Local realism and the Einstein-Podolsky-Rosen paradox. On concrete new tests

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Bohm's spin polarization modification of the paradox of Einstein, Podolsky, and Rosen has served as the basis for numerous analyses of the roles of hidden variables and Einstein locality in quantum mechanics. The present note points out how corresponding analyses can be carried out on the *original* formulation of Einstein, Podolsky, and Rosen (EPR) in terms of observations of position and momentum. Concrete conceptual experiments are proposed to furnish tangible illustrations of EPR tests of reality, and a position-momentum variant of Bell's inequality is formulated. Implications regarding joint measurements of position and momentum, delayed choice experiments, and superluminal communication are discussed briefly.

INTRODUCTION

A series of experimental tests[1] of Bell's inequality[2] and related theorems[3,4] has attracted popular attention.[4,5] At issue is not the clarification of ambiguous areas of quantum theory—orthodox quantum mechanics is perfectly definite in what it has to say about the tests—but the validity of quantum theory itself. Even though few physicists actively dispute the essential correctness of quantum mechanics in the domain under investigation, the rather bizarre character of reality implied by quantum mechanics invites questions and engenders doubt. Einstein's own rejection of orthodox interpretations of quantum mechanics is well known.[6]

At the bottom of this renewed interest in quantum interpretation is a 1935 paper[7] by Einstein, Podolsky, and Rosen (EPR). In this work the quantum-mechanical consequences of a brief encounter between two particles are considered. It is shown that, long after the interaction has ceased, at a time when the particles have become widely separated, a measurement of the state of one of the particles formally dictates the state of the other. For example, long after the interaction, depending upon whether the observer chooses to measure the momentum (*or* position) of particle No. 1, he can thereby seemingly govern whether particle No. 2 is currently in an eigenstate of momentum (*or* position) or whether it "shall have been" in an eigenstate of momentum (*or* position) at the time of the encounter. Inasmuch as a particle cannot simultaneously be in an eigenstate of position and momentum, this conclusion is disturbing, and not less so because of the control over the state of particle 2 without touching it. This paradox is the focal point of a voluminous literature in which diverse and contradictory opinions have been expressed.[8,9]

One of the aims of the present paper is to outline a concrete conceptual experiment illustrating how to carry out and test the implications of the original EPR experiment over the range of the possible alternative choices of measurements. The meaning of the seemingly incompatible outcomes can then be seen in a more intuitively appealing perspective than has been the case in many prior formal and abstract discussions.

Its enormous impact on physics notwithstanding, the original EPR paper attracted only moderate attention at the time of its publication. The idea did not begin to capture the imagination of a wide audience until Bohm[10] introduced a variant of the EPR experiment in which states of atomic spin replaced states of position or momentum as observables. Although the contrast between complementary pairs of observables was less dramatic than in the original EPR paper, Bohm's thought experiment was far easier to visualize and treat, and served as a model for conceptual experiments involving photon polarization or nuclear spin.[1] Subsequently, Bell[2] systematically analyzed the differing implications of quantum mechanics and local realistic theories. In particular, Bell's inequality and its violation (statistically) by quantum systems provided the basis for the aforementioned tests of local realism.

In the following it will be shown how to construct analogous inequalities and tests pertaining to the original EPR observables, position and momentum.

CONCEPTUAL EXPERIMENT

It was Furry[11] who first suggested a "microscope" with which the EPR potentiality, previously represented only by a mathematical formula, could be realized. Furry's design in slightly modified form will be used in the following as a basis for discussion; later it will be augmented in order to illustrate the meaning of and the statistics of the EPR options of measurement. Actual details of hardware will be omitted (as well as any analysis of the current feasibility of the requisite optics) in the interests of focusing attention upon concepts. In the same vein, intrinsic limitations imposed by the uncertainty relations will not be examined until Appendix A because they do not interfere seriously with the proposed observations.

For simplicity, we shall assume that the EPR particles (1 and 2) have equal masses and zero charges. Uncharged particles are preferred over charged to ensure, to within the limits of their very small scattering lengths r_0, that the particles share the same coordinate x when they scatter from each other. Although the EPR scatterings will be rare if r_0 is small, those that send a particle into the Furry microscope will illustrate the necessary points. The particles are prepared to have well-defined and virtually equal but opposite momenta before collision. By conservation, then, p_{x2} remains equal and opposite to p_{x1} after the collision as well as before it (although collision changes the magnitude). Furthermore, an inference of the value of x_1 at collision establishes the corresponding value of x_2.

