Comment on the 20-dominance model for charm decays

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From systematic studies of $D \rightarrow K\pi$ decay amplitudes, it is pointed out that vital damage to the conventional "mild" <u>20</u>-dominance model for charm decays is caused by an observation $B(D^0 \rightarrow \overline{K}^0 \pi^0)/B(D^0 \rightarrow \overline{K}^- \pi^+) > 0.5$ but not by an observation $\tau(D^+) \gg \tau(D^0)$.

A recent report¹ on *D*-meson decays from the Mark II detector at SPEAR gives a useful piece of information on charm decays: The preliminary measurements $B(D^+ \rightarrow eX) = 0.158 \pm 0.053$ and $B(D^0 \rightarrow eX) = 0.052 \pm 0.033$ lead to the ratio of the lifetimes

$$\tau(D^0)/\tau(D^+) = 0.33 \pm 0.24$$
 (1)

under the assumption $\Gamma(D^+ \rightarrow eX) = \Gamma(D^0 \rightarrow eX) \equiv \Gamma_{SL}$. From the ratio (1) and the branching ratios $B(D^+ \rightarrow \overline{K}{}^0\pi^+) = 0.021 \pm 0.005$ and $B(D^0 \rightarrow K^-\pi^+)$ = 0.028 ± 0.005, we obtain

$$R_{0}^{+} \equiv \Gamma(D^{+} \to \overline{K}^{0}\pi^{+}) / \Gamma(D^{0} \to K^{-}\pi^{+}) = 0.25 \pm 0.19.$$
(2)

The observation $B(D^0 \rightarrow \overline{K}{}^0\pi^0) = 0.021 \pm 0.009$ leads to

 $R_0^0 \equiv \Gamma (D^0 \to \overline{K}{}^0 \pi^0) / \Gamma (D^0 \to K^- \pi^+) = 0.75 \pm 0.35.$ (3)

The Cabibbo-suppressed decay mode $D^+ \rightarrow \overline{K}{}^0K^+$ is not as suppressed as expected: $B(D^+ \rightarrow \overline{K}{}^0K^+)/B(D^+ \rightarrow \overline{K}{}^0\pi^+) = 0.24 \pm 0.16$.

Furthermore, recent data² from DELCO yield the branching ratio $B(D^+ \rightarrow eX) = 0.24 \pm 0.04$ and the bound $B(D^0 \rightarrow eX) < 0.05$, which lead to $\tau(D^0)/$ $\tau(D^+) < 0.21$ and $R_0^+ < 0.16$ with the data of $B(D \rightarrow K\pi)$ from Mark II.

These data seem to contradict theoretical expectations based on the conventional "mild" 20dominance model for charm decays.^{3,4} Usual quantum-chromodynamics (QCD) calculation leads to $c_{-}/c_{+} \simeq 3.16$ ($c_{-} \simeq 2.15$ and $c_{+} \simeq 0.68$),⁴ where c_{-} and c_{+} are the enhancement factor for the SU(4) 20-plet part and the suppression factor for the SU(4) 84-plet in the effective $\Delta C = \Delta S = 1$ nonleptonic Hamiltonian, respectively.

From the studies of the data $B(D \rightarrow eX)$, $B(D^+ \rightarrow \overline{K}{}^0\pi^+)$, $B(D^0 \rightarrow K^-\pi^+)$, and $B(D^+ \rightarrow K^-\pi^+\pi^+)$, Katuya and the author⁵ have previously pointed out that the 20 part in the effective $\Delta C = \Delta S = 1$ nonleptonic Hamiltonian must be more strongly enhanced than usual theoretical expectations, that is, $c_-/c_+ \simeq 8$ ($c_- \simeq 4$ and $c_+ \simeq 0.5$), and they have predicted that $\tau(D^0)/\tau(D^+) \simeq 0.05$ and $\tau(D^+) \simeq 1 \times 10^{-12}$ sec, which are roughly consistent with experiments.^{1,2,6} However, their prediction $B(D^0 \rightarrow \overline{K}{}^0\pi^0) \simeq 0.07$ is in disagreement with the data from Mark II.

