General form of the β energy spectrum in charged-hyperon β decay

A. García*
Departamento de Física, Centro de Investigación y Estudios Avanzados (IPN), Apartado Postal 14-740, México 14, D.F., México

S. R. Juárez W.
Escuela Superior de Física y Matemáticas (IPN), Apartado Postal 14-740, México 14, D.F., México
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We have computed the radiative corrections to the β energy spectrum in the semileptonic decay of charged hyperons. Strong-interaction and intermediate-vector-boson effects on the electromagnetic corrections are retained in general by extending an approach that Sirlin introduced for neutron decay. Our result is quite accurate. It is valid up to corrections no larger than a few tenths of a percent. It is suitable for performing experimental analysis in a model-independent fashion.

INTRODUCTION

With the advent of hyperon beams, experiments that measure the energy spectrum of the electron or positron emitted in the β decay of charged hyperons can be performed. The analysis of such experiments will require a knowledge of radiative corrections. Our aim in the present paper is to obtain a theoretical expression for the energy spectrum of the electron or positron that includes the radiative corrections to first order in α, the fine-structure constant, and that is as general as possible.

There are several problems in computing those radiative corrections. First, they are divergent in the ultraviolet, second, they are affected by the strong interactions of the hadrons and the intermediate vector boson of the weak vertex, and third, the mass difference between the two hadrons involved is such that the four-momentum transfer q cannot be neglected. The first problem may already be solved by using a gauge-theory model.[1] The second problem requires a deep knowledge of strong interactions which is not yet available. The third one renders the first two more difficult. One is faced with a complicated situation. Choosing some particular gauge-theory model and making specific hypotheses about strong interactions does not really solve the problems. An experimental analysis based on some models for radiative corrections only produces experimental numbers for quantities of interest that are model dependent, and that would not guide or even misguide if used to check predictions of theoretical calculations based on different models. What is desirable is that the experimental analysis be performed without bias in favor of or against any model.

There is one approach[2] that meets this requirement. It was proposed by Sirlin for neutron β decay. It consists of separating a finite model-independent part from the most general form of the radiative corrections within the $V-A$ theory, allowing for the effects of strong interactions and the presence of an intermediate vector boson with all generality. We shall follow this approach to handle the radiative corrections in the β decay of charged hyperons. We shall end with an expression for the energy spectrum of the β that is very accurate and can be used in performing unbiased experimental analysis.

In Sec. I, we discuss the separation of the model-dependent part in the virtual radiative corrections and we compute the finite model-independent part. In Sec. II, we give the form of the model-dependent part and we show that effective form factors can be introduced. In Sec. III, we study the bremsstrahlung contribution. In Sec. IV, we show that if hard photons can be experimentally discriminated then there is no contribution from model-dependent terms in the bremsstrahlung part. Finally, we devote Sec. V to collecting our results to derive a formula for the β energy spectrum. After discussing it, we give a few suggestions on how to improve an experimental analysis and on how theoretical predictions could be better compared with the data.

I. GENERAL FORM OF THE VIRTUAL RADIATIVE CORRECTIONS

For definiteness, we shall study the radiative corrections to order α for the process

$$\Sigma^- \to ne\bar{\nu}_e$$

but our calculations will be valid for the β decays of other charged hyperons. The uncorrected transition amplitude is

$$M_0 = \frac{G_V}{\sqrt{2}}\langle n|J_\mu|\Sigma^-\rangle \bar{u}_e O_\mu v_\nu . \tag{1}$$

The four-momentum vectors of Σ^-, n, e, and $\bar{\nu}_e$ are p_1, p_2, l, and p_ν, respectively.[3] J_μ stands for both the vector and the axial-vector currents and $O_\mu = \gamma_\mu(1+\gamma_5)$. The hadronic part of M_0 is[4]

$$\langle n|J_\mu|\Sigma\rangle \equiv \bar{u}_n W_\mu(p_1,p_2)u_\Sigma = \bar{u}_n\left\{f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{f_3(q^2)}{M_1}q_\mu + \left[g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5\right\}u_\Sigma . \tag{2}$$

Here $q = p_1 - p_2$ is the four-momentum transfer. Since the electron mass is very small, the terms f_3 and g_3 can be ignored. The q_μ that accompanies them will become a factor of m when it is applied to the lepton covariant and the Dirac equation is used. Their contribution is then very much suppressed. We shall drop contributions proportional to m/M_1.

The virtual radiative corrections to M_0 are given by three amplitudes. The steps in their derivation are the same ones as in Ref. 2. It is not necessary to repeat them here. The resulting expressions are[5]

$$M_1 = -\frac{G_V}{\sqrt{2}}\frac{\alpha}{4\pi^3 i}\int d^4k\, D_{\mu\nu}(k)\bar{u}_e\frac{2l_\nu - \gamma_\nu k\!\!\!/}{k^2 - 2l\cdot k + i\epsilon}O_\lambda v_\nu \bar{u}_n\left[\frac{W_\lambda(p_1,p_2)(2p_{1\mu} - k_\mu)}{k^2 - 2p_1\cdot k + i\epsilon} + T_{\mu\lambda}\right]u_\Sigma \equiv M_1^i + M_1^d , \tag{3}$$

$$M_2 = \frac{\alpha}{8\pi^3 i}\frac{G_V}{\sqrt{2}}\bar{u}_n W_\lambda(p_1,p_2)u_\Sigma \int d^4k\, D_{\mu\nu}(k)\bar{u}_e\frac{(2l_\mu - \gamma_\mu k\!\!\!/)l\!\!\!/(2l_\nu - k\!\!\!/\gamma_\nu)}{2m^2(k^2 - 2l\cdot k + i\epsilon)^2}(l\!\!\!/ + m)O_\lambda v_\nu , \tag{4}$$

$$M_3 \equiv M_3^c + M_3' = \frac{\alpha}{8\pi^3 i}\frac{G_V}{\sqrt{2}}\bar{u}_n W_\lambda(p_1,p_2)u_\Sigma \bar{u}_e O_\lambda v_\nu \int d^4k\, D_{\mu\nu}(k)\frac{(2p_1 - k)_\mu(2p_1 - k)_\nu}{(k^2 - 2p_1\cdot k + i\epsilon)^2} + M_3' . \tag{5}$$

k is the virtual-photon four-momentum, the photon propagator in the Landau gauge is

$$D_{\mu\nu}(k) = -\frac{g_{\mu\nu} - k_\mu k_\nu/(k^2 - \lambda^2 + i\epsilon)}{k^2 - \lambda^2 + i\epsilon} .$$

