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Heavy Higgs bosons in the Weinberg-Salam model

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We search for a systematic way to characterize the low-energy sensitivity of the minimal Weinberg-Salam model to a heavy-Higgs-boson sector. We find that it is convenient to view this theory as the regulated version of a nonlinear σ model coupled to an $SU(2)_L \times U(1)$ Yang-Mills theory. Within this framework, M_H acts as the regulator. Using the symmetry properties of the nonlinear theory, along with a power-counting analysis, we are able to classify low-energy observables according to their sensitivity to M_H. We find that, at one loop, the greatest sensitivity is logarithmic. An illustration of these ideas is provided by a calculation of the one-loop, M_H-dependent corrections to the natural relation $M_W/M_Z \cos\theta = 1$. Finally, we discuss other possible applications of this technique.

I. INTRODUCTION

During the past decade, the trend in theoretical physics has been to describe all the interactions of elementary particles as manifestations of a single underlying dynamical principle—local gauge invariance. As products of this program, we have a candidate theory of the strong interactions, quantum chromodynamics, and the Weinberg-Salam model,[1] a rather successful unified theory of the weak and electromagnetic interactions [quantum flavor dynamics (QFD)].

Of course, despite any subjective prejudices which we may entertain concerning the credibility of this program, it will be experimental facts which will ultimately decide its fate. In particular, take the case of QFD as described by the $SU(2)_L \times U(1)$ Weinberg-Salam model with a minimal Higgs structure [i.e., the scalars are introduced as a single complex $SU(2)_L$ doublet]. In addition to preserving the predictions of quantum electrodynamics and accommodating existing charged-current weak-interaction phenomenology, it successfully predicted the existence of weak neutral currents and the violation of parity in deep-inelastic electron-nucleon scattering.[2] Another experimentally verified prediction of the minimal model is the relation $M_W/M_Z \cos\theta = 1$,[3] where M_W and M_Z are the masses of the charged and neutral vector mesons, respectively, and θ is the weak mixing angle.

Yet no existing experiment directly tests the basic gauge-theory formalism, or the attendant symmetry-breaking mechanism which provides the gauge mesons of the Weinberg-Salam model with their masses. The vector mesons have not been observed; their self-interactions, which are peculiar to this class of theory, have not been verified; the Higgs particle, the residual scalar of the mass-generation mechanism, has not been found.[4] Since present-day accelerators operate at energies below the masses expected for the vector mesons (about 90 GeV), the first two objections are probably premature, and might be resolved in the not too distant future. On the other hand, the problems presented by the scalar sector of the theory warrant further discussion.

The fact that the mass of the Higgs scalar, M_H, is not predicted by the Weinberg-Salam model (it has been estimated to lie anywhere between 4 GeV and 1 TeV) is essentially a manifestation of our ignorance of the dynamics triggering spontaneous symmetry breakdown. In the standard formalism, elementary scalar fields are coupled in a gauge-invariant manner to the vector mesons, which are described by a pure Yang-Mills Lagrangian. The strength of the scalar self-coupling, λ, is related to the mass of the Higgs particle by $M_H{}^2 = \sqrt{2}\lambda/G_F \simeq (350\ \text{GeV})^2\lambda$, where G_F is the Fermi coupling constant. If the self-coupling is weak (i.e., $\lambda \ll 1$), as is usually assumed, $M_H \ll 350$ GeV, and the Higgs particle can be relatively light. Presumably it could be observed, thereby verifying the standard picture of symmetry breakdown. Once this discovery is made, along with that of the vector mesons, there might be essentially no new physics until at least 10^{17} GeV, where the new phenomenology of grand-unified theories will begin to manifest itself.

A more interesting scenario results when symmetry breakdown occurs dynamically.[5] The massless Goldstone bosons required to drive the Higgs mechanism are assumed to be bound states of more fundamental particles. The natural scale of the Higgs sector is then given by $\langle 0|\Phi(x)|0\rangle = 1/(\sqrt{2}G_F)^{1/2} \simeq 250$ GeV, where $\Phi(x)$ is an "effective" scalar field. If the force binding these new quanta becomes strong at this mass scale, the typical masses of physical particles will be a few times $\langle 0|\Phi(x)|0\rangle$ or about 1 TeV.

Within the standard formalism, a mass of 1 TeV for the Higgs particle implies $\lambda \simeq 1$. This suggests that we can use the minimal Weinberg-Salam model with fundamental scalars as a phenomenological, low-energy description of a dynamically generated Higgs mechanism. (In this context, low energies are much less than 1 TeV.) In the limit $M_H \to 1$ TeV (or $\lambda \to 1$), we test the low-energy sensitivity of the minimal Weinberg-Salam model to strong 1-TeV Higgs-boson physics.

It is the purpose of this paper to study the impact which a strongly interacting, heavy-Higgs-boson sector has on the low-energy structure of the minimal Weinberg-Salam model.[6] We seek an answer to the question of whether there are any measurable low-energy quantities which are sensitive to M_H, and which can give us information about the 1-TeV Higgs-boson sector and its strong interactions.

In Sec. II, we review the conventional formulation of the minimal Weinberg-Salam model and its mass-generation mechanism (spontaneous symmetry breakdown). We also introduce an alternate formulation which is convenient for the description of the low-energy structure of the theory.

In Sec. III, we show that a convenient way of searching for low-energy sensitivity of the minimal Weinberg-Salam model to a heavy-Higgs-boson sector is to formally take the $M_H \to \infty$ limit at the outset. The resulting theory is the nonlinear σ model coupled to a pure $SU(2)_L \times U(1)$ Yang-Mills theory, and is nonrenormalizable within the context of perturbation theory. In this sense, the parameter M_H plays the role of a regulator when the linear theory is used beyond the tree approximation.

In Sec. IV we present a power-counting analysis which, when used in conjunction with the symmetry properties of the nonlinear Lagrangian, allows us to locate all the new cutoff dependence of the nonlinear theory. By interpreting this cutoff dependence as the M_H dependence of the linear theory, we are able to systematically isolate those low-energy observables which are most sensitive to the regulator M_H. We find that, at one loop, the greatest sensitivity is logarithmic.

As an illustration of these ideas, we calculate the one-loop, heavy-Higgs-boson corrections to the natural relation $M_W/M_Z \cos\theta = 1$ of the minimal Weinberg-Salam model in Sec. V. A natural relation is a constraint between coupling constants and masses, for example, which results from the symmetry structure of the Lagrangian before spontaneous breakdown. The counterterms needed to renormalize these parameters have the symmetry of this Lagrangian, and, consequently, not all of the counterterms are independent. This results in the fact that the radiative corrections to a natural relation are finite. This example is particularly interesting since it has been experimentally determined that $(M_W/M_Z \cos\theta)^2 = 0.981 \pm 0.037$.[3] As expected on the basis of the general analysis of Sec. IV, we find that the M_H dependence of these corrections is logarithmic, so that it will be rather difficult to learn anything about the strongly interacting 1-TeV sector described above by doing low-energy experiments.