A schematic illustration of how the Furry microscope is able to accomplish a measurement of x or p_x of any particle incident upon it is presented in Fig. 1. It is clear from the preparation of the states of the colliding particles how a measurement of p_{x1} (at plane A, Fig. 1) implies the value of p_{x2} or how a measurement of x_1 (at plane B) implies the value of x_1 at collision and, thereby, the value of x_2 at collision. However, it is much more instructive to extend the experimental design so that the following questions may be addressed. How reliable is a deduction about the state of particle 2 if it has been inferred from a measurement of particle 1? Is it experimentally verifiable that a collapse of the wave function induced by the detection of particle 1 brings into being[12,13] the correlated component state of particle 2 predicted by quantum mechanics? Or alternatively, did particles 1 and 2 have definite (but correlated) states of position (or momentum) all along, *before* the detection, as specified by hidden variables ? Insights into the answers can be provided by tests calling into play a system of coupled Furry microscopes each equipped with additional accessories.

One version of an augmented Furry microscope is illustrated in Fig. 2. Greatly amplifying its effectiveness in discriminations between quantum events and events biased by hidden variables is a double-slit accessory designed for insertion at the image plane I of its projector stage S. This accessory permits the observer to perceive interference effects arising in quantum events that would not be expected if local hidden variables shaped nature. Characteristic of quantum mechanics is its representation of any wave function in terms of a superposition of eigenfunctions of arbitrarily chosen observables (within limits). So, for example, the wave function for the scattered particles 1 and 2 can be expressed as a superposition of a great number of spherical wavelets each emanating from a different collision site near plane P in Fig. 1. Alternatively, it can be protrayed by a superposition of a great number of scattered plane waves each corresponding to a different p_x component. In either case the basis functions correlate particles 1 and 2 in the required EPR fashion, x_1 with x_2 (the collision site) or p_{x1} with p_{x2} (momentum conservation). By contrast, hidden-variable theory attributes a reality, preexisting before the detection, to the particular momentum value or position that ultimately registers in the measurement apparatus. Of course, it

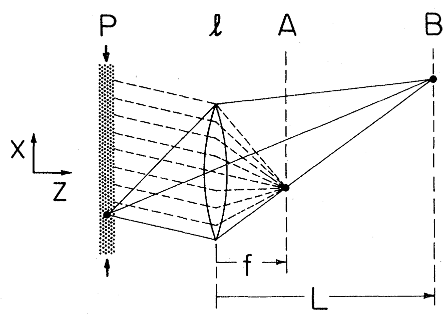

FIG. 1. Furry microscope for analyzing EPR collisions between particles 1 and 2 in region P. Lens l transmits spherical wavelets (cf. solid lines) issuing from collision sites x at P and focuses them to an image on plane B. The lens also focuses plane waves (cf. dashed lines), propagating with a definite momentum component p_x, to a point on plane A. Therefore, up until the moment of impact on plane A or B, a decision can be made to measure either x or p_x of the particle entering the microscope.

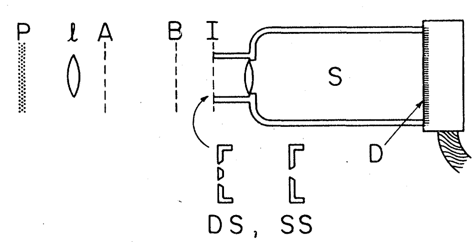

FIG. 2. Compound Furry microscope with enhanced powers of discrimination. An image of radiation received at plane I of the projector stage S is focused upon detector plane D which is densely packed with detectors. Plane I of the projector stage can be placed at plane B or plane A to measure x or p_x of the entering EPR particle. A single-slit or double-slit accessory (SS or DS) can be inserted at plane I to enable the observer to test various hypotheses (see Table I). It is advantageous to be able to place the slit accessories at an *image* (plane B) of the collision site instead of at the collision site itself where it could disturb the collisions.

is not possible, in a single measurement with the Furry microscope, to detect the delocalization of position (or momentum), corresponding to the simultaneous existence of the quantum components of superposition. Elementary particles are never detected simultaneously over a distribution of sites; either a whole particle is detected, or none. Therefore, just as in tests of the Bell inequality, a decision between the contending hypotheses of reality requires a statistical analysis.

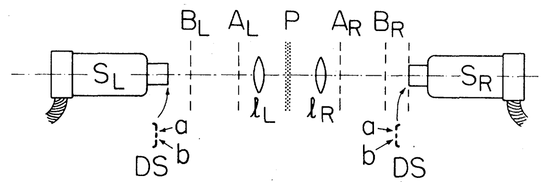

FIG. 3. System of coupled compound Furry microscopes equipped to test for hidden variables in EPR collisions (at collision region P) according to procedures outlined in Table I. For the functions of each microscope, see Figs. 1 and 2.