M_1 contains all the contributions of the vertex form, i.e., when the photon goes from the electron line to any hadron line or to the intermediate vector boson. All the model dependence due to the effects of the strong interactions and the intermediate vector boson are contained in the tensor $T_{\mu\lambda}$,

$$T_{\mu\lambda}(p_1,p_2,k) = \frac{R^\Sigma_{\mu\lambda}(p_1,p_2,k)}{k^2 - 2p_1\cdot k + i\epsilon} + G_{\mu\lambda}(p_1,p_2,k) + \frac{R^n_{\mu\lambda}(p_1,p_2,k)}{k^2 + 2p_2\cdot k + i\epsilon} . \tag{6}$$

In this expression, the first term corresponds to graphs where the photon emitted by the electron is absorbed by the fully dressed Σ^-, the second one corresponds to graphs where the photon is absorbed at the dressed weak vertex, and the third term corresponds to graphs where it is absorbed at the dressed neutron. A part that is independent of the details of strong interactions and the intermediate boson has been separated in M_1, it is free of the ultraviolet divergence and it contains the infrared-divergent contribution in this type of vertex graphs. We denote this part by M_1^i.

Following the reasoning of Sirlin,[5] our tensor $T_{\mu\lambda}(p_1,p_2,k)$ is "regular" as $k\to 0$ and is transverse in the sense that

$$k_\mu T_{\mu\lambda} = 0 .$$

These two properties follow from the essential differentiability assumptions on the electromagnetic vertex functions, which can be made plausible by a judicious use of the generalized Ward identities for them.

M_2 comes from the electron wave-function renormalization. M_3 corresponds to graphs where the virtual photon is emitted and reabsorbed by the hadron lines. It is split into two contributions. M_3^c is a convection-convection contribution[6] that arises from graphs where the photon is emitted and reabsorbed by the incoming Σ. It contains the infrared divergence due to the hadron lines. All other contributions that are not infrared divergent are in M_3'. Its general form is just like the form of M_0.

The model-independent part in the order-α virtual corrections is the sum of M_1^i, M_2, and M_3^c. It can be checked that it is gauge invariant, finite, and contains the infrared divergence.

Our expressions M_1, M_2, and M_3 formally match the corresponding ones of Ref. 2, except that our $W_\lambda(p_1,p_2)$ is no longer restricted to q being negligibly small, $q\simeq 0$. The reason for this is that the separation procedure is valid even where q cannot be ignored.

Let us now concentrate on the model-independent part and leave the model-dependent one for Sec. II. $W_\lambda(p_1,p_2)$ is given by Eq. (2). The form factors that appear there are independent of k, so that the integrations over k can be explicitly performed. Denoting by M_v the model-independent virtual radiative correction to be added to M_0, the result is

$$M_v = M_0\phi(E) + M_{p_1}\phi'(E),$$

where[7]

$$\phi(E) = \frac{\alpha}{\pi}\left\{\left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right)\ln\left(\frac{\lambda}{m}\right) - \frac{1}{2\beta}(\tanh^{-1}\beta)^2 + \frac{1}{2\beta}L\left(\frac{b-c}{-c}\right) - \frac{1}{2\beta}L\left(\frac{b-c}{1-c}\right) + \frac{1}{2\beta}\tanh^{-1}\beta\frac{M_1^2 - M_1E(1+\beta^2)}{d} + \frac{3}{4}\ln\left(\frac{M_1}{m}\right) - \frac{11}{16} - \frac{1}{2\beta}\ln\left(\frac{1-b}{1-c}\right)\left[\ln\left(\frac{M_1}{m}\right) - \tanh^{-1}\beta\right]\right\}, \tag{7}$$

$$\phi' = \frac{\alpha}{\pi}\frac{1-\beta^2}{\beta}\left[-\frac{1}{2}\tanh^{-1}\beta\left(1 + \frac{M_1E}{d}\right) + \frac{M_1 l}{2d}\ln\left(\frac{M_1}{m}\right)\right]. \tag{8}$$

We have used the following notation:

$$b, c = \frac{-E \pm l}{M_1 - 2E},$$

$$d = M_1^2 - 2M_1E,$$

$$\beta = l/E.$$

$l = |\vec{l}|$ is the three-momentum of the electron[8] and E is its energy. λ is the infrared cutoff. L is the Spence function.[2] The amplitude M_{p_1} is

$$M_{p_1} = \frac{G_V}{\sqrt{2}}\bar{u}_n W_\lambda(p_1, p_2)u_\Sigma \bar{u}_e \not{p}_1 O_\lambda v_v.$$

In these results we have neglected m/M_1, but have kept m otherwise, especially where it leads to mass divergences.

II. CONTRIBUTIONS OF THE MODEL-DEPENDENT PART OF THE VIRTUAL CORRECTION

In the virtual radiative corrections, the contributions that depend on the details of strong interactions and of the intermediate vector boson are all contained in the tensor $T_{\mu\lambda}(p_1, p_2, k)$ which was introduced in Sec. I.

What we propose to do now is to exploit as far as we can the general properties that $T_{\mu\lambda}$ has, avoiding involving ourselves with any specific model. At the end of this section we shall see that effective form factors can be introduced such that the theoretical formulas to be compared with experimental results are general and not biased by any model. Those formulas would then be useful in performing and finishing an experimental analysis.

The only approximation we shall make is to consider the contributions of $T_{\mu\lambda}$ up to first order in the four-momentum transfer q and the electron momentum l. The contributions of second and higher order in these momenta will be suppressed twofold. First, because of the factor of α, and second, because although the energy release in hyperon decays is noticeable it is not large enough to give an important contribution to the radiative corrections to higher orders in q and l. Apart from this approximation we shall only exploit the relativistic covariance of $T_{\mu\lambda}$.

