Covariant kinematics for the production of spacelike particles

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The kinematic problem connected with the production of particles with spacelike four-momentum is analyzed in its simplest form: the decay of an ordinary particle into b ordinary particles, luxons, and t spacelike particles. What, apart from four-momentum conservation, are the kinematic constraints that guarantee stable finite kinematic limits and a finite number $b + t$?

The problem of how particles with spacelike four-momentum might be produced is one of the deepest problems in tachyon physics. There is no theory for possible production mechanisms. However, before looking for such a theory the kinematic limits for the produced spacelike momenta must be known, since these should follow directly from the general kinematic structure and symmetry that is taken as the basis for any theory. We shall examine this problem in its simplest form: the decay of an ordinary particle into b ordinary particles, luxons, and t spacelike particles.

Let us begin by describing the various aspects of this problem. For $t=1$ the law of the conservation of the total four-momentum reads $P=q+p$, where P denotes the four-momentum of the decaying bradyon, q the total momentum of all final bradyons and luxons, and p the tachyon momentum. There is also the important constraint $Pq \geqslant Mm$, which is due to causality ($M=$mass of the decaying bradyon, $m=$invariant mass of all final bradyons and luxons). These four equations and this constraint yield

$$q_0=(M^2+m^2+m_{ta}{}^2)/2M\,,\qquad e_{ta}=(M^2-m^2-m_{ta}{}^2)/2M \tag{1}$$

for q_0 and the tachyon energy relative to the rest frame of the initial bradyon. The equations show that the unphysical limit $m\to\infty$, as well as the unphysical limit $m_{ta}\to\infty$, are kinematically permissible. In each limit q_0 tends towards plus infinity and the tachyon energy towards minus infinity.

It is important to remember that kinematic singularities also arise if the tachyon energies are non-negative. For example, if there are no luxons, $b=0$, and $t=2$, the two tachyon energies read

$$e_1=(M^2-m_1{}^2+m_2{}^2)/2M\,,\qquad e_2=(M^2+m_1{}^2-m_2{}^2)/2M \tag{2}$$

relative to the rest frame of P. Although both energies are positive for $|m_1{}^2-m_2{}^2|<M^2$, infinite momenta $|\vec{p}_1|$ and $|\vec{p}_2|$ are kinematically permissible in the limit of simultaneous $m_1\to\infty$ and $m_2\to\infty$.

The problem becomes yet more serious once one realizes that infinite momenta can arise even when all the masses are bounded, which is possible when more than two particles are produced. The spacelike momentum hyperboloid shows that for every spacelike momentum p_k there is another spacelike momentum p_l, so that $p_k+p_l=0$. That $p_k=-p_l$ tends to infinity in no way contradicts four-momentum conservation.

An infinite number of tachyons can clearly be produced. All of these can have zero energy, which looks like a great kinematic instability even if all tachyons only had positive energies (for instance in the frame $\vec{P}=0$). Although the sum of the infinitely many momenta is finite, an infinite number of particles with mass is unphysical simply because there would be two groups of infinitely many particles, each of which represents an infinite momentum, and infinite momenta cannot exist in nature.

The reason for these elementary kinematic singularities is easy to see. Whereas ordinarily the causality constraint $Pq_k\geqslant Mm_k$ (and $q_kq_l\geqslant m_km_l$) makes the kinematics finite, we have not used any constraint for the tachyon momenta.

The singularity related to Eqs. (1) will only disappear where e_{ta} has a lower bound. To introduce such a bound covariantly, the scalar Pp or qp has to be bounded. As P is the momentum of the tachyon's source, we need only limit the first scalar. The natural choice is

$$Pp_k\geqslant 0\,,\quad k=1,\ldots,t \tag{3}$$

since then and only then is the spacelike momentum hyperboloid symmetrically divided (by a spacelike cut through the origin) into a physical upper and an unphysical lower half. With this bound (3) all tachyon energies are positive definite in the $\vec{P}=0$ frame.