After the observation of $\tau(D^+) \gg \tau(D^0)$, many attractive models for charm decays have been proposed in order to understand the new data on D decays.^{7,8} These models are indeed worth taking into consideration. However, it is also worthwhile to check the following items: Do the new data truly rule out the mild 20-dominance model? Can we save the model from disagreement with the experiments by considering the symmetry-breaking effects on the form factors and the decay constants? What experiment causes vital damage to the model?

The purpose of the present paper is to check such a question by assuming only the factorization of the matrix elements, which is a traditional calculation method for the two-body mesonic decays. (Now under the new data on *D* decays, we need no aid of assumptions used in Ref. 5.) We point out that the vital damage to the mild 20dominance model is caused by the observation $R_0^0 > 0.5$ rather than by the observation $\tau(D^+)$ $\gg \tau(D^0)$, and the difficulty cannot be removed even by regarding f_D/f_{π} as a free parameter.

Before we discuss the details of the decay amplitudes, let us see the data (1)-(3) in the light of the $\Delta I = 1$ relationship⁹

$$A\left(D^{+} \rightarrow \overline{K}{}^{0}\pi^{+}\right) = A\left(D^{0} \rightarrow \overline{K}{}^{-}\pi^{+}\right) + \sqrt{2}A\left(D^{0} \rightarrow \overline{K}{}^{0}\pi^{0}\right).$$
(4)

The relationship (4) leads to

$$\eta_0^+ (R_0^+)^{1/2} = \mathbf{1} + \eta_0^0 (2R_0^0)^{1/2} , \qquad (5)$$

where η_0^+ and η_0^0 are relative phases of $A(D^+ \rightarrow \overline{K}{}^0\pi^+)/A(D^0 \rightarrow K^-\pi^+)$ and $A(D^0 \rightarrow \overline{K}{}^0\pi^0)/A(D^0 \rightarrow K^-\pi^+)$, respectively. If $\eta_0^0 = +1$, then the $\Delta I = 1$ relationship predicts $R_0^+ > 1$, so that the case $\eta_0^0 = +1$ is obviously ruled out by data (2). In the case of $\eta_0^0 = -1$, the cases $\eta_0^+ = -1$ and $\eta_0^+ = +1$ mean $R_0^0 > 0.5$

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and $R_0^0 < 0.5$, respectively. The experimental values (2) and (3) give $(R_0^+)^{1/2} = 0.50^{+0.17}_{-0.26}$ and $(2R_0^0)^{1/2} = 1.22^{+0.26}_{-0.33}$, so that the data are favorable to $\eta_0^+ = -1$, but we cannot yet rule out the case of $\eta_0^+ = +1$. Note that the data on $B(D \rightarrow eX)$ from DELCO lead to very small values of $\tau(D^0)/\tau(D^+)$ and R_0^+ in comparison with the values (1) and (2)

$$\begin{split} A\left(D^{+} \to \bar{K}^{0}\pi^{+}\right) &= G\left[X_{+} F_{\pi}^{DK}(m_{D}^{2} - m_{K}^{2}) + X_{-}F_{K}^{D\pi}(m_{D}^{2} - m_{\pi}^{2})\right],\\ A\left(D^{0} \to K^{-}\pi^{+}\right) &= G\left[X_{+} F_{\pi}^{DK}(m_{D}^{2} - m_{K}^{2}) - X_{-}F_{D}^{K\pi}(m_{K}^{2} - m_{\pi}^{2})\right],\\ \sqrt{2}A\left(D^{0} \to \bar{K}^{0}\pi^{0}\right) &= GX_{-}\left[F_{K}^{D\pi}(m_{D}^{2} - m_{\pi}^{2}) + F_{D}^{K\pi}(m_{K}^{2} - m_{\pi}^{2})\right], \end{split}$$

where $G = (G_F/\sqrt{2}) \cos^2\theta_C$, $X_{\pm} = (2c_{\pm} \pm c_{-})/3$, and $F_{\pi}^{DK} = f_{\pi} f_s^{DK} (m_{\pi}^2)$, and f_{π} and $f_s^{DK} (q^2)$ are the decay constant of the pion and the scalar form factor of the $D \rightarrow K$ current,

$$f_s^{DK}(q^2) \equiv f_+^{DK}(q^2) + q^2 f_-^{DK}(q^2) / (m_D^2 - m_K^2)$$

respectively. The amplitudes (6), of course, satisfy the $\Delta I = 1$ relationship (4).