We can always write the lepton covariant in the second term of Eq. (3) in the form $\bar{u}_e O_\mu v_v$, after relabeling the indices. After the integration over the virtual-photon four-momentum k, we must end with a one-index tensor which in general will be function of the available four-vectors, namely, p_1, p_2, l, and the γ matrices. In the case of M_3' we shall also have a one-index tensor but this time a function of p_1, p_2, and the γ matrices. The model-dependent part is

$$M_1^q + M_3' = \frac{\alpha}{\pi}\frac{G_v}{\sqrt{2}}\bar{u}_n T_\lambda(p_1, p_2, l)u_\Sigma \bar{u}_e O_\lambda v_v.$$

Within our approximation, the form of T_λ given by Lorentz covariance is

$$\bar{u}_n T_\lambda u_\Sigma = \bar{u}_n\left[a\gamma_\lambda + \frac{b}{M_1}\sigma_{\lambda\mu}q_\mu + \frac{c}{M_1}\sigma_{\lambda\mu}l_\mu + \frac{d}{M_1}q_\lambda + \frac{e}{M_1}l_\lambda + \left(a'\gamma_\lambda + \frac{b'}{M_1}\sigma_{\lambda\mu}q_\mu + \frac{c'}{M_1}\sigma_{\lambda\mu}l_\mu + \frac{d'}{M_1}q_\lambda + \frac{e'}{M_1}l_\lambda\right)\gamma_5\right]u_\Sigma. \tag{9}$$

We have kept terms that are first order in q and l only. The coefficients a, b, etc., are Lorentz scalars functions of the variable $p_+\cdot l \equiv (p_1 + p_2)\cdot l$ only, that is,

$$a = a(p_+\cdot l),$$

$$b = b(p_+\cdot l), \quad \text{etc.}$$

One can form other combinations in Eq. (9), but the Dirac equation and γ-matrix identities allow us to reduce them to the form given. To have form factors of the same dimensionality, we have used the same normalization as for the form factors in M_0. The same argument given by Sirlin[2] can be followed here to show that there are no contributions of order $1/l$, instead there will be terms of logarithmic order in E and m. There is no need to reproduce this argument here, since it can be traced in exact parallelism to the one of Ref. 2. It is this argument which permits the above normalization.

Since $M_1^q + M_3'$ must be added to the uncorrected transition amplitude M_0, we can absorb a, b, d, a', b', and d' into f_1, f_2, f_3, g_1, g_2, and g_3, respectively. In addition, the terms e and e' in Eq. (9) can be fused into f_3 and g_3, because the l_λ that goes with them can be replaced by q_λ, after it is applied to the lepton covariant and the Dirac equation for the antineutrino is used. Only the terms c and c' seem to stand by themselves. It turns out that they can be absorbed into f_2 and g_2, respectively.

To see this, let us consider their contribution to the transition probability. It is convenient to make the replacement $l_\mu = \frac{1}{2}q_\mu + \frac{1}{2}l_\mu - \frac{1}{2}p_{\nu\mu}$. Then we can absorb $\frac{1}{2}c$ and $\frac{1}{2}c'$ into f_2 and g_2, respectively. In our approximations, the terms with $\frac{1}{2}l_\mu - \frac{1}{2}p_{\nu\mu}$ will contribute to the transition probability through interference with the terms with f_1 and g_1 in M_0. Performing the trace calculation after summations over spins, we get

$$2M_2M_1[\,\mathrm{Re}(g_1c^*)(E-E_\nu)-\mathrm{Re}(g_1c'^*)(E+E_\nu)]\,l\cdot p_\nu\,. \tag{10}$$

E_ν is the antineutrino energy. The interference terms with f_1 are of second and higher order in q.

Working out the trace and spin summations for the uncorrected transition probability, one of the contributions is

$$4M_2M_1[\,\mathrm{Re}(g_1f_2^*)(E-E_\nu)-\mathrm{Re}(g_1g_2^*)(E+E_\nu)]\,l\cdot p_\nu\,. \tag{11}$$

A common factor to expressions (10) and (11) has not been displayed. We do not give the full expression for the uncorrected transition probability now, because it will be given later on in Sec. V. We invite the reader to inspect it there. The important point to notice is that the other contributions of f_2 and g_2 to Eq. (11) are all of second and higher order in q. This fact and the form of Eq. (10) allow us to absorb the remaining $\frac{1}{2}c$ and $\frac{1}{2}c'$ into f_2 and g_2, respectively. The effective form factors are then

$$\begin{aligned}
f_1'(q^2,p_+\cdot l) &= f_1(q^2)+\frac{\alpha}{\pi}a(l\cdot p_+)\,,\\
g_1'(q^2,p_+\cdot l) &= g_1(q^2)+\frac{\alpha}{\pi}a'(l\cdot p_+)\,,\\
f_2'(q^2) &= f_1(q^2)+\frac{\alpha}{\pi}b+\frac{\alpha}{\pi}c\,,\\
g_2'(q^2) &= g_2(q^2)+\frac{\alpha}{\pi}b'+\frac{\alpha}{\pi}c'\,,\\
f_3'(q^2) &= f_3(q^2)+\frac{\alpha}{\pi}d+\frac{\alpha}{\pi}e\,,\\
g_3'(q^2) &= g_3(q^2)+\frac{\alpha}{\pi}d'+\frac{\alpha}{\pi}e'\,.
\end{aligned} \tag{12}$$

In accordance with our approximation, b, c, d, e and their primed counterparts can be taken to be constant. For completeness, we have also given f_3' and g_3', but as we explained before, their contribution to the electron energy spectrum will be too small, and can be neglected.

To summarize, we have shown that the model-dependent part of the virtual radiative corrections can be handled by defining effective form factors in M_0. From now on we shall take for M_0 the same expression of Eq. (1), except that each form factor will be replaced by the corresponding primed one of Eqs. (12).

III. INNER BREMSSTRAHLUNG

The β decay of Σ^- will be accompanied by photon emission also and, thus, it is necessary to include in the radiative correction an inner-bremsstrahlung part. As is well known, the infrared divergence in the virtual part will be canceled by the infrared divergence of the inner-bremsstrahlung. We shall restrict our calculation to the case when only rather soft photons are undetected, i.e., we shall assume that the experimental setup is such that hard photons can be detected and separated from the proper β decay processes. This restriction need not be a drawback of our calculation. As a matter of fact, if we computed the bremsstrahlung contribution without allowing for a cutoff for the hard photons our calculation might not be directly applicable to fine experiments, where a provision has been made to discriminate against energetic photons.

If only photons with energy up to a certain Δk are undetected, we shall assume that this Δk is very small compared to the mass of either of the hadrons involved in the decay process. Typically, Δk should be around a few MeV, 4 say. It may be that in a given experimental setup Δk is not unique, it may depend on the direction of emission of the photon. We shall assume that, if that is the case, the range of values of Δk is such that any of the allowed values is still very small compared to the hadron masses. Then, such an angle dependence will give a negligible contribution to the bremsstrahlung and can be ignored. Δk would represent an average cutoff.