The results of our analysis are summarized and discussed in Sec. VI, where we also comment on other possible low-energy Higgs-boson effects.

II. FORMALISM OF THE WEINBERG-SALAM MODEL

In the standard description of the minimal Weinberg-Salam model, the elementary scalar fields are introduced as a weak-isospin complex doublet

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) - i\phi_2(x) \\ \sigma(x) + i\chi(x) \end{pmatrix}. \tag{2.1}$$

The fields $\phi_1(x)$, $\phi_2(x)$, $\sigma(x)$, and $\chi(x)$ are all Hermitian. Under a local $SU(2)_L \times U(1)$ gauge transformation,

$$\Phi(x) \to \Phi'(x) = e^{i[\epsilon_0(x) + \vec{\epsilon}(x)\cdot\vec{\tau}]/2}\Phi(x), \tag{2.2}$$

where $\vec{\tau} \equiv (\tau_1, \tau_2, \tau_3)$ are the usual Pauli matrices. The covariant derivative

$$D_\mu \Phi(x) = [\partial_\mu + \tfrac{1}{2} i g \vec{A}_\mu(x) \cdot \vec{\tau} + \tfrac{1}{2} i g' B_\mu(x)]\Phi(x) \tag{2.3}$$

obeys a similar transformation law

$$D_\mu \Phi(x) \to [D_\mu \Phi(x)]' = e^{i[\epsilon_0(x) + \vec{\epsilon}(x)\cdot\vec{\tau}]/2} D_\mu \Phi(x), \tag{2.4}$$

where the gauge field $B^\mu(x)$ and coupling constant g' are associated with the weak-hypercharge group $[U(1)]$, and the triplet gauge field $\vec{A}^\mu(x) \equiv (A_1^\mu(x), A_2^\mu(x), A_3^\mu(x))$ and coupling constant g are associated with the weak-isospin group $[SU(2)_L]$.

The scalar sector of the theory is described by the Lagrangian

$$\mathcal{L}_0(x) = [\partial_\mu \Phi(x)]^\dagger [\partial^\mu \Phi(x)] - V(\Phi^\dagger(x)\Phi(x)), \tag{2.5a}$$

where

$$V(\Phi^\dagger(x)\Phi(x)) = \lambda[\Phi^\dagger(x)\Phi(x) + \mu^2/2\lambda]^2 \tag{2.5b}$$

and λ measures the strength of the scalar self-coupling. In order to make the global $SU(2)_L \times U(1)$ gauge symmetry of $\mathcal{L}_0(x)$ a local symmetry, the substitution $\partial_\mu \Phi(x) \to D_\mu \Phi(x)$ is made in Eq. (2.5a), where D_μ is the covariant derivative defined in Eq. (2.3). Adding in the kinetic terms for the gauge mesons, we arrive at the Weinberg-Salam (WS) Lagrangian, minus fermions, and before symmetry breakdown has occurred

$$\mathcal{L}_{WS}(x) = -\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + [D_\mu\Phi(x)]^\dagger[D^\mu\Phi(x)] - \lambda[\Phi^\dagger(x)\Phi(x) + \mu^2/2\lambda]^2 , \quad (2.6a)$$

where

$$B_{\mu\nu} = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) , \quad (2.6b)$$

$$F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + ig[A_\mu(x), A_\nu(x)] , \quad (2.6c)$$

and $A_\mu(x) = \frac{1}{2}\vec{\tau}\cdot\vec{A}_\mu(x)$.

The Lagrangian of Eqs. (2.5a) and (2.5b) possesses an additional symmetry which can be made manifest by a change in notation. Instead of introducing the scalar fields as a complex doublet, as in Eq. (2.1), we can represent them by the 2×2 matrix field

$$M(x) = \sigma(x) + i\vec{\tau}\cdot\vec{\pi}(x) . \quad (2.7)$$

The field $\sigma(x)$ is identical to that appearing in Eq. (2.1), whereas the triplet $\vec{\pi}(x) \equiv (\pi_1(x), \pi_2(x), \pi_3(x)) \equiv (-\phi_2(x), \phi_1(x), -\chi(x))$. The connection between these two representations of the scalars can be made more explicit if, in addition to the isospin $\frac{1}{2}$, hypercharge +1 field $\Phi(x)$, we introduce the isospin $\frac{1}{2}$, hypercharge −1 field $\tilde{\Phi}(x) \equiv i\tau_2\Phi^*(x)$. Equation (2.2) indicates that under a local $SU(2)_L \times U(1)$ gauge transformation

$$\tilde{\Phi}(x) \to \tilde{\Phi}'(x) = e^{i[-\epsilon_0(x)+\vec{\epsilon}(x)\cdot\vec{\tau}]/2}\tilde{\Phi}(x) . \quad (2.8)$$

Writing $M(x)$ in terms of $\phi_1(x)$, $\phi_2(x)$, $\chi(x)$ by using the correspondence

$$(\pi_1(x), \pi_2(x), \pi_3(x)) \equiv (-\phi_2(x), \phi_1(x), -\chi(x)) ,$$

we obtain

$$M(x) = \sqrt{2}\begin{pmatrix} \phi^{0\dagger}(x) & \phi^+(x) \\ -\phi^-(x) & \phi^0(x) \end{pmatrix} \equiv \sqrt{2}(\tilde{\Phi}(x)\Phi(x)) . \quad (2.9)$$

Under a local $SU(2)_L \times U(1)$ gauge transformation

$$\begin{aligned} M(x) \to M'(x) &= \sqrt{2}(\tilde{\Phi}'(x)\Phi'(x)) \\ &= e^{i\vec{\epsilon}(x)\cdot\vec{\tau}/2} \times \sqrt{2}(e^{-i\epsilon_0(x)/2}\tilde{\Phi}(x)e^{i\epsilon_0(x)/2}\Phi(x)) \\ &= e^{i\vec{\epsilon}(x)\cdot\vec{\tau}/2}\sqrt{2}(\tilde{\Phi}(x)\Phi(x))e^{-i\epsilon_0(x)\tau_3/2} \\ &= e^{i\vec{\epsilon}(x)\cdot\vec{\tau}/2}M(x)e^{-i\epsilon_0(x)\tau_3/2} . \end{aligned} \quad (2.10)$$