TESTS FOR LOCAL HIDDEN VARIABLES

Experiments to test for hidden variables are more effective if designed to implement observation of both partners in an EPR collision, as shown in Fig. 3 and outlined in Table I. Note that straightforward design modifications can postpone measurements at either end of the arrangement of Fig. 3 relative to the other, as desired. Not only do coupled Furry microscopes allow tests of all

TABLE I. Experiments with coupled Furry microscopes to illustrate various aspects of EPR phenomena. Refer to FIG. 3 for notation. Particles L and R are those received by the left-hand and right-hand detectors.

(A) Standard EPR observations:

To observe x of particle L immediately after collision impact, move plane I of stage S_L to plane B_L.
To observe x of particle R immediately after collision impact, move plane I of stage S_R to plane B_R.
To observe p_x of particle L immediately after collision impact, move plane I of stage S_L to plane A_L.
To observe p_x of particle R immediately after collision impact, move plane I of stage S_R to plane A_R.

Can observe any combination of x_L or $(p_x)_L$ and x_R or $(p_x)_R$.

Expected results:

$[x_L = x_R]$ or $[(p_x)_L = -(p_x)_R]$ every time joint measurements are made.

$[(p_x)_L$ and $x_R]$ or $[x_L$ and $(p_x)_R]$ are chaotically related.

(B) EPR observations to illustrate meaning of $(p_x)_L$ when x_R is definite:

Move planes I of stages S_L and S_R to planes A_L and B_R, respectively.
Insert a double slit at A_L and a single slit at B_R. Record only those events for which genuine L, R coincidences are observed.

Can accumulate records of momenta associated with a definite position. [a]

Diagnosis of results at left-hand detector array:

If double-slit image is incoherent, momenta are definite prior to recording but chaotic when position is definite.
If double-slit image is coherent, momenta are distributed in a coherent quantum superposition when position is definite. [b] Hidden variables did not determine momentum before registration at detectors.

(C) EPR observations to illustrate meaning of x_L when $(p_x)_R$ is definite:

Move planes I of stages S_L and S_R to planes B_L and A_R, respectively.
Insert a double slit at B_L and a single slit at A_R. Record only those events for which genuine L, R coincidence are observed.

Can accumulate records of positions associated with a define momentum. [a]

Diagnosis of results at left-hand detector array:

If double-slit image is incoherent, positions are definite prior to recording but chaotic when momentum is definite.
If double-slit image is coherent, positions are distributed in a coherent quantum superposition when momentum is definite. [b] Hidden variables did not determine position before registration at the detectors.

[a] Or can remove lens from projector stage and determine whether or not double-slit interference fringes accumulate at detectors.

[b] Or, with projector lens out, interference fringes at detector signify that p_x (case B) [or x (case C)] was not definite prior to registration at detectors.

of the originally proposed EPR correlations, they make additional discriminations, as well. The original EPR experiment was devised not to discriminate against hidden variables, of course, but to suggest the incompleteness of quantum mechanics. The authors expressed their belief in hidden variables.

Particularly enlightening EPR experiments are those made upon the compatible, but seemingly disparate, observables x_1 and p_{x2} (or x_2 and P_{x1}). To see what is involved, consider, for example, that subset of measurements (out of a huge body of EPR events) for which x_1 has been observed to lie very close to some particular value, say x_1'. The associated broad distribution of p_{x2} values could be interpreted either in terms of quantum mechanics or in terms of individual events in which x_1 and p_{x2} were perfectly definite before observation but merely distributed in response to the operation of some (unknown) hidden parameter. These two interpretations would lead to characteristically different images in the coupled Furry microscopes, as follows.

Carry out such a subset of observations by placing a single slit in one Furry microscope (at a "B" plane) in order to screen out all events except those occurring very close to x_1'. Insert a double slit at the "A" plane of the other microscope to admit EPR particles with momenta of p_{xa} or p_{xb}, and record only genuine EPR coincidences. Although almost all events are eliminated by the slit system, some satisfy the conditions, activate the detectors, and thereby build up images of the slits at the detectors.

