normalization is ensured by taking $P_Q(1)=1-P_Q(0)$. This rather drastic approximation is still expected to be fairly good, because the probability that a constituent quark consists of a valence quark dressed by zero or one gluon is $P_Q(0)+P_Q(1)=0.9825$ and the probability that a constituent quark containing gluons contains only one gluon is $P_Q(1)/[1-P_Q(0)]=0.7978$.

The results of the calculations at c.m. energies between 10.7 and 62.8 GeV are shown in Tables II, III, and IV. Table II presents data on the average charged-hadron multiplicity in inelastic scattering events, which is henceforth denoted by $\bar{n}$. The data sources are Bromberg *et al.*,[15] Dado *et al.*,[16] and Thome *et al.*[17] The experimental values of the charged multiplicity are compared to the calculated values and to the Albini[14] parametrization

$$\bar{n}_{\text{Albini}}=2.5+0.28\ln(E-2m_N)+0.53[\ln(E-2m_N)]^2,$$

which can be used to represent the experimental data on average charged multiplicities in pp collisions at c.m. energies below 150 GeV. The results in Table II show that the calculated values of the average charged multiplicity are between 3 and 10% lower than the experimental values at the tabulated energies. When the Albini[14] parametrization was used to represent the experimental data on average charged-hadron multiplicities over the c.m. energy range 5–150 GeV, calculations A and B both gave

TABLE I. Percentage of different types of high-energy nucleon-nucleon scattering events. The first square bracket indicates the behavior of the beam quark and the second square bracket indicates the behavior of the target quark.

Elastic scattering (No fireballs)	$[P_Q{}^2(0)][P_Q{}^2(0)]$	=17.5%
One-fireball events		
Beam produces leading fireball	$[P_Q(0)\{1-P_Q(0)\}][P_Q{}^2(0)]$	= 9.6%
Target produces leading fireball	$[P_Q{}^2(0)][P_Q(0)\{1-P_Q(0)\}]$	= 9.6%
Beam produces central fireball	$[1-P_Q(0)][P_Q{}^2(0)]$	=14.8%
Target produces central fireball	$[P_Q{}^2(0)][1-P_Q(0)]$	=14.8%
	Total one-fireball events	=48.7%
Two-fireball events		
Beam and target produce leading fireballs	$[P_Q(0)\{1-P_Q(0)\}][P_Q(0)\{1-P_Q(0)\}]$	= 5.2%
Beam produces leading fireball, target produces central fireball	$[P_Q(0)\{1-P_Q(0)\}][1-P_Q(0)]$	= 8.1%
Target produces leading fireball, beam produces central fireball	$[1-P_Q(0)][P_Q(0)\{1-P_Q(0)\}]$	= 8.1%
Beam and target produce central fireballs	$[1-P_Q(0)][1-P_Q(0)]$	=12.5%
	Total two-fireball events	=33.8%

calculated multiplicities lying between ±10% of the Albini values, and calculations A and B never differed from each other by more than 1%. Because of the $E^{1/4}$ growth in the average multiplicity at high energy predicted by the model, the multiplicities calculated from the model are progressively higher than the values predicted by the Albini parametrization at c.m. energies above 150 GeV, in agreement with the trend observed in cosmic-ray data.[13] The fact that the calculated values of the charged multiplicity lie within 10% of the Albini values at energies as low as 5 GeV is somewhat surprising, since the model is not expected to be valid below 15 GeV in the c.m., where the experimental values of σ_{el}/σ_{tot} are greater than the constant value observed at higher energies and predicted by this model at all energies.

Table III provides information on the moments of

TABLE II. Average charged-hadron multiplicities. The first three rows show the c.m. energy, the corresponding laboratory momentum, and the experimental value of the average charged-hadron multiplicity. The fourth row shows the source of the charged-hadron multiplicity data: Bromberg *et al.* (Ref. 15), Dado *et al.* (Ref. 16), or Thome *et al.* (Ref. 17). The fifth row shows the value of the charged-hadron multiplicity predicted by the Albini *et al.* parametrization $\bar{n}_{\text{Albini}} = 2.5 + 0.28\ln(E-2m_N) + 0.53[\ln(E-2m_N)]^2$. The last four rows show the average charged-hadron multiplicities in inelastic scattering events and the percentage errors from the experimental results for the two model calculations discussed in the text.

