Regge slopes in dual topological expansion

Chung-I Tan
Physics Department, Brown University, Providence, Rhode Island 02912
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The structure of the dual topological expansion is studied up to the cylinder level by concentrating on the determination of Reggeon and Pomeron slopes. A precise formulation for the generation of Regge behavior in terms of an effective random walk is presented, and a well-defined meaning is provided for the trajectory slope in terms of average step lengths in the rapidity and the impact-parameter directions. The smallness of the Pomeron slope, $\alpha_P'/\alpha_0' \sim 0.3$, is shown to represent a nontrivial constraint for theories satisfying the requirement of short-range ordering; a topological phase consideration is shown to be the primary mechanism responsible for this phenomenon. The relation between our finding to the naive expectation $\alpha_P'/\alpha_0' \simeq 1/2$ based on a string picture and to the general phenomenon of the f/P identity is clarified.

I. INTRODUCTION

The topological expansion[1,2] based on the $N_c \to \infty$ limit has been advanced as a promising calculational scheme for gauge theories like quantum chromodynamics (QCD), where, one hopes, in the leading order quarks and antiquarks are permanently bound to form *zero-width* mesons. The dual topological program[2-9] which is motivated by a $1/N$ expansion with N_c, $N_f \to \infty$, and N_c/N_f fixed, on the other hand, has also been advocated recently as a unified approach to the study of soft hadronic phenomena.

Common to all topological-expansion (TE) approaches to strong interactions is the basic requirement that the planar component of the S matrix, i.e., the leading order of the expansion, must resemble closely the observed low-energy spectrum, e.g., the existence of a vector-tensor nonet of Regge families of particles with a Regge intercept $\alpha_0 \sim \frac{1}{2}$ and a slope $\alpha_0' \sim 1$ GeV^{-2}. One major advantage of a dual topological approach is the admission of resonance widths at the planar level, consistent with the remarkably successful Okubo-Zweig-Iizuka (OZI) rule for particle decays. Unlike the $1/N_c$ expansion for QCD, internal quark loops are not suppressed in a dual TE approach, leading to a unitarized OZI rule already at the leading order of the expansion. Furthermore, the presence of internal quark loops allows a simultaneous description of an elastic amplitude and, through a unitarity cut, particle production. Therefore, it is much more appropriate for making contact with high-energy reactions where soft hadronic production predominates. However, due partly to the difficulty of handling fermion degrees of freedoms, not much progress has been made in exploring consequences of the dual TE by directly working with the QCD Lagrangian. Nevertheless, because of planar unitarity, together with duality arguments, consistency relations among physical observables can be deduced, in the spirit of the analytic S-matrix approach to hadron dynamics.[3]

In this paper we study the structure of the dual TE up to the cylinder level by concentrating on the determination of Reggeon and Pomeron slopes. A *precise* formulation for the generation of Regge behavior in terms of an effective random walk is presented, and a well-defined meaning is provided for the relation

$$\alpha_0' = B_0/2L_0 , \tag{1.1}$$

where L_0 and $B_0^{1/2}$ are average planar elementary step lengths in the rapidity and impact-parameter directions, respectively. By analyzing the impact-parameter structure of a planar absorptive amplitude, (1.1) allows us to relate the Regge slope α_0' to dimensional parameters characterizing the multiparticle amplitude-phase structure and their $|p_\perp|$ distribution, i.e., those which define what is meant by soft hadronic phenomena. We next carry out an analogous analysis for the Pomeron sector; we identify the mechanism responsible for the smallness of the Pomeron slope,

$$\alpha_P'/\alpha_0' \sim 0.3 \tag{1.2}$$

(which, as we shall point out shortly, is a rather nontrivial result).

One major triumph of the TE approach to soft hadronic phenomena has been the observation[7] of Lee and Veneziano that the pattern of Regge intercepts at $t=0$ ($\alpha_P \sim 1$, $\alpha_0 \sim \frac{1}{2}$) can be understood by a general topological analysis together with a positivity requirement. However, the determination of Regge slopes must involve additional inputs. This can best be contrasted with the study of QCD where, at the classical level, the theory does not contain any dimensional parameter. Whereas Regge intercepts are "effective spins," which are dimensionless, a dimensional parameter such as

α' can only emerge through "dimensional transmutation" in the course of performing a renormalization (and, of course, also after the color-confinement mechanism is properly identified).

In a dual topological approach in terms of quark lines, color confinement is built-in and the dynamics is controlled by the nonlinear planar unitarity. At $t=0$ under a general factorization assumption for production amplitudes, a dynamical scheme for Regge intercepts can be constructed where only integrated dimensionless couplings are involved. In the nonforward limit, however, dimensional parameters must be introduced, e.g., to characterize the $p_\perp$ distribution. Therefore, a determination of α_0' corresponds to finding precise relations such as (1.1) which relate dimensional parameters of the problem (e.g., α_0' and $\langle p_\perp{}^2\rangle$). Past dynamical calculations, though crude, indicate that consistency at this level can indeed be achieved.[4,8,9]

However, divergent views exist on the proper treatment of cylinder diagrams at the nonforward limit, leading to a variety of schemes for generating the Pomeron. One popular mechanism has been Veneziano's two independent $q\bar{q}$ jets picture.[2,10,11] This mechanism is suggested by a string picture where the Pomeron is to be associated with a closed string. If a string $q\bar{q}$ jet, i.e., the planar structure, corresponds to a random walk, the Pomeron would then be generated by a simultaneous "double random walk." For instance, to reproduce (1.1), we need only to consider a random walk with an elementary step distribution

$$\tilde{\omega}(y,\vec{b}) \sim \frac{1}{\pi L_0 B_0}\, e^{-y/L_0} e^{-\vec{b}^2/B_0}\theta(y). \tag{1.3}$$

A straightforward analysis would then lead to a "Pomeron" with a slope[5]

$$\alpha_P' \sim \tfrac{1}{4}(B_0/L_0) \sim \tfrac{1}{2}\alpha_0', \tag{1.4}$$

a phenomenologically acceptable result. However, it is also easy to show that the Pomeron so generated actually is a logarithmic branch point in the J plane.[5] Since one expects the Pomeron to be a pole at the cylinder level, both from theoretical and phenomenological considerations, one apparently must remove this extremely simple mechanism from directly being responsible for a small Pomeron slope. (For completeness, the essential arguments involved are summarized in Appendix A.)

By insisting on the Pomeron being a pole, we can again analyze its generation by an effective random walk leading to a relation

$$\alpha_P' = B_P/2L_P, \tag{1.5}$$

where L_P and B_P are average *nonplanar* elementary step lengths. To provide a qualitative estimate, it is perhaps reasonable to at first expect

$$B_P \simeq B_0 \tag{1.6}$$

and

$$L_P \simeq \tfrac{1}{2} L_0. \tag{1.7}$$

The later relation (1.7) is suggested by the fact that a cylinder topology corresponds to a higher-multiplicity event than a planar topology would, and a factor of 2 follows from a rough estimate based on an analysis of the Lee-Veneziano type.[7] Substituting (1.6) and (1.7) into (1.5), one arrives at $\alpha_P' \simeq 2\alpha_0'$, in gross disagreement with the empirical result (1.2).

It is our contention that, in the dual TE, when effective random walks[12,13] responsible for generating α_0' and α_P' are constructed properly, both (1.6) and (1.7) are false. We demonstrate that the importance of both clustering and Regge behavior for particle production leads to contributions

$$B_0 = B_0^c + B_0^h, \quad L_0 = L_0^c + L_0^h, \tag{1.8}$$

$$B_P = B_P^c + B_P^h, \quad L_P = L_P^c + L_P^h, \tag{1.9}$$

where superscripts c and h stand for "cluster mass distribution" and "helicity pole," respectively. By explicitly constructing multiperipheral (MP) cluster kernels, we obtain

$$\alpha_0' = B_0^c/(2L_0) + (L_0^h/L_0)\alpha_c'(0), \tag{1.10}$$

$$\alpha_P' = B_P^c/(2L_P) + (L_P^h/L_P)\alpha_c'(0), \tag{1.11}$$

where $\alpha_c'(0) = \frac{1}{2}\alpha_0'$ is the slope of a two-Reggeon cut. We next identify the primary mechanisms leading to consistency for (1.10) on the one hand and leading to a much smaller value for α_P' in (1.11) on the other.