If hidden variables operated to make the momentum definite before the "A" particles encountered the momentum selection slits, the double-slit image would be incoherent[14] (coherence factor $\gamma = 0$) since slit intensities, not amplitudes, would be added. If quantum mechanics applies, however, the momentum is not merely unknown before observation, it is not yet definite. The different momentum components in the quantum superposition are more than abstract symbols incorporated to express indeterminacy. They correspond to different spatial elements in the EPR wavefront incident upon the double slits in the "A" plane. Their simultaneous existence is physical enough to produce observable interference effects. That is, the imaging of the double slits would be coherent[14] (with coherence factor $\gamma \approx 1$ if slits were arranged symmetrically) because quantum amplitudes from each slit are added before the recording of intensity.

The difference between coherent and incoherent images can be made more conspicuous by stopping down the projector lens of stage S until the diffraction broadening of the slit images approaches the image separation. A graphic representation of the difference between coherent and incoherent imaging of two pinholes is depicted in Fig. 4.

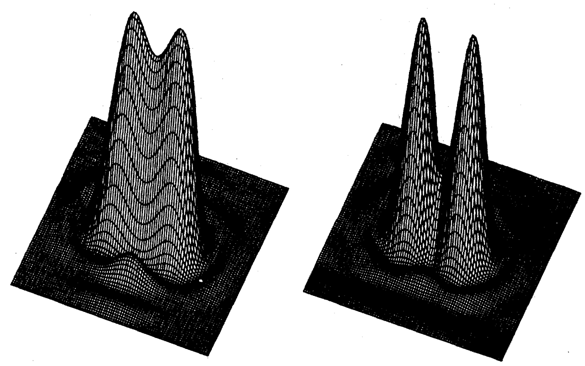

FIG. 4. Intensity distributions in images of two closely spaced pinholes. Left, coherent imaging, coherence factor $\gamma = 1$ (amplitudes added). Right, incoherent imaging (intensities added). Wavelength and pinhole spacing identical in both images.

Alternatively, instead of imaging the slits, the projector lens in the compound microscope could be removed in order to display the far-field diffraction patterns from the double slit. Double-slit interference fringes would demonstrate the quantum superposition.

TESTS OF EINSTEIN SEPARABILITY

More profound than simple tests of hidden variables are those which call into question Einstein separability. Einstein's insistence[15] that "the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former" is a key element of local realistic theory, a theory incompatible with quantum mechanics. Extensive discussions of this point have been published by Bell[2] and others[1-4] and will not be recited here. Instead, it will be shown how Bell's ideas, heretofore applied almost exclusively to modified EPR experiments involving spin or photon polarization, can be related to the original starkly dissimilar EPR observables position and momentum. In order to explain how the proper associations can be made, it is helpful to write the two-particle EPR wave function $\Psi(1,2)$ in such a way that features common to all EPR experiments are evident. In the original notation of Einstein, Podolsky, and Rosen, the wave function, after suitable interaction between the particles, is[7]

$$\Psi(1,2) = \sum_n \psi_n(2) u_n(1) \tag{1a}$$

$$= \sum_s \phi_s(2) v_s(1), \tag{1b}$$

where the functions u_n represent a set of eigenfunctions of some particular observable and the v_s represent eigenfunctions of another noncommuting observable. Functions ψ_n and ϕ_s, introduced as expansion coefficients, manifest EPR correlations between the particles. If the u_n and v_s are eigenfunctions of two continuous dynamical variables such as position and momentum (as in the original EPR experiment), the sums become integrals. If $\{u_n\}$ and $\{v_s\}$ are alternative basis sets of two-valued observables such as half-integral spin (as in the EPR-Bohm-Bell experiments upon singlet states), the sums, of course, reduce to just two terms, or[10]

$$\Psi(1,2) = \alpha_A(2)\beta_A(1) - \beta_A(2)\alpha_A(1) \quad (2a)$$

$$= \alpha_B(2)\beta_B(1) - \beta_B(2)\alpha_B(1), \quad (2b)$$

where A and B refer to axes corresponding to different orientations of the analyzers of the instruments of measurement.

The enormous conceptual simplification gained by going from a continuum to two eigenvalues proved to be compellingly attractive to philosophers and experimentalists. Still, the principles involved in EPR tests of local realism are the same, whether based on the original or the Bohm wave functions. Furthermore, as will be demonstrated, the theoretical treatment and (in principle) the physical tests, themselves, of the original position-momentum version, need be no more complicated than those of the Bohm version if advantage is taken of the capabilities of the Furry microscope.