The bremsstrahlung contribution is also affected by the details of strong interactions and the intermediate vector boson. This is not the case when the momentum transfer to the leptons is small enough to be neglected. In our case we cannot ignore q and, therefore, we must consider the model-dependent contributions. We are again faced with a complicated situation. It can be handled in a similar way as in the virtual correction.

We shall proceed to perform a separation of the bremsstrahlung amplitude into two parts. One will contain the infrared divergence and will be model independent. The other one will contain all of the model dependence. The separation is accomplished following the same steps of the virtual case. We obtain the following amplitude:

$$M_B = eM_0\left(\frac{2l\cdot\epsilon}{2l\cdot k+\lambda^2+i\epsilon}+\frac{2\epsilon\cdot p_1-\epsilon\cdot k}{\lambda^2-2p_1\cdot k+i\epsilon}\right)$$
$$+\frac{eG_V}{\sqrt{2}}\bar{u}_n W_\lambda(p_1,p_2)u_\Sigma \bar{u}_e \frac{\not\epsilon\not k}{2l\cdot k+i\epsilon}O_\lambda v_\nu$$
$$+\frac{eG_V}{\sqrt{2}}\bar{u}_n(\epsilon_\mu T_{\mu\lambda})u_\Sigma\bar{u}_e O_\lambda v_\nu$$
$$\equiv [1]+[2]+[3]. \qquad (13)$$

The coefficient e is the electron's electric charge, ϵ_λ is the polarization four-vector of the photon, and k is its four-momentum; otherwise, we have used the same notation as in Sec. I. The first two terms are model independent, the last one contains all of the model dependence. The form of $T_{\mu\lambda}(p_1,p_2,k)$ is exactly the same one as in Eq. (6), except that now k corresponds to a real photon. We refer the reader to the discussion after Eq. (6) for details on $T_{\mu\lambda}$, except that one must keep in mind that that discussion should now be limited to emission of a real photon.

In the remainder of this section we shall study the model-independent contribution, leaving the model-dependent part for Sec. IV. We shall exploit the restriction $\Delta k \ll M_1, M_2$. The phase-space factor and the matrix elements depend on k, but whenever such a dependence can be put as terms with a factor $1/M_1$, that is as k/M_1, it can be dropped off. This simplifies the calculation a good deal. We must square the sum of the first two terms in Eq. (13), sum over the different spins and the photon polarization, and integrate over the photon direction $\hat{k}$ and energy k_0. The upper limit of the k_0 integral is not simply Δk, because the real upper limit may be smaller than Δk. It can be written as

$$k_0 = (E_m - E)\Delta,$$

where

$$E_m = \frac{M_1^2 - M_2^2 + m^2}{2M_1} \approx \frac{M_1^2-M_2^2}{2M_1}.$$

Δ is such that if $(E_m - E)\le \Delta k$, $\Delta = 1$, and if $(E_m - E) > \Delta k$, then $\Delta = \Delta k/(E_m - E)$. Thus, Δ is something like a step function.

The contribution to the transition rate due to the model-independent part of the inner bremsstrahlung is

$$d\omega_B^I = \frac{G_V^2}{2\pi^3}\frac{lE(E_m-E)^2 dE}{(1-2E/M_1)^2}(|M_0'|^2\theta + |M_{p_1}''|^2\theta'),$$

where

$$\theta = \frac{\alpha}{\pi}\left\{\left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right)\left[\frac{E_m-E}{E\beta^2}\Delta_1 + \frac{(E_m-E)^2}{E^2\beta^2}\frac{\Delta_2}{2}+\frac{1}{2}-2\Delta+\frac{\Delta^2}{2}+\ln\frac{2(E_m-E)\Delta}{\lambda}\right] + 1 - \frac{1}{2\beta}(\tanh^{-1}\beta)^2 + \frac{1}{2\beta}L\left(\frac{2\beta}{1+\beta}\right)\right\}, \qquad (14)$$

$$\theta' = \frac{\alpha}{\pi}\left\{\left(1-\frac{1}{\beta^2}\right)\left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right)\left[\frac{E_m-E}{E}\Delta_1 + \frac{(E_m-E)^2}{E^2}\frac{\Delta_2}{2}\right] + \frac{(E_m-E)^2}{E^2}\frac{\Delta_2}{2}\right\}. \qquad (15)$$

We have used the following definitions:

$$\Delta_1 = \Delta - \Delta^2 + \frac{\Delta^3}{3}, \qquad \Delta_2 = \frac{\Delta^2}{2} - \frac{2\Delta^3}{3} + \frac{\Delta^4}{4}.$$

We stay close to the notation of Sec. I, except that we have arranged the phase-space factor after extracting a factor $M_1 E E_\nu$ from $|M_0|^2$ and $|M_{p_1}|^2$. We indicate this by putting a prime on them. M_{p_1} was defined in Sec. I for the virtual case; in the present case, M_{p_1}' is proportional to M_{p_1}. The proportionality constant is the mass of the electron (or minus the mass of the positron in the case of positive-hyperon decay). We indicate this by putting another prime on M_{p_1}'.

IV. CONTRIBUTION OF THE MODEL-DEPENDENT PART OF INNER BREMSSTRAHLUNG

We shall now study $T_{\mu\lambda}(p_1, p_2 k)$ of Eq. (13). Just as in the virtual-corrections case, we do not want to get involved with a particular model to compute the contributions of $T_{\mu\lambda}$. Instead, we want to exploit general properties of $T_{\mu\lambda}$ as far as we can.

Let us recall that we are concerned only with the case when the maximum energy of the unobserved photons is much smaller than the mass of either of the hadrons involved in the process, i.e., $\Delta k \ll M_1, M_2$. It must be noted that this restriction does not require that the momentum transfer to the leptons be small. $T_{\mu\lambda}$ is infrared convergent and gauge invariant. So, we can take $k^2 = 0$ and $\epsilon\cdot k = 0$ to be valid now. The most general form allowed by relativistic covariance and gauge invariance for $\epsilon_\mu T_{\mu\lambda}$ is