The ratios R_0^+ and R_0^0 can be expressed as a function of $x \equiv X_-/X_+$. As far as the ratios $R_0^+(x)$ and $R_0^0(x)$ are concerned, we need no aid of the relation $c_-c_+^2 = 1$, which has played an essential role in Ref. 5.

It is convenient to define parameters α and β which are the zeros of $A(D^+ \rightarrow \overline{K}{}^0\pi^+)$ and $A(D^0 \rightarrow K^-\pi^+)$, respectively:

$$\alpha = - (F_{\pi}^{DK} / F_{K}^{D\pi}) \times 0.93 , \quad \beta = (F_{\pi}^{DK} / F_{D}^{K\pi}) \times 14.4 .$$
(7)

Since we may consider that the difference between π and K relative to D is small, we suppose $-1 < \alpha < 0$. On the contrary, for the parameter β , we can design both the magnitude and sign quite freely because of the inclusion of f_D and $f_s^{K\pi}(m_D^2)$. For example, if we assume the simple scalarmeson pole dominance for $f_s(q^2)$, we can take $f_s^{K\pi}(m_D^2)/f_s^{DK}(m_{\pi}^2) < 0$. Therefore, according to the variation of the value of β , we classify the following four cases: (a) $\beta < -1$ or $\frac{1}{3} < \beta$, (b) $0 \leq \beta \leq \frac{1}{3}$, (c) $-1 < \alpha < \beta < 0$, and (d) $-1 < \beta < \alpha < 0$. The allowed regions of x for $\eta_0^0 = -1$ ($R_0^+ < 1$) are summarized schematically in Fig. 1.

As seen in Fig. 1, the solution x, which provides $\eta_0^+ = -1$ ($R_0^0 > 0.5$), must be $x < \alpha$ except for the solution $\beta < x < \frac{1}{3}$ in case (b) and the solution in case (c). If we suppose $F_K^{D\pi}/F_{\pi}^{DK} \simeq 1.28$, so that $\alpha \simeq -0.73$, the observations $R_0^+ < 1$ and $R_0^0 > 0.5$ ($\eta_0^0 = -1$ and $\eta_0^+ = -1$) demand an extremely "strong" 20-dominance model. Therefore, we must examine the promising solutions in cases (b) and (c) in the beginning.

from Mark II, respectively, so that as far as the data on $B(D \rightarrow eX)$ are concerned, the $\Delta I = 1$ relationship prefers those from DELCO to those from Mark II.

The assumption of the factorization of the matrix elements provides the $D \rightarrow K\pi$ decay amplitudes

(6)