c.m. energy (GeV)	10.7	13.8	19.7	23.6	27.6	30.8	45.2	53.2	62.8
Laboratory momentum (GeV/c)	60	100	205	296	400	505	1090	1510	2100
$\bar{n}$ (experiment)	5.60	6.39	7.67	8.12	8.83	9.54	11.01	11.77	12.70
Data source (Ref. No.)	15	16	16	17	16	17	17	17	17
$\bar{n}$ [Albini *et al.* (Ref. 14)]	5.62	6.44	7.70	8.78	9.00	9.44	11.08	11.82	12.60
Calculation A									
$\bar{n}$ (calculated)	5.42	6.06	7.10	7.70	8.26	8.68	10.35	11.15	12.05
% error	−3.2	−5.2	−7.4	−5.2	−6.5	−9.0	−6.0	−5.3	−5.1
Calculation B									
$\bar{n}$ (calculated)	5.39	6.03	7.06	7.65	8.21	8.63	10.27	11.07	11.96
% error	−3.8	−5.6	−8.0	−5.8	−7.0	−9.5	−6.7	−5.9	−5.8

TABLE III. The moments of the charged-hadron multiplicity distribution, $D_q = [\langle (n - \bar{n})^q \rangle]^{1/q}$. Data sources for D_2 (experiment) are the same as in Table II for the corresponding c.m. energy. D_q (Thome *et al.* fit) are the fits to experimental data presented by Thome *et al.* (Ref. 17): $D_2 = 0.576(\bar{n} - 0.968)$, $D_3 = 0.522(\bar{n} - 0.995)$, and $D_4 = 0.799(\bar{n} - 1.067)$. The calculated values of D_q are denoted D_q (calculation A) and D_q (calculation B), respectively. Least-squares linear fits through the calculation A points are $D_2 = 0.389(\bar{n} + 0.402)$, $D_3 = 0.308(\bar{n} + 0.736)$, and $D_4 = 0.506(\bar{n} + 0.648)$. The corresponding fits to the calculation B points are $D_2 = 0.395(\bar{n} + 0.319)$, $D_3 = 0.316(\bar{n} + 0.624)$, and $D_4 = 0.515(\bar{n} + 0.570)$.

c.m. energy (GeV)	10.7	13.8	19.7	23.6	27.6	30.8	45.2	53.2	62.8
D_2 (experiment)	2.50	3.21	3.82	4.05	4.59	4.83	5.90	6.39	6.92
D_2 (Thome *et al.* fit)	2.67	3.12	3.86	4.12	4.53	4.94	5.78	6.22	6.76
D_2 (calculation A)	2.24	2.50	2.92	3.16	3.38	3.54	4.18	4.49	4.82
D_2 (calculation B)	2.24	2.50	2.92	3.16	3.38	3.54	4.19	4.50	4.84
D_3 (Thome *et al.* fit)	2.40	2.82	3.48	3.72	4.09	4.46	5.23	5.62	6.11
D_3 (calculation A)	1.87	2.08	2.42	2.62	2.79	2.92	3.42	3.66	3.92
D_3 (calculation B)	1.87	2.09	2.44	2.63	2.81	2.94	3.45	3.69	3.95
D_4 (Thome *et al.* fit)	3.62	4.25	5.28	5.64	6.20	6.77	7.94	8.55	9.29
D_4 (calculation A)	3.03	3.38	3.94	4.25	4.54	4.75	5.58	5.97	6.40
D_4 (calculation B)	3.03	3.38	3.94	4.26	4.55	4.76	5.59	5.99	6.42

the multiplicity distribution $D_q = [\langle (n - \bar{n})^q \rangle]^{1/q}$. The experimental values of D_2 are taken from the same data sources as the charged multiplicity data in Table II. The columns labeled D_q^{fit} are calculated from the following fits to experimental data presented by Thome *et al.*[17]:

$$D_2^{\text{fit}} = 0.576(\bar{n} - 0.968),$$

$$D_3^{\text{fit}} = 0.522(\bar{n} - 0.995),$$

$$D_4^{\text{fit}} = 0.799(\bar{n} - 1.067).$$

The values for D_q obtained from calculations A and B are always within 1% of each other, and they show the same qualitative behavior as the experimental results, rising as an approximately linear function of $\bar{n}$. However, the calculated values of D_q rise more slowly than the experimental data, indicating that the calculated multiplicity distributions are narrower than the experimentally observed distributions. For example, the best linear fits to the values of D_q obtained from calculation B for the c.m. energies tabulated in Table III are $D_2 = 0.395(\bar{n} + 0.319)$, $D_3 = 0.316(\bar{n} + 0.624)$, and $D_4 = 0.515(\bar{n} + 0.570)$. Taking account of the distribution of constituent-quark and valence-quark momenta in the colliding nucleons should move the calculated values of D_q in the direction of better agreement with experiment.