In Sec. II we first explain how Regge behavior can be viewed as a diffusion process with the Regge slope serving as the "diffusivity". A microscopic random-walk model is then constructed allowing us to express α' in terms of microscopic parameters.[12] In Sec. III we demonstrate how a general MP cluster production model can always be recast in the form of a Markovian walk.[13] An effective random walk is then contructed out of a Markovian walk in Sec. IV, which allows us to calculate the Regge slope directly. In Sec. V properties of the elementary density for our effective random walk is investigated for both a general MP cluster model and for the case of a factorizable kernel. Our discussion up to this point applies to both $\alpha_0(t)$ and $\alpha_P(t)$, so that no subscripts 0 and P are used, and $\alpha(t) \equiv \alpha(0) + \alpha' t$. In Sec. VI we recast the dual TE dynamics in terms of our effective random-walk language and the resulting

structures for α_0' and α_P' are discussed in Sec. VII.

II. POLE DOMINANCE AS A RANDOM WALK

Under a Regge-pole-dominance assumption, an absorptive amplitude near the forward limit can be represented by

$$A_{i,j}(s,t) \sim \beta_i(t) s^{\alpha(t)} \beta_j(t), \tag{2.1}$$

where, for $|t|$ small, $\alpha(t) \sim \alpha(0) + \alpha' t$ and $\beta(t)$ is peaked. In an impact-parameter representation, (2.1) becomes

$$\tilde{A}_{i,j}(y,\vec{b}) \sim \int d\vec{b}'\, d\vec{b}''\, \tilde{\beta}_i(\vec{b}') \times e^{\alpha(0)y} P(y, \vec{b} - \vec{b}' - \vec{b}'')\, \tilde{\beta}_j(\vec{b}''), \tag{2.2}$$

with

$$P(y,\vec{b}) = (e^{-\vec{b}^2/4\alpha' y})/4\alpha' y, \tag{2.3}$$

where $y \simeq \ln s$ is the rapidity. Equation (2.2) formally corresponds to a signal propagation in the $(y,\vec{b})$ space where $e^{\alpha(0)y} P(y,\vec{b})$ is the propagator and $\tilde{\beta}_i$'s serve as the sources. From (2.3), one verifies that P satisfies a conventional two-dimensional diffusion equation:

$$(\partial_y - \alpha' \nabla_b^2) P(y,\vec{b}) = \delta(y)\delta^2(\vec{b}), \tag{2.4}$$

α' being the macroscopic diffusivity. The formal construct suggests that this macroscopic phenomenon can be explained in terms of microscopic local dissipations and α' can be related to microscopic parameters.

To provide an explicit realization, next we introduce a microscopic model in which (2.3) becomes valid when y and $|\vec{b}|$ are larger than certain microscopic parameters to be specified below. Consider a random-walk process in a three-dimensional $(y,\vec{b})$ space; let $\tilde{\omega}(y,\vec{b})$ be a normalized elementary probability density for the one-step distribution:

$$\int \tilde{\omega}(y,\vec{b})\, dy\, d^2\vec{b} = 1. \tag{2.5}$$

We assume axial symmetry so that $\tilde{\omega}(y,\vec{b})$ is a function of $\vec{b}^2$ only and we also assume that $\tilde{\omega}(y,\vec{b}) \propto \theta(y)$. We assume further that $\omega(y,\vec{b})$ is rapidly convergent so that

$$L = \int y\tilde{\omega}(y,\vec{b})\, dy\, d^2\vec{b}, \tag{2.6}$$

$$B = \int \vec{b}^2 \tilde{\omega}(y,\vec{b})\, dy\, d^2\vec{b} \tag{2.7}$$

are well defined, e.g., that given by (1.3).

Let $P_n(y,\vec{b})$ be a probability density corresponding to having traveled to the point $(y,\vec{b})$ from an origin in n steps; it can then be evaluated recursively from $\tilde{\omega}(y,\vec{b})$. We next define a total density

$$P(y,\vec{b}) = \sum_{n=1}^{\infty} P(y,\vec{b}), \tag{2.8}$$

which then satisfies an integral equation

$$P(y,\vec{b}) = \tilde{\omega}(y,\vec{b}) + \int dy'\, d^2\vec{b}'\, \tilde{\omega}(y',\vec{b}') P(y-y', \vec{b}-\vec{b}'). \tag{2.9}$$

Expanding $P(y-y', \vec{b}-\vec{b}')$ in the integrand as a power series in y' and $\vec{b}'$, and making use of (2.5), (2.6), and (2.7), one arrives at

$$[\partial_y - (B/2L)\nabla_b^2] P(y,\vec{b}) \simeq 0. \tag{2.10}$$

Corrections to (2.10) involve higher derivatives of $P(y,\vec{b})$, which are unimportant in the limit $y \gg L$, $\vec{b}^2 \gg B$. Comparing (2.10) with (2.4), one obtains

$$\alpha' = B/2L, \tag{2.11}$$

which is precisely the desired relation between our macroscopic and microscopic parameters.

III. MULTIPERIPHERAL MECHANISM AS MARKOVIAN WALK

Analyses of experimental data in the early seventies have revealed that the essential features of high-energy production processes can be understood in terms of a multiperipheral (MP) cluster production picture.[14] The key ingredients involved are local rapidity correlation and limited momentum transfers to neighboring clusters. These assumptions are fairly standard, and, through an inelastic unitarity relation, an overlapping-function calculation then leads to Regge behavior for the elastic absorptive part. It then seems possible to identify the micromodel introduced earlier with the actual cluster production process. If true, $P(y,\vec{b})$ in (2.8) can be related to the Fourier transform of the absorptive amplitude $A(s,t)$, and $P_n(y,\vec{b})$, when integrated over $\vec{b}$, becomes the cluster multiplicity distribution. For the moment, we keep our discussion general without distinguishing between planar and Pomeron sectors. Therefore, a Regge function $\alpha(t) = \alpha(0) + \alpha' t$ can apply to either $\alpha_0(t)$ or $\alpha_P(t)$.

Consider a standard MP equation (see Fig. 1),

$$A(p,p_0;Q) = C(p,p_0;Q) + \frac{2}{(2\pi)^4} \int d^4p'\, C(p,p';Q) R(p,p',p_0;Q) \times A(p',p_0;Q), \tag{3.1}$$

where the kernel is allowed to have a p_0 dependence

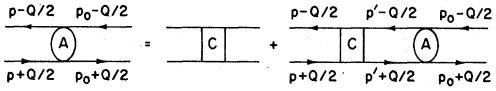

FIG. 1. Multiperipheral integral equation.

appropriate for a multi-Regge cluster model. Take Q to be a spacelike two-dimensional Euclidean vector $(0;\vec{q},0)$ so that we can occasionally replace Q by $\vec{q}$ where $Q^2 \equiv t = -\vec{q}^2$. Equation (3.1) can be expressed in terms of standard momentum-transfer invariants

$$t_\pm = (p \pm Q/2)^2\,, \quad t'_\pm = (p' \pm Q/2)^2\,, \quad t^0_\pm = (p_0 \pm Q/2)^2 \tag{3.2}$$

and "energy invariants"

$$s = (p - p_0)^2\,, \quad s' = (p' - p_0)^2\,, \quad M_1{}^2 = (p - p')^2\,. \tag{3.3}$$

However, to obtain a simpler expression for the phase space, we introduce, in place of $t_\pm$, $t'_\pm$, and $t^0_\pm$,[15,16]

$$u \equiv p^2\,, \quad u' \equiv p'^2\,, \quad u_0 \equiv p_0{}^2$$
$$\cos\phi \equiv z \equiv [p\cdot Q/(ut)^{1/2}], \quad \cos\phi' \equiv z' \equiv [p'\cdot Q/(u't)^{1/2}], \quad \cos\phi_0 \equiv z_0 \equiv [p_0\cdot Q/(u_0 t)^{1/2}] \tag{3.4}$$

and three rapidity variables $y \sim \ln s$, $y' \sim \ln s'$, $\eta' \sim \ln M_1{}^2$. In terms of these variables (3.1) can be rewritten as (see Appendix B)

$$A_{ij}(y;\vec{q}) = C_{ij}(y;\vec{q}) + \sum_k \int dy' K_{ik}(y-y';\vec{q}) A_{kj}(y';\vec{q})\,, \tag{3.5}$$

where

$$K_{ik}(y;\vec{q}) \equiv \int_0^y d\eta' \int_0^y d\xi' \delta(y-\eta'-\xi') e^{-\xi'} \times C_{ik}(\eta';\vec{q}) R_k(\xi';\vec{q})\,. \tag{3.6}$$