First, we discuss the solution $0 \le \beta \le x \le \frac{1}{3}$ in case (b), which occurs in the case $f_D/f_{\pi} \gg 1$ and $f_s^{K\pi}(m_p^2)/f_s^{DK}(m_{\pi}^2) > 0$. In the range $\beta < x \le \frac{1}{3}$, the ratio $R_0^0(x)$ has a minimum at $x = \frac{1}{3}$. In order to give $R_0^0(\frac{1}{3}) \leq 0.75$, we must require a surprisingly large value of f_D/f_{π} . For example, assuming $f_s(q^2) \simeq 1$ and $f_K / f_\pi \simeq 1.28$, we get $f_D / f_\pi \simeq 3.4 \times 10^2$. As an example, we illustrate the behavior of $R_0^0(x)$ for $f_D/f_{\pi} = 400$ in Fig. 2. However, such a large value of f_D/f_{π} brings about very strong enhancement of $D^+ \rightarrow \mu \nu$ decay [for example, f_D/f_{π} $\simeq 4 \times 10^2$ causes $\Gamma(D^+ \rightarrow \mu \nu) / \Gamma(D^+ \rightarrow \overline{K}^0 e^+ \nu) \simeq 3$ $\times 10^2$]. Obviously, such strong enhancement of $D^+ \rightarrow \mu \nu$ contradicts the relation $\Gamma(D^+ \rightarrow all)$ $> \Gamma(D^+ \rightarrow eX) + \Gamma(D^+ \rightarrow \mu \nu)$, that is, $\Gamma(D^+ \rightarrow \mu \nu)/(D^+ \rightarrow \mu \nu)$ $\Gamma(D^+ - eX) < [B(D^+ - eX)]^{-1} - 1 \simeq 5$. We estimate roughly the upper bound

$$f_D/f_{\pi} \lesssim 4 \times 10 . \tag{8}$$

Next we investigate the solutions in case (c), which occurs in the case $f_D/f_{\pi} \gg 1$ and $f_s^{K\pi}(m_D^2)/f_s^{DK}(m_{\pi}^2) < 0$. As an example, we illustrate the behavior of $R_0^0(x)$ for $F_K^{D\pi}/F_{\pi}^{DK} = 1.28$ and $F_D^{K\pi}/F_{\pi}^{DK}$ = -50 in Fig. 2. If we assume the simple scalar-

(a) $\beta < -1$ or $\frac{1}{3} <_{\beta}$ (b) $0 \le \beta \le \frac{1}{3}$ (c) $-1 < \alpha <_{\beta} < 0$ (d) $-1 <_{\beta} < \alpha < 0$ (e) $-1 <_{\beta} < \alpha < 0$ (f) $-1 <_{\beta} < \alpha < 0$ (g) $-1 <_{\beta$

d)
$$-1 < \beta < \alpha < 0$$

 $-1 \beta \rightarrow \alpha \leftarrow 0$ 1/3
 $x = X_{-}/X_{+}$

FIG. 1. Allowed regions of $x \equiv X_-/X_+ = (2c_+ - c_-)/(2c_+ + c_-)$ for $\eta_0^0 = -1$ $[R_0^+(x) < 1]$. The hatched and dotted regions give $\eta_0^+ = -1$ $[R_0^0(x) > 0.5]$ and $\eta_0^+ = +1$ $[R_0^0(x) < 0.5]$, respectively. An arrow shows the direction in which the solution x proceeds when $\tau(D^0)/\tau(D^+) \to 0$.



FIG. 2. The relative branching ratio $R_0^0 \equiv B (D^0 \to \overline{K}^0 \pi^0) / B (D^0 \to \overline{K}^- \pi^*)$ versus the parameter $x \equiv X_\perp / X_*$, where curves (a), (b), and (c) correspond to $(F_K^{D\pi}/F_\pi^{DK}, F_K^{D\pi}/F_\pi^{DK}) = (1.28, 0)$, (1.28, 400), and (1.28, -50), respectively. For the curve (a'), the input (1.61, -2.48) is used (see text).

meson pole dominance for $f_s(q^2)$, we get $f_s^{DK}(m_{\pi}^2) = f_+^{DK}(0) \times 1.003$, $f_s^{D\pi}(m_K^2) = f_+^{D\pi}(0) \times 1.051$, and $f_s^{K\pi}(m_D^2) = -f_+^{K\pi}(0) \times 1.296$, where we use $m_{D_s} = 2.27$ GeV and $m_{F_s} = 2.40$ GeV by assuming $m_{D_s} - m_{D^*} \simeq m_{F_s} - m_F * \simeq (m_{\delta} - m_{\rho} + m_{\chi} - m_{\psi})/2$. From the restriction (8), we estimate the lower bound $F_D^{K\pi}/F_{\pi}^{DK} \ge -5 \times 10$ and the upper bound $\beta \le -0.3$, where we suppose $f_+^{K\pi}(0)/f_+^{DK}(0) \simeq 1$. Therefore, as seen from Fig. 2, we obtain the upper bound $x \le -0.6$, so that we fail to get a mild 20-dominance solution as long as we consider $\overline{F_K}^{D\pi}/F_{\pi}^{DK} \simeq 1.3$.

