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Experimental constraint on widths of postulated low-mass dibaryons

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The consistency of a postulated $^3P_1$ resonance near $E_L = 390$ MeV with the experimentally determined nucleon-nucleon phase shifts at $E_L = 325$, 380, and 425 MeV is examined. It is found that the data require that the width of such a resonance $\Gamma \lesssim 0.3$ MeV. A similar restriction on the width of a postulated $^1S_0$ resonance near $E_L = 300$ MeV is implied by the accuracy of the data at $E_L = 210$, 325, and 380 MeV.

I. INTRODUCTION

Mac Gregor has recently noted[1] that the observed proton-proton resonances fall on a trajectory which would also pass through a $^3P_1$ resonance at proton laboratory energy $E_L = 390$ MeV and a $^1S_0$ resonance at $E_L = 300$ MeV. Such resonances of some tens of MeV width or less are consistent with the total-cross-section measurements[2] which have gaps between 388.0 and 406.5 MeV and between 267.5 and 315.0 MeV.

This note reports on an examination of the constraints imposed by all the experimental information in the energy range, through the phase shifts determined by the data, and finds the constraints to be much stronger than those of the total cross sections alone. In particular it is found that a $^3P_1$ dibaryon between $E_L = 380$ MeV and $E_L = 425$ MeV would be required to have a width $\Gamma \lesssim 0.3$ MeV. A $^1S_0$ resonance near 300 MeV would be similarly restricted as the phase shifts are as accurately known as the $^3P_1$ phases, and fall on a smooth curve.

The data which bear most directly on the postulated $^3P_1$ resonance are those of Bugg *et al.*[3] at $E_L = 325$, 380, and 425 MeV. Combined with older angular-distribution and total-cross-section data, these data have been independently analyzed at each energy into the partial-wave phase shifts.[3] The $^1S_0$ and $^3P_1$ phase shifts are given in Table I. The phases fit on smooth curves which also fit through lower- and higher-energy data. The accuracy of the phase shifts implies, as we will demonstrate below, that the width of any resonance in the region must be very much less than the 45-MeV gap (only 22.5 MeV in c.m.) between the bounding phase shifts. A Breit-Wigner–type resonance must decay many half-widths before it will not displace the bordering phases from a smooth curve by more than a standard deviation in opposite directions. The phase shift at the lowest energy is required to establish the slope in addition to the mean height of the smooth background.

II. CONSTRUCTION OF THE RESONANT PHASES

Any model which can produce Breit-Wigner resonances of given widths superimposed on an adjustable smooth background would suffice in this investigation. Here we apply a coupled-channel approach currently being used[4] in a study of the experimentally indicated nucleon-nucleon dibaryons and all the phase shifts up to 1 GeV. In this way we also learn the type of coupled-channel interactions which could be consistent with the resonances hypothesized by Mac Gregor[1] and with the data.[3] In the limit of narrow width, the resonances produced by the model have a Breit-Wigner shape, as one expects from any physical model. This is verified below for the case most relevant to the experimental situation.

In order to produce a reasonable energy dependence over a large range of energies it is advisable to use a model which is both phenomenologically and theoretically well founded. In the nucleon-nucleon channel we use the Feshbach-Lomon interaction[5] which satisfies these requirements for $E_L \lesssim 400$ MeV.

The effect of inelastic thresholds is obtained[4,6] by extending both the potential and the internal boundary condition of the model to a matrix that couples to nucleon-isobar and isobar-isobar channels. The most important channel for low-energy isospin triplet states is the $N\Delta$ system. For the purpose of this investigation the potential tail of the transition interaction can be ignored (as it would negligibly change the shape of a resonance) and the simple off-diagonal boundary condition coupling to the $N\Delta$ channel is employed.

