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Q^2 evolution of multihadron fragmentation functions

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In order to characterize jet structure more completely than is possible with the single-hadron fragmentation functions of partons, we deduce the evolution (in Q^2) equations for the fragmentation functions of a parton to fragment into an arbitrary number N of hadrons. The solution to the evolution equations is explicitly displayed in terms of multiple moments for the N-hadron fragmentation functions. Our procedure is simpler than using the rules of the jet calculus. We also show how the Q^2 dependence of these fragmentation functions can be understood in a recursive cascade-model framework in which both the momentum and the amount by which a parton is off its energy shell (as measured by Q^2) are reduced at each branching step during jet formation. This reformulation suggests an alternative way for solving the evolution equations in which moment inversion is not required.

I. INTRODUCTION

The experimental study of jets of hadrons produced in electron-positron annihilation, neutrino (and antineutrino) reactions, and hadron-hadron collisions has inspired considerable theoretical research into jet properties. From our present level of understanding of partons and quantum chromodynamics (QCD), we believe these hadron jets contain information on the parton confinement mechanism. Such information can be extracted when characterization of jet properties is complete. This requires more knowledge than that available from the single-particle fragmentation functions $D_{j\to h}(x, Q^2)$. These functions give the probability of finding in a jet a single hadron h with momentum fraction x of that possessed by the jet-initiating parton j (a quark, antiquark, or gluon) which is off mass shell as specified by Q^2. The evolution in Q^2 of this probability is given by equations deduced by Owens[1] and Uematsu,[2] which are analogous to the Altarelli-Parisi Q^2-evolution equations[3] for structure functions.

To aid in the complete characterization of jets when data becomes more complete in the future, we deduce in this work the corresponding evolution equations for the fragmentation of parton j into N hadrons. These hadrons carry fractions $x_1, x_2, \ldots, x_N$ of the momentum carried by the initial parton j at some Q^2. In Sec. II we discuss these evolution equations for multihadron fragmentation functions in detail, beginning with a brief review of the equations for single-hadron fragmentation.

In Sec. III we present explicit solutions of these evolution equations. The relation of our work to previous work is also discussed.

In Sec. IV, we show that since jet formation proceeds via repeated branching, it is possible to reformulate the Q^2-evolution equations in terms of a recursive cascade model.[4] Since such a model has proved useful in characterizing jet formation in the nonperturbative regime, one has a unified description of jets at any Q^2. The final section provides a brief summary of this paper.

II. EVOLUTION EQUATIONS FOR MULTIHADRON FRAGMENTATION FUNCTIONS

Before describing the general case of a parton fragmenting into N hadrons, let us briefly review the formulation of the evolution equations for single-hadron fragmentation functions $D_{i\to h}$. This will establish the notation and indicate clearly how the generalization to N hadrons is made.

The Q^2-evolution equation for a parton i fragmenting into a hadron h with momentum fraction x of that of the initial parton is[1,2]

$$\frac{d}{dY} D_{i\to h}(x;Y) = \int_x^1 \frac{dx'}{x'} P_{ji}(x') D_{j\to h}\left(\frac{x}{x'};Y\right), \qquad (1)$$

where

$$Y = \frac{1}{2\pi b} \ln[\alpha_0 b \ln(Q^2/\Lambda^2)], \quad 12\pi b = 11N_c - 2f \qquad (2)$$

for N_c colors and f flavors, with α_0 and Λ setting the strength and scale of the QCD running coupling constant. In Eq. (1) the parton indices i, j take on $(2f+1)$ values $(q_1, q_2, \ldots, q_f, \bar{q}_1, \bar{q}_2, \ldots, \bar{q}_f, G)$ corresponding to f quark flavors and the gluon G. There is an implied summation over the repeated index j and this summation convention is used throughout the paper. The $P_{ji}(x)$ are standard parton vertex functions[3] for the process $i(p)\to j(xp)$, with p the momentum of parton i. Equation (1) is very similar to the Altarelli-Parisi equation for the evolution of structure functions. The terms on the right-hand side can best be visualized pictorially as in Fig. 1 for the case of i = quark fragmentation. The two diagrams depict the initial quark branching into a quark and a gluon, and the ob-