When proposing thought experiments to discriminate between quantum mechanics and local realism *via* Bohm experiments, it is standard to select at least two different orientations of one of the particle polarization analyzers relative to the other. A family of Bell inequalities exists[1-4] of which we shall write only one member to illustrate the point involved, namely, in the notation and form of d'Espagnat,[4]

$$n[A^+B^+] \leq n[A^+C^+] + n[B^+C^+], \quad (3)$$

where, for example, $n[A^+B^+]$ denotes the ratio

$$n_T[A^+B^+]/\{n_T[A^+B^+] + n_T[A^+B^-]\},$$

of events registered with positive eigenvalues at both the left-hand and right-hand analyzers (set at orientations A and B, respectively) to the total number of EPR events with positive eigenvalues at the left analyzer. Inequality (3) is required by a theory of local realism embodying Einstein separability. It is violated for certain ranges of B and C for a given A, according to quantum mechanics.

Upon going from the simple case, in which observables can exhibit only two eigenvalues, to the original EPR case in which observables display a continuous distribution, a much greater range of possible tests can be imagined. The enormously increased complexity is not essential, however, as shown below, and an exact counterpart of Eq. (3) can be put to the test directly. How this can be done can be seen with the aid of Fig. 5. First note that the change in orientation of a given EPR-Bohm analyzer (e.g., $A \to B$) corresponds naturally to a transformation of basis functions in Eq. (2) (e.g., $\alpha_A, \beta_A \to \alpha_B, \beta_B$). Likewise, a change from one plane of detection in Fig. 5 to another (e.g., $A \to B$) corresponds naturally to a transformation of basis functions in Eq. (1) (e.g., $u_{xAi}, u_{xAj}, u_{xAk}, \ldots \to v_{xBl}, v_{xBm}, v_{xBn}, \ldots$). Clearly, the incoming wave focused upon x_i in plane A is evidence of the plane wave u_{xAi} leaving collision region P with momentum p_{xi}; alternatively, the wave focused upon x_n in plane B images the source of the outgoing spherical wave v_{xBn} exiting region P. Although detection planes A and B of Fig. 5 convey information about especially clear-cut observables they represent only two in an infinite range of apparatus settings. Conjugate planes C_R and C_L are equally valid, and can be set, within definite limits, over a continuous range of positions. Conjugate planes are related by a simple construction. Rays emitted from a given point on a plane at the right (e.g., from the circle, square, or diamond at A_R, B_R, or C_R) would be focused to a corresponding point on the conjugate plane at the left. Obviously, the basis sets corresponding to the C planes are less simple than those for A and B. They represent virtual sources outside of collision region P and intrinsically interrelate x and p_x in the outgoing waves from P. In any case, EPR events measured at conjugate planes should be found to be correlated in position as indicated in Fig. 5.

Rather than treat individual eigenvalues belonging to the continuous set detectable at a given plane (Fig. 5), it is enormously simpler to reduce to *two* possibilities, +1 (detection above the optic axis) and −1 (detection below the optic axis). In the case of detections at planes A and B, this choice of observation corresponds, of course, to determination of the *signs* of p_x and x after the impact of collision. Because there are now only two outcomes of each measurement, a direct analog of the Bell inequality, Eq. (3), can be formulated. The form of Eq. (3), although appropriate for EPR-Bohm observations, is not applicable to the present system, however. This is because the very different properties of position and momentum introduce a feature not encountered in the conjugate EPR spin measurements. For the colliding particles, note that the EPR signs of position, x_1

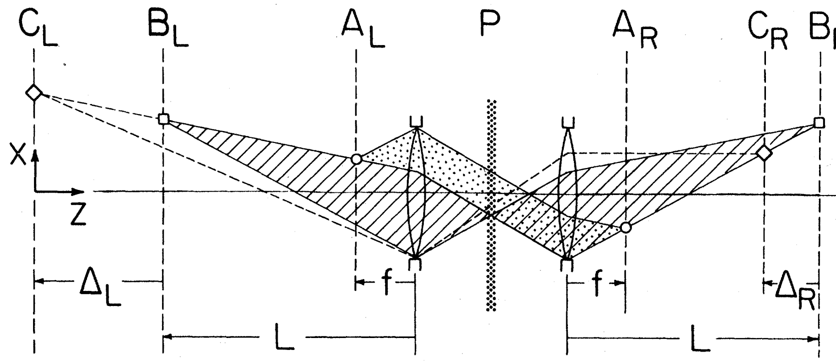

FIG. 5. Coupled microscopes to test Bell's inequalities as they pertain to position and momentum. The detection of a particle at a given value of x upon a particular plane implies the value upon collision of p_x, x, or some function of both (detections at A, B, or C, respectively). Conjugate EPR points are illustrated for planes A (circles), planes B (squares), and planes C (diamonds). (Analogous conjugate points corresponding to a different family of observables could be found in planes C' close to A_L and A_R.) A two-valued observable particularly appropriate for simple tests of Bell's inequalities is the *sign* of the x value of detection (+ for above, − for below the optic axis).