Note that the *W*-exchange dominance model⁷ is practically the same as the assumption $f_D/f_{\pi} \gg 1$ as far as the nonleptonic decays are concerned, but the *W*-exchange dominance is a mechanism peculiar to the nonleptonic decays, so that the model is free from the restriction (8). Although a promising case under the absence of the restriction (8) is case (c) [the case $f_s^{K\pi}(m_D^2) < 0$], the status is not as improved as long as we suppose $F_K^{D\pi}/F_{\pi}^{DK} \simeq 1.3$.

The $\eta_0^+ = -\eta_0^0 = +1$ solution, which is still not ruled out affirmatively from experiments, can exist in the range $\alpha < x \le 0$ except for case (c), but the solution, too, lies near to α when $\tau(D^+)$ $\gg \tau(D^0)$. [The $\eta_0^+ = -\eta_0^0 = +1$ solution in the range $0 \le x \le \frac{1}{3}$ in case (c) is obviously ruled out because $R_0^+(x, f_D/f_\pi) \ge R_0^+(\frac{1}{3}, 40) \simeq 0.46$.] Thus both solutions for $\eta_0^0 = \pm \eta_0^+ = -1$ lie near to α . If we want to get a mild 20-dominance solution, we make an effort to lower the value of $|\alpha|$, that is, to enlarge the value of $F_K^{D\pi}/F_{\pi}^{DK}$. For example, in order to get a typical value of the mild 20-dominance solution⁴ $x \simeq -0.23$ ($c_-/c_+ \simeq 3.16$), we must suppose $f_+^{D\pi}(0)/f_+^{DK}(0) \ge 2$ roughly, but such a large deviation from unity is unlikely considering the difference between K and π relative to D.

In order to select models, it is important to check the predicted lifetimes of *D* as well as the ratio R_0^0 and R_0^+ . If we assume that the QCD result $c_-c_+^2 = 1$ is applicable over the wide range $1 \le c_-/c_+ < \infty$, then we can describe the partial decay widths $\Gamma(D \to K\pi)$ by one parameter *x*. As long as f_D/f_{π} does not take extremely large value, the predicted value of $\Gamma(D^0 \to K^-\pi^+)$ has an upper bound.¹⁰ Generally, when $|\beta| \ge 3$, we can show $\Gamma(D^0 \to K^-\pi^+) \ge [f_s^{DK}(m_{\pi}^2)]^2 \times 1.865 \times 10^{11} \text{ sec}^{-1}$ for $-1 < x \le 0$, and from the experimental value¹ $B(D^0 \to K^-\pi^+) = 0.028 \pm 0.005$, we can derive

$$\tau(D^{0}) \leq (1.5 \pm 0.3) \times [f_{s}^{DK}(m_{\pi}^{2})]^{-2} \times 10^{-13} \text{ sec} .$$
(9)

If we use a theoretical value of $\Gamma_{\rm SL}$, we can predict the upper bound of $B(D^0 \rightarrow eX)$, or inversely if we use the experimental value of $B(D^0 \rightarrow eX)$, we can get the lower bound of $\Gamma_{\rm SL}$. If we suppose $f_s^{DK}(m_{\pi}^2) \simeq 1$, the value $B(D^0 \rightarrow eX) \simeq 5.3\%$ leads to $\Gamma_{\rm SL} \simeq 3.5 \times 10^{11} \text{ sec}^{-1}$, which is somewhat large as compared with usual estimates.¹¹ We may expect that further accumulation of data will lower the value of $B(D^0 \rightarrow eX)$.