Information on the correlation function $f_2 = \langle n(n-1) \rangle - \langle n \rangle^2$ and the reduced moments $\langle n^q \rangle / \langle n \rangle^q$ of the charged-hadron multiplicity distributions is presented in Table IV. The data sources at each value of the c.m. energy are the same as the data sources for the corresponding energy in Table II. The calculated values of the correlation function f_2 show the same qualitative behavior as experiment, but they rise more slowly with energy. The calculated values of the reduced moments fall very slowly as the energy increases instead of rising slowly as is observed experimentally. In both cases the results from calculations A and B lie within a few percent of each other. Once again, taking account of the distribution of constituent- and valence-quark momenta in the colliding nucleons should move the calculated results in the direction of better agreement with experiment.

(d) In this model, the average number of gluons contained in a constituent quark is $\mu = 0.436$. Based on the assumption that a gluon carries (on the average) twice as much momentum as a quark, the fraction of a proton's momentum carried by the electrically neutral gluons is $2\mu/(1+\mu) = 46.6\%$, on the average. Correspondingly, the average percentage of the proton's momentum carried by charged matter (the quarks) is 53.4%, in agreement with the result extracted from experiment.[18]

(e) Existing experimental data suggest that final-state hadron multiplicities at low rapidity in the overall c.m. frame are approximately constant as a function of energy. However, in its present form the model developed in this paper predicts a vanishing hadron multiplicity at low rapidity in the overall c.m. frame at asymptotic energies. Consequently, if the present trends in experimental data on rapidity distributions continue at higher energies, the model will have to be extended to take account of the distribution of quark momenta within the colliding hadrons in order to provide an adequate description of the longitudinal rapidity distribution of final-state hadrons.

The simplest way to see that the present version of the model leads to vanishing hadron multipli-

TABLE IV. Correlation function and reduced moments for charged-hadron multiplicity distributions. The data sources for the correlation function $f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$ and the reduced moments $\langle n^q\rangle / \langle n\rangle^q$ are the same as for the corresponding c.m. energies in Table II. The rows denoted calculation A and calculation B give the calculated results obtained from the two calculations described in the text.

c.m. energy (GeV)	10.7	13.8	19.7	23.6	27.6	30.8	45.2	53.2	62.8
$f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$									
Experiment	0.96	3.9	6.9	8.3	12.2	13.76	23.78	29.10	35.20
Calculation A	−0.38	0.21	1.42	2.27	3.14	3.85	7.12	8.98	11.23
Calculation B	−0.37	0.23	1.46	2.32	3.21	3.94	7.27	9.16	11.45
$\langle n^2\rangle / \langle n\rangle^2$									
Experiment	1.25	1.25	1.25	1.25	1.25	1.26	1.29	1.30	1.30
Calculation A	1.17	1.17	1.17	1.17	1.17	1.17	1.16	1.16	1.16
Calculation B	1.17	1.17	1.17	1.17	1.17	1.17	1.17	1.17	1.16
$\langle n^3\rangle / \langle n\rangle^3$									
Experiment	1.84	1.84	1.84	1.84	1.84	1.86	1.99	2.01	2.02
Calculation A	1.56	1.55	1.55	1.54	1.54	1.54	1.53	1.52	1.52
Calculation B	1.56	1.56	1.56	1.55	1.55	1.55	1.54	1.53	1.53
$\langle n^4\rangle / \langle n\rangle^4$									
Experiment	3.05	3.05	3.05	3.05	3.05	3.12	3.53	3.58	3.60
Calculation A	2.29	2.29	2.27	2.26	2.26	2.24	2.21	2.19	2.18
Calculation B	2.30	2.30	2.29	2.28	2.27	2.26	2.24	2.22	2.21

city at low rapidity for asymptotic energies is the following. Hadrons with low longitudinal rapidity in the overall c.m. frame are hadrons with longitudinal momentum near zero in the overall c.m. frame. If a fireball with invariant mass m^* decays isotropically into k pions in its rest frame, the magnitude of the velocity of the secondary pions in the fireball rest frame is