Since the predicted resonances are well below the Δ production threshold we may use the zero-width approximation for the Δ, for which the boundary condition may be written[4,6]

$$\frac{dU_{NN}}{dr_0} = f_{NN}U_{NN}(r_0) + f_{N\Delta}U_{N\Delta}(r_0)\,, \tag{1a}$$

$$\frac{dU_{N\Delta}}{dr_0} = f_{N\Delta}U_{NN}(r_0) + f_{\Delta\Delta}U_{N\Delta}(r_0)\,. \tag{1b}$$

TABLE I. Experimental and model pp phases (degrees).

	Case \ E_L	325 MeV	380 MeV	425 MeV	Res. position (MeV)	Res. width (MeV)
$\delta(^1S_0)$ experimental (Ref. 3)		-9.34 ± 0.51	-13.76 ± 0.66	-18.26 ± 0.53		
$\delta(^3P_1)$ experimental (Ref. 3)		-30.20 ± 0.52	-33.82 ± 0.56 (148.18)[a]	-35.26 ± 0.36 (144.74)[a]		
$\delta(^3P_1)$ $f_{NN}=22.53$, $f_{N\Delta}=3.9$, $f_{\Delta\Delta}=5.0$, $\Gamma_\Delta=115$ MeV	A	-22.6	61.7	119.4	377	15
$\delta(^3P_1)$ $f_{NN}=22.53$, $f_{N\Delta}=3.9$, $f_{\Delta\Delta}=5.0$, $\Gamma_\Delta=0$	B	-23.0	44.7	117.0	383	15
$\delta(^3P_1)$ $f_{NN}=6.2$, $f_{N\Delta}=0.53$, $f_{\Delta\Delta}=-1.665$, $\Gamma_\Delta=0$	C	-26.6	-21.8	138.9	396.3	2.5
$\delta(^3P_1)$ $f_{NN}=7.7$, $f_{N\Delta}=0.2$, $f_{\Delta\Delta}=-1.695$, $\Gamma_\Delta=0$	D	-29.8	-32.8	142.7	399.2	0.3
$\delta(^3P_1)$ $f_{NN}=7.3$, $f_{N\Delta}=0.1$, $f_{\Delta\Delta}=-1.7$, $\Gamma_\Delta=0$	E	-29.6	-33.1	143.6	398.1	0.08

[a] 180° should be added to the quoted phase shifts if the partial wave is presumed to have passed through resonance.

The f_{ij} $(i,j=N,\Delta)$ are the energy-independent components of the f matrix, U_{NN} and $U_{N\Delta}$ are the reduced radial wave functions $[\sim r h_1^{(\lambda)}(kr)$ asymptotically$]$ in the $NN(^3P_1)$ and the $N\Delta(^3P_1$ or $^5P_1$, $I=1)$ partial waves, respectively. The boundary radius $r_0=0.51\hbar/\mu c$, as determined by the low-energy data,[5] agrees with the theoretical value related to the onset of multiparticle exchange.[7] We use $M_\Delta=1232$ MeV.

The boundary conditions are easily extended to include the width of the isobars.[6,8] The effect of the width will be presented for the case in which it has the greatest effect, i.e., for the NN resonance of largest width. In that case we use $\Gamma_\Delta=115$ MeV.

The smooth background behavior of the phase shift is determined by the Feshbach-Lomon potential and by f_{NN}, which represents the short-range interaction in the NN channel. The effect of coupling to the $N\Delta$ channel through $f_{N\Delta}$ is to add an attraction which increases with energy up to the Δ meson threshold.[4,6] In the absence of a long-range transition potential the result is determined by the effective f matrix f_{eff} which acts in the NN channel after elimination of the $N\Delta$ channel (i.e., the NN-channel Schrödinger equation is solved with f_{eff} as the internal boundary condition):

$$f_{\text{eff}}=f_{NN}-\frac{(f_{N\Delta})^2}{[f_{\Delta\Delta}+\theta_{L'}(k')]} \qquad (2)$$