In (3.5) and (3.6), (u,ϕ), (u_0,ϕ_0), and (u',ϕ') have been replaced by a set of discrete indices i,j,k, where a discrete sum over k represents

$$\sum_k \to (1/32\pi^2) \int_{-\infty}^0 du' \int_0^{2\pi} d\phi'\,. \tag{3.7}$$

Introducing two-dimensional Fourier transforms $\tilde{A}_{ij}(y,\vec{b})$, $\tilde{C}_{ij}(y,\vec{b})$, and $\tilde{K}_{ij}(y,\vec{b})$, e.g.,

$$\tilde{A}_{ij}(y,\vec{b}) \equiv \int \frac{d^2\vec{q}}{2\pi} e^{-i\vec{q}\cdot\vec{b}} A_{ij}(y;\vec{q})\,, \tag{3.8}$$

Eq. (3.5) becomes

$$\tilde{A}_{ij}(y,\vec{b}) = \tilde{C}_{ij}(y,\vec{b}) + \sum_k \int dy'' d\vec{b}' \tilde{K}_{ik}(y'',\vec{b}') \tilde{A}_{kj}(y-y'',\vec{b}-\vec{b}')\,, \tag{3.9}$$

and now it resembles Eq. (2.9). However, because of its matrix nature, instead of a random walk, (3.9) can be shown to lead to a Markovian walk.[17] To complete this identification, we must construct an elementary step distribution $\tilde{\omega}_{ij}(y,\vec{b})$ where

$$\sum_i \int dy\, d\vec{b}\, \omega_{ij}(y,\vec{b}) = 1\,. \tag{3.10}$$

Equation (3.10) is a generalization of (2.8) and it must hold independent of j, i.e., $\tilde{\omega}_{ij}(y,\vec{b})$ being a probability density for taking a step of type i, after having previously taken a step of type j, must be asymmetric in i and j. Once $\tilde{\omega}_{ij}$ is constructed, a total probability density $P_{ij}(y,\vec{b})$ can then be found by solving an integral equation.

The primary difference between the solution of (3.9) and the desired probability density $P_{ij}(y,\vec{b})$ will be their asymptotic behavior in y; this suggests that $\tilde{\omega}_{ik}(y,\vec{b})$ is directly proportional to $\tilde{K}_{ik}(y,\vec{b})$ with a proportionality factor independent of $\vec{b}$. To determine this factor, it is sufficient to integrate (3.9) over $\vec{b}$, which, by the definition of inverse Fourier transform, is equivalent to treating the original integral equation (3.5) in the forward limit, $\vec{q}=0$.

Performing a partial-wave analysis at $\vec{q}=0$ by

$$A^J_{ij} \equiv \int_0^\infty dy\, e^{-Jy} A_{ij}(y;0)\,, \tag{3.11}$$

Eq. (3.5) at $\vec{q}=0$ can be diagonalized:

$$A^J_{ij} = C^J_{ij} + \sum_k K^J_{ik} A^J_{kj}\,. \tag{3.12}$$

Regge behavior for $A_{ij}(y;0)$ at large y

$$A_{ij}(y;0) \sim \beta_i e^{\alpha(0)y} \beta_j$$

follows if and only if there is a nonzero solution for the corresponding homogeneous equation

$$\beta_i = \sum_k K^{\alpha(0)}_{ik} \beta_k\,. \tag{3.13}$$

This is the central result of any $t=0$ MP dynamics—once the kernel $K_{ij}(y;\vec{q}=0)$ is known, $\alpha(0)$ can be found by solving (3.13) as an eigenvalue problem. As a by-product, we also obtain the forward Regge residue, playing the role of a wave function.

In general, $K^J_{ik} \neq K^J_{ki}$, however, it is sufficient for our purpose to consider the case

$$K^J_{ik} = S^J_{ik} f^J_k, \quad S^J_{ik} = S^J_{ki}. \tag{3.14}$$

It follows that the left eigenfunction of $K^{\alpha(0)}_{ik}$ is $\beta_i f^{\alpha(0)}_k$, i.e.,

$$\sum_i \beta_i f^{\alpha(0)}_i K^{\alpha(0)}_{ik} = \beta_k f^{\alpha(0)}_k. \tag{3.15}$$

It is then straightforward to show that if $\tilde{\omega}_{ij}(y, \vec{b})$ is given by

$$\tilde{\omega}_{ij}(y, \vec{b}) \equiv \beta_i f^{\alpha(0)}_i \tilde{K}_{ij}(y, \vec{b}) / (\beta_j f^{\alpha(0)}_j e^{\alpha(0)y}), \tag{3.16}$$

it satisfies the desired normalization condition (3.10).

As anticipated, $\tilde{\omega}_{ij}(y, \vec{b})$ and $\tilde{K}_{ij}(y, \vec{b})$ are related by factors which depend only on the solution of the MP dynamics in the forward limit, this independent of $\vec{b}$.

IV. EFFECTIVE RANDOM WALK AND REGGE SLOPE

We have just seen that when the transverse degree of freedom is treated properly, an MP mechanism can be cast in the form of a Markov process. We now demonstrate that, starting from (3.16), an effective random walk can be defined so that we can again evaluate Regge slopes by intuitively simple representations (1.1) and (1.5).

Define a probability density matrix

$$P_{ij}(y, \vec{b}) = \beta_i f^{\alpha(0)}_i \tilde{A}_{ij}(y, \vec{b}) \beta_j f^{\alpha(0)}_j e^{-\alpha(0)y}. \tag{4.1}$$

It satisfies an equation analogous to (3.8) with $\tilde{\omega}_{ik}(y, \vec{b})$ as the kernel. It then follows from (2.2) that, at large y and $\vec{b}$, $P(y, \vec{b})$ satisfies (2.9) with

$$\tilde{\omega}(y, \vec{b}) = \sum_i \sum_k \tilde{\omega}_{ik}(y, \vec{b}) \beta_k^2 f^{\alpha(0)}_k \Big/ \Big(\sum_i \beta_i^2 f^{\alpha(0)}_i \Big), \tag{4.2}$$

so that the analysis of Sec. II readily applies. Note, in particular, because of (3.15), $\tilde{\omega}(y, \vec{b})$ is properly normalized

$$\int \tilde{\omega}(y, \vec{b}) dy\, d\vec{b} = 1, \tag{4.3}$$

so that at large y and $|\vec{b}|$, $P(y, \vec{b})$ satisfies the diffusion equation (2.10), with L and B retaining their proper interpretation as average elementary step lengths for the distribution (4.2).

Defining an integral transform

$$\omega(\lambda, t) \equiv \int_0^\infty dy \int \frac{d^2b}{2\pi} e^{\lambda y} e^{i\vec{b}\cdot\vec{q}} \tilde{\omega}(y, \vec{b}), \tag{4.4}$$

it clearly serves as the generating function for moments of y and $\vec{b}^2$ with respect to $\tilde{\omega}(y, \vec{b})$. It is also clear from (3.16) that this transform is directly expressible in terms of partial-wave projection of the kernel (3.11), generalized to $t \neq 0$. That is, if we define

$$K(J, t) \equiv \Big(\sum_{ij} \beta_i f^{\alpha(0)}_i K_{ij}(J, t) \beta_j \Big) \Big/ \Big(\sum_k \beta_k^2 f^{\alpha(0)}_k \Big), \tag{4.5}$$

it follows from (3.16), (4.4), and (4.5) that

$$\omega(\lambda, t) = K(\alpha(0) - \lambda, t), \tag{4.6}$$

$$L = \partial_\lambda \omega(\lambda, 0)\big|_{\lambda=0} = -\partial_J K(J, 0)\big|_{J=\alpha(0)}, \tag{4.7}$$

$$B = 2\partial_t \omega(0, t)\big|_{t=0} = 2\partial_t K(\alpha(0), t)\big|_{t=0}. \tag{4.8}$$

V. ELEMENTARY DENSITY IN MP CLUSTER MODEL

A. General analysis

As we have emphasized before, our ability to diagonalize Eq. (3.6) by an appropriate O(2, 1) partial-wave analysis follows from the intrinsic short-range correlation property of a MP cluster integral equation. Although we have grossly simplified our actual group analysis, it is nevertheless correct to be able to write in general the kernel as a product

$$K_{ik}(J, t) = C_{ik}(J, t) H_k(J, t), \tag{5.1}$$

where C_{ik} is a transform for a cluster-mass distribution and H_k corresponds to a helicity-pole contribution. Furthermore, the cluster contribution can also be written as

$$C_{ik}(J, t) = S_{ik}(J, t) g_k(J, t), \tag{5.2}$$

where $S_{ij} = S_{ji}$. In particular, $g_j(J, t)$ contains information on the phase structure of multiparticle amplitudes. If we next define

$$f_k(J, t) = g_k(J, t) H_k(J, t), \tag{5.3}$$

(5.1) is then in agreement with our earlier notation (3.14) at $t = 0$.