$$
\begin{aligned}
\epsilon_\mu T_{\mu\lambda}(p_1,p_2,k) = {} & \frac{1}{-2p_1\cdot k+i\epsilon}\{2\epsilon\cdot p_1[(\omega_3 p_+\cdot k+\omega_3' q\cdot k)\gamma_\lambda+(\omega_7 p_+\cdot k+\omega_7' q\cdot k)p_{+\lambda} \\
& \qquad +(\omega_8 p_+\cdot k+\omega_8' q\cdot k)q_\lambda+\omega_1 k_\lambda+(\omega_2\gamma_\lambda+\omega_4 q_\lambda+\omega_5 p_{+\lambda}+\omega_6 k_\lambda)\not{k}] \\
& \qquad +(\epsilon\cdot p_1\not{k}-p_1\cdot k\not{\epsilon})(x_1 k_\lambda+x_2\gamma_\lambda+x_3 p_{+\lambda}+x_4 q_\lambda) \\
& \qquad +\not{\epsilon}\not{k}(x_5 k_\lambda+x_6\gamma_\lambda+x_7 p_{+\lambda}+x_8 q_\lambda)\} \\
& +\frac{1}{2k\cdot p_2+i\epsilon}[(\epsilon\cdot p_2\not{k}-k\cdot p_2\not{\epsilon})(x_1' k_\lambda+x_2'\gamma_\lambda+x_3' p_{+\lambda}+x_4' q_\lambda)+\not{\epsilon}\not{k}(x_5' k_\lambda+x_6'\gamma_\lambda+x_7' p_{+\lambda}+x_8' q_\lambda)] \\
& +\epsilon\cdot p_+[\omega_7 p_{+\lambda}+\omega_8 q_\lambda+u_4 k_\lambda+\omega_3\gamma_\lambda+(u_1' p_{+\lambda}+u_2' q_\lambda+u_4' k_\lambda+u_{10}'\gamma_\lambda)\not{k}] \\
& +\epsilon\cdot q[\omega_7' p_{+\lambda}+\omega_8' q_\lambda+u_8' k_\lambda+\omega_3'\gamma_\lambda+(u_3' p_{+\lambda}+u_6' q_\lambda+u_8' k_\lambda+u_{12}'\gamma_\lambda)\not{k}] \\
& +(u_{11} p_{+\lambda}+u_{13} q_\lambda+u_{15} k_\lambda+u_{16}\gamma_\lambda)\not{\epsilon}+\not{\epsilon}\gamma_\lambda u_{17}+\not{\epsilon}\gamma_\lambda\not{k}u_{17}'+(u_{11}' p_{+\lambda}+u_{13}' q_\lambda+u_{15}' k_\lambda+u_{16}'\gamma_\lambda)\not{\epsilon}\not{k}\,, \qquad (16)
\end{aligned}
$$

where $p_+ = p_1 + p_2$.

In constructing this expression we have used the property of $R^{\Sigma^-}_{\mu\lambda}$ and $R^{n}_{\mu\lambda}$ [see Eq. (6)] that they should go to zero when k goes to zero. There are alternative forms for this expression (16), but using the Dirac equation and γ-matrix identities they can be reduced to the above form. The model dependence is contained in the different form factors, i.e., ω_i, u_i, etc., with $i = 1, 2, \ldots$. They are functions of the scalars that can be formed with p_1, p_2, and k, e.g.,

$$\omega_1 = \omega_1(p_+\cdot k,\ q\cdot k,\ q^2)\,.$$

In order to match the dimensionality of the form factors in Eq. (16) with those of the uncorrected matrix element M_0, we shall introduce the same convention as in Eq. (2). We shall divide by M_1 those form factors whose dimensions do not match with the dimensions of f_1. For example, instead of u_{17}, we shall put u_{17}''/M_1. Now u_{17}'' has the same dimensions as f_1 and f_2. We shall not rewrite Eq. (16), but we shall understand that this convention is being followed from now on.

In a certain way, we are introducing a disguised assumption. We are assuming that strong interactions are well enough behaved so as not to induce terms in the electromagnetic corrections that are anomalously large. In other words, once we have chosen to normalize the form factors with M_1, then the resulting form factors shall all be of comparable size and none too large, very much as f_1 and f_2 are expected to be of the same order of magnitude. Most of the models we would like to use support this idea. But we must admit it as an assumption.

$T_{\mu\lambda}$ will contribute to the transition rate through the square of the third term, and its interference with the first two in Eq. (13). The full contribution is too long, so we prefer not to reproduce it here. It has one general feature which we want to stress. It only contains terms that can always be factorized into a product of $\Delta k/M_1$ times a factor which is of the same order as the contributions of the model-independent part. In other words, the contributions from $T_{\mu\lambda}$ to the transition rate will always be suppressed by at least a factor $\Delta k/M_1$ with respect to the contributions of the model-independent part.

To illustrate this point, let us pick up a term which is expected to give a potentially large contribution, the interference of $u_{17} \equiv u_{17}''/M_1$ with $1/l\cdot k$ in the first term of Eq. (13). The integral over the photon momentum will be proportional to

$$\int_0^{\Delta k} \frac{d^3k}{k}\frac{1}{l\cdot k}\frac{1}{M_1} \propto \frac{\Delta k}{EM_1} = \frac{1}{E}\frac{\Delta k}{M_1}\,,$$

while the interference of $1/l\cdot k$ with other terms of the model-independent part will be of order $1/E$. It can be checked that any other contribution from $T_{\mu\lambda}$ behaves in the same way or gets an extra power of $\Delta k/M_1$.

We can now draw our main conclusion in this section. Provided that only photons with energy much smaller than the mass of the hadrons are undetected, the model-dependent contributions of the inner bremsstrahlung are very small, of the order of $\Delta k/M_1 \ll 1$, and can therefore be ignored in the radiative correction. The only part that contributes is the model-independent one, given in Sec. III. Let us emphasize once more that our conclusion is not restricted to small q, as was the case in neutron β decay.

V. RESULTS AND DISCUSSION

We are now in a good position to obtain an expression for the energy spectrum of the emitted β in charged-hyperon decays. We must evaluate all traces, sum over spins, and integrate over all kinematical variables except the energy of the β. Since the expression we are going to obtain is very long, we shall organize it so that it can be read in steps. First we shall take constant form factors in W_λ and later on we shall discuss their q^2 and $p_+ \cdot l$ dependence. The traces can be evaluated more expediently if one uses[4]

$$F_1 = f_1 + (1 + M_2/M_1) f_2 ,$$
$$G_1 = g_1 - (1 - M_2/M_1) g_2 ,$$
$$F_2 = -2f_2 , \quad G_2 = -2g_2 ,$$
$$F_3 = f_2 + f_3 , \quad G_3 = g_2 + g_3 ,$$

instead of the Dirac form factors of W_λ. Our result for the energy spectrum of the β is[9]

$$d\omega(E)_{\Sigma^- \to ne\nu} = \frac{G_V{}^2}{2\pi^3} \frac{lE(E_m - E)^2 dE}{(1 - 2E/M_1)^2} \times [A(1 + \phi + \theta) + A'(\phi' + \theta')] , \quad (17)$$

where[8]

$$A = a_1 |f_1'|^2 + a_2 |f_2'|^2 + a_3 \operatorname{Re} f_1' f_2'^* + a_4 |g_1'|^2 + a_5 |g_2'|^2 + a_6 \operatorname{Re} g_1' g_2'^* + a_7 \operatorname{Re} f_1' g_1'^* + a_8 \operatorname{Re} f_1' g_2'^* + a_9 \operatorname{Re} f_2' g_1'^* + a_{10} \operatorname{Re} f_2' g_2'^* . \quad (18)$$