$$|\vec{V}_k| = |\vec{p}_k|/E_k = \frac{k}{m^*}[(m^*/k)^2 - m_\pi^2]^{1/2}.$$

If p_F is the momentum of the fireball in the overall c.m. frame, $\vec{\beta}_F$ for the fireball is given by

$$|\vec{\beta}_F| = |\vec{p}_F|/(p_F^2 + m^{*2})^{1/2}.$$

Then the kinematical analysis detailed in Hagedorn's book[19] shows that secondary hadrons will be found with zero longitudinal momentum in the overall c.m. frame if $|\vec{\beta}_F| \leq |\vec{V}_k|$. From Eqs. (14) and (15), p_F grows like $E^{1/2}$, where E is the beam energy in the laboratory system. From Eqs. (12) and (13), m^* grows as $E^{1/4}$. Therefore, at high laboratory energy, $|\vec{\beta}_F| \approx 1 - m^{*2}/2p_F^2$ and $|\vec{V}_k| \simeq 1 - k^2 m_\pi^2/2m^{*2}$. Thus, at high energy, the condition $\beta_F \leq v_k$ for finding secondary hadrons with zero longitudinal momentum (and zero longitudinal rapidity) in the overall c.m. frame becomes $m^{*2}/m_\pi p_F \geq k$. Since the left side of this expression approaches a constant at asymptotic energy, there is an upper limit k' to the fireball decay multiplicities which can generate hadrons at zero longitudinal momentum (rapidity) in the overall c.m. frame. Then, from Eq. (3), the probability of finding hadrons with zero momentum in the overall c.m. frame is proportional to

$$\sum_{n=0}^{k'} \xi^n e^{-\xi}/n!$$

and, since $\xi \sim m^* \sim E^{1/4}$, this probability goes to zero at asymptotic energies. A similar argument can be developed for any finite longitudinal rapidity in the overall c.m. system

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[1]Since the model is developed in the c.m. frame for simplicity, the model is not relativistically covariant.

[2]The assumption of purely longitudinal momentum transfer is made because the great majority of inelastic high-energy hadronic scattering events produce narrow jets of secondaries along the directions of the incident hadrons, and the transverse motion of these secondaries can be neglected in the first approximation.

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[5]The Poisson distribution was chosen because of its simplicity and because the following assumptions regarding the occurrence of gluons in the color field around a valence quark seem reasonable: (a) The probability of finding a gluon in an infinitesimal volume δV is approximately $\lambda\delta V$ where λ is a small parameter. (b) The probability of finding two gluons in a volume δV is negligible compared to $\lambda\delta V$. (c) The number of gluons in any infinitesimal volume element is independent of the number in any nonoverlapping volume element. These assumptions can be used to derive the Poisson distribution.

[6]The interaction between nucleons is modeled by an impulsive force in the c.m. frame because the colliding quarks are Lorentz-contracted disks of infinitesimal thickness moving at infinite momentum. Therefore, the collision time, defined as the time required for the disks to pass through each other, is infinitesimal and an impulsive force may provide a reasonable representation of the physical system. Furthermore, at all energies high enough so that the colliding hadrons (or quarks) are moving at essentially infinite momentum, the collision will look the same in the c.m. frame, and the same impulsive force may be used to describe the interaction between the colliding hadrons (or quarks) in all such collisions. The impulsive force represents the quark-quark scattering force which might be mediated, for example, by exchange of a gluon carrying no net color charge [Yoichiro Nambu, Sci. Am. 235 (No. 5), 48 (1976)]. The dominance of high-energy hadronic scattering by a zero mass spin-one object (e.g., a colorless gluon) is also suggested by Regge-pole theory.

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[8]A Poisson distribution is again assumed because of its simplicity and because the assumptions given in Ref. 5 also seem reasonable when applied to the probability of producing additional pions within a fireball.

[9]Two arguments based on the uncertainty principle can be used to indicate the approximate size of $|\vec{p}_H|$. The first argument rests on the assumption that the secondary hadrons are created within a fireball having typical hadronic dimensions. The second argument assumes that pions are created from the quarks and antiquarks in a fireball when these energetic quarks and antiquarks move apart, and additional $Q\overline{Q}$ pairs are created out of the potential energy of their interaction. Thus, the characteristic volume within which secondary pions are created is related to the distance between a quark and an antiquark at which it becomes energetically favorable to create a $Q\overline{Q}$ pair out of the potential energy between them.

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