Given a function O_{ij}, we can define an average

$$\langle O \rangle \equiv \sum_{i,j} [O_{ij} \beta_i f^{\alpha(0)}_i K_{ij}(\alpha(0), 0) \beta_j] \Big/ \Big(\sum_k \beta_k^2 f^{\alpha(0)}_k \Big). \tag{5.4}$$

With this notational simplification, (4.7) and (4.8) can be rewritten as

$$B = B^c + B^h, \quad L = L^c + L^h, \tag{5.5}$$

where

$$\begin{aligned} B^c &= 2\partial_t \langle \ln C(\alpha(0), t) \rangle_{t=0}, \\ B^h &= 2\partial_t \langle \ln H(\alpha(0), t) \rangle_{t=0}, \\ L^c &= -\partial_J \langle \ln C(J, 0) \rangle_{J=\alpha(0)}, \\ L^h &= -\partial_J \langle \ln H(J, 0) \rangle_{J=\alpha(0)}. \end{aligned} \tag{5.6}$$

Equation (5.6) is the key result of our analysis. We emphasize that to be able to evaluate (5.6), only the dynamical equation (3.13) at $t=0$ needs to be solved. Once Eqs. (5.6) have been evaluated, we can parametrize our elementary one-step random-walk density as

$$\tilde{\omega}(y,b)\sim(1/\pi LB)e^{-y/L}e^{-b^2/B}\theta(y) \tag{5.7}$$

for the purpose of generating α' and $P(y,\vec{b})$.

B. Factorizable kernel

Equation (3.13), in general, can only be solved numerically. Fortunately, for models of interest to us, the use of a duality argument allows us to perform a factorizable approximation, thus allowing us to solve (3.13) analytically. In particular, we shall shortly consider kernels of the form

$$C_{ik}(J,t)=\beta_i(t)\beta_k(t)g_k(J,t)\,. \tag{5.8}$$

For $t=0$, the eigenvalue condition (3.13) then reduces to

$$\sum_i \beta_i^2(0)f_i^{\alpha(0)}=1\,, \tag{5.9}$$

where we have made use of (5.3). Substituting (5.2) and (5.8) into (5.4) and making use of (5.9), we are led to a general expression for an average of a tensorial quantity $O_{i_1,i_2,\ldots}$:

$$\langle O\rangle\equiv\sum_{i_1,i_2,\ldots}\left(\prod_{l=1}^{n}(\beta_{i_l}^2 f_{i_l}^{\alpha(0)})\right)O_{i_1,i_2,\ldots}\,. \tag{5.10}$$

In particular, from (5.6) and (5.8), contributions to B and L from the cluster mass distribution are

$$B^c=4\partial_t\langle\ln\beta(t)\rangle_{t=0}+2\partial_t\langle\ln g(\alpha(0),t)\rangle_{t=0} \tag{5.11}$$

and

$$L^c=-\partial_J\langle\ln g(J,0)\rangle_{J=\alpha(0)}\,. \tag{5.12}$$

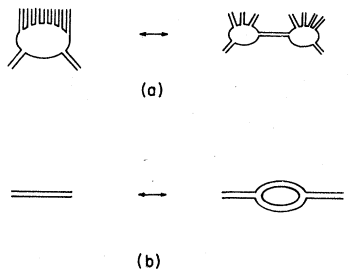

FIG. 2. (a) Planarity, (b) bootstrap condition.

VI. DUAL TOPOLOGICAL EXPANSION

We have shown in Sec. V. that Regge slopes can be found simply once (i) the kernel of MP cluster model is specified and (ii) the $t=0$ eigenvalue problem for the leading Regge pole is solved. However, our approach, up to this point, remains unsatisfactory due to the theoretical uncertainty on the structure of the MP kernel, i.e., both the phase structure and the dynamical origin of clustering are unspecified, and the strength of interaction remains unconstrained.

When we adopt the dual TE viewpoint, the situation is greatly improved. First of all, whereas clustering is an intrinsic feature in a dual model, the phase structure is specified by the topology. Furthermore, the strength of particle production can be constrained by a planar bootstrap condition.

In terms of quark-line diagrams, it is easy to see that when a planar cluster is subdivided into two subclusters, they again are planar [Fig. 2(a)]. When planar duality is applied to each subcluster, a planar bootstrap condition emerges [Fig. 2(b)]. This condition can be realized in terms of conventional invariants as[3,5,8,9]

$$\frac{N_f}{16\pi^2}\int dt_+'dt_-'\,\frac{[-\Delta(t,t_+',t_-')]^{1/2}\theta(-\Delta)}{[\alpha_0(t)-\alpha_c(t_\pm',t)]^2}\,\beta^2(t;t_+',t_-')\cos\{\pi[\alpha_0(t_+')-\alpha(t_-')]\}=1\,, \tag{6.1}$$

where N_f is the number of flavors, $\beta(t;t_+',t_-')$ is the triple-Regge coupling, and

$$\begin{aligned}\alpha_c(t_\pm',t)&\equiv\alpha_0(t_+')+\alpha_0(t_-')-1\\&\simeq(2\alpha_0-1)+(t_+'+t_-')\alpha_0'\,.\end{aligned} \tag{6.2}$$

A similar expression can also be written down for the Pomeron sector. It is our present purpose to clarify the structure of (6.1) near $t=0$ and that for the Pomeron from the viewpoint of an effective random walk introduced in Sec. IV.

A. MP kernel for planar sector

The process of planar subdivision illustrated by Fig. 2(b) can be continued indefinitely so that a recursive relation can be obtained, symbolically represented by $A_n\sim A_1\times\Phi\times R\times A_{n-1}$, where Φ is a phase factor, R is a propagator, and n denotes the number of subclusters. However, because of duality, the subscripts are redundant, provided that the mass M_1 of the A_1 cluster is limited so as to avoid double counting, i.e.,

$$A\sim A_1\times\Phi\times R\times A\,, \tag{6.3}$$

with $A_1\sim A\theta(s_0-M_1^2)$, for some fixed value s_0. Equation (6.3) is identical in structure to (B2), thus allowing us to identify its kernel as

$$\begin{aligned}K^0_{ik}(\eta',\xi';t)&=A_{ik}(\eta';t)\Phi_k(t)R_k(\xi';t)\\&\quad\times\Theta(\cosh\eta_0-\cosh\eta')\,,\end{aligned} \tag{6.4}$$

where we have switched from using conventional invariants to those introduced in (3.4) and (B1), and we have also replaced the cutoff in M_1^2 by one for $\cosh\eta'$.