Finally, we demonstrate that if we relinquish the derivation of $R_0^0 > 0.5$, we can understand the remaining data from a mild 20-dominance model although with difficulty. We use the result from the hard-meson technique¹² under the assumption that the decay constant of the scalar meson is negligible: $f_{+}^{K\pi}(0) = (f_K/f_{\pi} + f_{\pi}/f_K)/2, f_{+}^{DK}(0)$ = $(f_D/f_K + f_K/f_D)/2$, and so on. The experimental relation $f_K / f_{\pi} f_{+}^{K\pi}(0) = 1.28$ leads to $f_{+}^{K\pi}(0) = 1.04$ and $f_K/f_{\pi} = 1.33$. If we put $f_D/f_{\pi} \simeq 2$, we get $f_{+}^{D\pi}(0)$ $\simeq 1.25$ and $f_{+}^{DK}(0) \simeq 1.08$, so that we obtain $F_{K}^{D\pi}/T$ $F_{\pi}^{DK} \simeq 1.61$ and $F_{D}^{K\pi}/F_{\pi}^{DK} \simeq -2.48$. Then we get solutions $x = -0.83^{+0.07}_{-0.18}$ and $x = -(0.31^{+0.14}_{-0.10})$ from (2), and $x = -(0.70^{+0.18}_{-0.18})$ from (3). Direct observations in emulsions and bubble chambers have indicated $\tau(D^0) \simeq (0.6 - 1) \times 10^{-13}$ sec and $\tau(D^+) \simeq (5 - 10)$ $\times 10^{-13}$ sec.⁶ Since the solution $x \simeq -0.7$ predicts $au(D^0) \simeq 0.5 imes 10^{-13}$ sec, the $\eta_0^+ = \eta_0^0 = -1$ solution cannot be ruled out phenomenologically. However, let us dare to choose the $\eta_0^+ = -\eta_0^0 = +1$ solution. If we presume moderate 20-dominance $c_{-}/c_{+} \simeq 5 \ (c_{-} \simeq 2.9, \ c_{+} \simeq 0.58, \ x \simeq -0.43),$ we can

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predict $R_0^0 \simeq 0.26$, $R_0^+ \simeq 0.08$, $\tau(D^+) \simeq 8 \times 10^{-13}$ sec, and $\tau(D^0) \simeq 0.9 \times 10^{-13}$ sec. If we assume the weakboson mass $m_W = 84$ GeV, the charm-quark mass $m_c = 1.2$ GeV, and the number of quark flavors $n_F = 6$, the moderate enhancement $c_-/c_+ \simeq 5$ requires $\alpha_s(m_c) \simeq 1.1$. We may consider that the solution $c_-/c_+ \simeq 5$ does not so severely contradict QCD.

Note added. In the present paper, we have assumed that all the decay amplitudes $A(D \rightarrow K\pi)$ are real. Generally we must take into account the $K\pi$ scattering phase shifts in the final states:

$$\begin{split} A \left(D^{+} \rightarrow \overline{K}^{0} \pi^{+} \right) &= 3 a_{3} e^{i b_{3}} \,, \\ A \left(D^{0} \rightarrow K^{-} \pi^{+} \right) &= a_{3} e^{i b_{3}} + \sqrt{2} \, a_{1} e^{i b_{1}} \,, \\ A \left(D^{0} \rightarrow \overline{K}^{0} \pi^{0} \right) &= \sqrt{2} \, a_{3} e^{i b_{3}} - a_{1} e^{i b_{1}} \,, \end{split}$$