Because of bound (3) an electron or proton cannot spontaneously gain energy through interaction with tachyons. This was discussed by Antippa and

Everett[1] in connection with the severe experimental limits[2] for *elastic* decays of the electron and proton. One can thus say that bound (3) is experimentally confirmed with high precision.

The singularities related to Eqs. (2) show that bound (3) is insufficient. Here it should be pointed out that in considering electromagnetic tachyon-bradyon scattering quantum mechanically in Ref. 3, the restriction $pp' \leq 0$ was introduced for the final tachyon momentum p', which yielded finite kinematic limits, and also made it possible to interpret the free tachyon propagator as propagating the tachyon waves into the kinematically permissible half-space $pp' \leq 0$ of the spacelike momentum hyperboloid (in the frame $p_0 = 0$ this is the half-space $\vec{p}' \cdot \vec{p} \geq 0$ or $p_x' \geq 0$), and propagating the antitachyon waves into the negative half-space $pp' \geq 0$ which is kinematically impossible. Locally, this gave a propagator of the tachyon-antitachyon field, analogous to the propagator of the corresponding bradyon-antibradyon field with the role of the time axis transferred to the x axis. Two tachyon momenta macroscopically (globally) separated in spacetime cannot, of course, be related by the restriction $pp' \leq 0$; this can only refer to coherent tachyon momenta.

Since the final tachyon states of an elementary particle decay are coherent,

$$p_k p_l \leq 0 \tag{4}$$

should hold for all pairs (k, l) with $k = 1, \ldots, t$ and $l = 1, \ldots, t$. Writing this constraint in the form $p_{0k} p_{0l} \leq |\vec{p}_k| |\vec{p}_l| \cos\theta_{kl}$ and applying (3), we find that all tachyon three-momenta include angles $\theta_{kl} \leq 90°$. That is, all tachyon three-momenta point into a cone with an opening angle of 90° in the $\vec{P} = 0$ frame.

Before pointing out a possible uniform explanation of constraints (3) and (4), we will prove five theorems about the kinematics that consist of (3), (4), $Pq_k \geq Mm_k$, and four-momentum conservation. All the four-momentum components will relate to the rest frame of the decaying bradyon.

(1) *Without constraints (3) or (4) the kinematics cannot be finite.* This has been virtually proved above—there is no bound for Eqs. (1) without (3) and no bound for Eqs. (2) without (4). In considering decays into more than two particles, this theorem follows at once.

(2) *The total momentum of all spacelike products is spacelike; every group of spacelike products has also a spacelike momentum.* This follows from expanding the square $(p_1 + \cdots + p_n)^2$ with $n \leq t$. All terms of the result are negative because of $p_i^2 = -m_i^2$ and (4). Consequently, decays into tachyons only are impossible and, in particular, the case to which Eqs. (2) apply cannot be realized.

(3) *All bradyon and luxon momenta are bounded, as is their sum.* Because the total energy of the tachyons has a lower bound, the bradyon and luxon energies have an upper bound (M). This means that the bradyon masses are also bounded by M. The three-momenta of the bradyons and luxons, too, are then limited by M. Moreover, the sum of all bradyon and luxon four-momenta has a bounded energy (bounded by M) with the result that the number of produced bradyons cannot exceed $b_{max} = M/(m_1 + \cdots + m_b)$. Since this sum is a timelike vector, the spatial components of this sum are also bounded.

Subtracting the upper limit of the sum of all bradyon and luxon momenta from $P = (M, 0, 0, 0)$, Theorem 2 gives us this upper bound for the negative length squared of the sum of all spacelike four-momenta:

$$-(p_1 + \cdots + p_t)^2 \leq M^2 \,. \tag{5}$$

It also follows that the energy of a tachyon cannot be greater than $M/2$: $e_{ta} < M/2$.

(4) *The total number of tachyons produced is limited.* Bound (5) and the constraint (4) give us

$$\sum m_{ta}^2 \leq M^2 \,. \tag{6}$$

If the smallest tachyon mass has the value of m_{min}, then the tachyon number has the bound $t_{max} < M^2/m_{min}^2$.