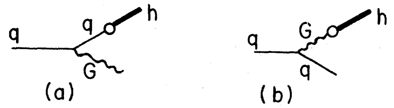

FIG. 1. Diagrams contributing to single-hadron quark fragmentation. In this and all successive diagrams, heavy lines represent hadrons, the small circle is the fragmentation function, and the thin lines represent partons (quarks or gluons).

served hadron h coming from either the quark [Fig. 1(a)] or the gluon [Fig. 1(b)]. Likewise, Fig. 2 represents the two terms on the right-hand side of Eq. (1) for the gluon fragmentation ($i=G$).

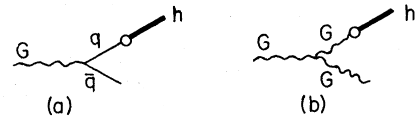

FIG. 2. Diagrams contributing to single-hadron gluon fragmentation.

For the fragmentation of a parton into two hadrons, the diagrams corresponding to those in Figs. 1 and 2 can readily be sketched for all possible ways in which i, a quark (Fig. 3) or gluon (Fig. 4), can fragment. Therefore, the evolution equations for two-hadron fragmentation functions $D_{i\to h_1 h_2}$ can then be deduced by analogy with Eq. (1):

$$\frac{d}{dY}D_{i\to h_1h_2}(x_1,x_2;Y)=\int_{x_1+x_2}^{1}\frac{dx'}{x'^2}P_{ji}(x')D_{j\to h_1h_2}\left(\frac{x_1}{x'},\frac{x_2}{x'};Y\right)+\int_{x_1}^{1-x_2}\frac{dx'}{x'(1-x')}\hat{P}_{i\to b_1b_2}(x')D_{b_1\to h_1}\left(\frac{x_1}{x'};Y\right)D_{b_2\to h_2}\left(\frac{x_2}{1-x'};Y\right), \quad (3)$$

where the hadrons h_1 and h_2 have momentum fractions x_1 and x_2, respectively, of that of the initial parton. Also, our notation consistently will suppress the symbol for summation over repeated parton indices j, b_1, b_2. The first term on the right-hand side corresponds to a branching $i\to j(x')$ followed by parton j fragmenting into hadrons h_1 and h_2 [see Figs. 3(a), 3(b), 4(a), and 4(b)]. The second term corresponds to a branching vertex $i\to b_1(x')+b_2(1-x')$ given by[5] $\hat{P}_{i\to b_1b_2}(x')$ followed by single-particle fragmentation [see Figs. 3(c), 3(d), 4(c), and 4(d)] of each parton to produce the hadrons h_1 and h_2. The limits on the x' integration in Eq. (3) come from requiring a fragmenting parton to have at least as much momentum as the fragmentation products.

As illustrated in Fig. 5, the diagrams for one- or two-hadron fragmentation generalize readily to N-hadron fragmentation. The evolution equations for these N-hadron fragmentation functions, $D_{i\to h_1,\ldots,h_N}(x_1,x_2,\ldots,x_N;Y)$, with hadrons $h_1,h_2,\ldots,h_N$ having momentum fractions $x_1,x_2,\ldots,x_N$ of the initial parton momentum, straightforwardly follow:

$$\frac{d}{dY}D_{i\to h_1,\ldots,h_N}(x_1,x_2,\ldots,x_N;Y)=\int_{\Sigma_1^N x_i}^{1}\frac{dx'}{x'^N}P_{ji}(x')D_{j\to h_1,\ldots,h_N}\left(\frac{x_1}{x'},\ldots,\frac{x_N}{x'};Y\right)$$
$$+\sum_{M=M_{\min}}^{N-1}\int_{\Sigma_1^M x_i}^{1-\Sigma_{M+1}^N x_i}\frac{dx'}{x'^M(1-x')^{N-M}}\hat{P}_{i\to b_1b_2}(x')D_{b_1\to h_1,\ldots,h_M}\left(\frac{x_1}{x'},\ldots,\frac{x_M}{x'};Y\right)$$
$$\times D_{b_2\to h_{M+1},\ldots,h_N}\left(\frac{x_{M+1}}{1-x'},\ldots,\frac{x_N}{1-x'};Y\right), \quad (4)$$