The coefficients a_i are defined in two steps: first,

$$a_1 = A_1 ,$$
$$a_2 = \left(1 + \frac{M_2}{M_1}\right)^2 A_1 + 4A_2 - 2\left(1 + \frac{M_2}{M_1}\right) A_3 ,$$
$$a_3 = 2\left(1 + \frac{M_2}{M_1}\right) A_1 - 2A_3 , \quad (19)$$
$$a_4 = A_4 ,$$
$$a_5 = \left(1 - \frac{M_2}{M_1}\right)^2 A_4 + 4A_5 + 2\left(1 - \frac{M_2}{M_1}\right) A_6 ,$$
$$a_6 = -2\left(1 - \frac{M_2}{M_1}\right) A_4 - 2A_6 ,$$
$$a_7 = A_7 ,$$
$$a_8 = -\left(1 - \frac{M_2}{M_1}\right) A_7 ,$$
$$a_9 = \left(1 + \frac{M_2}{M_1}\right) A_7 ,$$
$$a_{10} = -\left(1 - \frac{M_2^2}{M_1^2}\right) A_7 ;$$

then,

$$A_1 = \left(2 - \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) - H + \beta \times \left[J - \frac{l}{M_1}\left(\frac{E}{M_1} + \frac{M_2}{M_1}\right)\right],$$
$$A_2 = \tfrac{1}{2}\left(1 + \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left[1 - \frac{E}{M_1} - \beta \frac{l}{M_1}\right] - \tfrac{1}{2}H - \tfrac{1}{2}\beta J ,$$
$$A_3 = \left(1 + \frac{M_2}{M_1}\right)\left[1 - \frac{E}{M_1} - \beta \frac{l}{M_1}\right],$$
$$A_4 = \left(2 + \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) - H + \beta\left[J - \frac{l}{M_1}\left(\frac{E}{M_1} - \frac{M_2}{M_1}\right)\right], \quad (20)$$
$$A_5 = \frac{1}{2}\left(1 - \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left[1 - \frac{E}{M_1} - \beta \frac{l}{M_1}\right] - \tfrac{1}{2}H - \tfrac{1}{2}\beta J ,$$
$$A_6 = -\left(1 - \frac{M_2}{M_1}\right)\left[1 - \frac{E}{M_1} - \beta \frac{l}{M_1}\right],$$
$$A_7 = 2\left(1 - \frac{E}{M_1}\right)\frac{E}{M_1} - 2H + 2\beta\left(J + \frac{El}{M_1^2}\right).$$

A' stands for

$$A' = a_1' |f_1'|^2 + a_2' |f_2'|^2 + a_3' \operatorname{Re} f_1' f_2'^* + a_4' \operatorname{Re} f_1' f_3'^* + a_5' \operatorname{Re} f_2' f_3'^* + a_6' |f_3'|^2 + a_7' |g_1'|^2 + a_8' |g_2'|^2 + a_9' \operatorname{Re} g_1' g_2'^* + a_{10}' \operatorname{Re} g_1' g_3'^* + a_{11}' \operatorname{Re} g_2' g_3'^* + a_{12}' |g_3'|^2 + a_{13}' \operatorname{Re} f_1' g_1'^* + a_{14} \operatorname{Re} f_1' g_2'^* + a_{15}' \operatorname{Re} f_2' g_1'^* + a_{16} \operatorname{Re} f_2' g_2'^* . \quad (21)$$

Again, the a_i' are defined in two steps:

$$a_1' = A_1' ,$$
$$a_2' = \left(1 + \frac{M_2}{M_1}\right)^2 A_1' + 4A_2' - 2\left(1 + \frac{M_2}{M_1}\right) A_3' + \left(1 + \frac{M_2}{M_1}\right) A_4' - 2A_5' + A_6' ,$$
$$a_3' = 2\left(1 + \frac{M_2}{M_1}\right) A_1' - 2A_3' + A_4' ,$$
$$a_4' = A_4' ,$$
$$a_5' = \left(1 + \frac{M_2}{M_1}\right) A_4' - 2A_5' + 2A_6' ,$$
$$a_6' = A_6' ,$$
$$a_7' = A_7' ,$$

$$a_8' = \left(1 - \frac{M_2}{M_1}\right)^2 A_7' + 4A_8' + 2\left(1 - \frac{M_2}{M_1}\right)A_9' - \left(1 - \frac{M_2}{M_1}\right)A_{10}' - 2A_{11}' + A_{12}',$$

$$a_9' = -2\left(1 - \frac{M_2}{M_1}\right)A_7' - 2A_9' + A_{10}',$$

$$a_{10}' = A_{10}',$$

$$a_{11}' = -\left(1 - \frac{M_2}{M_1}\right)A_{10}' - 2A_{11}' + 2A_{12}',$$

$$a_{12}' = A_{12}',$$

$$a_{13}' = A_{13}',$$

$$a_{14}' = -\left(1 - \frac{M_2}{M_1}\right)A_{13}',$$

$$a_{15}' = \left(1 + \frac{M_2}{M_1}\right)A_{13}',$$

$$a_{16}' = -\left(1 - \frac{M_2^2}{M_1^2}\right)A_{13}', \qquad (22)$$

and

$$A_1' = \left(2 - \frac{M_2}{M_1} - \frac{2E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) - H - \beta \frac{El}{M_1^2},$$

$$A_2' = \tfrac{1}{2}\left(1 + \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) - \tfrac{1}{2}H,$$

$$A_3' = \left(1 + \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) - \beta \frac{El}{M_1^2},$$

$$A_4' = \frac{E}{M_1}\left(1 + \frac{M_2}{M_1} - \frac{2E}{M_1}\right)\left(1 - \frac{E}{M_1} + \beta \frac{l}{M_1}\right),$$

$$A_5' = \frac{E}{M_1}\left[\left(1 + \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1} + \beta \frac{l}{M_1}\right) - H + \beta J\right],$$