For large ζ', $R_k(\zeta';t)$ is power behaved in $e^{\zeta'}$, the usual leading helicity-pole approximation corresponds to keeping[15]

$$R_k(\zeta';t)\sim e^{[\alpha_0(t'_+)+\alpha_0(t'_-)]\zeta'}=e^{[\alpha_c(t;k)+1]\zeta'} \tag{6.5}$$

and a phase factor

$$\Phi_k(t)\sim e^{i\pi[\alpha_0(t'_+)-\alpha_0(t'_-)]}. \tag{6.6}$$

We next integrate out ζ' and η', with our asymptotic constraint $y\sim\eta'+\zeta'$, and we obtain

$$K^0_{ik}(y,t)=\left[\int_0^{\eta_0}d\eta' A_{ik}(\eta',t)e^{-\alpha_c(t;k)\eta'}\right]\times\Phi_k(t)H_k(y;t), \tag{6.7}$$

where $H_k(y;t)\equiv e^{-y}R_k(y;t)$. At large η',

$$A_{i,k}(\eta',t)\sim\beta_i(t)e^{\alpha_0(t)\eta'}\beta_k(t);$$

we assert that a moderate value of η_0 can be chosen so that the quantity in square brackets in (6.7) can be replaced by its asymptotic value. Under this *duality* assumption, the planar kernel becomes factorizable, and its Laplace transform is given by

$$K^0_{ik}(J,t)=C^0_{ik}(t)H_k(J,t), \tag{6.8}$$

where

$$C^0_{ik}(t)=\beta_i(t)\beta_k(t)g^0_k(t), \tag{6.9}$$

$$g^0_k(t)\simeq[\Phi_k(t)][\tfrac{1}{2}e^{(\alpha(t)-\alpha_c(t;k))\eta_0}]/[\alpha(t)-\alpha_c(t;k)], \tag{6.10}$$

and the helicity-pole propagator is

$$H_k(J,t)=[J-\alpha_c(t;k)]^{-1}. \tag{6.11}$$

Note that the only J dependence of $K^0_{ik}(J,t)$ is contained in $H_k(J,t)$. In what follows, we shall also simplify the discussion by setting the quantity in brackets in (6.10) containing the exponential term to be unity in order to be compatible with (6.1).[3,8,9]

B. Effective planar random walk

With (6.11), the eigenvalue condition at $t=0$ [Eq. (5.9)] becomes

$$\sum_i \beta_i^2(0)/[\alpha_0-\alpha_c(0;i)]^2=1. \tag{6.12}$$

An effective planar random walk can then be introduced with an elementary density $\tilde\omega_0(y,\vec b)$ given by (4.2) after appropriate substitutions, e.g., $\alpha(0)\to\alpha_0$. We can now define a "planar average" with respect to $\tilde\omega_0(y,\vec b)$ for a function $O_i(t)$:

$$\langle 0|O(t)|0\rangle\equiv\sum_i O_i(t)\beta_i^2(0)/[\alpha_0-\alpha_c(0;i)]^2 \tag{6.13}$$

and, in terms of invariants,

$$\langle 0|O(t)|0\rangle=\frac{N_f}{32\pi^2}\int_0^{2\pi}d\phi\int_{-\infty}^{0}du\,\frac{\beta^2(0;u,u)}{\{\alpha_0-[2\alpha_0(u)-1]\}^2}\times O(t;u,\phi), \tag{6.14}$$

where an extra factor of N_f associated with each loop integration has now been inserted. The eigenvalue condition $\langle 0|0\rangle=1$, when expressed in the form (6.14), is in exact agreement with the $t\to 0$ limit of (6.1).

The calculation of α_0' will be discussed in the next section, and for now we illustrate the structure of the planar random walk by calculating the average elementary step length in the rapidity direction. In general, we expect contributions from both C and H in (6.8); however, since the only J dependence of $K^0_{ik}(J,t)$ is contained in $H_k(J,t)$, we immediately obtain, upon applying (5.6) for the case of (6.13),

$$L_0=L_0^c+L_0^h,\quad L_0^c=0 \tag{6.15}$$

and

$$L_0^h=\langle 0|[\alpha_0-\alpha_c(0;i)]^{-1}|0\rangle\simeq\frac{1}{1-\alpha_0}\simeq 2. \tag{6.16}$$

Therefore, for $\alpha_0'\simeq 1\ \mathrm{GeV}^{-2}$, a value of

$$B_0=B_0^c+B_0^h\simeq 4\ \mathrm{GeV}^{-2} \tag{6.17}$$

must be obtained.

C. Pomeron kernel

The absorptive amplitude for the Pomeron sector can also be written in terms of cluster contributions so that

$$A_p\sim A\times R\times A_p. \tag{6.18}$$

Equation (6.18) is analogous to (6.3) with the exceptions that (i) A is the planar amplitude without a mass cutoff and (ii) the phase factor Φ is absent. Therefore, in (6.18), neighboring clusters do not resonate; they clearly correspond to configurations not included in the planar sector.

The Pomeron kernel can be directly obtained from (6.7) by first removing the factor $\Phi_k(t)$ and then setting the upper limit of the η' integral to be y. It then follows after applying a similar duality assumption that

$$K^P_{ik}(J;t)=C^P_{ik}(J,t)H_k(J,t), \tag{6.19}$$

where

$$C^P_{ik}(J,t)=\beta_i(t)\beta_k(t)g^P(J,t), \tag{6.20}$$

$$g^P(J,t)=\frac{1}{J-\alpha_0(t)}. \tag{6.21}$$

Note, in particular, the cluster contribution now represents the dominant J dependence with a pole at $\alpha_0(t)$, to the right of $\alpha_c(t)$, for t small.

D. Effective Pomeron random walk

The eigenvalue condition for the Pomeron pole at $t=0$ now reads as

$$\sum_k \beta_k^2(0) f_k^P(\alpha_P, 0) = 1 , \tag{6.22}$$

where

$$f_k^P(J,t) \equiv g_k^P(J,t) H_k(J,t) = \{[J-\alpha_0(t)][J-\alpha_c(t,k)]\}^{-1} . \tag{6.23}$$

In terms of invariants, (6.22) becomes

$$\frac{N_f}{32\pi^2}\int_0^{2\pi} d\phi \int_{-\infty}^{0} du \frac{\beta^2(0;u,u)}{[\alpha_P-\alpha_0]\{\alpha_0-[2\alpha_0(u)-1]\}} = 1 . \tag{6.24}$$

By comparing (6.24) with (6.14), it is easy to show that $\alpha_P > \alpha_0$ follows, which is the essence of the Lee-Veneziano relation. Equation (6.24) also embodies the phenomenon of f promotion, leading to the P/f identity hypothesis.[6] We assume that the triple-Regge coupling $\beta(0;u,u)$ is such that (6.13) and (6.24) are satisfied with $\alpha_0 \simeq \frac{1}{2}$ and $\alpha_P \simeq 1$.

An effective Pomeron random walk can then be introduced in a straightforward manner, whose elementary density $\tilde{\omega}_P(y;\vec{b})$ can be specified in terms of its transform, defined by (4.4), with

$$\omega_P(\lambda;t) = K_P(\alpha_P - \lambda, t) \tag{6.25}$$

and

$$K_P(J,t) = \sum_{i,k} \beta_i f_i^{\alpha_P} K_{ik}^P(J,t) \beta_k ,$$

where $f_i^{\alpha_P} \equiv f_i^P(\alpha_P, 0)$ and we have made use of (6.22). With respect to $\tilde{\omega}_P(y;\vec{b})$, a Pomeron average of a function $O_i(t)$ can be defined by

$$\langle P|O(t)|P\rangle \equiv \sum_i \beta_i^2 f_i^{\alpha_P} O_i(t) \tag{6.26}$$

and (5.6) can then be used to calculate α_P'. In particular, we find

$$L_P = L_P^c + L_P^h , \tag{6.27}$$

where

$$L_P^c = -\partial_J \langle P|\ln C^P(J,0)|P\rangle_{J=\alpha_P} = (\alpha_P - \alpha_0)^{-1} \simeq 2 , \tag{6.28}$$

$$L_P^h = -\partial_J \langle P|\ln H^P(J,0)|P\rangle_{J=\alpha_P} = \langle P|\{\alpha_P - [2\alpha(u)-1]\}^{-1}|P\rangle \simeq [\alpha_P - 2\alpha_0 + 1]^{-1} \simeq 1 . \tag{6.29}$$

From (6.16), (6.28), and (6.29), we find

$$L_P \gtrsim L_0 , \tag{6.30}$$

a result opposite to the naive expectation (1.7). Finally, we note that to achieve $\alpha_P' \sim 0.3\alpha_0'$ with $L_P \simeq 3$, we must have

$$B_P \sim 2 \text{ GeV}^{-2} . \tag{6.31}$$

When compared with (6.17), one obtains

$$B_P/B_0 \sim \tfrac{1}{2} , \tag{6.32}$$

a result again differing from one's naive expectation (1.6).