where a_n is the reduced matrix element of

- ¹V. Luth, in Proceedings of the 1979 International Symposium on Lepton and Photon Interaction at High Energy, Fermilab, edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1979); Report No. SLAC-PUB-2405, LBL-9851 (unpublished).
- ²J. Kirby, in *Proceedings of the 1979 International Symposium on Lepton and Photon Interaction at High Energy*, *Fermilab* (Ref. 1); Report No. SLAC-PUB-2419 (unpublished).
- ³J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. <u>B100</u>, 313 (1975); G. Altarelli, N. Cabibbo, and L. Maiani, Phys. Rev. Lett. <u>35</u>, 635 (1975); D. Fakirov and B. Stech, Nucl. Phys. <u>B133</u>, 315 (1978).
- ⁴N. Cabibbo and L. Maiani, Phys. Lett. <u>73B</u>, 418 (1978).
- ⁵M. Katuya and Y. Koide, Phys. Rev. D <u>19</u>, 2631 (1979).
 ⁶A. L. Read *et al.*, Phys. Rev. D <u>19</u>, 1287 (1979);
- A. Angelini et al., Phys. Lett. <u>80B</u>, 428 (1979); <u>84B</u>, 150 (1979); H. Fuchi et al., *ibid*. <u>85B</u>, 135 (1979);
 D. Allasia et al., *ibid*. <u>87B</u>, 287 (1979); H. C. Ballagh et al., *ibid*. <u>89B</u>, 423 (1980); M. I. Adamovich et al., *ibid*. <u>89B</u>, 427 (1980).
- ⁷S. P. Rosen, Phys. Rev. Lett. <u>44</u>, 4 (1980); I. Bigi and L. Stodolsky, Report No. SLAC-PUB-2410, 1979 (unpublished).

 $H_w(\Delta I=1)$, *n* denotes that the final $K\pi$ state has I=n/2, and δ_n is the $K\pi$ scattering phase shift for I=n/2 at the $K\pi$ c.m. energy equal to the *D* mass. Note that the zero of $A(D^+ \rightarrow \overline{K}{}^0\pi^+)$, which plays an essential role in our investigation, is still given by α in Eq. (7) independently of δ_n . Since what is of great interest to us is the investigation of the case where $R_0^+ \ll 1$, we may suppose $|a_3| \ll |a_1|$, so that

 $|\mathrm{Im}(a_3e^{i\delta})| \ll |\mathrm{Re}(a_3e^{i\delta})| \ll |a_1|$

where $\delta = \delta_3 - \delta_1$, if $|\delta|$ is not so large. Therefore, our results under the assumption $\delta = 0$ are also valid for $\delta \neq 0$ as far as R_0^+ is small. However, if we find that the sum rule (5) is in very poor agreement with experiment, then the experiment suggests that $|\sin\delta| \gg |\cos\delta|$, so that our investigation must be modified.

- ⁸M. Matsuda, M. Nakagawa, and S. Ogawa, Prog. Theor. Phys. <u>63</u>, 351 (1980) [also see T. Hayashi, M. Nakagawa, M. Nitto, and S. Ogawa, Prog. Theor. Phys. <u>49</u>, 351 (1973); <u>52</u>, 636 (1974)]; M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. <u>44</u>, 7 (1980); B. Guberina, S. Nussinov, R. D. Peccei, and R. Rückel, Phys. Lett. <u>89B</u>, 111 (1980); X. Y. Pham, Report No. PAR LPTHE <u>79/25</u>, 1979 (unpublished); V. Barger, J. P. Leveille, and P. M. Stevenson, Phys. Rev. D <u>22</u>, 693 (1980); W. Bernreuther, O. Nachtmann, and B. Stech, Report No. HD-THEP-79-17, 1979 (unpublished); A. I. Sanda, Report No. COO-2232B-191 (unpublished).
- ⁹R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D <u>11</u>, 1919 (1975).
- ¹⁰Y. Koide, Phys. Rev. D <u>18</u>, 1644 (1978).
- ¹¹D. Fakirov and B. Stech, Nucl. Phys. <u>B133</u>, 315 (1978); X. Y. Pham and R. P. Nabavi, Phys. Rev. <u>18</u>, 220 (1978); N. Cabibbo and L. Maiani, Phys. Lett. <u>79B</u>, 109 (1978); M. Suzuki, Nucl. Phys. <u>B145</u>, 420 (1978).
- ¹²S. L. Glashow and S. Weinberg, Phys. Rev. Lett. <u>20</u>, 224 (1978); Riazuddin, A. Q. Sarker, and Fayyazuddin, Nucl. Phys. B6, 515 (1968).