It is also apparent that the covariant derivative of $M(x)$, constructed to transform like $M(x)$ itself, is given by

$$\begin{aligned} \mathfrak{D}_\mu M(x) &= \sqrt{2}(i\tau_2[D_\mu\Phi(x)]^* D_\mu\Phi(x)) \\ &= [\partial_\mu + \tfrac{1}{2}ig\vec{A}_\mu(x)\cdot\vec{\tau}]\sqrt{2}[\tilde{\Phi}(x)\Phi(x)] - \tfrac{1}{2}ig'B_\mu(x)\sqrt{2}[\tilde{\Phi}(x)\Phi(x)]\tau_3 \\ &= \partial_\mu M(x) + \tfrac{1}{2}ig\vec{A}_\mu(x)\cdot\vec{\tau}M(x) - \tfrac{1}{2}ig'B_\mu(x)M(x)\tau_3 . \end{aligned} \quad (2.11)$$

The explicit presence of a τ_3 matrix in the last lines of Eqs. (2.10) and (2.11) can be traced to the fact that although $\tilde{\Phi}(x)$ and $\Phi(x)$ transform identically under the local $SU(2)_L$ group, they transform "oppositely" under the local U(1) group [compare Eqs. (2.2) and (2.8)].

Equations (2.5a) and (2.5b) may now be rewritten in terms of $M(x)$:

$$\mathcal{L}_0(x) = \tfrac{1}{4}\mathrm{Tr}[\partial_\mu M^\dagger(x)\partial^\mu M(x)] - \tfrac{1}{4}\lambda\{\tfrac{1}{2}\mathrm{Tr}[M^\dagger(x)M(x)] + \mu^2/\lambda\}^2 . \quad (2.12)$$

This is exactly the Lagrangian of the linear σ model, and it is now apparent that $\mathcal{L}_0(x)$ is invariant under the global $SU(2)_L \times SU(2)_R$ gauge transformation

$$M(x) \to M'(x) = e^{i\vec{\epsilon}_L\cdot\vec{\tau}/2}M(x)e^{-i\vec{\epsilon}_R\cdot\vec{\tau}/2} . \quad (2.13)$$

After gauging the global $SU(2)_L \times U(1)$ symmetry of $\mathcal{L}_0(x)$ by making the replacement $\partial_\mu M(x) \to \mathfrak{D}_\mu M(x)$, the scalar sector loses its additional global $SU(2)_L \times SU(2)_R$ symmetry, due to the presence of the τ_3 matrix in $\mathfrak{D}_\mu M(x)$ [see Eq. (2.11)]. In the limit $g' = 0$ (i.e., $\theta = 0$), the gauge group reduces to $SU(2)_L$, and the global chiral symmetry is restored. It is important to realize that the global chiral symmetry of the scalar sector is accidental and is not required of a general theory which is locally invariant under $SU(2)_L \times U(1)$. The fact that it is respected by $\mathcal{L}_0(x)$ has important consequences which we will discuss later.

The choice $\mu^2 < 0$ in the Weinberg-Salam Lagrangian

$$\mathcal{L}_{WS}(x) = -\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \tfrac{1}{4}\mathrm{Tr}\{[\mathfrak{D}_\mu M(x)]^\dagger\mathfrak{D}^\mu M(x)\} - \tfrac{1}{4}\lambda\{\tfrac{1}{2}\mathrm{Tr}[M^\dagger(x)M(x)] + \mu^2/\lambda\}^2 \quad (2.6a')$$

forces the potential

$$V(M^\dagger(x)M(x)) = \tfrac{1}{4}\lambda\{\tfrac{1}{2}\mathrm{Tr}[M^\dagger(x)M(x)] + \mu^2/\lambda\}^2$$

to develop an asymmetric minimum, which is located at

$$M^\dagger(x)M(x) = M(x)M^\dagger(x) = -\mu^2/\lambda \equiv f^2 \quad (2.14)$$

in the representation space of the scalars. This implies that $M(x)$ has a nonzero vacuum expectation value $\langle 0|M(x)|0\rangle = f$. In order to perform perturbation theory about the stable, asymmetric vacuum state, we define a new field, $M'(x) \equiv M(x) - f$, such that $\langle 0|M'(x)|0\rangle = 0$. If the substitution $M(x) = M'(x) + f$ is made in Eq. (2.6a′), and the Higgs mechanism is invoked, the particle spectrum of the theory may be read off the resulting Lagrangian. The would-be Goldstone bosons $\pi_1(x)$, $\pi_2(x)$, and $\pi_3(x)$ are gauged away and become the longitudinal components of three vector mesons which become massive. Two of these are charged

$$W^{\mu}_{\pm}(x)=(1/\sqrt{2})[A^{\mu}_{1}(x)\mp iA^{\mu}_{2}(x)] \tag{2.15a}$$

and one is neutral

$$Z^{\mu}(x)=\cos\theta A^{\mu}_{3}(x)-\sin\theta B^{\mu}(x)\,, \tag{2.15b}$$

where $\theta\equiv\tan^{-1}(g'/g)$ is the weak mixing angle. At the level of the Lagrangian, the W's both have mass $M_W=\frac{1}{2}gf$, whereas the Z has mass $M_Z=\frac{1}{2}Gf$, where $G\equiv(g^2+g'^2)^{1/2}$. The combination of fields orthogonal to that appearing in Eq. (2.15b),

$$A^{\mu}(x)=\sin\theta A^{\mu}_{3}(x)+\cos\theta B^{\mu}(x)\,, \tag{2.15c}$$

remains massless, and is identified with the photon. The remaining scalar field $\sigma'(x)$ develops a tree-level mass $M_H=\sqrt{2\lambda}f$, and is the physical Higgs particle.

In the renormalizable R_ξ gauge, the Goldstone-boson fields $\vec{\pi}(x)$ remain explicit in the Lagrangian obtained from Eq. (2.6a′) by shifting the field $M(x)\to M(x)+f$. In order to quantize the theory, we append the gauge-fixing (GF) term

$$\mathcal{L}_{GF}(x)=-\frac{1}{2\xi}\mathrm{Tr}\left[\partial_\mu B^{\mu}(x)-\frac{ig'f\xi}{4}[M(x)\tau_3-\tau_3M^{\dagger}(x)]\right]^2 -\frac{1}{\xi}\mathrm{Tr}\left[\partial_\mu A^{\mu}(x)+\frac{igf\xi}{8}[M(x)-M^{\dagger}(x)]\right]^2 \tag{2.16}$$

to the shifted Lagrangian. The parameter ξ varies continuously from zero (Landau gauge) to infinity (unitary gauge). Defining $\eta(x)\equiv\vec{\eta}(x)\cdot\vec{\tau}/2$ and $\omega(x)\equiv\vec{\omega}(x)\cdot\vec{\tau}/2$, we associate the following Faddeev-Popov (FP) Lagrangian with the $\xi\to 0$ limit of $\mathcal{L}_{GF}(x)$:

