Sketch of a composite quark-lepton model based merely on SU(2) symmetry

H. P. Dürr and H. Saller
Max-Planck-Institut für Physik und Astrophysik, Munich, Federal Republic of Germany
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The U(1,loc) × SU(2,loc) × G(loc)$_{\text{strong}}$-invariant effective Lagrangian of quarks and leptons with electroweak and strong gauge fields can be reduced to an SU(2,loc)-invariant Lagrangian involving only one basic noncanonical SU(2)-doublet Weyl-type spinor field. A condensation of fermion pairs resulting from a self-interaction of the spinor fields leads to a spontaneous breakdown of dilatation and SU(2) symmetry and causes different local dressings of the basic field which can induce the larger variety of phenomenological fields. Hypercharge U(1,loc) reflects the SU(2,loc) (third component) property of the dressing. The strong-interaction gauge group G(loc)$_{\text{strong}}$ = SO(3,loc)—usually taken as SU(3,loc)$_{\text{color}}$—and the flavor group SU(2,loc) are interpreted to arise from the same basic SU(2,loc) similar to the manner in which orbital and spin rotations relate to the same rotation group.

With the ever-increasing proliferation of quarks and leptons there is at present a certain readiness to search for unification schemes where the number of basic dynamical degrees of freedom is drastically reduced.[1] The original flavor-triplet scheme [SU(3)$_{\text{flavor}}$] merged into a fundamental electroweak SU(2)-doublet scheme with different "generations."[2] Strong interactions are generally interpreted as a consequence of a color-SU(3, loc)-invariant gauge-type interaction.[3] To unify hadron physics with lepton physics at least a fourfold property with regard to strong interaction is required to distinguish between the three colored quarks participating in the strong-color interaction and the colorless leptons. Phenomenologically, there seems to exist a correlation between the color and hypercharge properties: The colorless leptons have half-integer and the colored quarks sixths-integer hypercharge. (Hypercharge is here defined as the average charge of an isospin multiplet; i.e., $Q = I_3 + Y$. This differs by a factor $\frac{1}{2}$ from the more common definition.) Furthermore, the fermions can be grouped into generations for which the hypercharge Y fulfills the mysterious rule $\sum Y = 0$ for the fermions in each generation.[2] Hypercharge, on the other hand, has a strong affinity to the flavor properties because it conspires with the third component of isospin to give rise to the mixed combinations of neutral weak and electromagnetic interaction.[4] All this strongly suggests that color and flavor are not really as uncorrelated as commonly assumed. In the present letter we will propose that color and flavor actually arise from one and the same fundamental SU(2, loc) symmetry group. Their effective distinction is a consequence of a structural symmetry,[5] analogous to the situation in atoms where a separate validity of an SO(3) = SU(2)/Z_2 symmetry group connected with orbital angular momentum and the SU(2) symmetry group connected with spin angular momentum results in the limit of very weak spin-orbit coupling. In our proposal color properties will relate, loosely speaking, to orbital SU(2) and flavor properties (isospin I, hypercharge Y, charge Q) to intrinsic SU(2).

The fundamental SU(2) symmetry group is assumed to be spontaneously broken which forces all asymptotic states (particles) to occur as singlets.[6] For the "intrinsic properties" (flavors), this leads to the "freezing mechanism," well known as the appropriate interpretation[6-12] of the spontaneous symmetry breakdown producing all aspects of the usual spontaneously broken U(1) × SU(2) models.[4,11,12] For the "orbital properties" (colors), however, the singlet condition seems to be established by a special combination with similar consequences as the usual SU(3)$_{\text{color}}$-singlet condition if, in addition, certain uniqueness conditions implying integer charge are imposed on the physical states.

As in our last Letter concerning the electroweak interactions[11] we outline in the present paper essentially only the formal decomposition of the effective local fields into a much smaller number of fundamental fields, a procedure which will be called *deflation of dynamical degrees of freedom*. The inverse procedure, the *inflation of dynamical degrees of freedom* referring to a dynamical deduction of the phenomenologically established local fields and their particular properties as effective local compounds of the fundamental fields, of course, constitutes an extremely complicated problem. For the electroweak interaction, first attempts on this have been initiated.[12] For the present case the inflation procedure consists at present in major parts only of a program which still leaves many open questions and hence ultimately may also fail. Except for a few preliminary sketches at the end of this paper, it has to be deferred to future investigations.

To simplify matters we suppress in the present note modifications and generalizations to account for the generations and parity, features which have been treated to some extent in earlier papers[8-12] and probably could be incorporated in a similar fashion. Therefore, we start with a simplified version of a unified phenomenological Lagrangian based on three (in usual terminology "colored") left-handed quark isodoublets $Q(x)$ and a left-handed lepton isodoublet $L(x)$ with the former interacting strongly via (color) gauge fields $\hat{G}_\mu(x)$ and all electroweakly via (flavor) gauge fields $\hat{A}_\mu^{\text{eff}}(x)$, $B_\mu^{\text{eff}}(x)$ in a $U(1)_q \times U(1)_l \times G(\text{loc})_{\text{strong}} \times SU(2,\text{loc})_I \times U(1,\text{loc})_Y$-invariant fashion:

$$\mathcal{L}^{(1)} = Q^*\overline{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{G}_\mu - \hat{A}_\mu^{\text{eff}} + \tfrac{1}{6}B_\mu^{\text{eff}})Q + L^*\overline{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{A}_\mu^{\text{eff}} - \tfrac{1}{2}B_\mu^{\text{eff}})L - \frac{1}{2g_s^2}\text{Tr}(\hat{G}_{\mu\nu})^2 - \frac{1}{2g^2}\text{Tr}(\hat{A}_{\mu\nu}^{\text{eff}})^2 - \frac{1}{4g'^2}(B_{\mu\nu}^{\text{eff}})^2, \tag{1}$$

$$\begin{aligned}\hat{G}_{\mu\nu} &= \partial_\nu\hat{G}_\mu - \partial_\mu\hat{G}_\nu - i[\hat{G}_\mu, \hat{G}_\nu],\\ \hat{A}_{\mu\nu}^{\text{eff}} &= \partial_\nu\hat{A}_\mu^{\text{eff}} - \partial_\mu\hat{A}_\nu^{\text{eff}} - i[\hat{A}_\mu^{\text{eff}}, \hat{A}_\nu^{\text{eff}}],\\ B_{\mu\nu}^{\text{eff}} &= \partial_\nu B_\mu^{\text{eff}} - \partial_\mu B_\nu^{\text{eff}},\end{aligned} \tag{2}$$