VII. WHY IS α_P'/α_0' SMALL?

By inspecting (6.19), (6.20) and (6.21), it is clear that the Pomeron kernel is completely specified once the planar problem is solved. Although our parametrizations for K^0 and K^P are highly simplified, the above observation reflects the fact that in a dual TE approach, one always proceeds in a hierarchical fashion by isolating all nonlinearities in the planar level and then derive nonplanar properties by linear iterations. Therefore, to be able to ask the question posed for this section quantitatively, one must first solve the planar eigenvalue equation e.g., (6.2).

Whereas the triple-Regge coupling $\beta(t;t_+,t_-)$ can in principle be determined by a planar bootstrap,[9] its small $t_\pm$ dependence can be more reliably extracted from inclusive experiments. However, $\beta(t;t_+,t_-)$ is measurable only at $t=0$, where one normally parameterizes

$$\beta(0;u,u) \sim \beta_0 e^{b_0 u} \tag{7.1}$$

with $b_0 \simeq 1$ GeV^{-2}.[9] As we shall see shortly, to be able to calculate α_0' and α_P', we must evaluate $\partial_t \langle 0|\ln\beta|0\rangle_{t=0}$ and $\partial_t \langle P|\ln\beta|P\rangle_{t=0}$. Therefore, at the minimum, we must obtain $\beta(t;t_+,t_-)$ from equations like (6.1) or other bootstrap equations at small t where we can parametrize

$$\beta(t;t_+,t_-) \sim \beta_c(t) e^{b_0(t_+ + t_-)/2} , \tag{7.2}$$

with $\beta_0(0) = \beta_0$. To delay this task (since our current formulation can at best provide semiquantitative results), we first discuss the aspects which are least model dependent before we discuss those requiring a detail knowledge of $\beta(t;t_+,t_-)$.

A. Phase contribution

We have seen that, with $L_0 \simeq 2$ and $L_P \simeq 3$, the constraint $\alpha_P'/\alpha_0' \simeq \frac{1}{3}$ requires [in units of $\alpha_0' = 1$ GeV^{-2}], $B_0 \simeq 4$ GeV^{-2}, $B_P \simeq 2$ GeV^{-2} [Eqs. (6.17) and (6.31)]. Therefore, our task has been reduced to evaluating

$$B_0 = 2\partial_t \langle 0|\ln K^0(\alpha_0, t)|0\rangle_{t=0}, \tag{7.3}$$

$$B_P = 2\partial_t \langle P|\ln K^P(\alpha_P, t)|P\rangle_{t=0}. \tag{7.4}$$

From (6.8)–(6.11) and (6.19)–(6.21) we note that the crucial difference between (7.3) and (7.4) is the addition of a phase factor $\Phi(t)$ in $g_k^0(t)$, whereas the remainders are structurally similar.

The contribution to B_0 from this phase factor is

$$B_0(\phi) \equiv 2\partial_t \langle 0|\Phi(t)/\Phi(0)|0\rangle_{t=0} = 2\partial_t \langle 0|\Phi(t)|0\rangle_{t=0}. \tag{7.5}$$

In terms of invariants,

$$\Phi_t(t) = e^{i\pi\alpha_0'(t_+ - t_-)} = e^{2i\pi\alpha_0'\sqrt{ut}\cos\phi}. \tag{7.6}$$

Substituting (7.6) into (7.5) to evaluate $B_0(\phi)$ explicitly by (6.14), one obtains

$$B_0(\phi) = 2\pi^2 \alpha_0'^2 \langle -u\rangle, \tag{7.7}$$

where

$$\langle -u\rangle = \frac{N_f}{32\pi}\int_{-\infty}^{0} du(-u)\beta^2(0;u,u)/\{\alpha_0 - [2\alpha(u)-1]\}^2. \tag{7.8}$$

Note that to evaluate $\langle -u\rangle$, only the $t=0$ behavior of $\beta(0;u,u)$ is needed. Previous estimates for $\beta(0;u,u)$, e.g., (7.2), would yield a value for $\langle -u\rangle$ around 0.3–0.5 GeV^2, leading to $B_0(\phi) \simeq 6$–9 GeV^{-2}. Therefore, the phase contribution to B_0 is very large, and is the primary source for the difference of B_0 and B_P needed for leading to a ratio $\alpha_P'/\alpha_0' \simeq 0.3$. It is in this sense we claim that the smallness of the Pomeron slope is essentially a topological phenomenon.

We must remark that a value of $B_0(\phi) \simeq 6$–9 GeV^{-2} is in fact such a large contribution that $\partial_t \langle 0|\ln\beta|0\rangle_{t=0}$ is forced to be negative for consistency. This behavior near $t=0$ is intimately tied to the phenomenon of P/f identity.[6] We shall return to this point shortly. We turn next to an investigation on the contributions to α_0' and α_P' from the helicity-pole propagator first.

B. Helicity-pole contributions

The generation of a Regge pole by a MP mechanism can formally be thought of as an infinite iteration of a J-plane singularity of the MP kernel. Within current accepted leading helicity-pole approximation formalism,[15] the basic J-plane singularity is the two-Reggeon cut $\alpha_c(t;t_\pm')$, with $t=0$ slope $\alpha_c'(0) = \frac{1}{2}\alpha_0'$. Therefore, it would not be surprising that both α_0' and α_P', generated by their respective random-walk mechanisms, contain pieces reflecting the presence of the two-Reggeon cut.

With $H_k(J;t)$ given by (6.11), its contributions to B_0 and B_P are

$$B_0^h = 2\partial_t \langle 0|\ln H(\alpha_0, t)|0\rangle_{t=0} = 2\alpha_c' L_0^h \simeq 2 \text{ GeV}^{-2}, \tag{7.9}$$

$$B_P^h = 2\partial_t \langle P|\ln H(\alpha_P, t)|p\rangle_{t=0} = 2C_c' L_P^h \simeq 1 \text{ GeV}^{-2}. \tag{7.10}$$

Combining (7.7) and (7.8) with (6.16), (6.28), and (6.29), we find that the contributions to α_0' and α_P' from the helicity-pole propagator are

$$\alpha_0'(h) = B_0^h/2L_0 = \alpha_c' \simeq \tfrac{1}{2} \text{ GeV}^{-2}, \tag{7.11}$$

$$\alpha_P'(h) = B_P^h/2L_P = (L_P^h/L_P)\alpha_c' \simeq \tfrac{1}{6} \text{ GeV}^{-2}. \tag{7.12}$$

Whereas the Regge slope α_0' receives contribution from α_c' at full strength, the contribution of α_c' to α_P' is suppressed by a factor $(L_P^h/L_P) \simeq \frac{1}{3}$. This suppression factor comes about due to the presence of massive resonating clusters sequentially arranged in the rapidity direction. With this suppression effect, we note that, from (7.11) and (7.12), the helicity-pole contributions, $\alpha_0'(h)$ and $\alpha_P'(h)$, can account for half of the observed values of α_0' and α_P', respectively.

C. α_P'/α_0' and phase contribution

With (7.11) and (7.12), the consistency check for $\alpha_P'/\alpha_0' \simeq \frac{1}{3}$ has now been reduced to demonstrating that the cluster distributions satisfy

$$B_0^c \simeq 2 \text{ GeV}^{-2}, \quad B_P^c \simeq 1 \text{ GeV}^{-2}. \tag{7.13}$$

However, due to the uncertainty on the contributions from $\beta(t;t_+,t_-)$, a direct verification can only be performed after the planar bootstrap is under control quantitatively. Therefore, instead trying to check (7.13) directly, by eliminating the contributions from $\beta(t;t_+,t_-)$, we obtain another estimate for $B_0(\phi)$ in terms of the ratio α_P'/α_0'. We can then establish the overall consistency by showing that with $\alpha_P'/\alpha_0' \simeq \frac{1}{3}$, $B_0(\phi)$ so obtained agrees with the earlier estimate [Eq. (7.7)] $B_0(\phi) \sim 6$–9 GeV^{-2}.