$$\mathcal{L}_{FP}(x)=\partial_\mu\eta_0(x)\partial^{\mu}\omega_0(x) +2\,\mathrm{Tr}\{\partial_\mu\eta(x)\partial^{\mu}\omega(x)-ig\partial_\mu\eta(x)[\omega(x),A^{\mu}(x)]\}\,, \tag{2.17}$$

where the fields $\eta_a(x)$ and $\omega_a(x)$, $a=0,\ldots,3$ are anticommuting scalars (Faddeev-Popov ghosts). In later sections of this paper, we will exploit the fact that the ghosts decouple from the $\vec{\pi}(x)$'s in the Landau gauge. From Eqs. (2.6a′), (2.16), and (2.17), we find the Landau-gauge effective Lagrangian

$$\mathcal{L}_{EFF}(x)=\mathcal{L}_{WS}(x)+\mathcal{L}_{GF}(x)+\mathcal{L}_{FP}(x)\,, \tag{2.18}$$

where it is assumed that $M(x)$ has been eliminated in favor of $M'(x)+f$. Fermions can be included in $\mathcal{L}_{EFF}(x)$ in the standard way, and we shall have occasion to do so in Sec. V.

III. THE HIGGS SECTOR AS AN EFFECTIVE LOW-ENERGY THEORY

In order to study the strongly interacting, 1-TeV region through experiments which are done at energies $E\ll 1$ TeV, it is necessary to find some low-energy observable which is sensitive to the heavy-Higgs-boson sector of the theory. The problem is that there are many quantities which satisfy this criterion, and we have no guidelines to help us determine which observable is most useful as a probe of the heavy-Higgs-boson sector. To motivate the solution to this problem, let's examine the typical behavior of the one-loop corrections to an observable which depends on M_H in the limit of M_H approaching infinity.

In general, the one-loop corrections to an observable will contain divergences which, due to the renormalizability of the Weinberg-Salam model, can be removed by adding counterterms to the original, tree-level Lagrangian. In particular, the counterterms used to carry out the renormalization program must have the same form as the terms appearing in the original Lagrangian; no new structures are required. Renormalization, therefore, amounts to a rescaling of the bare fields and parameters of the theory and does not result in any new physical effects.

However, if M_H is gradually increased and is finally made to approach infinity, the renormalized observable being calculated will grow in magnitude and ultimately diverge. In this sense, M_H acts as a regulator for the Weinberg-Salam model. Although the theory resulting from the removal of the Higgs particle from the physical spectrum is nonrenormalizable, it is reasonable to expect that, at one loop, the new divergences can be removed by introducing a small number of new counterterms into the theory. If the $M_H\to\infty$ limit is taken in a way which keeps the $SU(2)_L\times U(1)$ symmetry of the theory intact, the construction of the one-loop counterterms is limited only by the fact that it, too, must respect this symmetry. Furthermore, the cutoff dependence of these counterterms is calculable, since the counterterms must cancel the divergences which arise when $M_H\to\infty$. In contrast to the situation in the case of the Weinberg-Salam model, the nonrenormalizability of the no-Higgs-boson theory implies that some of the new symmetric counterterms will be different in form from the terms already present in the tree-level Lagrangian. Consequently, the cutoff dependence of the new counterterms is, in principle, measurable. Once the coefficients of the new counterterms have been determined, we may reinstate the Higgs particle by identifying M_H with the cutoff.

The strength of this technique[7] lies in the fact that it enables us to systematically isolate those observables which are most sensitive to M_H. Beginning with the tree-level, no-Higgs-boson Lagrangian, we may determine, *a priori,* all possible symmetric structures which can be generated as counterterms for an L-loop calculation. As will be shown in Secs. IV and V, the dependence of the coefficients of these counterterms on M_H may be found from a power-counting analysis. Since the counterterms will typically contain some of the vertices of the original Weinberg-Salam theory, the M_H dependence of some of the quantities calculated in this theory reflects the presence of the symmetric counterterms. Furthermore, these counterterms contain all the one-loop, heavy-Higgs-boson effects.[7]

At the level of the Lagrangian of Eq. (2.6a′), taking $M_H \to \infty$ (or, equivalently, $\lambda\to\infty$) means that the potential

$$V(M^\dagger(x)M(x))=\tfrac{1}{4}\lambda\{\tfrac{1}{2}\,\mathrm{Tr}[M^\dagger(x)M(x)]+\mu^2/\lambda\}^2$$

acquires an infinite, positive curvature at its minimum, determined by the $SU(2)_L\times U(1)$-invariant condition

$$M(x)M^\dagger(x)=M^\dagger(x)M(x)=f^2\,. \qquad (2.14')$$

This implies that the scalar fields $\sigma(x)$ and $\vec{\pi}(x)$ are constrained to lie on the four-dimensional hypersphere $\sigma(x)^2+\vec{\pi}(x)^2=f^2$. Imposing this constraint on the Lagrangian of Eq. (2.6a′), we find that the no-Higgs-boson theory, obtained by taking the limit $M_H\to\infty$, is an $SU(2)_L\times U(1)$ Yang-Mills theory coupled in a gauge-invariant manner to the nonlinear σ model. An interesting consequence of this is that in the unitary gauge limit $(\vec{\pi}(x)=0)$, we obtain the massive Yang-Mills theory[8]

$$\mathcal{L}(x)=-\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}-\tfrac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})+M_W{}^2W^-_\mu(x)W^\mu_+(x)+\tfrac{1}{2}M_Z{}^2Z_\mu(x)Z^\mu(x) \qquad (3.1)$$

with $M_W=\tfrac{1}{2}gf$ and $M_Z=\tfrac{1}{2}Gf$.

It is important to note that within this framework, the Higgs field $\sigma(x)$ is realized nonlinearly by Eq. (2.14), and the gauge mesons obtain their masses from the nonlinearity of the realization. The $SU(2)_L\times SU(2)_R$ global symmetry of Eq. (2.14) guarantees that the chiral symmetry of the Lagrangian of Eq. (2.6a′) is broken only by the U(1) portion of the covariant derivative, as before. Since Eq. (2.14) is locally invariant under $SU(2)_L\times U(1)$, the gauge symmetry of Eq. (2.6a′) remains intact when we impose $M^\dagger(x)M(x)=M(x)M^\dagger(x)=f^2$. Consequently, the counterterms required to renormalize the nonlinear theory must be constructed to be $SU(2)_L\times U(1)$ invariants. We indicate how this construction can be accomplished in the next section.