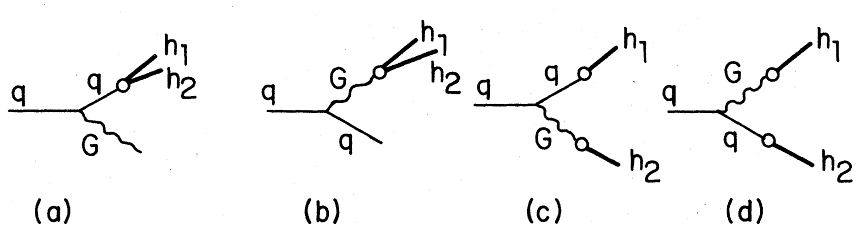

FIG. 3. Various contributions to the two-hadron fragmentation function for quarks.

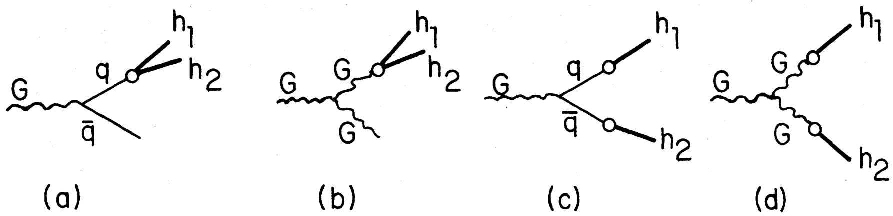

FIG. 4. Various contributions to the two-hadron fragmentation function for gluons.

where

$$M_{\min} = \tfrac{1}{2}N \quad (N \text{ even})$$
$$= \tfrac{1}{2}(N+1) \quad (N \text{ odd}). \tag{5}$$

Note that Eq. (4) is also correct for the production of N partons, if the labels $h_1, \ldots, h_N$ are interpreted as parton indices. In fact, *at the parton level* both Eq. (4) and the N-parton evolution equations given by Kirschner[6] are equivalent—the only difference being that in our procedure the fundamental branchings P_{ji}, $\hat{P}_{i\to b_1 b_2}$ occur first, not last. However, this difference is crucial when one is considering *N-hadron* fragmentation functions since our Eq. (4) is applicable, but Kirschner's results are not.

To go from the parton level to final-state hadrons, it is necessary to convert the partons into color singlets. This could occur, e.g., by further fragmentation, as in the cascade model,[4] or by recombination.[7] It is preferable to discuss fragmentation functions into hadrons directly since these are experimental observables.

III. SOLUTION OF THE EVOLUTION EQUATIONS

A. Single-hadron fragmentation functions

Let us first review how the evolution equations for single-particle fragmentation [Eq. (1)] are solved. The moments of the single-particle fragmentation functions and of the anomalous dimensions are defined, respectively, by

$$D_{i\to h}(n;Y) \equiv \int_0^1 dx\, x^{n-1} D_{i\to h}(x;Y)\,, \tag{6}$$

$$A^n_{ji} \equiv \int_0^1 dx\, x^{n-1} P_{ji}(x)\,. \tag{7}$$

Explicit expressions for the moments of these vertex functions $P_{ji}(x)$ are[3]

$$A^n_{q_jq_i} = \delta_{ij}A^n_{qq} \equiv \delta_{ij}\frac{N_c^{\,2}-1}{2N_c}\left(-\frac{1}{2}+\frac{1}{n(n+1)}-2\sum_{j=2}^{n}\frac{1}{j}\right),$$