where the fields transform as

$$\begin{aligned}Q(x) &\to \exp[-i\delta_q - i\hat{\gamma}(x) - i\hat{\beta}(x) + \tfrac{1}{6}i\alpha(x)]Q,\\ L(x) &\to \exp[-i\delta_l - i\hat{\beta}(x) - \tfrac{1}{2}i\alpha(x)]L(x),\end{aligned} \tag{3}$$

$$\begin{aligned}\hat{G}_\mu(x) &\overset{\text{inf}}{\Rightarrow} \hat{G}_\mu(x) - i[\hat{\gamma}(x), \hat{G}_\mu(x)] + \partial_\mu\hat{\gamma}(x),\\ \hat{A}_\mu^{\text{eff}}(x) &\overset{\text{inf}}{\Rightarrow} \hat{A}_\mu^{\text{eff}}(x) - i[\hat{\beta}(x), \hat{A}_\mu(x)] + \partial_\mu\hat{\beta}(x),\\ B_\mu^{\text{eff}}(x) &\to B_\mu^{\text{eff}}(x) + \partial_\mu\alpha(x),\end{aligned} \tag{4}$$

$$\hat{\beta} = \frac{\tau_i}{2}\beta_i, \quad \hat{A}_\mu^{\text{eff}}(x) = \frac{\tau_i}{2}A_{i\mu}^{\text{eff}}(x), \quad \hat{A}_{\mu\nu}^{\text{eff}}(x) = \frac{\tau_i}{2}A_{i\mu\nu}^{\text{eff}}(x) \quad (i = 1, 2, 3). \tag{5}$$

Usually $G(\text{loc})_{\text{strong}}$ is chosen to be the $SU(3,\text{loc})_{\text{color}}$ in which case the G_μ refer to the eight gluon fields

$$\hat{\gamma} = \frac{\lambda_i}{2}\gamma_i, \quad \hat{G}_\mu(x) = \frac{\lambda_i}{2}G_{i\mu}(x), \quad \hat{G}_{\mu\nu}(x) = \frac{\lambda_i}{2}G_{i\mu\nu}(x) \quad (i = 1, \ldots, 8)\ [SU(3)\text{ case}] \tag{6}$$

with λ_i the eight 3×3 Gell-Mann matrices.

We imagine now the Lagrangian (1) to be augmented by appropriate terms—conventionally constructed with the scalar Higgs field[4]—such that the global $SU(2)_I \times U(1)_Y$ flavor symmetry gets spontaneously broken including $U(1)_Y$. As shown earlier,[11,12] this allows the elimination of the hypercharge degree of freedom by shielding this property with a unitary dressing operator $w(x)$ (constructed from the Goldstone fields which arise in consequence of the asymmetric ground state). Since $w(x)$ varies as

$$w(x) \to \exp[\tfrac{1}{2}i\alpha(x) + \tfrac{1}{2}i\kappa(x)]w(x), \tag{7}$$

$$\kappa(x) = [\beta_1(x)\cos\varphi + \beta_2(x)\sin\varphi]\tan\frac{\theta}{2} + \beta_3(x), \tag{8}$$

the hypercharge variation is compensated at the price of a nonlinear phase variation under isospin transformations involving the SU(2) Goldstone fields $\theta_\pm(x) = \theta(x)e^{\mp i\varphi(x)}$. For the redressed fields

$$\begin{aligned}q(x) &= w^{*1/3}(x)Q(x)\\ &\to \exp[-i\delta_q - i\hat{\gamma}(x) - i\hat{\beta}(x) - \tfrac{1}{6}i\kappa(x)]q(x),\\ l(x) &= w(x)L(x)\\ &\to \exp[-i\delta_l - i\hat{\beta}(x) + \tfrac{1}{2}i\kappa(x)]l(x),\\ C_\mu^{\text{eff}}(x) &= -B_\mu^{\text{eff}}(x) - iw^*\overleftrightarrow{\partial}_\mu w(x) \to C_\mu^{\text{eff}}(x) + \partial_\mu\kappa(x),\end{aligned} \tag{9}$$

which transform nonlinearly under the isospin group, we obtain the $U(1)_Y$ deflated Lagrangian

$$\mathcal{L}^{(1)} = q^*\overline{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{G}_\mu - \hat{A}_\mu^{\text{eff}} - \tfrac{1}{6}C_\mu^{\text{eff}})q + l^*\overline{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{A}_\mu^{\text{eff}} + \tfrac{1}{2}C_\mu^{\text{eff}})l - \frac{1}{2g_s^2}\text{Tr}(\hat{G}_{\mu\nu})^2 - \frac{1}{2g^2}\text{Tr}(\hat{A}_{\mu\nu}^{\text{eff}})^2 - \frac{1}{4g'^2}(C_{\mu\nu}^{\text{eff}})^2. \tag{10}$$

It will be our goal now to reconstruct this Lagrangian involving the fields q, l, $\hat{G}_\mu$, $\hat{A}_\mu^{\text{eff}}$, C_μ^{eff} solely in terms of one basic left-handed $SU(2)_I$-doublet $\chi(x)$ (2×2 complex components) and one $SU(2)_I$-triplet gauge field $\vec{A}_\mu(x)$ with the transformation properties (fermion number $F = \frac{1}{2}$)

$$\begin{aligned}\chi(x) &\to \exp[-\tfrac{1}{2}i\delta - i\hat{\beta}(x)]\chi(x),\\ \hat{A}_\mu(x) &= \frac{\vec{\tau}}{2}\cdot\vec{A}_\mu(x) \overset{\text{inf}}{\Rightarrow} \hat{A}_\mu(x) - i[\hat{\beta}(x), \hat{A}_\mu(x)] + \partial_\mu\hat{\beta}(x).\end{aligned} \tag{11}$$