IV. CONSTRUCTION OF THE COUNTERTERMS

The construction of $SU(2)_L\times U(1)$-invariant counterterms begins with the scalar field $M(x)$. By operating on $M(x)$ with an arbitrary number of covariant derivatives $\mathfrak{D}_\mu$, we obtain a set of tensors, $T_{\alpha\beta\ldots}$, which transform like $M(x)$ under $SU(2)_L\times U(1)$,

$$T_{\alpha\beta\ldots}\to T'_{\alpha\beta\ldots}=e^{i\vec{\epsilon}(x)\cdot\vec{\tau}/2}T_{\alpha\beta\ldots}e^{-i\epsilon_0(x)\tau_3/2}\,. \qquad (4.1)$$

If we let $M(x)\to M(x)\tau_3$ in each of these tensors, we obtain a second set of tensors which transform according to the law given in Eq. (4.1). From the elements of the union of these two sets of tensors we can construct a complete list of objects which transform like $M(x)$ under $SU(2)_L\times U(1)$. We will denote the members of this list by $V_{\alpha\beta\ldots}$. As an illustration, $\mathfrak{D}_\alpha M(x)$ and $\mathfrak{D}_\alpha M(x)\tau_3$ are two tensors which obey the transformation law of Eq. (4.1).

It is convenient to build $SU(2)_L\times U(1)$-invariant counterterms out of objects which are bilinears in the tensors $V_{\alpha\beta\ldots}$. A bilinear like $V_{\alpha\beta\ldots}V^\dagger_{\alpha\beta\ldots}$ transforms covariantly under $SU(2)_L$, and is a U(1) singlet

$$V_{\alpha\beta\ldots}V^\dagger_{\mu\nu\ldots}\to V'_{\alpha\beta\ldots}V'^\dagger_{\mu\nu\ldots}=e^{i\vec{\epsilon}(x)\cdot\vec{\tau}/2}V_{\alpha\beta\ldots}V^\dagger_{\mu\nu\ldots}e^{-i\vec{\epsilon}(x)\cdot\vec{\tau}/2}\,. \qquad (4.2)$$

Two typical examples are $\mathfrak{D}_\alpha M(x)\tau_3M^\dagger(x)$ and $\mathfrak{D}_\alpha M(x)[\mathfrak{D}_\beta M(x)]^\dagger$. Similarly, a bilinear like $V^\dagger_{\mu\nu\ldots}V_{\alpha\beta\ldots}$ transforms covariantly under U(1), and is an $SU(2)_L$ singlet

$$V^\dagger_{\mu\nu\ldots}V_{\alpha\beta\ldots}\to V'^\dagger_{\mu\nu\ldots}V'_{\alpha\beta\ldots}=e^{i\epsilon_0(x)\tau_3/2}V^\dagger_{\mu\nu\ldots}V_{\alpha\beta\ldots}e^{-i\epsilon_0(x)\tau_3/2}\,. \qquad (4.3)$$

An object obeying this transformation law is $[\mathfrak{D}_\alpha M(x)]^\dagger M(x)$. If we include the $SU(2)_L\times U(1)$ singlet $B_{\mu\nu}$, and $F_{\mu\nu}$, which transforms covariantly under $SU(2)_L$ and is a U(1) singlet, we have all the objects which can be used to build $SU(2)_L\times U(1)$-invariant counterterms. A moment's thought suggests that the way to construct these invariants is to take the trace of a sequence of bilinears like $V_{\alpha\beta\ldots}V^\dagger_{\mu\nu\ldots}$ and $F_{\mu\nu}$'s, or to take the trace of a sequence of bilinears like $V^\dagger_{\mu\nu\ldots}V_{\alpha\beta\ldots}$. Either type of trace may be multiplied by $B_{\mu\nu}$'s without changing its invariant character. The imposition of the $SU(2)_L\times U(1)$-invariant constraint $M(x)M^\dagger(x)=M^\dagger(x)M(x)=f^2$ on these traces leaves us with the $SU(2)_L\times U(1)$-invariant counterterms of the nonlinear theory.

In order to decide exactly which counterterms are relevant for a particular calculation, we can use a power-counting analysis developed by Appelquist and Bernard[7] and arrive at a set of relations which link the detailed structure of a counterterm (e.g., its dimension, or its cutoff dependence) to topological quantities which characterize Feynman diagrams (e.g., the number of loops in a graph). Assuming that a typical counterterm is accompanied by n powers of f^2, r powers of a cutoff Λ (to be later identified with M_H), and D gauge fields and derivatives, dimensional analysis implies

$$D+2n+r=4\,. \tag{4.4}$$

The authors of Ref. 7 deal with the dimensionless scalar field $U(x)\equiv M(x)/f$, which is why D only counts derivatives and gauge fields. In terms of $U(x)$, the Landau-gauge[9] nonlinear (NL) effective Lagrangian is [see Eqs. (2.6a′), (2.16)–(2.18)]

$$\mathcal{L}_{\rm EFF}^{\rm NL}(x)=\tfrac{1}{4}f^2\,{\rm Tr}\,\{[\mathfrak{D}_\mu U(x)]^\dagger \mathfrak{D}^\mu U(x)\}+\mathcal{L}_G(x)\,, \tag{4.5a}$$

where

$$\mathcal{L}_G(x)=-\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}-\tfrac{1}{2}\,{\rm Tr}(F_{\mu\nu}F^{\mu\nu}) +\mathcal{L}_{\rm GF}(x)+\mathcal{L}_{\rm FP}(x)\,. \tag{4.5b}$$

Implicit in these equations are the $\xi\to 0$ limit, and the constraint $U(x)U^\dagger(x)=U^\dagger(x)U(x)=1$. The quantity n of Eq. (4.4) can be related to the number of loops, L, appearing in a Feynman graph by noting that f^2 essentially plays the role of a loop-counting parameter in Eq. (4.5a). Consequently, the number of powers of f^2 associated with an L-loop Feynman graph is

$$n=1-L+(I_G-V_G)\,, \tag{4.6}$$

where V_G and I_G are, respectively, the number of vertices and internal lines of the graph whose origin is $\mathcal{L}_G(x)$ [see Eqs. (4.5a) and (4.5b)]. Using the fact that $I_G-V_G\geq 0$ (except when $L=0$), we derive

$$D\leq 2L+2-r \tag{4.7}$$

from Eqs. (4.4) and (4.6). The counterterms which are relevant for an L-loop calculation can now be classified by the quantities D, r, and n.