$$A^n_{q_iG} = A^n_{qG} = \frac{n^2+n+2}{2n(n+1)(n+2)}\,,$$

$$A^n_{Gq_i} = A^n_{Gq} = \frac{N_c^{\,2}-1}{2N_c}\;\frac{(n^2+n+2)}{n(n^2-1)}\,, \tag{8}$$

$$A^n_{GG} = N_c\left[-\frac{1}{6}+\frac{2}{n(n-1)}+\frac{2}{(n+1)(n+2)}-\frac{f}{3N_c}-2\sum_{j=2}^{n}\frac{1}{j}\right].$$

As written, Eqs. (8) are for integer n, but the analytic continuation to noninteger values follows from

$$\sum_{j=2}^{n}\frac{1}{j} = \sum_{j=2}^{\infty}\frac{(n-1)}{j(j+n-1)}\,. \tag{9}$$

For later use we present (Fig. 6) graphs of the n dependence of the anomalous dimensions [Eqs. (8)].

Taking moments of Eq. (1) gives

$$\frac{d}{dY}D_{i\to h}(n;Y) = A^n_{ji}D_{j\to h}(n;Y) \tag{10}$$

and[5]

$$D_{i\to h}(n;Y) = [e^{(Y-Y_0)A^n}]_{ji}D_{j\to h}(n;Y_0)\,, \tag{11}$$

where $D_{j\to h}(n;Y_0)$ is the boundary value of the fragmentation function. This solution is valid for any matrix A. However, since A is flavor independent [see Eq. (8)], the matrix $\exp[(Y-Y_0)A^n]$ is particularly simple[8] ($y = Y - Y_0$),

$$[e^{yA^n}]_{q_jq_i} = e^{yA^n_{qq}}\delta_{ij} + \frac{1}{2f(\sigma^n_+-\sigma^n_-)}\left[\sigma^n_+(e^{\lambda^n_- y}-e^{A^n_{qq}y}) + \sigma^n_-(e^{A^n_{qq}y}-e^{\lambda^n_+ y})\right],$$

$$[e^{yA^n}]_{Gq_i} = \frac{1}{2f(\sigma^n_+-\sigma^n_-)}(e^{\lambda^n_+ y}-e^{\lambda^n_- y})\,,$$

$$[e^{yA^n}]_{q_iG} = \frac{(-\sigma^n_-\sigma^n_+)}{(\sigma^n_+-\sigma^n_-)}(e^{\lambda^n_+ y}-e^{\lambda^n_- y})\,, \tag{12}$$

$$[e^{yA^n}]_{GG} = \frac{1}{(\sigma^n_+-\sigma^n_-)}(\sigma^n_+e^{\lambda^n_+ y}-\sigma^n_-e^{\lambda^n_- y})\,.$$

The quantities $\sigma^n_\pm$, $\lambda^n_\pm$, and A^n_{qq} appear in Eqs. (12), since $\lambda_\pm$ and A_{qq} are eigenvalues of the anomalous-dimension matrix A, and $\sigma_\pm$ appear in the eigenvectors. They are plotted in Fig. 7 and are given by[1]

$$\sigma^n_\pm = \frac{(A^n_{GG}-A^n_{qq}) \pm [(A^n_{GG}-A^n_{qq})^2+8fA^n_{qG}A^n_{Gq}]^{1/2}}{4fA^n_{Gq}}\,,$$

$$\lambda^n_\pm = A^n_{qq}+2fA^n_{Gq}\sigma^n_\pm\,. \tag{13}$$

Combining Eqs. (11) and (12) gives explicit known expressions for the gluon (G), singlet (S), and nonsinglet (NS) fragmentation functions,[1,2,8-10] e.g., for a specific quark (q_i),