The basic field χ is assumed to interact with itself in such a way as to produce a pair condensation of its SU(2) down components reflected by the asymmetric ground-state condition[11,12]

$$\langle\Omega|\chi_2^T(x)c_\sigma\chi_2(x)|\Omega\rangle \neq 0, \quad \chi = \begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix}, \quad c_\sigma = i\sigma_2, \tag{12}$$

which effects a breakdown of $SU(2)_I \times U(1)_F$ (F = fermion number) up to a $U(1)_{F_1}$ (F_1 = number of up components). The fermion-pair condensation (12) allows[11,12] the construction of unitary I_1, I_2, F_2 dressing operators $s(x)$ and $u(x)$ involving the corresponding Goldstone fields

$$s(x) \to \exp[\tfrac{1}{2}i\delta - i\hat{\beta}(x)]s(x)\,, \quad u(x) \to \exp[-\tfrac{1}{2}i\delta + \tfrac{1}{2}i\kappa(x)]u(x)\,, \tag{13}$$

where $\kappa(x)$ is constructed like (7), however, now with the explicit stipulation that the Goldstone fields $\theta_\pm(x)$ involved are produced by the fermion-pair condensate (12) and *not* by any other mechanism as, e.g., by the conventional Higgs-field condensate which hence will be unnecessary. The dressing operator $u(x)$ can be used to transmute the cumbersome (nonlinear) SU(2) transformation property into a Goldstone-field deflated global fermion-number property. With the $u(x)$-redressed fields

$$\Psi(x) = u^{1/3}(x)q(x) \to \exp[-i\delta_q - \tfrac{1}{6}i\delta - i\hat{\gamma}(x) - i\hat{\beta}(x)]\Psi(x)\,, \quad \psi(x) = u^*(x)l(x) \to \exp[-i\delta_l + \tfrac{1}{2}i\delta - i\hat{\beta}(x)]\psi(x)\,, \tag{14}$$

the Lagrangian (10) obtains the form

$$\mathfrak{L}^{(1)} = \Psi^*\bar{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{G}_\mu - \hat{A}^{\rm eff}_\mu)\Psi + \psi^*\bar{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{A}^{\rm eff}_\mu)\psi - \frac{1}{2g_s^2}\,{\rm Tr}(\hat{G}_{\mu\nu})^2 - \frac{1}{2g^2}\,{\rm Tr}(\hat{A}^{\rm eff}_{\mu\nu})^2 - \frac{1}{4g'^2}\,(C^{\rm eff}_{\mu\nu})^2 - (C^{\rm eff}_\mu - U_\mu)(\tfrac{1}{6}\Psi^*\bar{\sigma}^\mu\Psi - \tfrac{1}{2}\psi^*\bar{\sigma}^\mu\psi)\,, \tag{15}$$

$$U_\mu(x) = iu\overleftrightarrow{\partial}_\mu u^*(x) \to U_\mu(x) + \partial_\mu\kappa(x)\,. \tag{16}$$

Also the gauge fields $\vec{A}^{\rm eff}_\mu, C^{\rm eff}_\mu$ will be rearranged with the dressing operators $u(x)$, $s(x)$ and their derivatives (16) and

$$\vec{B}_\mu(x) = is^*\vec{\tau}\overleftrightarrow{\partial}_\mu s(x) \to [1 + \vec{\beta}(x)x]\vec{B}_\mu(x) + \partial_\mu\vec{\beta}(x) \tag{17}$$

to reduce to the basic gauge field $\vec{A}_\mu(x)$ ($\beta^\perp, \beta''$ = rearrangement parameters)

$$\vec{A}^{\rm eff}_\mu = \vec{A}_\mu - \beta^\perp(\vec{K}_\mu - \vec{S}K_\mu)\,, \quad C^{\rm eff}_\mu = U_\mu - \beta''K_\mu\,. \tag{18}$$

Here the gauge-invariant operators

$$\vec{K}_\mu(x) = \vec{B}_\mu(x) - \vec{A}_\mu(x) \overset{\rm inf}{\to} [1 + \vec{\beta}(x)x]\vec{K}_\mu(x)\,, \quad K_\mu(x) = \vec{S}(x)\vec{K}_\mu(x) \to K_\mu(x), \quad \vec{S}(x) = s^*\vec{\tau}s(x) \tag{19}$$

were used. The Lagrangian (15) then can be written as

$$\mathfrak{L}^{(1)} = \mathfrak{L}^{(2)} + \mathfrak{L}^{(21)} \tag{20}$$

with the gauge-invariant kinetic terms

$$\mathfrak{L}^{(2)} = \Psi^*\bar{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{G}_\mu - \hat{A}_\mu)\Psi + \psi^*\bar{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \hat{A}_\mu)\psi - \frac{1}{2g_s^2}\,{\rm Tr}(\hat{G}_{\mu\nu})^2 - \frac{1}{2g_w^2}\,{\rm Tr}(\hat{A}_{\mu\nu})^2, \tag{21}$$

and a part involving the dressing operators

$$\mathfrak{L}^{(21)} = \beta''K_\mu[\tfrac{1}{6}\Psi^*\bar{\sigma}^\mu\Psi - \tfrac{1}{2}\psi^*\bar{\sigma}^\mu\psi] + \beta^\perp(\vec{K}_\mu - \vec{S}K_\mu)\left[\Psi^*\bar{\sigma}^\mu\frac{\vec{\tau}}{2}\Psi + \psi^*\bar{\sigma}^\mu\frac{\vec{\tau}}{2}\psi\right] - \frac{\beta^\perp(2+\beta^\perp)}{4g_w^2}[\vec{S}(\vec{K}_\mu \times \vec{K}_\nu)]^2 - \frac{N}{4}(\partial_\nu K_\mu - \partial_\mu K_\nu)^2\,. \tag{22}$$

The constants g_w, N are related to β'', $\beta^\perp$ and g, g' as

$$g_w^2 = \frac{g^2}{(1+\beta^\perp)^2}\,, \quad N = \frac{\beta''^2}{g'^2} - \frac{\beta^\perp(2+\beta^\perp)}{g^2}\,. \tag{23}$$

The $\mathfrak{L}^{(21)}$ (22) should be considered to be exactly canceled by the rearrangement of the self-interaction term of the basic spinor field $\chi(x)$ in consequence of the broken symmetry as shown earlier[11,12] and essentially determine β'', $\beta^\perp$, and g_w^2. Derivatives of $\vec{A}_\mu$ should only arise in the curvature form $\vec{A}_{\mu\nu}$ which requires $N=0$. Hence, g and g' will be given as

$$g^2/g_w^2 = (1+\beta^\perp)^2, \quad g'^2/g^2 = \frac{\beta''^2}{\beta^\perp(2+\beta^\perp)} = \tan^2\varphi_w\,. \tag{24}$$

Up to now we have proceeded very much along the lines of the last Letter[11] deflating Weinberg's model. Now we make a first dramatic step: We assume the triplet Ψ, usually taken as an $SU(3)_{\rm color}$ triplet, to be an $SU(2)/Z_2 = SO(3)$ triplet of the *same* basic SU(2, loc), i.e., we consider the 3×3 matrix $\hat{\gamma}$ in transformations (14) and earlier to depend only on the three isospin Lie parameters $\vec{\beta}$

$$\hat{\gamma}(x) = \vec{t}\cdot\vec{\beta}(x) = \begin{pmatrix} 0 & -i\beta_3 & i\beta_2 \\ i\beta_3 & 0 & -i\beta_1 \\ -i\beta_2 & i\beta_1 & 0 \end{pmatrix}, \quad (t_j)_{AB} = i\epsilon_{AjB} \ [\text{SO(3) case}]\,, \tag{25}$$

which constitute the purely imaginary SU(3) λ matrices.