A number of rather obvious statements concerning D, r, and n can be made. Because D only counts gauge fields and derivatives, each of which carries one spacetime index, Lorentz invariance restricts D to be an even positive integer, or zero. Since r indicates the cutoff dependence of the counterterm, it too must be an even positive integer, or zero. (If $r=0$, the cutoff dependence is either logarithmic, or trivial.) Taking into account these facts, Eq. (4.4) allows us to conclude that the parameter f always appears an even number of times (in counterterms or graphs).

The one-loop calculation to be presented in Sec. V requires, *a priori*, all the $SU(2)_L\times U(1)$-invariant counterterms of the nonlinear theory characterized by $D\leq 4$. As will be seen there, the only counterterms which are actually required are those for which $D=2$. However, to illustrate the ideas outlined above, we construct all the $SU(2)_L \times U(1)$-invariant counterterms of the nonlinear theory which are characterized by $D=0$ and $D=2$. A future publication[10] will contain a complete list of the counterterms characterized by $D=4$.

Construction of the $D=0$ counterterms is limited by the fact that only the scalar field $U(x)$ may be used. After imposing the constraint $U(x)U^\dagger(x) =U^\dagger(x)U(x)=1$, the only invariant left is ${\rm Tr}\,[U(x)\tau_3 U^\dagger(x)]^m$, where m is a positive integer. Since this is a constant for any m, there are no $D=0$ counterterms for the nonlinear theory. A quartic cutoff dependence can only accompany a $D=0$ counterterm, so we also conclude that there are no one-loop quartic divergences in this theory.

The counterterms characterized by $D=2$ can only contain two Lorentz indices, and they must be contracted. This excludes the antisymmetrical tensors $B_{\mu\nu}$ and $F_{\mu\nu}$ from consideration. A detailed analysis[10] indicates that the only independent $D=2$ nonlinear invariants are ${\rm Tr}\{\mathfrak{D}_\mu U(x)[\mathfrak{D}^\mu U(x)]^\dagger\}$, which appears in $\mathcal{L}_{\rm EFF}^{\rm NL}(x)$ [see Eq. (4.5a)], and $\{{\rm Tr}[\mathfrak{D}_\mu U(x)\tau_3 U^\dagger(x)]\}^2$, which is new. [Note that the condition $U(x)U^\dagger(x)=U^\dagger(x)U(x)=1$ is implicit.] Equation (4.7) tells us that at one loop these structures are accompanied by quadratic cutoff dependence at most.

An explicit calculation is required to determine the exact cutoff dependence of each of these two structures. First, note that the structure ${\rm Tr}\{\mathfrak{D}_\mu U(x)[\mathfrak{D}^\mu U(x)]^\dagger\}$ contains a wave-function-renormalization counterterm proportional to $\partial_\mu\vec{\pi}(x)\cdot\partial^\mu\vec{\pi}(x)$, whereas the structure $\{{\rm Tr}[\mathfrak{D}_\mu U(x)\tau_3 U^\dagger(x)]\}^2$ contains a wave-function renormalization counterterm proportional to $\partial_\mu\pi_3(x)\partial^\mu\pi_3(x)$. Let $-i\Sigma_a'(q^2)$ be the first derivative of the mass operator of $\pi_a(x)$, $a=1,2,3$. A calculation of this quantity in the nonlinear theory indicates that it has a symmetric (i.e., independent of a) quadratic divergence. This divergence can only be removed by a symmetric wave-function-renormalization counterterm which, as we have just seen, is contained in the structure ${\rm Tr}\{\mathfrak{D}_\mu U(x)[\mathfrak{D}^\mu U(x)]^\dagger\}$. Hence, the cutoff dependence of ${\rm Tr}\{\mathfrak{D}_\mu U(x)[\mathfrak{D}^\mu U(x)]^\dagger\}$ is quadratic, while that of $\{{\rm Tr}[\mathfrak{D}_\mu U(x)\tau_3 U^\dagger(x)]\}^2$ is logarithmic.

The important point to note is that although ${\rm Tr}\{\mathfrak{D}_\mu U(x)[\mathfrak{D}^\mu(x)]^\dagger\}$ has a quadratic cutoff dependence, it corresponds to a rescaling of the original nonlinear Lagrangian, and is, therefore, not measurable. But the heavy-Higgs-boson effects signaled by the logarithmic cutoff dependence of

$\{\mathrm{Tr}[\mathfrak{D}_\mu U(x)\tau_3 U^\dagger(x)]\}^2$ are measurable, as will be explicitly shown in Sec. V. The only $D=2$ counterterms are, therefore,

$$\mathcal{L}_0(x)=\tfrac{1}{4}\beta_0\,\mathrm{Tr}\{\mathfrak{D}_\mu U(x)[\mathfrak{D}^\mu U(x)]^\dagger\}\,, \qquad (4.8a)$$

$$\mathcal{L}_1(x)=\tfrac{1}{4}\beta_1 f^2\{\mathrm{Tr}[\mathfrak{D}_\mu U(x)\tau_3 U^\dagger(x)]\}^2\,. \qquad (4.8b)$$

Before we end our discussion of these counterterms, it is interesting to note that since it is characterized by $D=2$ and $r=0$, the counterterm of Eq. (4.8b) must be accompanied by one power of f^2. It must, therefore, act to cancel a logarithmic divergence of the nonlinear $SU(2)_L\times U(1)$ theory which is proportional to f^2. There is, indeed, a graph of the nonlinear theory with one internal gauge meson line and one internal scalar line which satisfies these requirements. In contrast, the nonlinear $SU(2)_L$ theory studied by the authors of Ref. 7 contains no such diagram, and, in fact, there is no analog of the counterterm displayed in Eq. (4.8b) for that theory. The heavy-Higgs-boson effects signaled by this counterterm are, therefore, not present in the $SU(2)_L$ theory, as we will see in Sec. V.

The remaining one-loop counterterms are characterized by $D=4$, and they are at most logarithmically sensitive to the cutoff. In contrast to the situation when $D=0$ and $D=2$, we cannot present any simple criteria to limit the number of counterterms and there is a large number of candidates. Because we will not need these counterterms for the calculation of Sec. V, we defer a complete construction of them to a future publication.[10]

The $SU(2)_L\times U(1)$-invariant counterterms contain all the heavy-Higgs-boson effects of the theory, and, as we have seen, the only measurable one-loop M_H dependence is logarithmic. Motivated by the fact that the ratio $M_W/M_Z\cos\theta$ has been experimentally measured, we turn to a calculation of the one-loop heavy-Higgs-boson corrections to the natural relation $M_W/M_Z\cos\theta=1$ in the next section.