with

$$\theta_{L'}(k')=-r_0[h_{L'}^{(1)}(k'r_0)]^{-1}\frac{d}{dr_0}[h_{L'}^{(1)}(k'r_0)],$$

where L' is the orbital angular momentum and $k'(k,M_\Delta)$ is the relative momentum in the $N\Delta$ channel. Below inelastic threshold, k' is imaginary; $\theta_{L'}(k')$ is real and decreases monotonically from its value at elastic threshold to L' at the inelastic threshold. Therefore $f_{\text{eff}}<f_{NN}$, i.e., it is more attractive and decreases with increasing energy up to inelastic threshold (providing that $f_{\Delta\Delta}>-L'$). If $f_{N\Delta}$ is sufficiently large, f_{eff} becomes small enough at some energy to bring the phase shift through resonance. The resonance first appears near the inelastic threshold and then moves to lower energies as $f_{N\Delta}$ increases. We call this a coupled-channel resonance of the general type because its existence and position depend directly on the strength of coupling. The width and inelasticity of such resonances are insensitive to $f_{\Delta\Delta}$ and consequently are determined by $f_{N\Delta}$, which in turn is determined by the position. In fact, as we shall see below, such a general coupled-channel resonance at $E_L=400$ MeV is always too broad to fit the nearby phase shifts.

However, there is another type of coupled-channel resonance when $f_{\Delta\Delta}<-L'$, arising from the Dalitz-Tuan mechanism.[9] This special type of resonance is present under the condition that the diagonal $N\Delta$-channel interaction, represented here by $V_{\Delta\Delta}$ and $f_{\Delta\Delta}$, is attractive enough to cause a bound state in the uncoupled $N\Delta$ system. Coupling to the NN channel allows the bound state to leak into that channel so that it appears as a resonance in the NN channel, the width being narrow if the coupling is weak. In f_{eff} this mechanism is evidenced by the pole it develops below the inelastic threshold when $f_{\Delta\Delta}<-L'$. If in addition $f_{N\Delta}$ is small, the resonance energy is very near the energy at which $f_{\Delta\Delta}+\theta_{L'}(k')=0$. For these special coupled-channel resonances the position is determined by $f_{\Delta\Delta}$ and the width by $f_{N\Delta}$. Such a resonance can be made arbitrarily narrow and we can use it as our guide to the maximum width compatible with the known phase shifts. Although we will neglect the long-range transition potential and $N\Delta$ diagonal potential, $V_{N\Delta}$ and $V_{\Delta\Delta}$, the achievable Breit-Wigner plus smooth background curves will not be affected over a moderate energy range.

III. RESULTS

A. Coupled-channel resonances of the general type

A coupled-channel resonance of the general type was induced in the $^3P_1$ channel, which is coupled to the $N\Delta$ P-wave channel, by keeping $f_{\Delta\Delta}>-1$ and increasing $f_{N\Delta}$ until a resonance was obtained in the vicinity of $E_L=390$ MeV. The resonance so obtained has a width of 15 MeV for $f_{\Delta\Delta}=0$ or 5 (representing moderate attraction to strong repulsion in the $N\Delta$ channel). Such a width may possibly be compatible with the total cross-section measurements[2] given the limited accuracy and fluctuations in those values. However, as shown in cases A and B of Table I, δ (325 MeV) and δ (380 MeV) are much too large and δ (425 MeV) is much too small compared with the experimental values. Including the effect of a Δ width (case A) worsens the agreement. For the tabulated phase, case A and B, the f-matrix choice puts the resonance near $E_L=380$ MeV and provides a background that matches the 210 MeV phase shift. At the same time, the average value of the phase shifts between 325 and 425 MeV is correct, a consequence of the suitable NN-channel behavior of the Feshbach-Lomon interaction.[5] Increasing the energy of the resonance from 380 MeV towards 425 MeV would certainly improve its fit to the phase shift at 380 MeV but would also considerably worsen its already very bad fit at 425 MeV.