The color group in this reduced form looks like an orbital part of isospin. There are only three gluon fields which are identical to the basic $\vec{A}_\mu$:

$$\hat{G}_\mu(x) = \vec{t}\cdot\vec{A}_\mu(x), \quad \hat{G}_{\mu\nu}(x) = \vec{t}\cdot\vec{A}_{\mu\nu}(x)\,. \tag{26}$$

This deflates the Lagrangian (21) to

$$\mathfrak{L}^{(2)} = \Psi^*\bar{\sigma}^\mu[\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - (\vec{t} + \tfrac{1}{2}\vec{\tau})\cdot\vec{A}_\mu]\Psi + \psi^*\bar{\sigma}^\mu[\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu]\psi - \frac{1}{4g_0^2}(\vec{A}_{\mu\nu})^2, \tag{27}$$

$$\frac{1}{g_s^2} = \frac{1}{g_0^2} - \frac{1}{g_w^2}\,, \tag{28}$$

i.e., with g_0, g_w, $\beta^{\perp}$, β'' given the effective gauge coupling constants g_s (color), g and g' (flavor) can be in principle derived.

Relating the color gauge field to the same basic isospin gauge field to which the flavor gauge fields were deflated requires that the color index $A = 1, 2, 3$ of the $\Psi_{A\alpha}$ triplet connects to the same isospin degrees of freedom which are labeled by the other index $\alpha = 1, 2$. To make this connection manifest we assume in a second dramatic step that the fermion fields Ψ (quarks) and ψ (lepton) are *local compounds* of the fundamental spinor field χ. By definition, χ is an anticommuting operator quantized with anticommutator conditions. Pauli's principle allows maximally four χ's and four χ^*'s at each space-time point x. Composite fields with half-integer spins can be locally formed (besides the trivial case with one) with three, five, or seven χ, χ^*'s. By reasons to be offered below we concentrate on the possible three-field compounds

$$\begin{aligned} &\chi\chi\chi, \chi^*\chi^*\chi^* \text{ spin, isospin} = (\tfrac{1}{2}, \tfrac{1}{2}), \\ &\chi\chi^*\chi^*, \chi^*\chi\chi \text{ spin, isospin} = (\tfrac{1}{2}, \tfrac{1}{2}), (\tfrac{1}{2}, \tfrac{3}{2}); (\tfrac{3}{2}, \tfrac{1}{2}), \end{aligned} \tag{29}$$

and here only on the spin-$\frac{1}{2}$ combinations. The compound $\chi^3(x)$ [and the antifield $\chi^{*3}(x)$] leads to an $SU(2)_l$ doublet which will be used to construct ψ (with n_l some normalization constant, l = lepton):

$$\psi(x) = n_l \vec{\tau}\vec{\Delta}(x)\chi(x), \tag{30}$$

$$\begin{aligned} &\vec{\Delta}(x) = \tilde{\chi}^*\vec{\tau}\chi(x) \rightarrow \exp[-i\delta - i\gamma(x)]\vec{\Delta}(x), \\ &\vec{\Delta}^*(x) = \chi^*\vec{\tau}\tilde{\chi}(x) \rightarrow \vec{\Delta}^*(x)\exp[i\delta + i\gamma(x)], \end{aligned} \tag{31}$$

$$\tilde{\chi}(x) = c_\sigma c_\tau \chi^{*T}(x), \quad c_\sigma = -i\sigma_2, \quad c_\tau = -i\tau_2, \tag{32}$$

where T indicates transposition. The fermion number F (related to δ) has to be identified with half the fermion number F_l (related to δ_l):

$$F = \tfrac{1}{2} F_l \tag{33}$$

to secure the same variation $\delta = \delta_l/2$. Hence, one obtains from (14) for the variation of the lepton field under the basic $U(1)_F \times SU(2, \text{loc})_l$ group

$$\psi(x) \rightarrow \exp[-\tfrac{3}{2}i\delta - i\beta(x)]\psi(x). \tag{34}$$

The antisymmetry of three spinor operators of the same sort forces the isospin vector $\vec{\Delta}$ in (30) to antialign its isospin (and spin) to χ which simulates a strong "spin-orbit coupling." This situation is quite different for the remaining 2×3 spin-$\frac{1}{2}$ fields $\chi\chi^*\chi^*$ (and their antifields $\chi^*\chi\chi$) in (29) because here the dressing is composed of the antifields and therefore is free in its alignment. Hence, there is no spin-orbit coupling arising from the Pauli principle. It is therefore reasonable to identify the isovector-isospinor fields with the Ψ-triplet-doublet (with n_q some normalization constant, q = quarks)

$$\begin{aligned} \Psi(x) &= n_q \vec{\Delta}^*(x)\chi(x) \\ &\rightarrow \exp[\tfrac{1}{2}i\delta - i\hat{\gamma}(x) - i\hat{\beta}(x)]\Psi(x), \end{aligned} \tag{35}$$

where one identified

$$F_q = -\tfrac{2}{3}F = -\tfrac{1}{3}F_l \tag{36}$$

to secure $\delta_q = -\frac{2}{3}\delta$.