We now go back to our basic equations (1.1) and (1.5). From (6.8)–(6.11), we obtain

$$\alpha_0' = (1/L_0)[\partial_t \langle 0|\ln\beta|0\rangle_{t=0} + \tfrac{1}{4}B_0(\phi)] + \alpha_c'(0), \tag{7.14}$$

where $L_0 \simeq 2$ is given by (6.16). Similarly, from (6.19)–(6.21) we obtain

$$\alpha_P' = (2/L_P)[\partial_t \langle P|\ln\beta|P\rangle_{t=0}] + (L_P^c/L_P)\alpha_0' + (L_P^h/L_0)\alpha_c', \tag{7.15}$$

where L_P^c and L_P^h are given by (6.28). With parametrization (7.2), we have $\partial_t\langle 0|\ln\beta|0\rangle_{t=0} = \partial_t\langle p|\ln\beta|p\rangle_{t=0}$; therefore, we can eliminate them from (7.14) and (7.15). We then arrive at our crucial relation

$$B_0(\phi) = [(2L_0 + L_P^c + 1) - (2L_P)(\alpha_P'/\alpha_0')]\alpha_0'. \tag{7.16}$$

That is, given $\alpha_0 \simeq \frac{1}{2}$ and $\alpha_P \simeq 1$, the consistency requirement for $\alpha_P'/\alpha_0' \simeq \frac{1}{3}$ has been reduced to the compatibility between Eqs. (7.7) and (7.16). With $L_0 \simeq 2$, $L_P^c = 2$, and $L_P \simeq 3$, a value of $\alpha_P'/\alpha_0' \simeq \frac{1}{3}$ corresponds to $B_0(\phi) \simeq 7$ GeV^{-2}, and $\alpha_P'/\alpha_0' \simeq 0$ corresponds to $B_0(\phi) \simeq 9$ GeV^{-2}, in excellent agreement with our early estimate. On the other hand, $\alpha_P'/\alpha_0' \simeq 1$ would lead to a value $B_0(\phi) \simeq 3$ GeV^{-2}, a factor of 2 to 3 times smaller than estimated.

VIII. DISCUSSION

Hadronic phenomena at small transverse momenta have traditionally been grouped into three categories: (i) low-energy spectroscopy, (ii) diffraction scattering, and (iii) particle production. Nearly two decades of efforts have taught us that, up to current machine energies, regularities within each category can be characterized respectively by (1) a family of linear Regge trajectories obey quark-line duality with intercept and slope $\alpha_0 \sim \frac{1}{2}$, $\alpha_0' \sim 1$ GeV^{-2}, (2) a Pomeron pole with $\alpha_P \sim 1$, $\alpha_P' \sim 0.2$–0.3 GeV^{-2}, and (3) a limited $|p_\perp|$ distribution and short-range ordering in rapidity. A well-posed challenge has been to find theoretical links among properties (1)–(3).

Enough encouraging results have been obtained in the dual TE approach so that an attempt has been made to formalize this framework from general S-matrix principles.[3,18] In this approach, in place of the phenomenological groupings (i)–(iii), hadron regularities are organized so as to manifest the topological complexities of the hadron S matrix. Instead of trying to interrelate (1)–(3) as separate pieces of hadronic puzzles, one proceeds in a hierarchical fashion by first understanding planar S-matrix elements, then explaining nonplanar effects by iterations.

Diffraction scattering as the shadow of inelastic production has been a long-held physical picture for the occurrence of a forward elastic peak. Analyses of experimental results in the early seventies have revealed that the essential features of high-energy production processes can be understood in terms of a MP cluster production picture. Phenomenological cluster models can easily be constructed to account for features such as limited $|p_\perp|$, short-range ordering, etc. This phenomenological picture, however, has its limitation. For instance, the dynamical origin of clustering is unknown. In particular, both the strength and the phase of the cluster kernel are unconstrained theoretically.

With the advent of the dual TE, the deficiencies of earlier cluster models have been remedied. Whereas clustering is natural in a dual model, the strength for cluster production is determined by planar unitarity and the phase structure of multiparticle amplitudes is unambiguously specified in terms of topologies. A dynamical equation determining the planar trajectory $\alpha_0(t)$ can be constructed and a particular version corresponds to Eq. (6.1). Under a similar set of assumptions, an equation determining the Pomeron trajectory $\alpha_P(t)$ can be written as

$$(N_f/16\pi^2)\int dt_+' dt_-' \frac{[-\Delta(t,t_+',t_-')]^{1/2}\Theta(-\Delta)}{[\alpha_P(t)-\alpha_0(t)][\alpha_P(t)-\alpha_c(t_\pm')]} \times \beta^2(t;t_+',t_-') = 1. \tag{8.1}$$

Note that from (6.1) and (8.1), both intercepts α_0 and α_P and their slopes α_0' and α_P' can in principle be determined.

We have explained in the Introduction that, in principle, additional dynamical assumptions are necessary in order to calculate Regge slopes than that for their intercepts. It is therefore, in general, useful to have alternative yet equivalent formalisms which help to clarify the essential physics involved. In particular, various technical assumptions have been made in deriving (6.1) and (8.1); it is not at all clear how sensitive are results such as $\alpha_P'/\alpha_0' \simeq 0.3$ to the technical, rather physical assumptions involved.

In this paper, a *precise* formulation for a general MP mechanism as a Markovian walk is presented, from which an effective random walk is constructed for generating Regge behavior. In terms of this random-walk picture, a Regge slope can be expressed as $\alpha' = B/2L$ where $\sqrt{B}$ and L are elementary step lengths in the impact parameter and the rapidity, respectively.

By studying the structure of the dual TE up to the cylinder level, we emphasize that the naive expectations (1.6) and (1.7) are definitely false; the empirical ratio α_P'/α_0' can then be used as a test on the viability of a given dynamical scheme. We next show that once one accepts the multi-Regge hypothesis for production amplitudes, the following decomposition emerges:

$$B_0 = B_0^c + B_0^h, \quad B_P = B_P^c + B_P^h,$$

where superscripts refer to cluster mass distribution and helicity-pole contribution, respectively. A similar decomposition for L_0 and L_P also follows. In particular, under the same assumptions which lead to (6.1) and (6.2), we find

$L_0 \simeq 2$, $L_P \simeq 3$, $B_0^c \simeq B_0^h \simeq 2$ GeV^{-2}, and $B_P^c \simeq B_P^h \simeq 1$ GeV^{-2}.

Independent of the detailed assumptions needed for leading to (6.1) and (6.2), we emphasized that the primary mechanism responsible for $B_0^c > B_P^c$ is the phase structure associated with a planar kernel and the primary mechanism for $B_0^h > B_P^h$ is the presence of larger resonating cluster mass for the Pomeron sector than that of the planar sector. In principle, therefore, our approach can serve as an efficient starting point for making systematic improvement to existing models for the Reggeon and the Pomeron.

Our intuitively simple random-walk picture can easily make contact with a conventional t-channel picture for generating Regge behavior. In a usual integral equation language, the generation of a pole comes about due to an infinite iteration of the singularity of the basic kernel, which in our case, corresponds to a two-Reggeon cut. Therefore, α_0' approaches $\alpha_c'(0)$ in a weak-coupling limit. This is exhibited in (7.12) where α_0' consists of that from $\alpha_c'(0)$ and that from an effective interaction strength. On the other hand, (7.13) indicates that both α_0' and $\alpha_c'(0)$ contribute to α_P', with statistical weights (L_P^c/L_P) and (L_P^h/L_P), respectively. This is not really surprising, being a restatement for the presence of large resonating cluster in a Pomeron kernel.

How reliable is our estimate on the phase contribution $B_0(\phi)$? With $B_0(\phi) \simeq 6$–9 GeV^{-2}, we note that consistency can be achieved only if $\partial_t \langle 0|\ln\beta|0\rangle_{t=0} \simeq -1$ GeV^{-2}, a result which might cause some discomfort for the whole dual TE approach. If the phase structure is modified from that given by (6.6), one can easily obtain a positive value for $\partial_t \langle 0|\ln\beta|0\rangle_{t=0}$. However, it then follows from (7.13) that $\alpha_P' > \alpha_c'(0)$, a phenomenologically unacceptable result. Therefore, once again we have demonstrated the intimate connection between our topological phase structure (6.6) and the smallness of α_P'/α_0'. Furthermore, if we treat the Pomeron as a promoted f_0 trajectory, it follows from (7.13) that the trajectory difference $\Delta\alpha(t) \equiv \alpha_P(t) - \alpha_0(t)$ must increase as t decreases[19] (and is negative). This is then in consonant with the asymptotic planarity requirement[20] that $\Delta\alpha(t) \to 0$ as $t \to +\infty$.