$$D_{q_i\to h}(n;Y)=e^{A^n_{qq}(Y-Y_0)}D_{q_i\to h}(n;Y_0)+\frac{D_{G\to h}(n;Y_0)}{2f(\sigma^n_+-\sigma^n_-)}(e^{\lambda^n_+(Y-Y_0)}-e^{\lambda^n_-(Y-Y_0)})$$
$$+\frac{D_{S\to h}(n;Y_0)}{2f(\sigma^n_+-\sigma^n_-)}[\sigma^n_+(e^{\lambda^n_-(Y-Y_0)}-e^{A^n_{qq}(Y-Y_0)})+\sigma^n_-(e^{A^n_{qq}(Y-Y_0)}-e^{\lambda^n_+(Y-Y_0)})]\,. \tag{14}$$

The fragmentation functions in x space can be obtained by moment inversion.[8,10,11] Note that Eq. (14) becomes considerably simpler at large Y. For example, if n is large (i.e., $x\to 1$), then $\lambda^n_+=A^n_{qq}>\lambda^n_-$ (see Figs. 6 and 7) and at large Y the terms involving $\exp[\lambda^n_-(y-Y_0)]$ do not contribute. This leads to the $x\to 1$ limits of q, G, NS, and S fragmentation functions discussed in Refs. 1, 5, and 12.

B. Multihadron fragmentation functions

The solution of the evolution equations (3) and (4) for the two-hadron and multihadron fragmentation functions can also be obtained by taking moments. In general, the moments of an N-particle fragmentation function are defined by[5]

$$D_{i\to h_1,\ldots,h_N}(n_1,\ldots,n_N;Y)=\prod_{k=1}^{N}\int_0^1 dx_k\, x_k^{\,n_k}\,\theta\Big(1-\sum_1^N x_j\Big)D_{i\to h_1,\ldots,h_N}(x_1,\ldots,x_N;Y)\,. \tag{15}$$

Before proceeding to the solution for the multihadron fragmentation functions, we first examine the two-hadron functions. We take moments of Eq. (3), using $N=2$ in Eq. (15), to obtain a differential equation for the double moments,

$$\frac{d}{dY}D_{i\to h_1h_2}(n_1,n_2;Y)=(A^{n_1+n_2})_{ji}D_{j\to h_1h_2}(n_1,n_2;Y)+P^{b_1b_2,i}_{n_1n_2}D_{b_1\to h_1}(n_1;Y)D_{b_2\to h_2}(n_2;Y)\,, \tag{16}$$

where[5]

$$P^{b_1b_2,i}_{n_1n_2}\equiv\int dz\, z^{n_1}(1-z)^{n_2}\hat{P}_{i\to b_1b_2}(z) \tag{17}$$

are known functions (see Table III in Ref. 5). These double-moment equations (16) are a set of first-order, linear-differential equations. Hence, the moments of the two-hadron fragmentation functions are

$$D_{i\to h_1h_2}(n_1,n_2;Y)=\int_{Y_0}^{Y}dy[e^{(Y-y)A^{n_1+n_2}}]_{ji}P^{b_1b_2,j}_{n_1n_2}D_{b_1\to h_1}(n_1;y)D_{b_2\to h_2}(n_2;y)$$
$$+[e^{(Y-Y_0)A^{n_1+n_2}}]_{ji}D_{j\to h_1h_2}(n_1,n_2;Y_0)\,. \tag{18}$$

Note that Eq. (18) provides explicit expressions for quark and gluon fragmentation into two hadrons, since the matrix $\exp[(Y-Y_0)A^{n_1+n_2}]$ is given by Eqs. (12) and $D_{b\to h}(n,y)$ by Eq. (11).