$$A_6' = \frac{E^2}{M_1^2}\left[\left(1 + \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1} + \beta \frac{l}{M_1}\right) - H + \beta J\right]$$

$$A_7' = \left(2 + \frac{M_2}{M_1} - \frac{2E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) - H - \beta \frac{El}{M_1^2}, \qquad (23)$$

$$A_8' = \tfrac{1}{2}\left(1 - \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) - \tfrac{1}{2}H,$$

$$A_9' = \left(-1 + \frac{M_2}{M_1} + \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1}\right) + \beta \frac{El}{M_1^2},$$

$$A_{10}' = \frac{E}{M_1}\left(-1 + \frac{M_2}{M_1} + \frac{2E}{M_1}\right)\left(1 - \frac{E}{M_1} + \beta \frac{l}{M_1}\right),$$

$$A_{11}' = \frac{E}{M_1}\left[\left(1 - \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1} + \beta \frac{l}{M_1}\right) - H + \beta J\right],$$

$$A_{12}' = \frac{E^2}{M_1^2}\left[\left(1 - \frac{M_2}{M_1} - \frac{E}{M_1}\right)\left(1 - \frac{E}{M_1} + \beta \frac{l}{M_1}\right) - H + \beta J\right],$$

$$A_{13}' = -2H + 2\beta \frac{El}{M_1^2}.$$

We have used the definitions

$$H = \frac{(E_m - E)[3(1 - E/M_1)^2 + l^2/M_1^2]}{3(1 - 2E/M_1)M_1},$$

$$J = -\frac{4(E_m - E)(1 - E/M_1)l}{3(1 - 2E/M_1)M_1^2},$$

$$\beta = \frac{l}{E}, \quad \text{as before.}$$

l stands for the three-momentum of the β, $l = |\vec{l}|$.

The functions ϕ, θ, ϕ', and θ' were defined in Eqs. (7), (14), (8), and (15), respectively. We shall not reproduce them again. Let us just remark that in the sum $\phi + \theta$ the infrared divergence is canceled out, as was expected. We emphasize that our result is restricted to the emission of photons whose maximum energy is much smaller than either of the hadron masses, a few MeV, say. This is taken into account in the term Δ in θ and θ'. For completeness, we give Δ in terms of a step function θ

$$\Delta = \left(1 - \frac{\Delta k}{E_m - E}\right)\theta(\Delta k - E_m + E) + \frac{\Delta k}{E_m - E},$$

Δk is the maximum energy of the unobserved photons in the inner bremsstrahlung, as mentioned before. We have dropped all the terms of order m/M_1. The primed form factors were defined in Sec. II. Their q^2 and $p_+ \cdot l$ dependence can be handled by expanding them in a power series. For example,

$$f_1'(q^2, p_+ l) = f_1'(0) + \lambda_1^f \frac{q^2}{M_1^2} + \frac{\alpha}{\pi} \lambda_f^+ \frac{p_+ \cdot l}{M_1^2} + \cdots. \qquad (24)$$

All the other form factors can be expanded in a similar fashion. It is $f_1'(0)$, $f_2'(0)$, etc., that appear in the above formulas. Actually, according to our approximations, as explained in Sec. II, the form factors can be taken to be constant in A', and f_2 and g_2 can be taken to be independent of $p_+ \cdot l$ in A. We shall assume that contributions of $(q^2/M_1^2)^2$ and higher powers in the series expansion can be neglected. These contributions should be of order 10^{-3} at best. To complete our formulas we must obtain the contributions of λ_1^f, λ_2^f, λ_1^g, λ_2^g, λ_f^+, and λ_g^+. The contributions of the first four slopes are[9]

$$d\omega(E)(q^2 \text{ slopes}) = \frac{G_V^2}{2\pi^3} \frac{lE(E_m - E)^2 dE}{(1 - 2E/M_1)^2} A'', \qquad (25)$$

where

$$A''=2a_1''\,\mathrm{Re}\,\lambda_1^f f_1^* + 2a_2''\,\mathrm{Re}\lambda_2^f f_2^* + a_3''\,\mathrm{Re}(\lambda_1^f f_2^* + \lambda_2^f f_1^*) + a_4''\,\mathrm{Re}(\lambda_3^f f_1^* + \lambda_1^f f_3^*) + a_5''\,\mathrm{Re}(\lambda_3^f f_2^* + \lambda_2^f f_3^*) + 2a_6''\,\mathrm{Re}\lambda_3^f f_3^*$$
$$+ 2a_7''\,\mathrm{Re}\lambda_1^g g_1^* + 2a_8''\,\mathrm{Re}\lambda_2^g g_2^* + a_9''\,\mathrm{Re}(\lambda_2^g g_1^* + \lambda_1^g g_2^*) + a_{10}''\,\mathrm{Re}(\lambda_3^g g_1^* + \lambda_1^g g_3^*) + a_{11}''\,\mathrm{Re}(\lambda_3^g g_2^* + \lambda_2^g g_3^*) + 2a_{12}''\,\mathrm{Re}\lambda_3^g g_3^*$$
$$+ a_{13}''\,\mathrm{Re}\,(\lambda_1^g f_1^* + \lambda_1^f g_1^*) + a_{14}''\,\mathrm{Re}\,(\lambda_2^g f_1^* + \lambda_1^f g_2^*) + a_{15}''\,\mathrm{Re}(\lambda_1^g f_2^* + \lambda_2^f g_1^*) + a_{16}''\,\mathrm{Re}(\lambda_2^g f_2^* + \lambda_2^f g_2^*)\,.$$

The coefficients a_i'' are the same functions as the a_i' of Eq. (22), but with the A_i' replaced, respectively, by the following A_i'':

$$A_1''=\left(1+\frac{\beta^2}{3}\right)\left[\left(2-\frac{M_2}{M_1}-\frac{E}{M_1}\right)H_1-J_1\right]-\frac{2\beta^2}{3}H_1\,,$$
$$A_2''=\tfrac{1}{2}\left(1-\frac{\beta^2}{3}\right)\left[\left(1+\frac{M_2}{M_1}-\frac{E}{M_1}\right)H_1-J_1\right],$$
$$A_3''=\left(1-\frac{\beta^2}{3}\right)\left(1+\frac{M_2}{M_1}\right)H_1\,,$$
$$A_7''=\left(1+\frac{\beta^2}{3}\right)\left[\left(2+\frac{M_2}{M_1}-\frac{E}{M_1}\right)H_1-J_1\right]-4\beta H_2 -\frac{2\beta^2}{3}H_1\,, \tag{26}$$
$$A_8''=\tfrac{1}{2}\left(1-\frac{\beta^2}{3}\right)\left[\left(1-\frac{M_2}{M_1}-\frac{E}{M_1}\right)H_1-J_1\right],$$
$$A_9''=-\left(1-\frac{\beta^2}{3}\right)\left(1-\frac{M_2}{M_1}\right)H_1\,,$$
$$A_{13}''=2\left(1+\frac{\beta^2}{3}\right)\left(\frac{E}{M_1}H_1-J_1\right),$$
$$A_4''=A_5''=A_6''=A_{10}''=A_{11}''=A_{12}''\simeq 0\,.$$