It is noteworthy that, although the physical value of Γ_Δ (case A) introduces inelasticity below the Δ threshold, the minimum value of $\eta\simeq 0.97$ near $E_L=380$ MeV. In contrast the large inelasticity of the $^1D_2$ state for $E_L>500$ MeV (Ref. 10) is well reproduced by this general coupled-channel mechanism. This illustrates that, as one would expect, the inelasticity achievable in this model decreases rapidly for $E_L<E_T-\frac{1}{2}\Gamma_\Delta$, where E_T is the threshold for production of the central mass of the Δ. The good fits obtained by this model to the $^1D_2$ and $^3F_3$ resonances[10] provide a reason for confidence in the predictions obtained here by its application to the $^3P_1$ state at somewhat lower energies.

B. Coupled-channel resonance of the Dalitz-Tuan type

Switching to the Dalitz-Tuan–type coupled-channel resonance, one chooses $f_{\Delta\Delta}<-1$, so as to produce a pole in f_{eff} at $E_L\approx 400$ MeV. This corresponds to $f_{\Delta\Delta}\approx -1.7$. The width obtained is proportional to $(f_{N\Delta})^2$ for small $f_{N\Delta}$. Several choices of $f_{N\Delta}$ were compared with the data. For each $f_{N\Delta}$ a value of f_{NN} was found which gave the correct 210-MeV phase shifts. The mean value of the phase shifts between 325 and 425 MeV is simultaneously fitted as before.

As shown in Table I, case C, corresponding to $f_{N\Delta}=0.53$, induces a resonance only 2.5 MeV wide. Nevertheless, the phase shifts at 325 and 380 MeV are 7 and 21 standard deviations too high, respectively, and the 425-MeV phase shift is 17 standard deviations too low. When $f_{N\Delta}=0.2$, case D, a width of 0.3 MeV is obtained. The phase shifts at 325 and 380 MeV are in this case, 1 and 2 standard deviations too high, respectively, and the 425 MeV phase shift is 6 standard deviations too low. Finally for $f_{N\Delta}=0.1$, case E, a width of only 0.08 MeV is obtained. The 325 and 380-MeV phase shifts are each about 1 standard deviation high and the 425-MeV phase shift is 3 standard deviations low. Cases A–E are depicted in Fig. 1 together with the experimental data.

One may try to minimize χ^2 by changing the position of the resonance. However, decreasing the position of the 0.3-MeV-wide resonance to 391 MeV improves the fit at 425 MeV by only 0.5 stand-

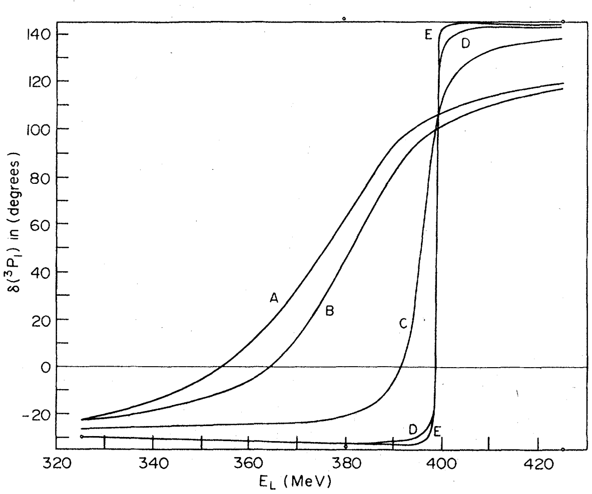

FIG. 1. The $^3P_1$ experimental and resonant model phase shifts. The circles represent the data values and errors (Ref. 3). The curves are labeled by letters that correspond to the cases in Table I.

ard deviations. We estimate that, for that case, $\chi^2 \gtrsim 25$; while for the 0.16-MeV-width case, $\chi^2 \gtrsim 8$ for any position of the resonance between 380 and 425 MeV.