Proceeding in a similar manner we can solve Eqs. (4) in the general case for the N-hadron fragmentation functions. The result (after some manipulations) is

$$D_{i\to h_1,\ldots,h_N}(n_1,\ldots,n_N;Y)=\int_{Y_0}^{Y}dy[e^{(Y-y)A^{n_1+\cdots+n_N}}]_{ji}$$
$$\times\sum_{M=M_{\min}}^{N-1}P^{b_1b_2,j}_{n_Ln_U}D_{b_1\to h_1,\ldots,h_M}(n_1,\ldots,n_M;y)D_{b_2\to h_{M+1},\ldots,h_N}(n_{M+1},\ldots,n_N;y)$$
$$+[e^{(Y-Y_0)A^{n_1+\cdots+n_N}}]_{ji}D_{j\to h_1,\ldots,h_N}(n_1,\ldots,n_N;Y_0)\,, \tag{19}$$

where

$$n_L=n_1+n_2+\cdots+n_M,\quad n_U=n_{M+1}+\cdots+n_N\,. \tag{20}$$

This notation is obviously suggested by Fig. 5, and Eq. (19) can be readily checked by the reader for $N=3$.

In Eq. (19) we have presented the solutions for N-particle fragmentation functions by a straightforward generalization of the single-particle Q^2-evolution equations. This procedure, in our opinion, is considerably simpler than using the machinery of the jet calculus.[5]

IV. CONNECTION BETWEEN THE EVOLUTION EQUATIONS AND A CASCADE MODEL

The current view of jet formation is that one starts with a jet-initiating parton i which has a

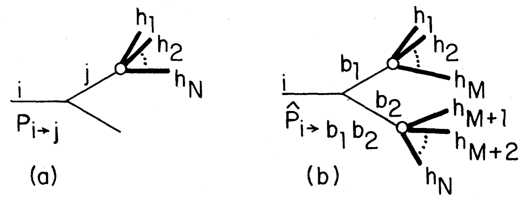

FIG. 5. Pictorial representation of a typical contribution to the N-hadron fragmentation function of a parton i.

large momentum p and is far from its mass shell (large Y or Q^2). Initially, perturbative QCD is applicable and the parton branches repeatedly via the vertices $P_{ji}(x)$. This produces many partons of lower momentum and less far off the energy shell (lower Y). Eventually, a point $Y=Y_0$ is reached[13] where one enters the nonperturbative stage of jet evolution and the partons convert into observable hadrons. This last stage, which is not understood from a fundamental viewpoint, can be well-described phenomenologically by a recursive cascade model based on repeated branching (or breakups) quark → meson + quark, gluon → meson + gluon, and gluon → quark + antiquark.[4]

Since the perturbative QCD stage of jet evolution is a branching process, it must be possible to describe the process in a cascade-model framework. This connection can be made manifest by integrating the evolution equations for single-hadron fragmentation functions [Eq. (1)] for $Y \geqslant Y_0$. Thus,

$$D_{i\to h}(x;Y)=D_{i\to h}(x;Y_0)+\int_{Y_0}^{Y}dy\int_x^1\frac{dx'}{x'}\mathcal{P}_{ji}(x',y)D_{j\to h}\left(\frac{x}{x'};y\right), \tag{21}$$

where

$$\mathcal{P}_{ji}(x,y)=P_{ji}(x). \tag{22}$$

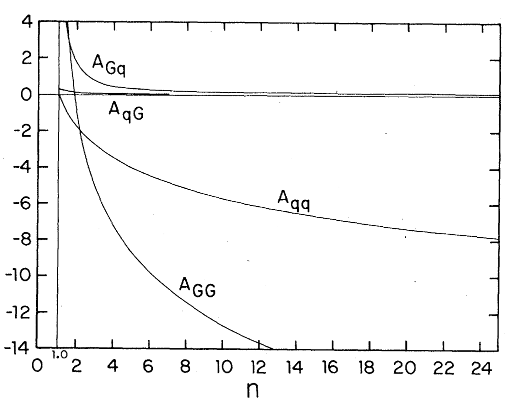

FIG. 6. The n dependence of the QCD anomalous dimensions A_{qq}, A_{qG}, A_{Gq}, A_{GG}.