With representation (30) and (35) the Lagrangian (27) gets deflated to

$$\begin{aligned} \mathcal{L}^{(2)} = &(n_q{}^2 + n_l{}^2)\vec{\Delta}\cdot\vec{\Delta}^*[\chi^*\bar{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu)\chi] \\ &- n_l{}^2 i\vec{\Delta}\times\vec{\Delta}^*\cdot[\chi^*\bar{\sigma}^\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu\vec{\tau} - \tfrac{1}{2}\mu\vec{A}_\mu)\chi] \\ &+ (n_q{}^2 - n_l{}^2)[\vec{\Delta}\cdot(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - \vec{t}\cdot\vec{A}_\mu)\vec{\Delta}^*](\chi^*\bar{\sigma}^\mu\chi) \\ &+ n_l{}^2[i\vec{\Delta}\times(\tfrac{1}{2}i\overleftrightarrow{\partial} - \vec{t}\cdot\vec{A}_\mu)\vec{\Delta}^*]\cdot(\chi^*\bar{\sigma}^\mu\tfrac{1}{2}\vec{\tau}\chi) - \frac{1}{4g_0{}^2}\vec{A}_{\mu\nu}{}^2 . \end{aligned} \tag{37}$$

Since $\vec{\Delta}$, $\vec{\Delta}^*$ are constructed from χ and χ^*, all $\vec{\Delta}$, $\vec{\Delta}^*$ terms can be expressed in terms of χ, χ^* as

$$\begin{aligned} &\vec{\Delta}\cdot\vec{\Delta}^* = (\chi^*\bar{\sigma}_\nu\chi)(\chi^*\bar{\sigma}^\nu\chi), \\ &-i\vec{\Delta}\times\vec{\Delta}^* = (\chi^*\bar{\sigma}_\nu\vec{\tau}\chi)(\chi^*\bar{\sigma}^\nu\chi), \end{aligned} \tag{38}$$

$$\begin{aligned} \vec{\Delta}\cdot\overleftrightarrow{\partial}_\mu\vec{\Delta}^* &= (\chi^*\bar{\sigma}_\nu\chi)(\chi^*\bar{\sigma}^\nu\overleftrightarrow{\partial}_\mu\chi), \\ -i\vec{\Delta}\times\overleftrightarrow{\partial}_\mu\vec{\Delta}^* &= (\chi^*\bar{\sigma}_\nu\vec{\tau}\chi)(\chi^*\bar{\sigma}^\nu\overleftrightarrow{\partial}_\mu\chi) \\ &\quad + (\chi^*\bar{\sigma}_\nu\chi)(\chi^*\bar{\sigma}^\nu\vec{\tau}\overleftrightarrow{\partial}_\mu\chi) . \end{aligned} \tag{39}$$

The quark and lepton fields Ψ and ψ are treated as canonical fields obeying $\{\psi(x), \psi^*(x')\}_{t=t'} \sim \delta(\vec{x} - \vec{x}')$ which characterizes them as (mass) dimension-$\frac{3}{2}$ fields under dilatation:

$$\begin{aligned} &\Psi(x) \rightarrow \exp(\tfrac{3}{2}\lambda)\Psi(e^{-\lambda}x), \\ &\psi(x) \rightarrow \exp(\tfrac{3}{2}\lambda)\psi(e^{-\lambda}x). \end{aligned} \tag{40}$$

If simple additivity of dimension is valid, the basic field $\chi(x)$ with our constructions (30) and (35) has to be assumed to have dimension $\frac{1}{2}$ (it is what we earlier called a spinor potential[13,14]), i.e., it varies under dilatation as

$$\chi(x) \rightarrow \exp(\tfrac{1}{2}\lambda)\chi(e^{-\lambda}x). \tag{41}$$

The dimension $\frac{1}{2}$ of $\chi(x)$ is reflected by a c-number part in the bilinear bilocal product (with a convenient normalization)

$$\begin{aligned} &\langle\Omega|\chi(x_+)\chi^*(x_-)|\Omega\rangle = \frac{i}{8\pi^2}\frac{\sigma_\mu\xi^\mu}{\xi^2}, \\ &x_\pm = x \pm \xi/2, \quad \xi \rightarrow 0. \end{aligned} \tag{42}$$

In this case $\vec{\Delta}$, $\vec{\Delta}^*$ can be regarded as a combined dimension and SU(2) dressing operator[15]

$$\vec{\Delta}(x) \to \exp(\lambda)\, \vec{\Delta}(e^{-\lambda} x)\ . \tag{43}$$

The symmetry-breaking condition (12) reveals itself as an isospin and dilatation breaking condition

$$\begin{aligned} \langle \Omega | \vec{\Delta}(x) | \Omega \rangle &= M \vec{\delta}_- \neq 0, \\ \langle \Omega | \vec{\Delta}^*(x) | \Omega \rangle &= M \vec{\delta}_+ \neq 0, \end{aligned} \qquad \vec{\delta}_\pm = \frac{1}{2} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}\ , \tag{44}$$

by which simultaneously a direction in isospace and a mass scale M is selected.

The dimension-$\frac{1}{2}$ property of the basic field χ now allows us to make a third dramatic step, namely, to consider even the "basic" gauge fields to be compounds as suggested earlier and to be represented as the tetrad-averaged finite part of the bilinear product[13-16]

$$\begin{aligned} \vec{A}_\mu(x) &= -16\pi^2 \colon\ \chi^* \bar{\sigma}_\mu \vec{\tau} \chi \colon (x)\ , \\ \hat{A}_\mu(x) &\xrightarrow{\text{inf}} \hat{A}_\mu(x) - i[\hat{\beta}(x), \hat{A}_\mu(x)] + \partial_\mu \hat{\beta}(x)\ , \end{aligned} \tag{45}$$

which reflects the second term in the expansion of the bilocal product (42)

$$\begin{aligned} \chi(x_+)\chi^*(x_-) &= \frac{i}{8\pi^2} \frac{\sigma \cdot \xi}{\xi^2} - \sum_{n=0}^{\infty} \frac{1}{n!} \colon\ \chi \left(\frac{\xi}{2} \cdot \overleftarrow{\partial} \right)^n \chi^* \colon (x) \\ &= \frac{i}{8\pi^2} \frac{\sigma \cdot \xi}{\xi^2} [1 - i \xi^\mu \hat{A}_\mu(x) + \dots]\ . \end{aligned} \tag{46}$$

Actually, one may also turn the argument around by postulating the fundamental field $\chi(x)$ according to (45) as the "square root" of the gauge field which then fixes naively its dimension to $\frac{1}{2}$. Canonical fermion fields then have to be constructed as local products of three such fields[13] which would explain the preference of the three constructions (29).