and x_2, are the *same* (sign eigenvalues +1, +1 or −1, −1) whereas those for the momenta are opposite (eigenvalues +1, −1 or −1, +1). For conjugate planes C chosen to be near B, the signs are the same, just as for planes B (if conjugate planes C' near A had been adopted instead, the signs would have been opposite). A little reflection confirms that the Bell-type inequalities pertaining to measurements in planes A, B, and C include the members

$$n[A^+B^+] \leq n[A^+C^+] + n[B^+C^-] \tag{4}$$

and

$$n[C^+B^+] \leq n[A^-B^+] + n[A^+C^+] \,. \tag{5}$$

Once this correspondence between the position-momentum measurements and polarization-polarization measurements has been made, it is elementary to test the outcome. The fractions $n[A^+B^+]$, etc., are easily visualized in terms of constructions from geometric optics. So, for example, the distribution of counts at plane A associated with the firing of a given detector x_{B_i} at plane B is easily visualized in terms of a beam focused at B_L, P, and B_R but spread out in between. In order to calculate the ratio $n[C^+B^+]$, then, it is necessary to average the distribution at plane C over the positive range of x in plane B, weighting properly to take into account the different cutoffs, by the lens diaphragms, of EPR events originating from different regions of the source at P (see Fig. 5). Even though it is assumed that the source of EPR events is continuously distributed in x over a considerable range (much greater than the lens diameters, Fig. 5), the EPR radiation is received over only a limited area of the planes A, B, and C. As indicated above, this is because bonafide EPR events, i.e., events for which counts at the right are genuinely correlated with counts at the left, can pass through *both* lens openings only when originating within a limited range of x at P and within a limited range of p_x. All other events recorded at one end or the other must be rejected.

Straightforward calculations based on the laws of thin lenses (cylindrical lenses, or lenses in the two-dimensional space of Fig. 5) yield the results

$$\begin{aligned} n[A^+B^+] &= n[B^+A^+] \\ &= n[A^+B^-] \text{ etc.} \\ &= \tfrac{1}{2} \,, \end{aligned} \tag{6}$$

$$\begin{aligned} n[A^+C^+] &= n[C^+A^+] \\ &= 1 - n[A^+C^-] \text{ etc.} \\ &= \tfrac{1}{2} - \gamma \,, \end{aligned} \tag{7}$$

and

$$\begin{aligned} n[B^+C^-] &= n[B^-C^+] \\ &= n[C^+B^-] \\ &= 1 - n[C^+B^+] \text{ etc.} \\ &= \zeta \,, \end{aligned} \tag{8}$$

where γ and ζ are positive numbers related to the system geometry as shown in Appendix B. For all legitimate optical parameters of the system, ζ is smaller than γ and, indeed, ζ approaches $\gamma/2$ as L/f becomes large.

A noteworthy correspondence between the transformations in the original EPR and the EPR-Bohm experiments is brought to light by the 50-50 statistics of Eq. (6). In the former, the p_x and x "apparatus settings" (i.e., those placing detectors at planes A and B) are related to each other as are Stern-Gerlach polarizer settings differing by

90° (for spin-$\frac{1}{2}$ particles) or Wollaston prism settings differing by 45° (for linearly polarized photons).

The necessary ingredients are now at hand for testing Einstein separability in the original Einstein, Podolsky, and Rosen conceptual experiment, with the help of a hypothetical system of coupled Furry microscopes. Insertion of the information from the preceding paragraphs into Eqs. (4) and (5) can be seen at once to lead to a violation of the Bell inequalities. If quantum mechanics is correct, local realistic theory is not.

DISCUSSION

From the foregoing heuristic treatment it may not be immediately obvious how a simple exercise in geometric optics can have much to say about realism. Certainly the role of the coupled microscopes in Fig. 5 appears more pedestrian than that of the compound microscopes fitted with double slits in Table I (and Figs. 2 and 4), yet its implications are even more fundamental. To be sure, the flavor of quantum mechanics is included in the geometric exercise partly through the quantum origin of the physical optics governing the particle trajectories. The uniquely quantum feature at the bottom of the inequality violations, however, is the classically incomprehensible nature of the EPR radiation generated by the collisions. As embodied in the EPR wave function[7] $\Psi(1,2)$ used to constrain the optical intensities mutually received at the planes A, B, and C, the radiation source behaves as if collision wavelets were generated over the whole x range of the collision region, with their amplitudes superposed upon each other. This delocalization is, of course, consistent with the assumed careful preparation of the momenta of the particles.