Finally, we remark that our Markovian walk picture can provide a natural language for making contact with attempts to interpret small-$|\vec{p}_\perp|$ physics using parton concepts. In Ref. 11, a tentative identification of valence parton distribution in a dual TE framework was made, which has enjoyed surprising phenomenological successes. Conceptually, however, it remains undesirable since the model utilized a two-independent-chain picture for the Pomeron,[21] which has been shown in Ref. 5 to correspond to a Pomeron being a branch point with slope $\alpha_c'(0)$. However, from (7.10) it is seen that in a treatment where a Pomeron always remains an isolated pole, it is possible to treat the two-chain model as the first term of a systematic expansion for the Pomeron. Whereas the introduction of the parton interpretation is unaffected, it remains to be seen whether the success of Ref. 11 will be improved or weakened in our theoretically more consistent formulations.

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APPENDIX A

In a string model, a meson trajectory and a Pomeron are supposed to correspond to excitations of open and closed strings, respectively; in this picture, Eq. (1.4) is to be expected. To the extent that the $N_c \to \infty$ limit of QCD should closely resemble a string picture, Eq. (1.4) can be expected to hold in this limit as well. However, as we have explained in the Introduction, in a QCD context the relevant limit corresponds to N_c, $N_f \to \infty$ with $N_f/N_c \neq 0$ and fixed; the connection between (1.4) and the empirical relation (1.2) is far from being clear.

With quark-loop insertion allowed, a string can break leading to the production of a jet, where particles produced are short-range correlated. A simple realization for this planar production is a random-walk picture. It has been suggested that the Pomeron configuration corresponds to the production of two independent jets. In Ref. 5, it has been shown that this picture leads to a Pomeron which is actually a J-plane branch point in what follows. The essential reasonings involved are repeated.

A Pomeron as the production of two uncorrelated jets, at first sight, seems to follow from the Lee-Veneziano[7] picture where, under a one-dimensional phase-space approximation, a one-jet production is approximately governed by a Poisson distribution, and a planar trajectory is generated by

$$s^{\alpha_0 - 1} \propto \sum_{n=0}^{\infty} \sigma_n^{(1)} \sim \sum_{n=0}^{\infty} \frac{(g^2 \ln s)^n}{n!}\, s^{2\alpha_0 - 2} \sim s^{2\alpha_0 - 2 + g^2}. \tag{A1}$$

The cross section for a two-jet process, if produced independently, would also obey a Poisson distribution, with $n = 0, 1, 2, \ldots$,

$$\sigma_n^{(2)} \propto \sum_{n_1+n_2=n} \sigma_{n_1}^{(1)} \sigma_{n_2}^{(1)} \propto \sum \frac{(g^2 \ln s)^{n_1+n_2}}{n_1! n_2!} = \frac{(2g^2 \ln s)^n}{n!} . \tag{A2}$$

In going from (A1) to (A2), the average multiplicity has been doubled; this, in turn, leads to the result that if $\alpha_0 \sim \frac{1}{2}$, $\alpha_P \sim 1$.

To determine the precise nature of the Pomeron singularity, we must incorporate the transverse dimension. We shall employ an impact-parameter analysis where each jet is treated as a multiperipheral chain, approximated as a simultaneous random walk in both the impact parameter and the rapidity direction. For simplicity, the elementary one-step distribution is chosen to be Eq. (1.3). The probability density after $n+1$ steps for our one-jet event is, with $n = 0, 1, 2, \ldots$,

$$P_{n+1}^{1\text{-jet}}(y, \vec{b}) = \frac{1}{\pi L_0 B_0} \frac{1}{(4+1)} \frac{(y/L_0)^n}{n!} \times e^{-y/L_0} e^{-b^2/(n+1)B_0} . \tag{A3}$$

[To check that (A3) is correct, we integrate (A3) over $\vec{b}$ to obtain $\sigma_n^{(1)}$; it is found to be in agreement with (A1) with $g^2 = L_0$.] For a two-jet process, with jets uncorrelated, the relative probability for n-cluster production is

$$P_n^{2\text{-jet}}(y, \vec{b}) \propto \sum_{n_1+n_2=n} P_{n_1+1}^{1\text{-jet}}(y, \vec{b}) P_{n_2+1}^{1\text{-jet}}(y, \vec{b}) \tag{A4}$$

and the corresponding production cross section is, for $n = 0, 1, 2, \ldots$,

$$\sigma_n^{(2)}(s) \propto \int P_n^{2\text{-jet}}(y, \vec{b}) \propto \frac{(2 \ln s/L_0)^n}{(n+2)n!} . \tag{A5}$$

From (A3) and (A4), one can verify that

$$\langle \vec{b}^2 \rangle^{(1\text{-jet})} = 2 \langle \vec{b}^2 \rangle^{(2\text{-jet})} , \tag{A6}$$

leading to the interesting result that

$$\alpha_0' \simeq 2\alpha_P' . \tag{A7}$$

However, comparing (A5) with (A2), the presence of the extra $(n+2)$ factor in (A5) renders it non-Poissonian, and the low-multiplicity events are now enhanced.

It was pointed out in Ref. 5 that the proper interpretation for (A5) corresponds to having

$$\sigma_{\text{tot}}^{(2)} \propto \int_{-\infty}^{0} dg^2 s^{\alpha_P + g^2 - 1} \sim s^{\alpha_P - 1}/\ln s ,$$

that is, α_P so generated is actually a branch point. Furthermore, $\alpha_P = 2\alpha_R - 1$, so that α_P is actually the Reggeon-Reggeon cut; therefore, (A7) is fortuitous.

APPENDIX B

In addition to (3.4), we introduce O(2, 1) boost parameters y, y', ζ', and η' by

$$\begin{aligned}
\cosh y &\equiv [-2p\cdot k - (uu_0)^{1/2} z z_0]/[2(uu_0)^{1/2}(1-z^2)^{1/2}(1-z_0^2)^{1/2}] , \\
\cosh y &\equiv (-2p'\cdot k - (u'u_0)^{1/2} z' z_0)/[2(u'u_0)^{1/2}(1-z'^2)^{1/2}(1-z_0^2)^{1/2}] , \\
\cosh \eta' &\equiv [-2p\cdot p' - (uu')^{1/2} z z']/[2(uu')^{1/2}(1-z^2)^{1/2}(1-z'^2)^{1/2}] , \\
\cosh \zeta' &\equiv \frac{(s-u-u_0) + [(uu_0)(1-z^2)(1-z_0^2)]^{1/2} \cosh\eta \cosh y'}{[(uu_0)(1-z^2)(1-z_0)^2]^{1/2} \sinh\eta \sinh y'} .
\end{aligned} \tag{B1}$$

The phase space d^4p' in (3.1) can be expressed exactly as $(-u') \sin^2\phi' du' d\phi' d\eta' ds'$, where y' is treated as a dependent variable. With this choice of variables, (3.1) can be diagonalized by a O(2, 1) partial-wave analysis (Ref. 15) leading to Eq. (3.12). However, the inversion formula is no longer given by (3.11) and a more complicated "transform" function is involved. For our present purpose, a short cut is taken by employing asymptotic relations among y, y', η', and ζ'. To be precise, writing (3.1) first as

$$A_{ij}(y; \vec{q}) = C_{ij}(y; \vec{q}) + \sum_k \int d\cosh\eta' ds' \times K_{ik}(\eta', \zeta'; \vec{q}) A_{kj}(y'; \vec{q}) , \tag{B2}$$

where y' is expressed in terms of η', ζ', and y as well as momentum-transfer invariants. However, for η', ζ', y, large, we obtain

$$y \sim \eta' + \zeta' + y' , \tag{B3}$$

so that (B2) can be written as (3.5) with $K_{ik}(z; \vec{q})$ given by (3.6).

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