With the insertion of (45) into (37) one obtains a gauge-invariant Lagrangian which only contains χ, χ^* and their first derivatives in a fashion which does not involve any scale parameter. We content ourselves giving its general structure[17]

$$\mathcal{L}^{(2)} = \{ \partial(\chi^*\chi)\partial(\chi^*\chi),\ (\chi^*\chi)^2 \partial(\chi^*\chi),\ (\chi^*\chi)^2 (\chi^* \overleftrightarrow{\partial} \chi),\ (\chi^*\chi)^4 \}\ . \tag{47}$$

This Lagrangian is dilatation invariant and hence formally renormalizable. Because of the anomalous (subcanonical) dimension of χ, the theory requires an indefinite metric in the quantum-mechanical state space[18] which is clearly reflected by (42) and is the price to be paid for constructing gauge fields from spinor fields. The local formulation of gauge theories in any case requires an indefinite metric (scalar modes, Faddeev-Popov ghosts) but the indefinite metric involved here seems to be more severe. The $\chi(x)$ in the simplest case, however, is a dipole ghost field relating only to zero-norm states,[19] similar as the plus and minus combinations of the longitudinal and scalar photons in QED which therefore do not correspond to asymptotic states (particles). This may eventually help to secure unitarity.[19,20]

Digging for an even deeper level the Lagrangian (47) can be further simplified by regarding it as the local tetrad-averaged limit of a bilocal Lagrangian represented by the determinant

$$\mathcal{L}(x) = \overline{\lim_{\xi \to 0}}\ \det[\chi(x_+)\chi^*(x_-)] = \tfrac{1}{24} \overline{\lim_{\xi = 0}}\ [\epsilon^{\alpha\beta\gamma\delta} \chi_\alpha \chi_\beta \chi_\gamma \chi_\delta (x_+) \epsilon_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} \chi^{*\dot{\alpha}} \chi^{*\dot{\beta}} \chi^{*\dot{\gamma}} \chi^{*\dot{\delta}}(x)],\quad x_\pm = x \pm \xi/2\ , \tag{48}$$

where $\alpha, \dots \dot{\alpha}, \dots = 1, 2, 3, 4$ now combines both the $SL(2, C)$ Lorentz and SU(2) isospin index. With dimension-$\frac{1}{2}$ fields χ the Lagrangian (48) is obviously dilatation invariant but also invariant under the large local group $SL(4, C, \text{loc})$. The condition

$$\begin{aligned} \chi(x_+)\chi^*(x_-) &= \frac{i}{8\pi^2} \frac{\sigma \cdot \xi}{\xi^2} \\ &\quad - \frac{1}{4} \sum_{n=0}^{\infty} \frac{(-)^n}{n!} \sigma^\mu \tau^\nu \colon \chi^* \bar{\sigma}_\mu \bar{\tau}_\nu \left(\frac{\xi}{2} \cdot \partial \right)^n \chi \colon (x) \end{aligned} \tag{49}$$

describes the continuum limit. It fixes the dimension and reduces the local group to $SL(2, C) \times SU(2, \text{loc.})$. The continuum limit procedure will essentially lead to a Lagrangian (47) but also involves terms up to three derivatives. Since the product $\chi^*\chi^*$ and $\chi\chi$ have no singular terms for $\xi \to 0$, these operators can be taken at the same space-time point. Their antisymmetry is explicitly taken into account by the ansatz (48). This ansatz appears interesting in many respect—in a certain approximation it corresponds to Heisenberg's nonlinear spinor theory[20]—and will be discussed in more detail elsewhere.[21]
In a certain way it is the spinor counterpart of the pure scale-invariant, renormalizable non-Abelian SU(2, loc) gauge theory which involves A^4 as highest nonderivative terms where the gauge field A of dimension one is replaced by bilinear forms of a spinor field of dimension $\frac{1}{2}$.

The spontaneous breakdown of the global SU(2) symmetry usually effected by a nonlinear Higgs-field Lagrangian involving a scale-invariant term ϕ^4 and a scale-breaking term $M_1^2\phi^2$ is supposed to arise in our case automatically from the nonlinear interaction term with $(\chi^*\bar{\sigma}_\mu\chi)^2$ essentially playing the role of ϕ^2.

It is also interesting to note that $(\chi^*\bar{\sigma}_\mu\chi)(\chi^*\bar{\sigma}_\nu\chi)$ constitutes a symmetric tensor of rank two of dimension two $[(\chi^*\bar{\sigma}_\mu\chi)(\chi^*\bar{\sigma}_\nu\chi)dx^\mu dx^\nu$ is dimensionless] which may be eventually related to the metric tensor and gravitation.[17,22] Also, the occurrence of a local canonical spin-$\frac{3}{2}$ field in (29) may be of importance in this context. This, however, requires more thorough investigations.

The field-deflation scheme is summarized in Fig. 1. It proceeds essentially in four steps. With the number of the step we have indicated the corresponding rearrangement of the Lagrangian, the tools (operator, etc.) which effect this rearrangement and the "properties" one loses in this step. In the first step local hypercharge is eliminated; steps 2a–2c reduce the gauge groups to SU(2, loc); step 3 introduces an indefinite metric to deflate the canonical quark and lepton fields to the basic field; and step 4 finally destroys even the continuous space-time description and the basic gauge fields.

In closing, let us shortly comment on the inverse procedure, the *inflation procedure*. This would mean to start from the fundamental $U(1)_F \times D(1) \times SL(4, C\text{loc})$-invariant Lagrangian (48) involving only a basic complex noncanonical 2×2 component $SU(2)\times SL(2, C)$ isospinor operator[23] occupying each space-time point maximally (χ and χ^* at alternating neighboring points). Bridging the distance between neighboring points by a Laurent expansion (49), with the quantization condition related to the leading c-number singular term, a Lagrangian of type (47) or (37) may evolve where the original SU(2) invariance at each point will give rise to effective SU(2) gauge fields $A_\mu(x)$ with certain normalizations by taking appropriate "plaquette limits."[12]

The canonical rearrangement with the isospin-covariant dilatation dressing operators $\vec{\Delta}(x)$ and $\vec{\Delta}^*(x)$ establishes the canonical spin-$\frac{1}{2}$ fields $\psi(x)$ and $\Psi(x)$ where the lepton doublet $\psi(x)$ is constructed only from strongly correlated $\chi(x)$ of the same point whereas the quark sextet $\Psi(x)$ contains uncorrelated $\chi(x)$ and $\chi^*(x)$ of neighboring points which suggest treating it on a triplet-doublet footing. The nonlinear interaction is supposed to lead to a condensation of $\vec{\Delta}(x)$, $\vec{\Delta}^*(x)$ which violates $D(1)$ and $U(1)_F\times SU(2)_I$ invariance with the latter leading to dressing operators $u(x)$, $s(x)$ constructed from the corresponding Goldstone fields. The gauge-invariant dynamical rearrangement of the nonlinear term[11,12] is supposed to generate terms of the form $\mathcal{L}^{(21)}$ in (22) involving parameters β'' and $\beta^\perp$. In particular,[12] a pure Goldstone-field term $\sim[\vec{S}\cdot(\vec{K}_\mu\times\vec{K}_\nu)]^2$ with parameter γ according to the decomposition