On a different note, considerations of the present conceptual experiments shed some light on other propositions about quantum behavior. For example, Park and Margenau,[16] in seeking to demonstrate that noncommuting observables are compatible, invoke the infallible EPR correlation between, say, x_1 and x_2 to legitimize the measurement of x_1 by means of the EPR observation of x_2. These authors then claim that a simultaneous EPR measurement of x_2 and p_{x1} would constitute a valid joint measurement of x_1 and p_{x1}. Although this approach might be taken as a metaphysical definition of what is meant by a joint measurement, entries B and C in Table I suggest that such a definition is empty of physical content. Certainly it cannot be understood to mean that the values x_1 and p_{x1} so obtained correspond to unique physical attributes characterizing the trajectory of particle 1.[17] The indeterminacy of x_1 when p_{x1} is measured signifies more than a chaotic distribution observable in x_2 when p_{x1} is measured (cf. entry A, Table I). A physical manifestation of the coherent superposition of x_2 eigenfunctions could presumably be observed, *via* interference effects, when p_{x1} is definite (cf. entry C, Table I).

Other authors have drawn attention to counterintuitive implications of quantum measurements relating to what is meant by "the past." John Archibald Wheeler,[18] in particular, has stressed the far from neutral role of the observer who has the option to decide what type of observation he wishes to make *after* an elementary event has taken place. In a sense, the observer "participates" *ex post facto* in what has happened. For example, in a double-slit experiment, after a photon has traversed the slit region, the observer can choose whether to swing into place an interference screen or a pair of directional detectors and, thereby, to decide whether the photon "shall have passed through" both slits, or only one.[18] However one chooses to look at this interpretation of the past, it is clear that the foregoing EPR experiments (Table I) carried out in Wheeler's "delayed choice" mode, offer a much richer and more physical view of the past. After an observer has carried out his delayed measurement upon particle 1 to infer "where the particles collided" (or "with what momentum they recoiled from each other"), he has by no means played out his hand—as he had in the double-slit experiment. He can observe the phenomenon again through the behavior of particle 2, and from the variety of perspectives sketched in Table I. As far as timing is concerned, it is a trivial matter to adjust apparatus parameters in such a way as to postpone arbitrarily the final measurement either at the right-hand side, or the left (Fig. 4).

Finally, it is worthwhile to examine some aspects of the apparently instantaneous collapse of the EPR wave function[13] when one particle is detected. This feature has attracted a great deal of attention recently, and some workers are seriously investigating the possibility of faster-than-light communication.[19] Beguiling situations are suggested, some of them related to the experimental arrangement of Fig. 4 as follows. The operator R of the right-hand Furry microscope endeavors to send signals to observer L at the left-hand microscope. Let projector stage S_L be moved to plane B_L; insert a transmission diffraction grating at its projector lens. A mask with a thin slit at X_R^0 is placed over the detector array of the right-hand microscope at D_R. Operator R places his stage S_R some of the time at plane B_R, and some of the time at A_R. Observer L examines the EPR events common to both his detectors and those at D_R. If observer L sees interference fringes, he knows that operator R has placed his projection stage at B_R. If the interference fringes wash out, observer L knows that the stage is at A_R. Thereby, signals can be transmitted, and the agency transmitting them is superluminal, as far as is known. What prevents superluminal communication in the above arrangement is that observer L must sort the bonafide EPR registrations out from the enormously larger number of events he detects. In order to do this, he must receive, subluminally, information from the detectors at D_R to identify which of the events he received were authentic coincidences with those at D_R. Other configurations and accessories can be introduced to compound the possibilities but all are bound by the above limitations.

CONCLUSION

It has been shown how statistical tests of quantum theory can be carried out with coupled "Furry microscopes" within the framework of the Einstein-Podolsky-Rosen conceptual experiment as originally formulated in terms of observations of position and momentum. Local realistic theory based on Einstein's principle of separability leads to predictions at variance with quantum mechanics, a fact not evident when the EPR experiment was first proposed. The concreteness with which outcomes of EPR experiments can be visualized lends fresh perspectives to old abstract arguments.