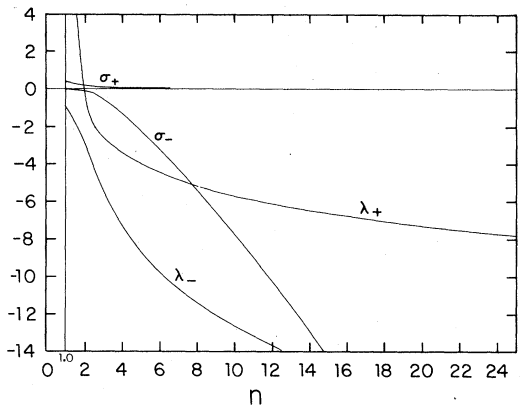

FIG. 7. The n dependence of the quantities $\sigma_\pm$ and the eigenvalues $\lambda_\pm$ defined in the text.

The second term in Eq. (21) shows that the perturbative QCD stage of jet evolution is a cascade process in which *both the momentum and the amount off the energy shell* are reduced at each breakup, since $\mathcal{P}_{ji}(x,y)$ may be identified as the probability for $i(p,Y)\to j(xp,y)$. For $Y<Y_0$, one can use the standard cascade model in which only the momentum is reduced in each breakup (via the momentum-sharing function).[4] Thus, one has a unified way of describing jet formation in both the perturbative and nonperturbative regimes of jet formation. This should be no surprise, if jet evolution is indeed a branching process.

Note that in the leading-logarithm approximation, the vertex $\mathcal{P}_{ji}(x,y)$ is particularly simple since it depends only on the variable x [see Eq. (22)]. Going *beyond* this approximation will in general produce a y dependence.[14] However, if the branching characteristics of jet formation remain true, then integral Eq. (21) will still be valid, and can be used for determining jet properties.

Finally, let us note that it is possible to solve Eq. (21) by successive iteration. This procedure is convergent, and often convenient, since the problem of taking and inverting moments is avoided. The solution is

$$D_{i\to h}(x;Y)=\sum_{k=1}^{\infty}D_{i\to h}^{(k)}(x;Y),$$

where

$$D_{i\to h}^{(1)}(x;Y)=D_{i\to h}(x;Y_0),$$
$$D_{i\to h}^{(k)}(x;Y)=\int_{Y_0}^{Y}dy\int_x^1\frac{dx'}{x'}\mathcal{P}_{ji}(x',y)D_{j\to h}^{(k-1)}\left(\frac{x}{x'};y\right) \quad (k=2,3,\ldots). \tag{23}$$

This method is quite suitable for numerical work since the multiple integrations involved can be well handled by Monte Carlo techniques.[15]

V. SUMMARY AND DISCUSSION

In this paper we have presented the Q^2-evolution equations for the fragmentation functions describing parton (quark or gluon) fragmentation into N hadrons. These equations were solved in terms of multiple moments of the N-hadron fragmentation functions. Such functions are, in fact, measurable quantities which can serve to characterize properties of jets and complement the information available from the usual (single-hadron) fragmentation functions. Data presently in existence do not have sufficient statistical significance or range in Q^2 to test whether or not the multihadron fragmentation functions evolve according to our QCD predictions; it is likely that future data may.[16] Nevertheless, our new Q^2 dependence, e.g., for two-particle fragmentation functions, is of immediate theoretical interest. We note the relevance to the Q^2 variation displayed by Jones *et al.*[7] in their calculation of $D_{q\to h}$ in a model in which partons produced in the perturbative QCD stage of jet evolution are recombined at Q_0^2. In such a scheme, the Q^2 evolution of $D_{q\to h}$ is quite different from Eq. (14), since it is governed by the two-parton distributions [Eq. (18)] with the subscript h_j interpreted as discussed in Sec. II.

We have also demonstrated that the Q^2 dependence of the multihadron fragmentation functions could be readily understood in terms of a recursive cascade model for jet formation. Each step in the branching process degrades both momentum and the amount by which a parton is off its energy shell as measured by Q^2. Consequently, iteration of these cascade-type equations is suggested as a practical alternative to moment inversion.

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