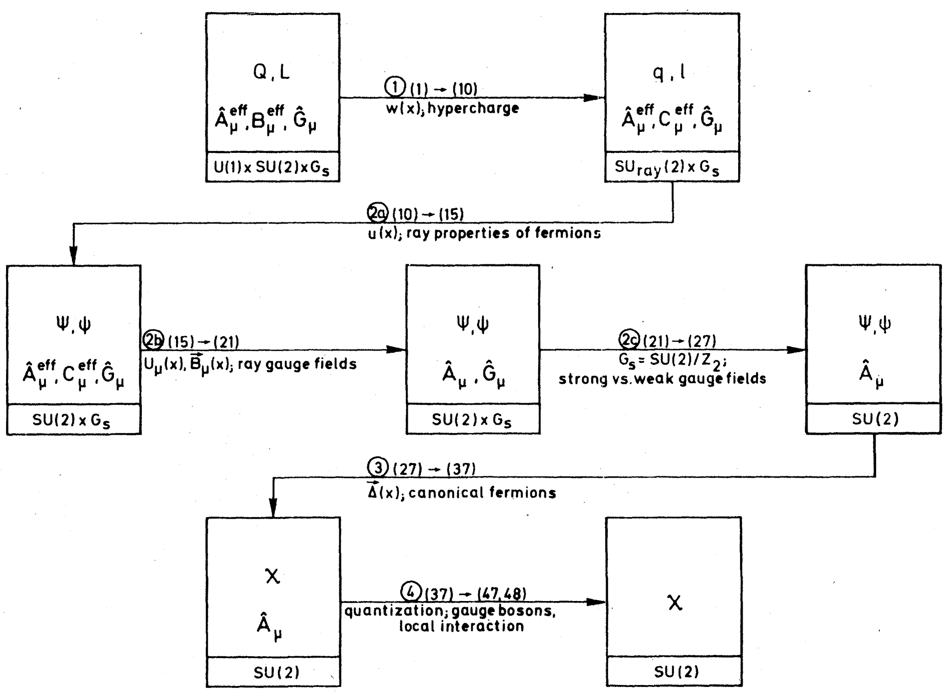

FIG. 1. Deflation scheme.

$$\frac{1}{4g_0^2}\{\vec{A}_{\mu\nu}^2+\gamma^2[\vec{S}\cdot(\vec{K}_\mu\times\vec{K}_\nu)]^2\}$$
$$=\frac{1}{4g_s^2}\,{}^0\vec{A}_{\mu\nu}^2+\frac{1}{4g^2}\vec{A}_{\mu\nu}^2+\frac{1}{4g'^2}C_{\mu\nu}^2 \quad (50)$$

in connection with the extractions, will trigger a proliferation of the gauge fields

$$\vec{A}_\mu\to\{{}^0\vec{A}_\mu,\ \vec{A}_\mu^{\text{eff}},\ C_\mu^{\text{eff}}\}\,, \quad (51)$$

where the parameters g_0, β'', $\beta^\perp$, and γ will determine the coupling constants $g_s\, g$, and g':

$$g_0^2/g_s^2=1-\frac{\gamma^2}{\beta^\perp(2+\beta^\perp)}\,,\quad g_0^2/g^2=\frac{\gamma^2}{(1+\beta^\perp)^2\beta^\perp(2+\beta^\perp)}\,,$$
$$g'^2/g^2=\frac{\beta''^2}{\beta^\perp(2+\beta^\perp)}\,. \quad (52)$$

At this point some argument has to be produced why the "left-over" part ${}^0\vec{A}_\mu$ of the gauge field couples only to the triplet degree freedom in Ψ ("orbital" isospin). This constitutes a first barrier in inflation procedure. If it can be successfully cleared we reach the point where Ψ, ψ, $\hat{A}_\mu^{\text{eff}}$, C_μ^{eff}, $\hat{G}_\mu$ are effectively established.

Now the correct $u(x)$ dressings have to be motivated which effectively convert fermion number into isospin property I_3 and phenomenologically show up as hypercharge. For the leptons the decoupling of the massless neutrino from the massless gauge field (photon) enforces the dressing

$$\Psi(x)\to l(x)=u(x)\psi(x)$$
$$=n_l\vec{\tau}\chi(\tilde{\chi}^*\vec{\tau}\chi)u(x)$$
$$=n_l\vec{\tau}(u^{1/3}\chi)[(u^{1/3}\chi)^T*\vec{\tau}(u^{1/3}\chi)](x)\,, \quad (53)$$

where the superscript T indicates transpose. By this dressing the fermion number $F=\frac{3}{2}$ of $\psi(x)$ is lifted by $\frac{1}{2}$ to $F=2$ [which, according to (33), corresponds to lepton number $F_l=+1$] connected with the generation of a hypercharge $Y=-\frac{1}{2}$ (I_3 property of the dressing). For the quarks we do not have an argument for a particular u dressing but apparently we have to demand

$$\Psi(x)\to q(x)=u^{*1/3}(x)\Psi(x)$$
$$=n_q\chi(\chi^*\vec{\tau}\chi)u^{*1/3}(x)$$
$$=n_q(u^{1/3}\chi)[(u^{1/3}\chi)^*\vec{\tau}(u^{1/3}\chi)^T](x)\,, \quad (54)$$

which in comparison to (53) suggests that $(u^{1/3}\chi)$ acts as an effective building block. The fermion number $F=-\frac{1}{2}$ with this dressing gets decreased by $\frac{1}{6}$ to $F=-\frac{2}{3}$ to generate a hyperchange $Y=+\frac{1}{6}$. With these arguments, therefore, the change shift (hypercharge) would ultimately be determined by the neutrality of the neutrino and the "universality" of the local building blocks. Clearly, the latter requirement is still rather unsatisfactory.