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APPENDIX A

Because it is impossible to constrain the EPR collisions exactly to plane P of Figs. 1, 2, 3, and 5, it is prudent to inquire whether the delocalization of the incident packets in the z direction spoils any conclusions arrived at in the text. The principal conclusions were based on geometric optics with the assumption that the source is in plane P. They are weakened by limitations of the apparatus variables to the extend that information is lost in measurements of position or momentum. The two main sources of uncertainty in position are Δx_l, the diffraction blurring by the lens and Δx_z, the blurring of the image at plane B by virtue of the fact that the collision may occur at a distance Δz from focal plane P. In order of magnitude, the indeterminacy in z coordinate over a region d (the diameter of the Furry lenses) is

$$\Delta z \approx [(h/\Delta p_z)^2 + (\Delta p_z/p)^2 d^2]^{1/2} \approx (2\lambda d)^{1/2} \text{ at minimum.} \quad \text{(A1)}$$

This imprecision in z introduces an associated blur at image plane B which, when expressed in terms of the indeterminacy of measurement of position at plane P, becomes

$$\Delta x_z \approx d\Delta z/L\,. \quad \text{(A2)}$$

This blur can be made small by making d small. The lens resolving power is

$$\Delta x_l \approx (h/d)[(Lf/p(L-f)] \approx [\lambda Lf/d(L-f)]\,, \quad \text{(A3)}$$

which is minimized by making d large. A compromise is to select a value of d adequate according to Eq. (A3) and to make L large.

The principal uncertainties in momentum are $(\Delta p_x)_l$, arising from the diffraction blurring due to the lens, and $(\Delta p_x)_z$, the x component of momentum uncertainty perhaps not present in the beam in its original state of preparation but introduced by scattering when Δz is limited by a diaphragm. The analog of Eq. (A3) for the indeterminacy in focus at plane A is simply

$$(\Delta p_x)_l \approx h/d\,, \quad \text{(A4)}$$

while

$$(\Delta p_x)_z \approx (\Delta p_z)(\Delta p_z/p) \approx h\lambda/(\Delta z)^2\,, \quad \text{(A5)}$$

which, if we use Eq. (A1), becomes

$$(\Delta p_x)_z \approx h/d\,. \quad \text{(A6)}$$

The way to decide whether these uncertainties are tolerable or not is to consider the product

$$[(\Delta x_z)^2 + (\Delta x_l)^2]^{1/2}[(\Delta p_x)_l{}^2 + (\Delta p_x)_z{}^2]^{1/2}\,.$$

According to the foregoing, this product is of the order of

$$h\{[(\lambda d)^{1/2}/L] + \lambda f/d^2\}\,.$$

By making $L \gg d \gg \lambda$ it is possible to make the product arbitrarily small compared with h, and thereby to make the individual measurements much more precise than allowed in a joint measurement. Therefore, the uncertainties are innocuous. EPR correlations of x_1, x_2 or p_{x1}, p_{x2} may be imperfect but by no more than the foregoing equations suggest.

APPENDIX B

The quantities γ and ξ of Eqs. (7) and (8) of the text are derived from considerations of geometric optics applied to the symmetric system of Fig. 5. Note that, depending upon the x position in plane A, B, or C, only part of the lens aperture can transmit EPR signals to both conjugate planes if $x \neq 0$. It is useful to define variables

$$\phi = [f\Delta_R/(L-\Delta_R)(L-f-\Delta_R)]\,, \quad \text{(B1)}$$

$$\rho = [(L-\Delta_R)(L-f)/f\Delta_R]\,, \quad \text{(B2)}$$

and

$$\sigma = [L(L-\Delta_R) - f(L+\Delta_R)]/f\Delta_R \quad \text{(B3)}$$

in terms of the quantities f, L, and Δ_R identified in Fig. 5. Then it can be shown that, if the lens aperture is small compared with f,

$$\gamma = \tfrac{1}{2}(G-1)/(G+1)\,, \quad \text{(B4)}$$

where

$$G = \phi(2+\phi) + \rho(1-\phi)^2/\sigma \quad \text{(B5)}$$

and

$$\zeta = \frac{1+(\rho-1)\ln(1-\rho^{-1})}{3+(\rho-\sigma-2)\ln(1-\rho^{-1})}\,. \quad \text{(B6)}$$

The above expressions pertain to positive values of Δ_R over its legitimate range $(L-f)/2 > \Delta_R > 0$. Analogous expressions can be derived for negative values of Δ_R, Fig. 5, but can be seen to correspond to values derived when positive Δ_L values are inserted into Eqs. (B1) and (B2) in place of Δ_R; positive Δ_L corresponds to plane C_L being placed inside plane B_L, Fig. 5.

Notice that the diameter of the lens does not enter the ratios needed for equalities (6)–(8) of the text. As L is made large compared with f, γ approaches the value $f\Delta_R/2L^2$, and ζ, which is characteristically about half as large as γ for all reasonable instrument settings, approaches $f\Delta_R/4L^2$.

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