V. HEAVY-HIGGS-BOSON CORRECTIONS TO A NATURAL RELATION

The measurement of the quantity $\rho\equiv(M_W/M_Z\cos\theta)^2$ does not have to await the production of the W and Z mesons and a determination of their masses.[11] In fact, ρ parametrizes the relative strength of the neutral- and charged-current interactions which occur in low-energy neutrino scattering experiments, and has already been determined to be 0.981 ± 0.037.[3] A calculation of the heavy-Higgs-boson corrections to the natural relation $\rho=1$ of the minimal $SU(2)_L\times(1)$ Weinberg-Salam model is, therefore, timely and may give us some insight into the strongly coupled, 1-TeV regime described in the Introduction.

In order to determine which counterterms of the nonlinear theory are responsible for the one-loop corrections to the tree-level prediction $\rho=1$, we will define ρ in terms of the zero momentum transfer limit ($q^2\to0$) of the purely leptonic processes $\nu_e+\mu^-\to e^-+\nu_\mu$ and $\nu_e+\nu_\mu\to\nu_e+\nu_\mu$. At the level of the Born approximation, the amplitude for $\nu_e+\nu_\mu\to\nu_e+\nu_\mu$ is given by

$$M^B=\frac{-ig^2}{4M_Z{}^2\cos^2\theta}g_{\alpha\beta}M^{\alpha\beta}\,, \qquad (5.1)$$

where $M_Z{}^2$ is the $q^2\to0$ limit of an inverse propagator, and $M^{\alpha\beta}$ contains the currents for the external lepton lines. Note that $q_\alpha M^{\alpha\beta}q_\beta=0$, so that this result is gauge independent. In the same approximation, the amplitude for $\nu_e+\mu^-\to\nu_\mu+e^-$ is

$$N^B=\frac{-ig^2}{2M_W{}^2}g_{\alpha\beta}N^{\alpha\beta}\,, \qquad (5.2)$$

where $N^{\alpha\beta}$ contains the external currents. If we assume that the leptons are massless, $q_\alpha N^{\alpha\beta}q_\beta=0$ and Eq. (5.2) is gauge invariant. The ratio $2M^B/N^B$ yields the quantity ρ, with $M_W{}^2$ and $M_Z{}^2$ the $q^2\to0$ limits of inverse propagators. The use of elastic neutrino scattering for the neutral-current process is, of course, meant only as an illustration.

The tree-graph relation $\rho=1$ is ensured by the $SU(2)_L\times SU(2)_R$ chiral symmetry of the ungauged scalar sector, as described in Sec. II. Since the full theory is only $SU(2)_L\times U(1)$ invariant, this natural relation receives finite radiative corrections. Extending the definition of ρ given above to a one-loop calculation in the linear Higgs theory, we obtain

$$\frac{M_W{}^2}{M_Z{}^2\cos^2\theta}=1+\frac{a^W(0)}{M_W{}^2}-\frac{a^Z(0)}{M_Z{}^2}+\cdots\,. \qquad (5.3)$$

The function $a(q^2)$ is defined by

$$\pi_{\mu\nu}(q)=g_{\mu\nu}a(q^2)+q_\mu q_\nu b(q^2)\,, \qquad (5.4)$$

where $\pi_{\mu\nu}(q)$ is the vacuum-polarization tensor of a gauge meson. The ellipsis in Eq. (5.3) is meant to represent the radiative corrections due to external lepton lines or vertices. The reason for separating the corrections in this manner is that heavy-Higgs-boson effects appear only in gauge particle lines, since the Higgs-boson coupling to leptons is proportional to the lepton mass which we are assuming is negligible.

Although the calculation suggested by Eq. (5.3) can be done quite easily in the linear theory, it is interesting to see how the heavy-Higgs-boson correction can be found in the context of the formalism presented in the earlier sections of this paper. At one loop, all the counterterms characterized by $D \leq 4$ are, in principle, needed. But the $D=4$ structures which can act as corrections to gauge-meson lines are $B_{\mu\nu}B^{\mu\nu}$, $\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$, $B_{\mu\nu}\mathrm{Tr} \times[F^{\mu\nu}U(x)\tau_3 U^\dagger(x)]$, and $\{\mathrm{Tr}[F_{\mu\nu}U(x)\tau_3 U^\dagger(x)]\}^2$. These only yield wave-function-renormalization pieces, which are proportional to $(q^2 g_{\mu\nu} - q_\mu q_\nu)$ and vanish as $q^2 \to 0$. Thus, the only relevant counterterms are the $D=2$ structures constructed in Sec. IV, which we list below:

$$\mathcal{L}_0(x) = \tfrac{1}{4}\beta_0 \mathrm{Tr}\{\mathcal{D}_\mu U(x)[\mathcal{D}^\mu U(x)]^\dagger\}, \tag{5.5a}$$

$$\mathcal{L}_1(x) = \tfrac{1}{4}\beta_1 f^2 \{\mathrm{Tr}[\mathcal{D}_\mu U(x)\tau_3 U^\dagger(x)]\}^2 . \tag{5.5b}$$

Explicitly displaying those terms relevant to our discussion, we have

$$\mathcal{L}_0(x) = \frac{\beta_0}{4}\left[\frac{G^2}{2} Z_\mu(x) Z^\mu(x) + g^2 W_\mu^-(x) W_+^\mu(x) + \frac{2}{f^2}\partial_\mu \vec{\pi}(x)\cdot\partial^\mu\vec{\pi}(x) + \cdots\right], \tag{5.6a}$$

$$\mathcal{L}_1(x) = -\frac{\beta_1 f^2}{2}\left[\frac{G^2}{2} Z_\mu(x) Z^\mu(x) + \frac{2}{f^2}\partial_\mu \pi_3(x)\partial^\mu \pi_3(x) + \cdots\right]. \tag{5.6b}$$

The coefficients β_0 and β_1 are determined by the fact that they must cancel the divergences which arise when M_H becomes large. Thus, including the counterterms of Eqs. (5.5a) and (5.5b), Eq. (5.3) becomes

$$\frac{M_W^2}{M_Z^2\cos^2\theta} = 1 + \frac{a^W(0)}{M_W^2} - \frac{a^Z(0)}{M_Z^2} - 2\beta_1 + \cdots, \tag{5.7}$$

which indicates that only β_1 leads to a measurable effect. As $M_W \to \infty$, $a^W(0)$ and $a^Z(0)$ both become quadratically divergent, but in a "symmetrical" way so that these divergences are removable by the counterterm of Eq. (5.5a). That is why only β_1 contributes to Eq. (5.7). We can also see this in another way. The deviation of ρ from one is due to the breaking of the chiral symmetry of the scalar sector and, as noted in Sec. II, the B gauge field is responsible for this. We therefore expect the Z, and not the W, to be responsible for the correction to one in Eq. (5.7).