Up to now we have concentrated only on the dynamical rearrangement of the local structure triggered by the fermion condensate of the ground state and have disregarded questions concerning the structure of the asymptotic states which manifest themselves as physical particles. Since SU(2) is broken *effectively*, *only SU(2) singlets* can arise as physical states. For the flavor properties, this is accomplished by the local freezing of the corresponding degrees of freedom or by a subsequent gauge-field dressing.[4,11,12] For the "orbital" SU(2), which was connected to the color property, another mechanism appears to be at work: The singlet condition can be established by constructing isoscalar combinations of the isospin vectors such as

$$\vec{q}^*\cdot\vec{q},\quad \vec{q}\cdot\vec{q},\quad (\vec{q}\times\vec{q})\cdot\vec{q},\quad (\vec{q}\times\vec{q})\cdot\vec{q}^*,\quad \text{etc.}, \quad (55)$$

which according to (54) have the $u(x)$ dressings

$$1,\quad u^{*2/3},\quad u,\quad u^{*1/3},\quad \text{etc.}\,, \quad (56)$$

respectively. One may even turn the argument around by stating that this new type of isospin shielding (55) is the main reason for the phenomenologically separate character of color and flavor.

The possible isoscalar combinations (55) can be further restricted by the observation that $u(x)$ contained in these expressions as (56) has a nontrivial phase-transformation property under global SU(2) transformations which, on the basis of the topological structure of the group, demands certain periodicity conditions, in particular,

$$\frac{\beta_3}{2}\triangleq\frac{\beta_3}{2}+2n\pi\quad n=\text{integer}\,. \quad (57)$$

If one requires the physical states to be unique[24] this would eliminate the second and fourth combination in (55). For physical states, therefore, only the combinations

$$\vec{q}^*\cdot\vec{q},\quad (\vec{q}\times\vec{q})\cdot\vec{q}=\epsilon_{ABC}q_Aq_Bq_C \quad (58)$$

survive. These are exactly the combinations which constitute the physical states also in the conventional quark model. In contrast to the usual quark-confinement condition, however, the selection of singlets in the asymptotic region would be, in our case, a direct consequence of the spontaneous breakdown of the basic SU(2) symmetry.

The interpretation of the color group as an SO(3) rather than an SU(3) does not invalidate the ultraviolet asymptotic-freedom property[25] of such a theory. The relevant function

$$\beta(g_s) = -\frac{g_s^3}{16\pi^2}\left[\tfrac{11}{3} C_2(G) - \tfrac{4}{3} T(R)\right]$$

in this context which for SU(3) with n_F fermion triplets (n_F = number of flavors) yields [C_2(SU(3)) = 3, $T(R) = \frac{1}{2} n_F$] the well-known

$$\beta(g_s) = -\frac{g_s^3}{16\pi^2}\left[11 - \tfrac{2}{3} n_F\right]$$

for the present case of SU(2) [C_2(SU(2)) = 2; $T(R) = 2n_F$] we obtain

$$\beta(g_s) = -\frac{g_s^3}{24\pi^2}\left[11 - 4n_F\right] . \tag{59}$$

Since $\beta(g_s)$ has to be negative to secure asymptotic freedom only maximally two flavors ($n_F \leq 2$) can be admitted which is completely exhausted by our assumption of *a single SU(2) flavor doublet*. As a consequence, the various observed generations cannot be accommodated on the basic level which would be an indication that they also should be interpreted as (soft) proliferations.[8-10]

[1] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); S. L. Glashow, Harvard Report No. HUTP-77/A005; Y. Ne'eman, Phys. Lett. 82B, 69 (1979); H. Harari, *ibid.* 86B, 83 (1979).

[2] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); H. Harari, Phys. Rep. 42C, 235 (1978).

[3] M. Han and Y. Nambu, Phys. Rev. 139B, 1006 (1965); Y. Nambu, *Preludes in Theoretical Physics*, edited by A. deShalit, L. Van Hove, and H. Feshbech (North-Holland, Amsterdam, 1966); H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973); S. Weinberg, Phys. Rev. Lett. 31, 494 (1973).

[4] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).

[5] H. P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki, Z. Naturforsch. 14a, 441 (1959); H. P. Dürr and W. Heisenberg, *ibid.* 16a, 726 (1961); H. P. Dürr, in *Properties of Matter under Unusual Conditions*, edited by H. Mark and S. Fernbach (Interscience, New York, 1969); and in Group Theoretical Methods in Physics, Springer Lecture Notes 79, 259 (1977).

[6] H. Umezawa, Nuovo Cimento 38, 1415 (1965); 40, 450 (1965).

[7] H. P. Dürr and H. Saller, Nuovo Cimento 39A, 31 (1977).

[8] H. P. Dürr and H. Saller, Nuovo Cimento 41A, 677 (1977).

[9] H. P. Dürr and H. Saller, Nuovo Cimento 48A, 505 (1978).

[10] H. P. Dürr and H. Saller, Nuovo Cimento 48A, 561 (1978).

[11] H. P. Dürr and H. Saller, Phys. Lett. 84B, 336 (1979).

[12] H. P. Dürr and H. Saller, Nuovo Cimento 53A, 469 (1979).

[13] I. I. Bigi, H. P. Dürr, and N. J. Winter, Nuovo Cimento 23A, 420 (1974).

[14] H. P. Dürr and N. J. Winter, Nuovo Cimento 70A, 467 (1970).

[15] H. Saller, Nuovo Cimento 34A, 99 (1976).

[16] H. Saller, Nuovo Cimento 24A, 391 (1974).

[17] H. Saller, Nuovo Cimento 42A, 189 (1977).

[18] S. N. Gupta, Proc. Phys. Soc. London A63, 681 (1950); K. Bleuler, Helv. Phys. Acta 23, 567 (1950); W. Heisenberg, Nucl. Phys. 4, 532 (1957); C. C. Chiang and H. P. Dürr, Nuovo Cimento 28A, 89 (1975).

[19] H. P. Dürr and E. Rudolph, Nuovo Cimento 62A, 411 (1969); 65A, 423 (1970); and 10A, 597 (1972); W. Karowski, *ibid.* 23A, 126 (1974); H. P. Dürr and E. Seiler, *ibid.* 66A, 734 (1970).

[20] W. Heisenberg, *Introduction to the Unified Field Theory of Elementary Particles* (Wiley, New York, 1966).

[21] H. P. Dürr and H. Saller, in preparation.

[22] H. P. Dürr, Gen. Relativ. Gravit. 4, 29 (1973); Nuovo Cimento 4A, 187 (1971).

[23] W. Heisenberg, Nachr. Akad. Wiss. Göettingen IIa, 111 (1953).

[24] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950); D. Saint-James, G. Sarma, and E. J. Thomas, *Type II Superconductivity* (Pergamon, New York, 1969).

[25] D. J. Gross, in *Methods in Field Theory*, proceedings of Les Houches Summer School, 1975, edited by R. Balian and J. Zinn-Justin (North-Holland, New York, 1976).