Alternately, this can be proved as follows. If there were infinitely many tachyons the total spacelike momentum would be infinite because all individual three-momenta include angles not greater than 90° and given theorem 3, then four-momentum cannot be conserved.

(5) *All the momenta of the spacelike particles are finite.* According to constraint (4), we also have

$$p_{0l} \sum_{k \neq l} p_{0k} - |\vec{p}_l| \sum_{k \neq l} p_{\parallel k} < 0 \,, \tag{7}$$

where the $p_\parallel$ are three-momentum projections on $\vec{p}_l$. The energy term is positive-definite. Let the momentum $|\vec{p}_l|$ tend towards infinity. According to theorem 3, the bradyons and luxons cannot compensate for this momentum. Hence, momentum conservation requires that $\sum' p_{\parallel k}$ becomes negative, which violates (7). Now, if the tachyon three-momenta cannot become infinite, then neither can the tachyon energies and masses.

Thus, the constraints (3) and (4) lead to finite kinematic limits.

Recently, I have argued in favor of constraint (3) within causality considerations.[4] According to Ref. 4, one would say that the decay products do not move backward in time for their source. Let us now turn to the problem connected with Eqs. (2). Using the notion of a superluminal reference frame, we assume that tachyon 1 cannot move backward in time for tachyon 2, and vice versa. The two tachyon four-momenta have opposite directions. Each direction defines (in the t-x plane) the direction of time in the superluminal rest frame of the respective tachyon. Let us denote the rest frame of tachyon 1 by S' and the $\vec{P}=0$ frame by S, relative to which S' should move in the positive x direction. The planes of constant time t' are given by the equation $t=\text{const}+(p_1/e_1)x$. Tachyon 2 moves towards negative x, that is, it intersects these planes in the negative direction. This means that it moves backward in time t'. To avoid this, the four-momentum of tachyon 2 must at least lie parallel to these planes, that is, $e_2/p_{2x} \leqslant (p_1/e_1)$ must be valid, which is constraint (4). Thus, we arrive at a uniform explanation of (3) and (4).[5]

Finally, I cannot but remark that neither constraint (3) nor (4) can be regarded as theoretically well founded. Moreover, the kinematics do not give stability of the electron or proton against tachyonic decays; a process such as $e_{br} \to e_{ta} + \gamma$ could happen on (and only on) the kinematic bound introduced by (3).

It is, however, pleasing to see that the electron and proton become stable if processes happening *only on* the kinematic bound are regarded as impossible. Lepton or baryon number conservation requires either $q_i^2 \geqslant M^2$ or $-p_i^2 \geqslant M^2$.[6] In the former case not all the tachyons can have a positive energy; in the latter case one sees from (6) that only one tachyon with mass M can be emitted and its energy also cannot be positive. Both situations could not occur if $Pp_k > 0$ were satisfied.

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[1]A. F. Antippa and A. E. Everett, Phys. Rev. D 8, 2352 (1973).

[2]J. S. Danburg and G. R. Kalbfleisch, Phys. Rev. D 5, 1575 (1972); A. Ljubicic *et al.*, *ibid.* 11, 696 (1975).

[3]H. Lemke, Lett. Nuovo Cimento 17, 209 (1976); Nuovo Cimento 35A, 181 (1976) [note that the symbols of this paper's Eq. (3), etc., imply a d^4x integration]; Int. J. Theor. Phys. 16, 307 (1977).

[4]H. Lemke, Phys. Lett. 72A, 409 (1979).

[5]It could be of interest to know whether the kinematic limits would remain finite if the scalars in (3) or (4) had a somewhat greater interval of variation. Let us, for instance, substitute

$$Pp_k \geqslant 0 - \epsilon M .$$

for (3) with a given positive ϵ. One can then show that the kinematic limits remain finite for every $\epsilon \leqslant m_{\min}$, where $m_{\min}$ is the smallest rest mass of the tachyons with negative or zero energy.

[6]On the basis of the so-called extended principle of relativity, we can assume that an electron-tachyon has the same mass as an ordinary electron.