We have used the abbreviations

$$H_1=\frac{2E(E_m-E)}{M_1^2}\,,$$
$$J_1=\frac{2E(E_m-E)^2}{M_1^3}\,,$$

and

$$H_2=-\frac{8El(E_m-E)}{3M_1^3}\,.$$

We can rearrange $p_+\cdot l/M_1^2$ in Eq. (24) as

$$\frac{p_+\cdot l}{M_1^2}\simeq\frac{M_1-M_2}{M_1}+\frac{E-E_\nu}{M_1}\,.$$

The constant terms can be reabsorbed into f_1' and g_1', so we redefine

$$f_1''(0)=f_1'(0)+\frac{\alpha}{\pi}\frac{M_1-M_2}{M_1}\lambda_f^+ \tag{27}$$

and

$$g_1''(0)=g_1'(0)+\frac{\alpha}{\pi}\frac{M_1-M_2}{M_1}\lambda_g^+\,. \tag{28}$$

We should replace f_1' and g_1' in Eq. (18) by f_1'' and g_1'', respectively. The other contributions of λ_f^+ and λ_g^+ to the energy spectrum are[9]

$$d\omega(E)(p_+\cdot l\text{ slopes}) =\frac{G_V{}^2}{2\pi^3}lE(E_m-E)^2dE\,\frac{\alpha}{\pi}\,[2\,\mathrm{Re}\lambda_f^+ f_1^* + 6\,\mathrm{Re}\lambda_g^+ g_1^*] \times\frac{2E-E_m}{M_1}\,. \tag{29}$$

To summarize, our complete result is given by the sum of Eqs. (17), (25), and (29), with f_1' and g_1' replaced by Eqs. (27) and (28). The primed form factors were defined in Sec. II, Eqs. (12).

Although our result is lengthy and tedious, it is basically simple. All of the radiative correction to first order in α is given by only two functions, $\phi+\theta$ and $\phi'+\theta'$, that are universal in the sense that they do not depend on the details of the strong interactions and the intermediate vector boson, plus model-dependent terms whose only effect is to modify the already existing form factors due to strong interactions alone without introducing new form factors. Their energy dependence is modified, though, by the introduction of a dependence on the variable $p_+\cdot l$. Such a dependence should be noticeable mainly for $f_1'(q^2,p_+\cdot l)$ and $g_1'(q^2,p_+\cdot l)$, while practically negligible for the other form factors.

The $p_+\cdot l$ dependence has an interesting feature. As we have seen above, part of it goes into the constant parts of f_1' and g_1'. The remaining part does depend on the β energy, but in such a way that it is very small in the central part of the energy spectrum and only at its tails does it become noticeable. This behavior should be contrasted to the q^2 dependence of the form factors, which becomes very small at the tails of the spectrum and largest at its central part. So to speak, we could say that the q^2 and $p_+\cdot l$ dependences do not interfere with one another.

Within our approximation of neglecting terms of order $q^2\alpha/\pi$, we can also neglect the $p_+\cdot l$ dependence when the factor $(2E-E_m)/M_1$ in Eq. (29) is of order q^2/M_1 or smaller. For $\Sigma^-\to ne\nu$, this means that a range of about 70 MeV is free of λ_f^+ and λ_g^+ from Eq. (29); this represents 30% of the energy spectrum. This range is centered around the maximum of the bell-shaped spectrum, and is therefore the statistically favored part.

For other charged-hyperon decays this range should be proportionally wider because our approximations become more accurate.

In using our result to perform an experimental analysis, the primed form factors should be used, since it is these form factors only that can be determined experimentally.[9] It is this fact that permits the experimental analysis to be concluded once and for all, in a model-independent fashion. After experimental values for the primed form factors have been determined, one can proceed to determine quantities of theoretical interest by computing partial contributions to the primed form factors using some model; that part of the form factors that cannot be computed, such as the bare coupling constants for example, can be attributed an "experimental value" within the model used.[10] Of course, that "experimental value" would change, in general, if another model were chosen, while the experimental values for the primed form factors would remain.[11]

A curious point is that f_3 and g_3 are enhanced through the radiative correction. They cannot simply be dropped altogether as our neglect of m/M_1 could have suggested. This opens the possibility of determining them experimentally even in electron decays, although it would require very high statistics.

Finally, let us note that our expression for the β spectrum is accurate up to terms that are not larger than a few tenths of a percent. It is valid equally well for the decay of positively charged hyperons.

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*Also at Escuela Superior de Física y Matemáticas (IPN).

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); Rev. Mod. Phys. 46, 255 (1974); A. Salam, in *Elementary Particle Theory: Relative Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[2]A. Sirlin, Phys. Rev. 164, 1767 (1967).

[3]The masses will be denoted by M_1, M_2, m, and $m_\nu = 0$, respectively.

[4]Our conventions for γ matrices and metric are those of D. R. Harrington, Phys. Rev. 120, 1482 (1960).

[5]In order to stay close to the notation of Ref. 2 we call the virtual amplitudes M_1, M_2, and M_3. This should not lead to confusion with our notation for the masses of the hyperons.

[6]N. Meister and D. R. Yennie, Phys. Rev. 130, 1210 (1963).

[7]We are very grateful to A. Sirlin for having given us a copy of his unpublished results for some of the more difficult integrals.

[8]To facilitate the writing of our lengthy expressions we use the same symbol l for the β four-momentum and three-momentum. This is clarified through the text.

[9]We introduced some terms of order α^2 in Eq. (17) to make it more concise. We could do the same in the following expressions so that only primed form factors appear. The error committed would be negligible.

[10]An example of this approach can be found in D. H. Wilkinson and D. W. Alburger, Phys. Rev. C 13, 2517 (1976).

[11]If the upper part of the spectrum is being studied, one can make the approximation $E \simeq l$. This simplifies all of the expressions of Sec. V substantially.