We can compare these results with those of adding Breit-Wigner resonant phases δ_{BW} to the smooth background phases δ_{BG} obtained by setting $f_{N\Delta}=0$. For a standard Breit-Wigner shape one has

$$\tan(\delta_{BW}-\delta_{BG}) = \Gamma_{res}/(E_L - E_{res}) \qquad (3)$$

(the absence of a familiar factor of $\frac{1}{2}$ is due to the use of laboratory rather than c.m. energies). We compare this with case D which provides an upper limit on the width of the suggested $^3P_1$ resonance. At 425 MeV (86 half-widths from the resonance) both the left- and right-hand sides of Eq. (3) are 1.2×10^{-2}, while at 325 MeV (248 half-widths from resonance) the left-hand side is 3.3×10^{-3} and the right-hand side is 4.0×10^{-3}. Therefore the resonance shape is of Breit-Wigner form to a very high degree. In spite of the special model used here the results can be considered to be very general.

Because little inelasticity is expected at such low energies, we have not examined the case of a very inelastic resonance in detail. But $\eta(^3P_1)$ cannot be much less than unity, by comparison with the total inelastic cross section. An energy-dependent analysis of elastic and inelastic data[11] indicates that the total inelastic cross section at $E_L=400$ MeV is about 1.5 mb. If all the inelasticity were due to the $^3P_1$ channel then

$$3K^{-2}[1-\eta^2(^3P_1)] = 0.15\ \text{fm}^{-2}$$

which requires $\eta(^3P_1)\gtrsim 0.96$. In fact much of the inelasticity is expected to be in the $^1D_2$ channel. This is in accord with the phase-shift analysis of Arndt,[12] who finds that $\eta\approx0.98$ for $^1S_0$, $^3P_0$, $^3P_1$, and $^1D_2$ states. But the $^1D_2$ has a statistical weight five times that of $^1S_0$ and $^3P_0$ and $\frac{5}{3}$ that of $^3P_1$, so that it alone accounts for half the inelasticity.

To modify our results for the real phase shift substantially, it would require that δ not pass through 90°. This requires that $\Gamma(\text{elastic}) \leqslant \frac{1}{2}\Gamma(\text{total})$, which in turn implies that $\eta \ll 1$ at the resonance peak. Even for models which emphasize inelasticity, such as case A, $\eta > 0.97$ near 400 MeV. In addition to the theoretical difficulty of producing such a small η, the resonance would again have to be separated by many half-widths from the energies of the known phase shifts and the data determining the inelastic cross section of Ref. 11 in order to be consistent with $\eta\approx1$ at those energies. The Breit-Wigner resonance form requires that $\Gamma(\text{total})\approx(1-\eta^2)^{1/2}(E_L-E_R)$ for inelastic resonances. Using the above limit, established by Ref. 11, for $E_L=380$ MeV (but allowing $\eta\ll1$ at $E_R=400$ MeV), we obtain $\Gamma(\text{total})<6$ MeV. The more likely limit at 380 MeV of $\eta(^3P_1)\gtrsim0.98$ indicated by the phase-shift analyses, which apportion a share of inelasticity to the $^1D_2$ partial wave, implies $\Gamma(\text{total})<4$ MeV.

We conclude that an elastic resonance of width greater than 0.1 MeV is difficult to reconcile with the data. We doubt that a width greater than 0.3 MeV could be reconciled even after a more complicated, but reasonable, background variation is permitted and one allows for systematic experimental errors, etc. Very large inelasticity of the resonance ($\eta_{min}<0.5$) would allow a width as large as 6 MeV but is difficult to reconcile with theory. The small $^1S_0$ phase-shift errors (see Table I) imply very similar restrictions on the width of a $^1S_0$ resonance near 300 MeV. It follows that a search for the dibaryons postulated by Mac Gregor[1] must be designed to observe resonances of width less than, perhaps a good deal less than, 0.5 MeV, or, if the resonance is very inelastic, less than 6 MeV.

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