A powerful consequence of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ symmetry-enforced counterterm structure is that we may calculate β_1 completely within the scalar sector of the nonlinear theory, since Eqs. (5.6a) and (5.6b) indicate that β_0 and β_1 act as wave-function-renormalization counterterms for the $\vec{\pi}(x)$'s. Let $-i\Sigma_a(q^2)$ be the mass operator for $\pi_a(x)$, $a=1,2,3$. In the linear theory, the divergences in $-i\Sigma_a'(q^2)$ are independent of a. This must be the case because the counterterm which is available to eliminate the wave-function divergences in $\Sigma_a'(q^2)$ is symmetric. As M_H gets large, each of $\Sigma_1'(q^2)$ and $\Sigma_2'(q^2)$ develops the same divergence, whereas the new divergence arising in $\Sigma_3'(q^2)$ is different. The "extra" divergence in $\Sigma_3'(q^2)$ can be removed by the wave-function counterterm resulting from Eq. (5.6b), thus determining β_1 to be

$$\beta_1 = \frac{g^2}{16\pi^2}\,\tfrac{3}{8}\tan^2\theta\left(\frac{2}{4-n}\right), \tag{5.8}$$

where we have used dimensional regularization. If we make the correspondence

$$\frac{2}{4-n} \to \ln\frac{M_H^2}{\mu^2}, \tag{5.9}$$

where μ is a typical low-energy mass scale which we take to be M_W^2, we find that

$$\frac{M_W^2}{M_Z^2\cos^2\theta} = 1 - \frac{3g^2}{32\pi^2}\tan^2\theta\ln\frac{M_H}{M_W} + \cdots \tag{5.10}$$

or

$$\frac{M_W^2}{M_Z^2} = \cos^2\theta - \frac{3\alpha}{8\pi}\ln\frac{M_H}{M_W} + \cdots . \tag{5.11}$$

This is the correction we have been looking for. Equation (5.10) indicates exactly how the heavy-Higgs-boson sector effects a low-energy observable. Unfortunately, as the general analysis of Sec. IV anticipated, the sensitivity to M_H is only logarithmic. Even if we assume $M_H \simeq 1$ TeV, the correction to one in Eq. (5.10) is only about 0.006, so that it will be difficult to probe the heavy-Higgs-boson sector by measuring ρ precisely.[12]

We have checked the process independence of our result in the sense that, for zero momentum transfer $(q^2 \to 0)$, the counterterm of Eq. (5.5b) is the only one which contributes to any neutral-current process between leptons. All other possible counterterms are proportional to $(q^2 g_{\mu\nu} - q_\mu q_\nu)$, and the only ones which survive as $q^2 \to 0$ are those which renormalize the Z-γ or γ-γ vacuum polarization tensors. The one-loop graphs which contribute to these tensors are the same in the linear and nonlinear theories, indicating that the surviving counterterms do not signal any heavy-Higgs-boson effects.

The author of Ref. 13 calculates the Higgs-boson contribution to ρ and arrives at a coefficient of $-11\alpha/24\pi$ (Ref. 13) instead of $-9\alpha/24\pi$ in Eq. (5.11). However, in his definition of ρ, M_W^2 and M_Z^2 are the physical masses of the W and Z vector bosons, respectively, and $\cos^2\theta \equiv 1 - e^2/g^2$, where g^2 is defined in terms of W decay (at a scale $q^2 = M_W^2$) and e^2 is defined in the conventional manner. We can account for his result within our framework by noting that at $q^2 = M_W^2$, the W wave-function correction due to the counterterm structure

$\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ no longer vanishes, and must contribute to the renormalization of g^2, and, consequently, $\cos^2\theta$. In addition, $M_W{}^2$ and $M_Z{}^2$ are renormalized by the counterterms listed in Eqs. (5.5a) and (5.5b). The result quoted in Ref. 13 is thus calculable within our formalism and indicates that the heavy-Higgs-boson corrections to ρ are q^2 dependent, which is not unreasonable.

VI. SUMMARY

In searching for a systematic description of the sensitivity of the low-energy observables of the minimal Weinberg-Salam model to a heavy-Higgs-boson sector, we discovered that it was very useful to view this theory as a regulated version of a nonlinear σ model coupled to a pure $\mathrm{SU}(2)_L\times \mathrm{U}(1)$-symmetric Yang-Mills theory. Within this framework, the Higgs-boson mass M_H plays the role of the regulator.

The key point of this procedure is that the counterterms needed to renormalize the divergences which arise in the linear theory when the regulator is removed are restricted to be $\mathrm{SU}(2)_L\times \mathrm{U}(1)$ invariants. By using power-counting arguments, the counterterms appearing in an L-loop calculation can be classified by their "dimension" D and cutoff dependence r. The regulator M_H may then be reinstated, and the counterterms used to locate those observables of the linear theory which are most sensitive to M_H.

In the minimal $\mathrm{SU}(2)_L\times \mathrm{U}(1)$ Weinberg-Salam model, we found that at one loop the only measurable M_H dependence was logarithmic. The accessibility of the ratio ρ to present-day experiments led us to consider the one-loop calculation of the heavy-Higgs-boson corrections to the natural relation $\rho=1$ as illustrative of the ideas presented above. As expected, the correction to this quantity is logarithmically dependent on M_H, and rather small, even for M_H approaching 1 TeV. The conclusion is that it will be difficult to learn anything about the heavy-Higgs-boson sector through low-energy ($E\ll 1$ TeV) experiments.

This paper has only given us an introduction to the low-energy structure of the Weinberg-Salam model. In fact, we should be able to completely characterize the low-energy regime of this theory by an "effective Lagrangian," composed of the tree-level nonlinear Lagrangian and those counterterms needed to renormalize it at one loop. This entails the construction of all $\mathrm{SU}(2)_L\times \mathrm{U}(1)$-invariant counterterms which arise at one loop, and the computation of their coefficients. We can then use these counterterms to locate all the heavy-Higgs-boson effects which arise in the Weinberg-Salam model. Unfortunately, we will have to await the actual production of the W and Z gauge mesons in order to measure the remaining Higgs-boson effects. We expect these to include corrections to the numerous natural relations of the theory. As an example, the ratio of the coupling of the Z to two W's, and the coupling of the Z to two left-handed leptons, is constrained to equal $\cos^2\theta$ at the tree level. At one loop, this natural relation should exhibit logarithmic dependence on M_H through the radiative corrections to the gauge-sector coupling g_{ZWW}. Presumably, this effect could be measured in Z-decay experiments. We might also expect to generate gauge-particle interactions which are logarithmically dependent on M_H, and not of the standard Yang-Mills type.[7] It might be possible to measure these new interactions through the scattering of gauge mesons. This program is currently being pursued, and its results will appear in a future publication.[10]

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